EQUAL-TEMPERED SCALE: Some Notes

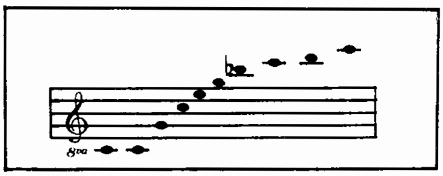
The musical scale now in common use isn't arbitrarily constructed, but is based on the characteristics of an acoustical note. Bill Markwick looks at its derivation.

IN THIS ISSUE you'll find a list of the frequencies to which the Polyphonic organ should be tuned. These are frequencies of the notes of the equal-tempered scale, also known as the scale of equal temperament; this is the international standard for musical instruments. Primitive man started it all off by being charmed with the various sounds he could get by thumping on a hollow log or his friend's head; present technology has it to the point where you have to listen to canned music when you're put on hold. In between, many centuries of fiddling occurred before arriving at the standard we use today. It's a marvel of mathematical simplicity, and a miracle of compromise when you see how it solves the problems arising from trying to get flexibility out of musical acoustics. Here's a very much condensed explanation in question-and-answer form:

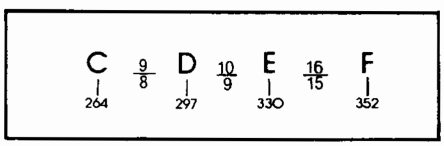
Does a musical scale have to be any specific form?
Not really; various cultures have used musical scales that were arbitrary - the pitch of the notes may have been based on drilling random holes in a bamboo flute, or something similar. However, the human brain happens to be able to detect the octave very readily; that's a doubling or halving of pitch. Next in ease of detection comes the fifth, a pitch increase of 50%. The octave and fifth turn up in all folk musics; we're on our way to a formal musical scale.

Is there a difference between pitch and frequency?
Frequency is the repetition rate of a note in cycles per second, or Hertz. Pitch is our subjective perception of frequency; more of this later when we come to the piano keyboard.

So how did the scale happen at all?
It's all based on the fact that a vibrating string or pipe doesn't produce a pure tone; that is, the note produced will be composed of the fundamental, which determines the pitch, and the various harmonics, which determine the colour or timbre of the sound. In the example of a string, for instance, the fundamental vibration first divides in two - this produces the first harmonic, the octave. Then it divides up into a ratio of three to two, and gives us the fifth. As the vibration continues dividing up in simple integer ratios, the notes of a musical scale will be generated. Here's the harmonic series based on a string vibrating at the pitch C.



That's all there is to it?
We've just begun. The fundamental and its harmonics have given us a series of pitches known as the natural scale, so-called because it occurs in any naturally vibrating string or pipe. This natural scale is quite useable; in fact, it was used for centuries. The problem arises with the peculiar spacing between the notes. This spacing results from the fact that the string or pipe divides up its fundamental in simple ratios. Here are a few of those ratios for the key of C:



The ratios between notes and note frequencies for the natural scale based on A = 440 Hz.

Why should this cause problems?
It doesn't really, as long as you stay in the key of C. The fun begins if we try to play in another key. You'll notice that there are three types of tones: the major, the minor and the semitone. For example, C to D is a major tone, D to E is a minor and E to F is a semitone. These three tones, by the way, are peculiar to the natural scale; you won't find them in our equal-tempered scale, but I've shown them anyway as part of the explanation.

Imagine that you're playing away on your natural-scale perforated gourd, and your friend arrives with his natural-scale nose flute which is pitched in the key of D. Your D to E is fixed as a minor tone (10/9). His D to E, however, is fixed as a major tone (9/8) because his D to E is the second note of his scale, i.e. "do" to "re".

It's going to sound terrible when you play together.

If the scale is determined by fixed simple ratios, then each key must have its own set of frequencies!
Exactly. A note, say C, would be different pitches depending on what key you were in.

There's obviously a solution here.
Some centuries before Bach wrote his musical treatise, The Well-Tempered Clavier, someone hit on the idea of dispensing with the three types of tones and constructing the scale out of one building block: the semitone. A whole tone, for instance C to D, would now be made up of two semitones.

And?
It worked like a charm.

How was this arrived at?
Start with any pitch, F, and its octave, 2F. Now divide the interval between them into 12 equal parts, i.e. the percentage change in frequency from any note to its neighbour is always the same throughout the octave. The percentage change from C to C#, for instance, is the same as the change from F to F#, and so on.