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Secrets of Transmission Lines

Part 6: The Smith Chart.

*In the last chapter, we looked at the behavior of transmission lines under steady-state conditions, and paid particular attention to the variation of impedance along the line for various termination conditions. In the experiment, we noted the repetition of the termination impedance every half-wave as well as the cycling of the impedance between Zo*VSWR and Zo/VSWR. In this chapter, we are going to look at what happens between the pure resistance points.*

In the previous work, we showed the expressions for the voltage and current along the line. From these expressions previously given, you can obtain the expression:

$Zx = Ex/ix$
eqn (6-1)

This simply says that at a point x along the line, the impedance is given by the voltage at that point divided by the current at that point. Bear in mind that both voltage and current can have forward and reflected components and may have real and imaginary parts.

The previously stated expressions for Ex and ix can be substituted into this expression to develop the equation for Zx. The actual manipulation is too long-winded for this treatment; however, for those interested, my version of it may be found in *Exploring Antennas and Transmission Lines by Personal Computer,* published by Van Nostrand Reinhold, New York. This book is now out of print, but still is in the possession of a number of hams and libraries. Other more current texts also carry the discussion.

As another shorthand notation to simplify the printing of the equations we define

$\beta = 2*\pi/\lambda$
(6-2)

where
λ = wavelength in the medium

It is also common to refer to the position j along the line as " ℓ " rather than "x" as we have been doing; however, for the purposes of the computer program to follow, we will retain the "x".

Using these conventions, we may write:

$$Z_x = Z_0*\{\frac{[Z_1*\cos(\beta x)]+[j*Z_0*\sin(\beta x)]}{[Z_0*\cos(\beta x)]+[j*Z_1*\sin(\beta x)]}\}$$
(6-3)

where
Z_1 is the terminating impedance
Z_x is as defined in (6-1)
β is as defined in (6-2)

This is the complete expression for the impedance at any point "x" on the transmission line. Note that it has real and imaginary (reactive; remember chapter 2!) parts and that the terminating element Zi can have real and imaginary parts as well.

For a particularly interesting case, let us set Zi = 0 (that is, the end of the line is perfectly short-circuited). For the shorted line Zi = 0, the upper left and lower right terms are zero. Therefore

$$Z_x = Z_0*\{\frac{[0+[j*Z_0*\sin(\beta x)]}{[Z_0*\cos(\beta x)]+0}\}$$
(6-4)

$$Z_x = j*Z_0*\tan(\beta x)$$
(6-5)

Equation (6-5) follows from the fact that sin(a)/cos(a) = tan(a).

The tangent function is such that if (b*x) = 45 degrees, then Zx = j*Zo. In other words, a shorted section of line an eighth of a wave long behaves like an inductor with a reactance equal to Zo ohms. At the half-wave point, the value of the tangent goes to infinity and the line section or stub looks like an open circuit, as we noted with the experiment in the previous chapter. As a matter of fact, the tangent changes algebraic sign just beyond a half wave and the stub looks like a parallel resonant circuit. These properties of the shorted stub are widely used in impedance matching.

The Smith Chart

Before the advent of the programmable calculator and the personal computer, the principal tool for solving transmission line problems was the Smith Chart, introduced by P.H. Smith of Bell Labs in 1939. This graphical solution was a boon to telephone and radio engineers.

For the power utility engineer, the transmission line equations had to be taken into account only when working with very long transmission lines of hundreds of miles or more. Also, there was usually only one frequency to be considered. In these infrequent cases, the transmission line equations were not too onerous.

For the telephone or radio engineer, on the other hand, the matter was more pressing. The telephone man had to deal with a wide range of frequencies and lines of moderate length, and, for radio work, even cables a few feet in length could show considerable impedance transformation, as we saw in our experiment. Having to solve the transmission line equations for a large number of frequencies using only a pad, pencil, and slide rule was tedious and time-consuming. The simple graphical solutions made possible by the Smith Chart were a welcome relief. Even today, when computing facilities are common features of nearly any antenna or RF lab, the Smith Chart is still used as the common way of displaying impedance/frequency plots.

An example of the usefulness of the Smith Chart can be obtained by considering the following questions.

1. With a given impedance or admittance termination, at what point on the line will a lossless reactance cancel the reflected wave? What size reactance is required?

2. Having measured the impedance-frequency plot at one point on a transmission line, what does the plot look like at another point on the line?

3. What is a given impedance when transformed into an admittance?

With the Smith Chart, questions 1 and 3 can be answered with a draftsman's compass and a straightedge, and question 2 requires only a small amount of calculation.

The Smith Chart is presented in all its glory in **Fig. 1**. At first glance, it can be a bit terrifying; however, we will look at the makeup a step at a time, and it will be a bit more simple to understand.

To begin with, you will note that there is only one straight line on the chart, right up the center. All the rest are circles, and technically the center line is also a circle of infinite radius. Smack dab in the middle of the chart is the characteristic impedance of the chart. If we are working with 50-ohm coax, then the center of the chart is 50 ohms. (For other characteristic impedances, they also print normalized charts with the center labeled one. Then, you multiply all the readings on the chart by the characteristic impedance. For instance, with our 300-ohm twinlead, you would multiply all readings by 300.)

The center line represents the locus of all pure resistances. Anyplace else on the chart has a reactive element. The center or pivot point is very important for the chart. All constant VSWRs pivot about the center of the chart. For example, if we have a 2:1 VSWR on the line then we know that as we move along the line, the impedance will pass through 25 ohms and 100 ohms A circle centered on the 50-ohm point will describe all the

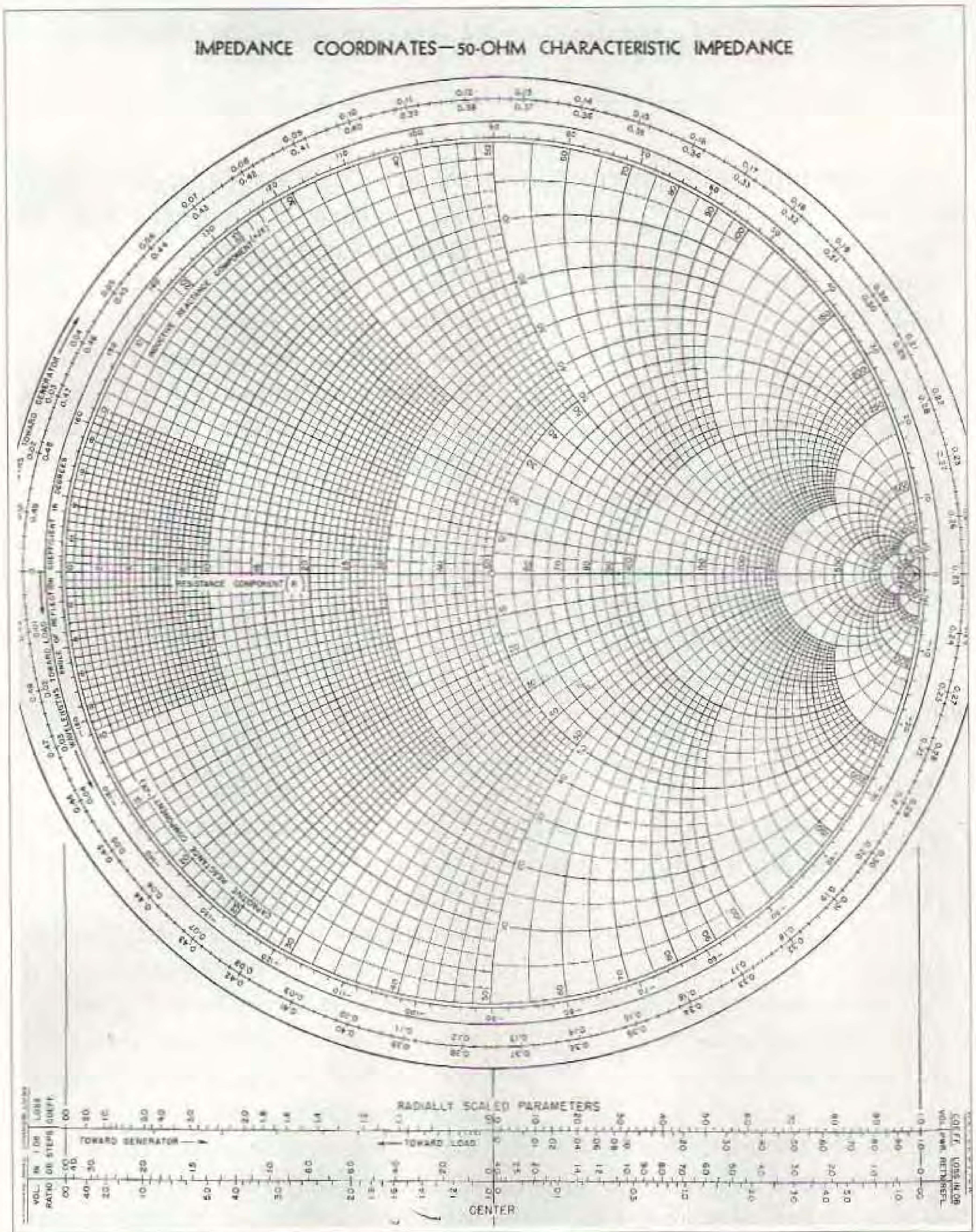

Fig. 1. The Smith Chart, impedance coordinates, 50-ohm characteristic impedance.

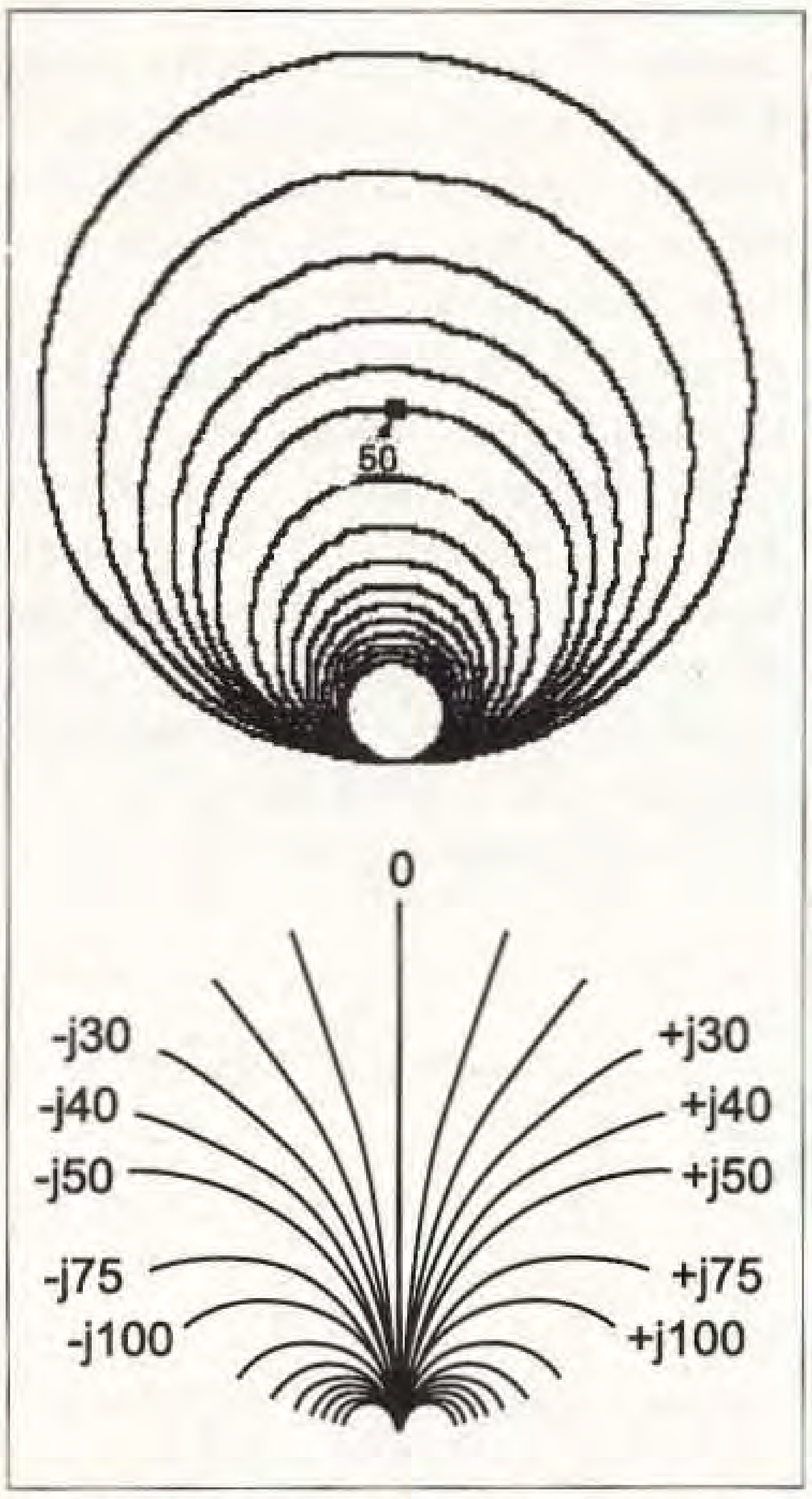

Fig. 2. The Smith Chart makeup. Top: Resistance curves are shown in 10-ohm steps for R<50 and 25-ohm steps for R>50. Bottom: Reactance curves.

impedances the line goes through. A quarter wave on the transmission line represents a half turn on the chart; the full circle represents a half wave, and the impedance repeats itself just as we saw in the last chapter.

Just think about this for the moment. By simply drawing the VSWR circle, we solved the transmission line equations for that load or termination for all possible line lengths.

Fig. 2 shows some of the resistance circles and some of the reactive circles on the chart. You will note that the zero reactance curve is the centerline and the reactance has a non-zero value everywhere else. The resistance circles and the reactance circles are said to be orthogonal, meaning that they always cross at right angles. Also note that all of the circles pass through the R/Zo = infinity point.

Fig. 3 shows how the circles are generated. Looking at **Fig. 3(a)**, we can see that if we terminate the line in a short circuit at R/Zo = zero, then a quarter of a wavelength down the line we will have R/Zo = infinity, and another quarter wave takes us back to the zero point. The R = 0 circle is the outer periphery of the chart. All possible impedances with real and reactive parts ranging from zero to infinity can be plotted on the Smith chart.

Also given on **Fig. 3** are the formulas for generating a Smith Chart in admittance terms. It is often convenient to work in admittance terms, since it is usually easier to place corrective or impedance matching elements in shunt across a coaxial cable rather than in series. From a practical standpoint, you can cut the cable and install a "tee" and hang a shunt element there more easily than you can insert a series element. The reciprocal of 50 ohms is 0.020 mhos or siemens.

An example of impedance matching with the Smith Chart

Next, let us consider an example of impedance matching using the Smith Chart. We will work in admittance because I intend to do the matching with a short-circuited stub of line of the same Yo (1/Zo) as the line. The example is shown in **Fig. 4**. We start with a chart having a 20 millimho center and plot the load on it, which is given as 10 + j2 mmhos. Next, we draw the VSWR circle. Only part of it is shown, for clarity. We rotate the arc until it meets the 20 mmho circle. The rotation is clockwise toward the generator and counterclockwise away from the generator. Next, we lay a straightedge from the center to the circular scale on the outside of the chart. The original load point reads .02 wavelengths and the point where

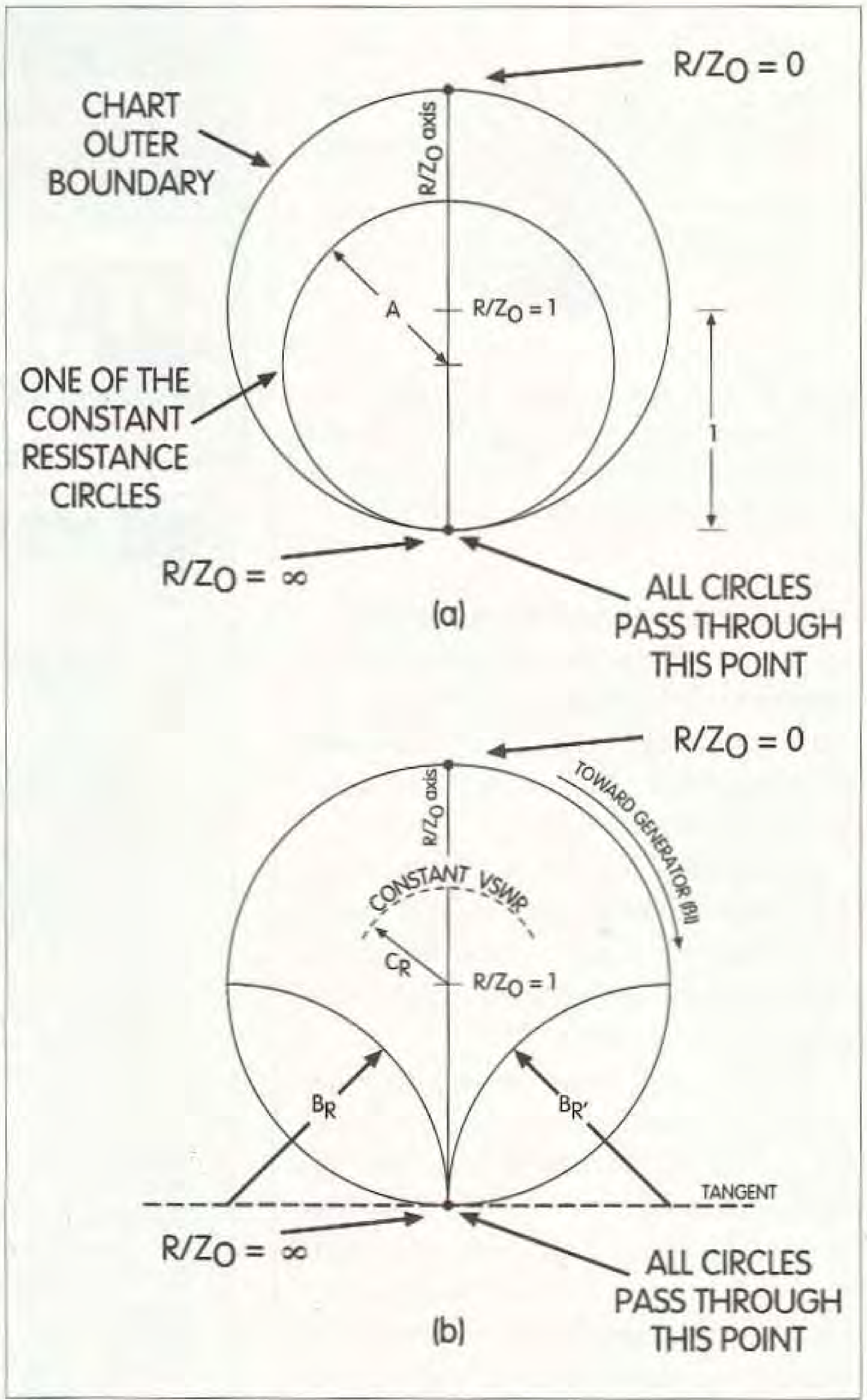

Fig. 3. Further makeup of the Smith Chart.

$$A = \frac{1}{1+\frac{R}{Z_0}} \quad \text{or} \quad A = \frac{1}{1+\frac{G}{Y_0}} \qquad C_R = 2A - 1 = \frac{2}{1+\frac{R}{Z_0}} - 1$$

$$B_R = \frac{Z_0}{X} \quad \text{or} \quad B_R = \frac{Y_0}{B}$$

the VSWR curve meets the 20 mmho circle is 0.154 wavelengths toward the generator. Therefore, the point to place the stub is 0.154 - .02 = 0.134 wavelengths toward the generator.

At this point, the admittance is 20 + j14.2 mmho, so we need to supply a shunt element of -j14.2 mmho to match the line. Note that when working in admittance, the signs of the suscep- tances are reversed with respect to impedances; that is, inductance is -j and capacitance is +j. So for our matching stub, we want an inductance. We can find the length required by going to the infinite conductance point (a short circuit) and rotating toward the generator around the periphery of the chart until we reach the -j14.2 location. Since we started at 0.25 wavelengths, the final point 0.402 wavelengths toward the generator means that the stub should be 0.402 - 0.25 = 0.152 wavelengths long. The stub thus applied will yield an impedance of .20 + j0 mmho or 50 + j0 ohms. Of course, both of these lengths are in line wavelengths. If polyethylene cable is used, the physical length will be only about 65% of free space wavelength.

Transforming from impedance to admittance is the equivalent of going from a series circuit to a parallel circuit. On a Smith Chart, it is easily performed graphically. Use a normalized chart marked unity at the center. Plot the impedance point by dividing each component by the Zo. For example, 50 - j100 would become 1 - j2. Plot the result on the normalized chart. Next, draw the VSWR circle centered on the chart and through the point. Draw the diameter through the point. Read the values at the other side of the circle and multiply the result by Yo, in this case 20 mmhos The result for the example will be 4 + j8 mmho.

The experiment

Using the transmission line setup constructed for the previous chapter, terminate the line in a 150-ohm resistor. Next, take a piece of aluminum foil about 2 inches wide, and wrap it

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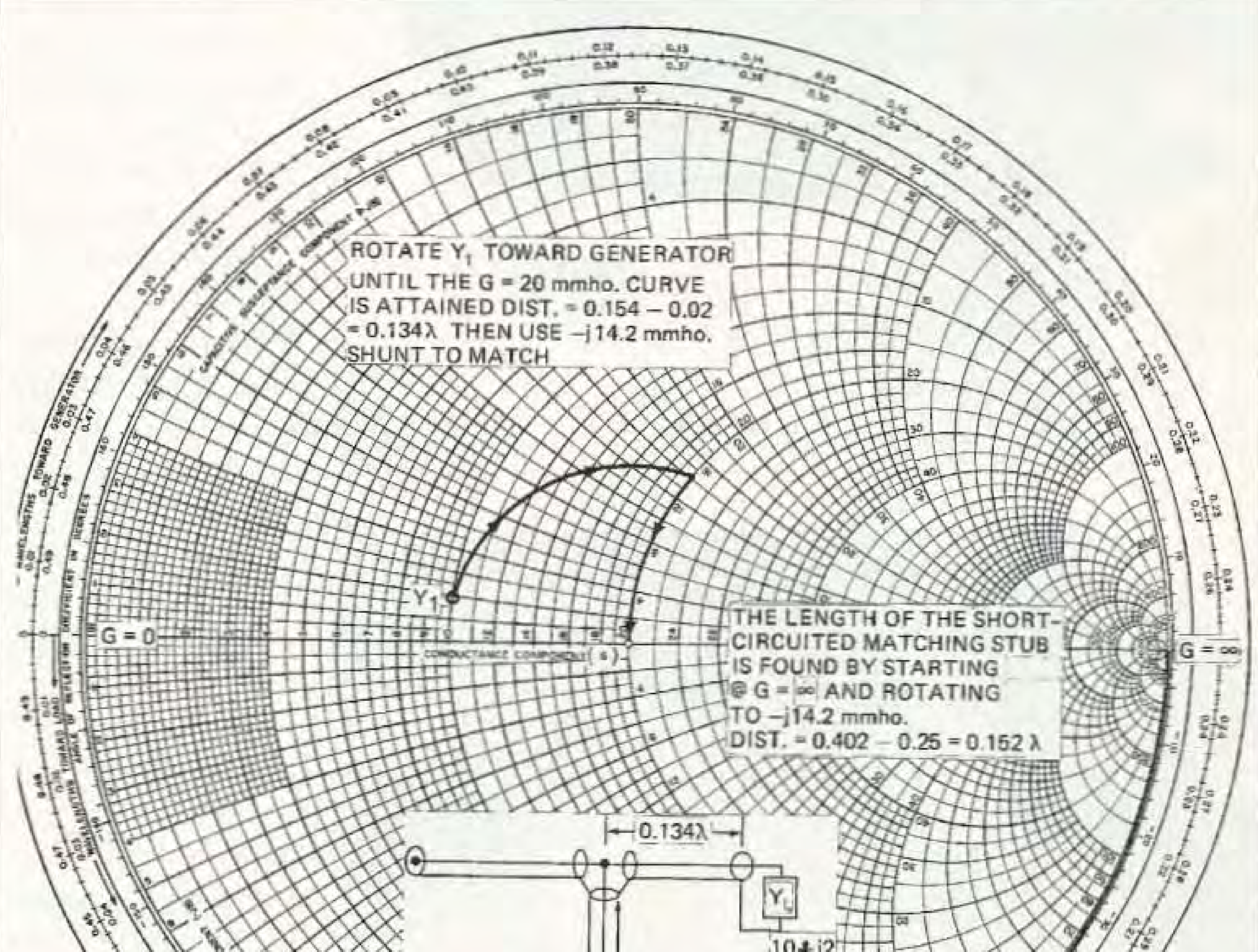

Fig. 4. Admittance coordinates, 20-millimho characteristic admittance.

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around the twinlead. This makes a capacitor. See whether you can find a location along the twinlead where you can make the line impedance match or flatten out on the generator side. Try the same trick with a 600-ohm termination.

Conclusion

Next time, we will conclude the series and present some computer programs suited to transmission line work and impedance matching, as well as have a general discussion on which circuits are appropriate for which impedances. 73