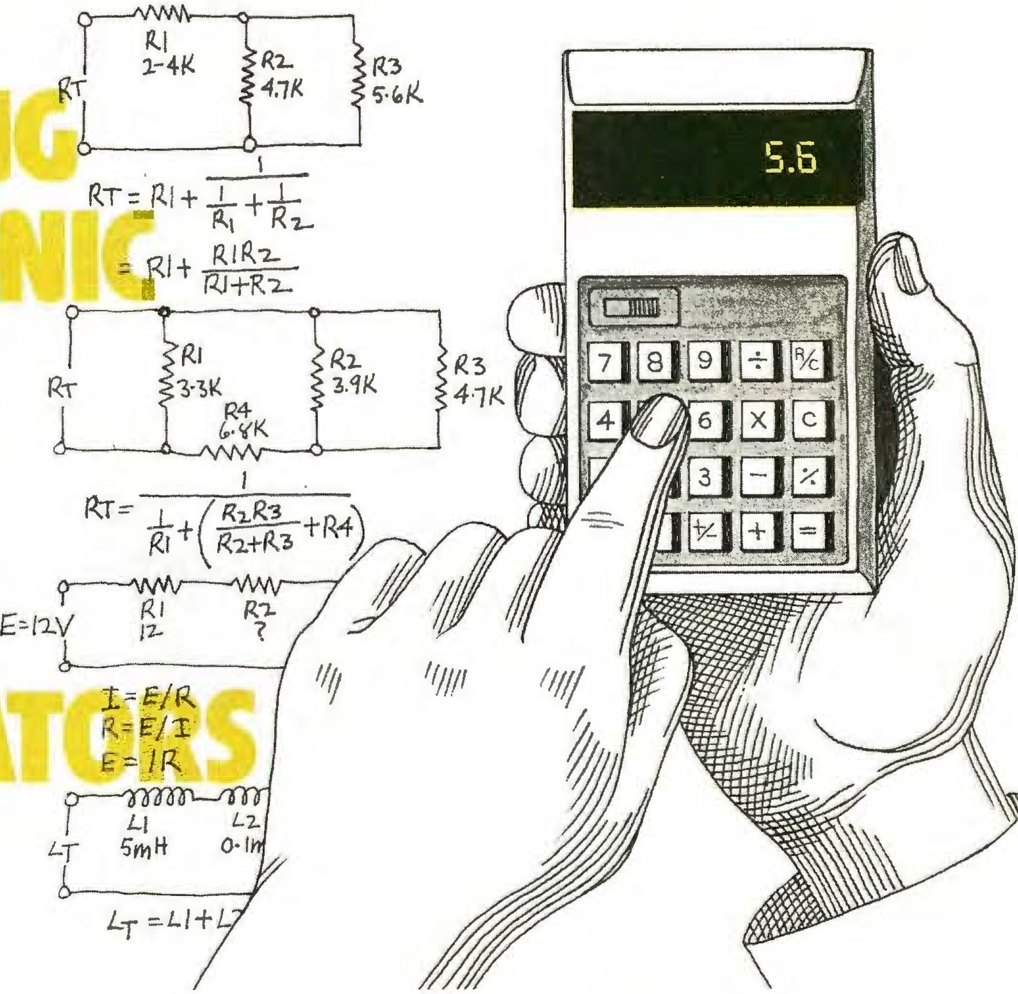
LEARNING ELECTRONIC THEORY WITH HAND CALCULATORS



BY EDWARD M. NOLL

PART TWO: Reactance, Time Constants, and AC Calculations

IN THIS second installment on using the hand-held scientific calculator to learn electronics, we cover reactance, RC time-constants, and phasor calculations. As you will soon see, it is in the realm of ac mathematics that the scientific calculator is especially useful. Calculations involve finding squares and square roots of numbers. The numbers used in calculations are often very small, in the micro (10^{-6}) and pico (10^{-12}) ranges; while frequencies are in the megahertz (10^{6}) range. Right-triangle and phase calculations involve voltages, currents, and impedances in ac circuits.

Calculating Reactance. Simply defined, reactance (impedance) is the opposition an inductor (coil, transformer, etc.) or capacitor has to the flow of ac charges. It is expressed in ohms. Reactance is similar to the opposition to the flow of current presented by a resistor in a dc circuit but is considerably more complex in nature.

Inductive reactance can be calculated from the equation $X_L = 2\pi fL$, where f is the frequency in hertz and L is the inductive value in henrys. Using this equation, what is the reactance of a 20-H choke at 60 Hz? Your keyboard entries for solving this problem would be:

2 × π × 60 × 20 =
Display: 7539.82237

After rounding off, the answer would be approximately 7540 ohms.

Now, what is the inductive reactance of a 12-μH coil at 4 MHz? The solution is:

2 × π × 4 EE 6 × 12 EE +/− 6 = Display: 301.5928948

After rounding off, the answer would be 302 ohms.

To calculate capacitive reactance, you use the formula $X_C = 1/(2\pi fC)$. Again, f is frequency in hertz, while C is capacitance in farads. Now, what is the reactance of an 8-μF capacitor at 60 Hz?

2 × π × 60 × 8 EE +/− 6 = 1/x Display: 331.572798

The answer is roughly 332 ohms.

What is the reactance of a 100-pF capacitor at 4 MHz? The key sequence for solution of this problem is:

2 × π × 4 EE 6 × 100 EE +/− 12 = 1/x
Display: 397.8873577

Rounded off, the answer is 398 ohms.

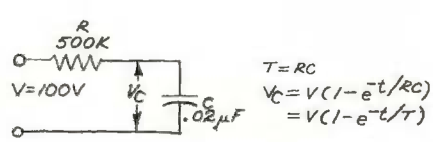


Fig. 1.

RC Time Constant. The time constant of a circuit is the product of resistance and capacitance and is expressed as T = RC, where T is in seconds, R is in ohms, and C is in farads. Therefore, if you wish to know the time constant for the component values shown in the Fig. 1, your keyboard entries would be:

5 EE 5 × .02 EE +/− 6 =
Display: 0.01 (second)

The answer can also be expressed as 10 ms (milliseconds).

The instantaneous voltage across a capacitor is an exponential related to the base e (2.718281828 obtained by operating the e^x key twice). The formula $V_C = V(1 - e^{-t/RC})$ is used for determining the voltage across a capacitor at any given time after power is initially applied to the circuit. Therefore to determine the voltage across the capacitor in Fig. 1 50 ms (0.05 second) after power is applied, the keyboard sequence would be:

100 × [1 − (+/− .05 ÷ 5
EE 5 ÷ .02 EE +/− 6) e^x]
= Display: 99.3262053

Note however that if you already know the time constant, you can simplify keyboard entry using the formula $V_C = V(1 - e^{-t/T})$:

100 × [1 − (+/− .05 ÷ .01
) e^x] = Display: 99.3262053

In both cases, you obtain the same answer—about 99.3 volts.

If time constant T and time period t for the Fig. 1 circuit were both 0.01 second, $-t/RC = -0.1/0.1 = -1$. Using −1 as the exponent of e, we get:

100 − (100 × +/− 1 e^x) =
Display: 63.2120559

This calculation verifies the rule that states that, in one time constant, a capacitor charges to 63.2% of the maximum voltage in the circuit.

Now, what is the voltage across the resistor when $-t/RC = -1$? Since $V = V_C + V_R$, $V_R = V - V_C$:

100 − 63.2120559 =
Display: 36.7879441

Hence, the voltage across the resistor at the end of one time period is approximately 36.8 volts.

Ac Time Constant. Time constant is important when sine waves are

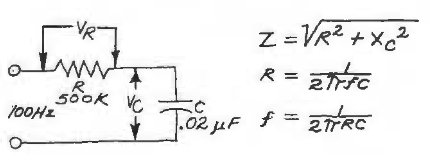


Fig. 2.

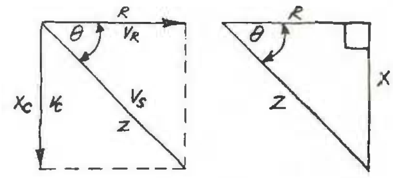


Fig. 3.

applied to resistor, capacitor, and inductor combinations. In the case of a sine wave, you cannot consider the voltage being applied as a constant as you do in dc circuits. Instead, the potential builds up gradually to peak amplitude. The reaction of the resistor-capacitor combination shown in Fig. 2 to the sine wave is a factor of frequency and the time constant of the network.

When the time constant is long compared to the period of the sine wave, the capacitor does not charge and discharge fully because the sine-wave variations are much faster than the time required to charge and discharge the capacitor by any appreciable amount. Consequently, the ac variations appear in their entirety across the resistor and not across the capacitor.

At low frequencies, where the sine-wave period is longer than the time constant, the capacitor does charge and discharge an appreciable amount. Therefore, there is likely to be a significant or even a great voltage variation across the capacitor, with a correspondingly lower variation across the resistor. To demonstrate this relationship, refer to Fig. 3. Note that capacitive reactance X_C lags resistance R by 90°. Therefore, the total impedance, Z, in the series RC circuit is a vector value somewhere between the resistive and reactive values. When you graph the values, a right triangle will result, with impedance Z becoming the hypotenuse.

To determine the impedance of the Fig. 2 circuit, use the first equation shown. Start by solving for X_C:

2 × π × 100 × .02 EE +/−
6 = 1/x Display: 79577.47151

Round off and convert the result to 79.6 k so that R and C are both in the same number system. Then continue to work the problem:

79.6 x^2 + 500 x^2 = $\sqrt{x}$
Display: 506.2965139

Hence, the circuit's impedance is roughly 506,300 ohms.

Now, at what frequency will $X_C = R$? The second equation in Fig. 2 allows us to derive the third equation, which is the one used for determining the frequency. Note that the denominator in the third equation contains the RC time-constant statement. Therefore:

2 × π × .01 = 1/x
Display: 15.9154943 (hertz)

If a 10-volt, 15.9-Hz signal is applied to the Fig. 2 circuit, how will the voltage divide? Inasmuch as reactance and resistance are basically the same, one might jump to the conclusion that 5 volts would appear across the resistor and 5 volts across the capacitor. This would be an incorrect assumption. Actually, 7.07 volts would appear across both elements. This does not mean that the applied voltage is 14.14 volts because a vector relationship exists.

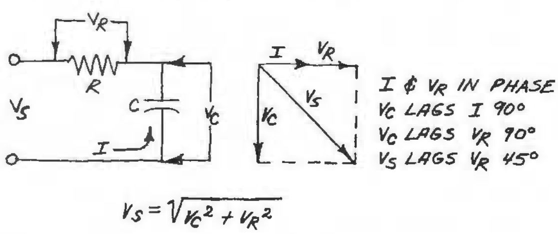


Fig. 4.

The angular relationship can be better understood by examining Fig. 4. The same current flows through both components. The voltage across the resistor is in-phase with the current, but the voltage across the capacitor lags the current by 90°. Therefore, the capacitor voltage lags the resistor voltage by 90°, which means that the resultant must fall between the two. To determine the value of applied voltage V_S, use the formula given in Fig. 4. So, assuming $V_C = V_R = 7.07$ volts:

7.07 x^2 + 7.07 x^2 = $\sqrt{x}$
Display: 9.998489885

This proves that with a source potential of 10 volts, the voltages across the resistor and capacitor are both 7.07 volts when $X_C = R$.

The same phase relationships exist between resistance, impedance, and reactance as shown in Fig. 5. Again, the right triangle is the basis of the phasor. With this in mind, calculate the reactance of the capacitor in Fig. 2 when f = 15.9 Hz:

2 × π × 15.9 × .02 EE +/− 6
= 1/x Display: 500487.2423

Note that the answer, after rounding off, is very close to the 500,000-ohm value of the resistor. Assuming R = X_C = 500 k, calculate Z:

500 x^2 + 500 x^2 = $\sqrt{x}$
Display: 707.1067811 (kilohms)

Ac Trigonometry. The modern hand-held scientific calculator permits the student of electronics to dispense with trigonometry tables when calculating angles. We find extensive use of sin, cos, and tan functions in ac vector calculations, as well as their arc functions ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$). The important relationships for the impedance vector are: sin θ = X/Z; cos θ = R/Z; and tan θ = X/R.

Knowing that reactance X_C is equal to resistance R, both being 500 kilohms, for the Fig. 2 circuit, tan θ = X/R = 500 k/500 k = 1. From this, we can calculate the phase angle, θ = $\tan^{-1}$ 1:

1 $\tan^{-1}$ Display: 45 (degrees)

Once you know angle θ and resistance R, you can find the impedance from the formula Z = R/cos θ:

5 EE 5 ÷ 45 cos =
Display: 707106. 7813 (ohms)

This proves that the impedance of the circuit is approximately 707,000 ohms and not the simple arithmetical summation of X and R.

The angle at which X = R is 45°. For other frequencies and other reactance ratios, the same equations can be

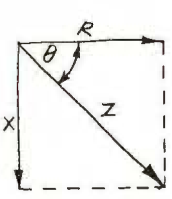


Fig. 5.

used. If any two quantities (X, R, Z) are known, angle θ can be calculated. The impedance can then be calculated. Likewise, when any two of the voltages are known, the third can be determined. A few examples will help to demonstrate these relationships in a series RC circuit.

First, in a series RC circuit that is to have an angle θ of 30°, X_C is 10,000 ohms. What resistor value is required? Using the formula R = X_C/tan θ, the calculator procedure would be:

10000 ÷ 30 tan =
Display: 17320.50811 (ohms)

The answer is approximately 17,320 ohms.

Next, calculate the impedance of a series RC circuit when X_C is 2000 ohms and R is 4000 ohms. Using the formula θ = $\tan^{-1}$ (X/R), we get:

2000 ÷ 4000 = $\tan^{-1}$
Display: 26.5650511

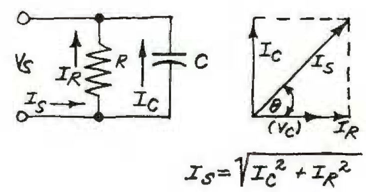


Fig. 6.

Then we find impedance using the formula Z = R/cos θ:

4000 ÷ 26.5650511 cos =
Display: 4472.135954 (ohms)

Now, prove that the above answer is correct by using the impedance formula:

2000 x^2 + 4000 x^2 = $\sqrt{x}$
Display: 4472.135954

As you can see, the answer checks out down to the last decimal place.

Finally, in a series RC circuit, R is 150 kilohms and f is 12,000 Hz. At what value of C will X_C be 500 kilohms? Start by solving for angle θ using the formula θ = $\cos^{-1}$ (R/Z):

150 ÷ 500 = $\cos^{-1}$
Display: 72.5423969

The tangent equation can then be used to determine the required reactance to obtain a 72.54° angle:

150 × 72.5423969 tan =
Display: 476.969601

The final step is to determine the value of C that will produce a 476.97-kilohm reactance at 12,000 Hz. Use the formula X_C = 1/(2πfC):

2 × π × 12 EE 3 × 476.96901
EE 3 = 1/x
Display: 2.780659563 −11

Hence, after rounding off the result, we know that the capacitor must have a value of 0.0278 μF when used with a 150-kilohm resistor to provide the required impedance at 12,000 Hz.

Parallel RC Circuits. In a parallel resistor-capacitor circuit, the applied voltage appears across both elements. (In any parallel circuit, the same voltage appears across each parallel leg.) The current, however, divides into separate components as a function of resistance and reactance. Likewise, the source current and the angle are related to the absolute and relative values of resistance and reactance.

In the resistive leg, the voltage and current are in-phase. Since source voltage V_S and capacitor voltage V_C are in parallel with resistor voltage V_R, they are in-phase with resistor current I_R as shown in Fig. 6. Capacitive current I_C must lead capacitor voltage V_C by 90° as shown. Therefore, in the parallel RC circuit, the source current must also lead resistor current.

The above relationship demonstrates that source current is not a simple summation of the resistor and capacitor currents. It is the vector sum:

$I_S = \sqrt{I_R^2 + I_C^2}$

Since the voltages across R and C are the same as the source, we can deal simply with impedances, so that

$(I/Z)^2 = (I/R)^2 + (I/X_C)^2$
or $Z = RX_C / \sqrt{R^2 + X_C^2}$

Impedance, therefore, is a product over sum relationship, much as is the case in which two resistors are connected in parallel. However, note that instead of a simple sum, a vector sum is required in the denominator.

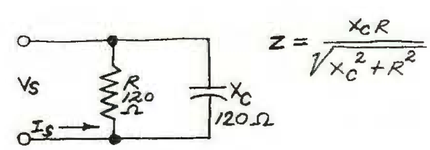


Fig. 7.

Calculate the impedance of the Fig. 7 circuit:

120 1/x x^2 + 120 1/x x^2 =
$\sqrt{x}$ 1/x Display: 84.85281381
or 120 × 120 ÷ (120 x^2 + 120
x^2) $\sqrt{x}$ = Display: 84.85281378

For all practical purposes, both answers are identical. Since R and X_C are equal, the calculations could have been simplified as follows:

120 1/x x^2 × 2 = $\sqrt{x}$ 1/x
or 120 x^2 ÷ (120 x^2 × 2)
$\sqrt{x}$ =

Answers in all cases will be within six-digit accuracy.

What is the source current if the source potential is 24 volts in Fig. 7? From Ohm's Law, we know that I_S = V_S/Z. Therefore

24 ÷ 84.8528138 =
Display: 2.828427123 −01

Translated, I_S is approximately equal to 283 mA.

A right triangle shows the vector relationship with angle θ between source current I_S and resistor current I_R, as in Fig. 6. The appropriate equations are: $\tan\theta = I_C/I_R = R/X_C$; $\cos\theta = I_R/I_S = Z/R$; and $\sin\theta = I_C/I_S = Z/X_C$. Now, using the tan θ formula, determine angle θ for the Fig. 7 circuit:

120 ÷ 120 = $\tan^{-1}$ Display: 45

Again, as in the series RC circuit, = 45° when X_C = R.

Calculate I_C and I_R for the Fig. 7 circuit. Use the sin θ and cos θ formulas — $I_C = I_S \sin\theta$:

.282842712 × 45 sin =
Display: 1.999999993 −01

and $I_R = I_S \cos\theta$:

.282842712 × 45 cos =
Display: 1.999999996 −01

Now find the vector sum of I_C and I_R:

.199999999 x^2 + .199999999 x^2
= $\sqrt{x}$ Display: .282842711

When X_C and R are not equal, the current in the circuit divides in accordance with the ohmic values of the reactance and resistance. With this in mind, calculate Z and I_S for the Fig. 8 circuit:

100 x^2 1/x + 10 x^2 1/x =
$\sqrt{x}$ 1/x Display: 9.950371903

or 100 × 10 ÷ (100 x^2 + 10 x^2) $\sqrt{x}$ =
Display: 9.950371903

Source current I_S can then be calculated:

100 ÷ 9.950371903 =
Display: 10.04987562

Phase angle θ becomes:

100 ÷ 10 = $\tan^{-1}$
Display: 84.2894069

From the answer obtained, it is obvious that this circuit is highly capacitive, since angle θ is much greater

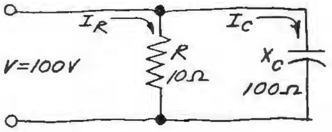


Fig. 8

than 45°. Reactive current I_C is 10 times greater than resistive current I_R and source current I_S leads source voltage V_S by a substantial amount.

Coming Up. This ends the second part of our series on using the handheld scientific calculator for learning electronics. In the third and final part of the series, we will be covering frequency response and resonance, RC coupling, basic amplifier calculations, and RLC circuits. ◈