FEATURE

Basic Electricity Part 3

In which we examine series and parallel circuits.

Ron C. Johnson

Well, here we are again in this land of great potential, charging upwards, full power. Though significant currents continue to oppose our forces I remain steadfast in my resistance to their rise. In fact, I charge you...

Sorry. Just practising my speech for the Electronics Club elections, next week.

Where were we?

Ah, yes! Last month we learned about Scientific Notation, Engineering Units, and the resistor color code before we looked at series and parallel together as well as looking at some practical aspects of meters, how they work and how to use them.

Let's get series, I mean serious.

Series And Parallel

Up to this point we have been learning about the very basic concepts of electricity and how we deal with the quantities involved. As we get into series-parallel networks we are finally talking about some practical aspects of the field. We can actually apply some of this stuff to real life situations. Here is an example.

Those of you who have an interest in stereo equipment have probably run into the situation where a number of speakers wee to be connected together to one power amp output. The power amp specs say that the amp is rated to drive into eight ohms. How should the speakers be connected to provide the correct load to the amp? This is an important consideration (often ignored), and the consequences of doing it wrong range from inefficient operation, to distortion, to outright damage to the amp.

We won't worry about the technical reasons why it is so critical to keep the load resistance close to the specified value. But our job is to make sure it is done.

Let's assume that the available speakers are all eight ohms and that we have four of them to connect. Obviously, one of them connected to the amp by itself would be easy. To connect four is a little more complex. Connected all in series, (Figure 1), they would be additive as we saw in last month's segment. The total resistance would be:

8ohms + 8ohms + 8ohms + 8ohms = 32ohms

This is obviously too much.

On the other hand, if they were all connected in parallel (Figure 2) the equivalent resistance seen by the amplifier would be:

Adding conductances in parallel

$G_T = 1/R_T = 1/8ohm + 1/8ohm + 1/8ohm + 1/8ohm = 4/8ohm = .5$ Siemens

or

$R_T = 2ohms$

And this is much too low.

Now let's mix the two up. In Figure 3 we see a series-parallel combination where two speakers in series are connected in parallel with two other speakers in series. Each branch of two speakers will have:

8ohms + 8ohms = 16ohms

If we redraw the diagram substituting the equivalent 16ohm resistances we now have them in parallel. (Figure 4). Combining them we get:

Again adding conductances

$G_T = 1/R_T = 1/16ohms + 1/16ohms = .125$ Siemens

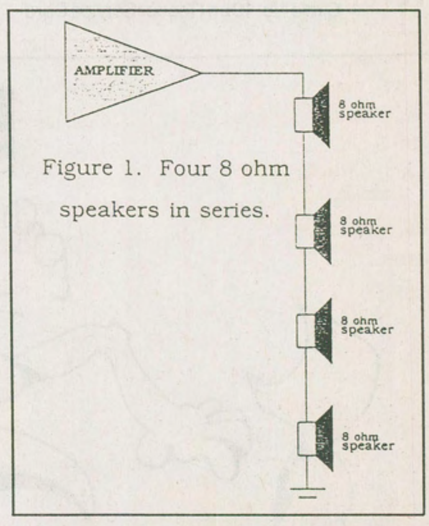

Figure 1. Four 8 ohm speakers in series.

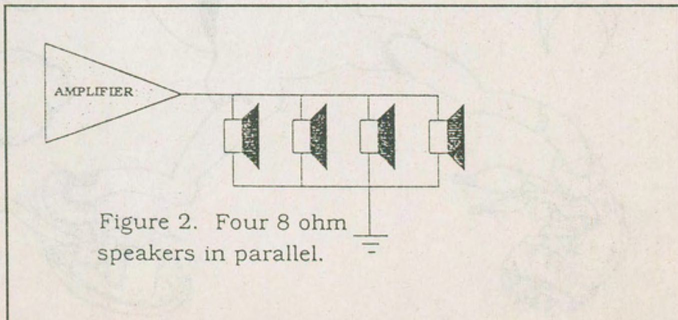

Figure 2. Four 8 ohm speakers in parallel.

Some Useful Rules And Formulae

Ron C. Johnson

In last month's segment we talked about how to calculate equivalent resistances. We said that resistances in series are just added together but to determine the equivalent resistance or resistors in parallel the simplest way was to use conductances. So:

$G_T = 1/R_1 = 1/R_2 + ... 1/R_N$

where G_T is conductance in Siemens, S, and N is the number of resistors.

This will calculate the equivalent resistance for any number of parallel resistors. But if you have just two resistors in parallel it may be simpler to use what is called the "product over sum" method. (This is just a derivation of the conductance method.)

$R_{eq} = R_1 X R_2 / R_1 + R_2$

If you have more than two resistors in parallel, any two can be reduced to an equivalent which can then be combined with the third.

In the case where resistors of the same value are in parallel the value of the resistors divided by the number of resistors will give the equivalent:

$R_{eq} = R/N$

Voltage Divider Rule

Kirchoff tells us that around any loop the voltage rises must equal the voltage drops. Last month we calculated the total resistance in a series loop and found the total current. We then multiplied the value of a particular resistor times the current to find the voltage drop across that resistor. Voltage divider rule is just a quicker way to do the same thing.

Given a series circuit with two resistors (R_1 and R_2) we can find the voltage across either one by using the following formula:

$V_{R1} = E X R_1/(R_1+R_2)$
or
$V_{R2} = E X R_2/(R_1+R_2)$

What the formula is saying is that the voltage across the resistor will be proportional to the ratio of that resistor to the total resistance of the circuit.

Current Divider Rule

Current divider rule is used to determine how much current splits down the branches of a parallel circuit and as in VDR it uses a ratio. In this case, however, the current through a resistor in one branch is proportional to the ratio of the resistance of the other branch to the total resistance.

$I_{R1} = I_T X R_2/(R_1+R_2)$
and
$I_{R2} = I_T X R_1/(R_1+R_2)$

and
$R_T = 8ohms$

The same could be accomplished by configuring the speakers as in Figure 5 where two sets of 8ohm speakers in parallel are connected in series. Each parallel set has an equivalent resistance of 4ohms. When the two equivalents are added together we get 8ohms.

Some Theory

So series-parallel networks can be useful and, in fact, most practical circuits have complex combinations of series and parallel branches. Quite often we need to be able to analyze these networks so we can predict the voltages across components, currents through them, power dissipated in specific devices or the equivalent resistance of a combination of components.

And most of it can be done with the help of our old friends, Ohm and Kirchoff.

Figure 5 shows a basic series-parallel circuit with a DC voltage source. The total current flowing out of the voltage source is I_T. All of this current flows through R_1. It then splits and some flows down through R_2 and some through R_3. We will call these currents I_1 and I_2 respectively.

The sixty-four thousand dollar question is: how much will flow through each resistor? Also, how do we calculate R_T, I_T, the power dissipated in each resistor... Okay, so it's more than one question. What is the approach?

Usually the first step is to find the total resistance in the circuit; the total load presented to the power source. This will allow us to determine the total current flowing in the circuit. The trick is to combine the resistances in the correct order. Any time the current in the circuit splits we must have a parallel section in the circuit. In this case it is R_2 and R_3. We can't combine R_1 with them until we know their equivalent resistance. So we either add their conductances and then convert back to resistance or we can use the product over sum rule described in the Rules and Formula section. Once we have an equivalent resistance we can redraw the circuit (Figure 6) with the R_2 - R_3 combination as a single resistor and place it in series with R_1. From there we simply add the two resistances together to get R_T.

Try this example. It's easy and it will help you get comfortable with the procedure. Again referring to Figure 5 let's make R_1 a 5.6k ohm resistor, R_3 a 3.3k ohm and R_3 a 2.2k ohm. (These are all standard EIA resistor values so you could set this up and confirm that it works.) The voltage source is a 12 volt battery but practically any low voltage DC source would do for experimentation. We said we would have to determine the equivalent resistance of the parallel branch first. Using the product over sum rule we get:

$R_{eq} = 3.3kohm X 2.2kohm / 3.3kohm + 2.2kohm = 1.32kohm$

We then put the R_{eq} in series with R_1 and add them together.

$R_T = R_1 + R_{eq} = 5.6k ohm + 1.32k ohm = 6.92k ohm$

We could now redraw this again as a simple series circuit: a voltage source, one resistance and a current carrying path. But let's just calculate the total current flowing out of the voltage source:

$I_T = E/R_T = 12v/6.9k ohm = 1.73 mAmps$

So now we know the source voltage, the total resistance in the circuit and the total current drawn from the supply. The purpose of our analysis will determine where we go from here. We could find the total power dissipated in the circuit using $P = I_T X E$ and the information we have so far. Quite often though we want to know how the current splits through R_2 and R_3. There are a couple of ways of finding this.

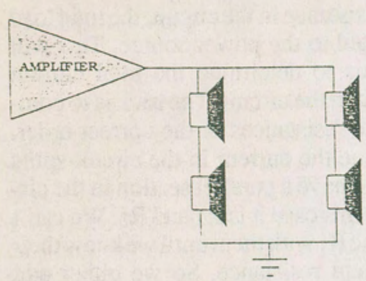

Figure 3. Four 8 ohm speakers in series-parallel.

We know one way using the tools we already have. You can check out the Rules and Formulae section for another way called *current divider rule*.

Take a look, again, at Figure 6 where we have redrawn the circuit diagram showing the R_2 - R_3 parallel combination as an equivalent resistance. We have seen this circuit before and know how to find the voltage across R_{eq}. V_{eq} would be equal to R_{eq} times the current through it since R_{eq} is the equivalent resistance of the parallel section and I_T flows into that combination.

If V_{Req} is across R_{eq} then the same voltage is across the parallel combination of R_2 and R_3 which means that each resistor has the same voltage, V_{Req}, across it. To determine the current through either one of those resistors we just use Ohm's Law:

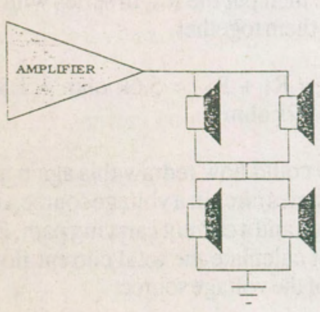

Figure 4. Four 8 ohm speakers in series-parallel.

$I_{R2}=V_{Req}/R_2$
and
$I_{R3}=V_{Req}/R_1$

These two values of current should add up to I_T which was calculated before. (Just check for practice; go ahead and calculate these currents and check.)

All of this brings up some rules which, while they are not absolutely necessary to know, can be useful in this process of circuit analysis.

I know, more rules and formulae, but they are pretty simple and can help streamline your analysis technique.

KCL

Kirchoff had two good ideas. Kirchoff's Current Law states that the algebraic sum of all the currents entering and leaving a node will equal zero. What this really means is: "what goes in has to come out". A node is an electrical connection of two or more components. If current flows in from one branch the same amount has to flow out somewhere. In our previous example the current, I_T flowed into Node A, where R_1 connects to R_2 and R_3. The same amount of current—the total of I_2 and I_3—must equal I_T. Another way of calculating how much flows in each branch (Current Divider Rule) is shown in the Rules and Formulae section along with a method of finding the voltage across a resistor in a series circuit (Voltage Divider Rule).

Meters

Probably most of you have had opportunity to use a meter at one time or another. Perhaps you have one of your own. Our purpose here is not to cover the use of meters as much as to talk about how meters relate to this subject of series-parallel circuits. Even so, we'll take a general look at meters as a way of introduction.

We could categorize meters in several ways. We could differentiate between analog and digital meters, bench meters and portables, specialty meters versus general purpose, or high accuracy versus economy units.

For our purposes let's talk about functions. The basic meter we are considering measures voltage, current and resistance, the quantities we have been dealing with in this series. In addition to DC values most meters of this type will measure alternating currents and voltages. Older units were called VOMs (Volt Ohm Meters) while others were TVMs (Transistorized Volt Meters), VTVMs (Vacuum Tube Voltmeters), and more recently DVMs (Digital Voltmeters). In all cases, though, the same quantities were measured. Before the advent of digital technology and the availability of digital displays, bench and portable test meters used various kinds of electromechanical meter movements for indication. Many were excellent pieces of test equipment considering the delicacy of their meter

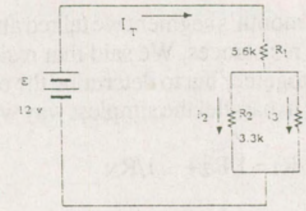

Figure 5. A basic series-parallel circuit.

movements, difficulty of interpreting scales and limited specifications. (More on that later.)

Digital meters, which have replaced analog for most applications, have benefited, not just by their solid state displays but also by the improved technology used in their input sections. In addition to being more rugged, generally, they can fit more functions in a smaller package and give better specifications.

We'll take a look at some actual products and their use in another segment.

The important concept that must be understood is that although the equipment available is generally very good, meters do have an effect on the circuit they measure. Figure 7 shows a simple series circuit with two resistors, R_1 and R_2 and a 20 volt source. Let's imagine that you have been asked to measure the voltage

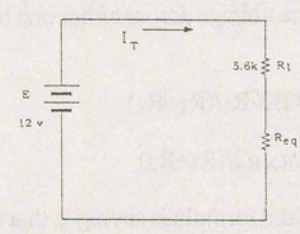

Figure 6. The parallel combination replaced by an equivalent resistance.

across R_2 with the meter shown. If the meter was perfect it could be connected across R_2 and it would indicate the voltage dropped there. We could predict what the voltage should be by using the voltage divider rule to calculate it.

A perfect or ideal meter would look like an infinite resistance and so would draw zero current from the circuit and consequently have no effect on it. However,

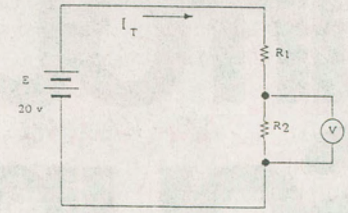

Figure 7. Voltmeter loading.

meters are never ideal. Such is life. In reality meters have an internal resistance which, while very high, is a finite value. So in order to use our meter intelligently, we need to know under what conditions it will "load down" the circuit. If it loads down the circuit it will give significantly erroneous readings.

For example, digital meters often have an input resistance specification of over 1M ohm which is very high — much higher than most analog VOMs. If R_1 and R_2 in Figure 7 were 100ohm resistors we could calculate V_{R2} to be 10 volts. If the meter (1M ohm) were connected across R_2 we would have to consider a 1M ohm resistor to have been connected across R_2. We would then re-calculate V_{R2} based on an R_{eq} of 100ohms in parallel with 1M ohm. In this case the change in R_{eq} would be negligible.

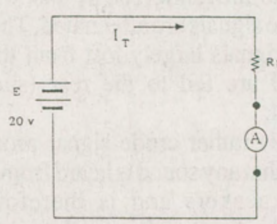

Figure 8. Ammeter insertion loss.

But what if R_1 and R_2 were 1Mohm each?

In that case V_{R2} should still be 10 volts, because the ratio of the resistors is still the same, but now R_{eq} would be a 1M ohm resistor in parallel with a 1M ohm meter resistance which would equal 500k ohm. Using the voltage divider rule and the equivalent resistance we would get:

V_{R2} = (500k ohm/1.5M ohm) X 20volts=6.67volts

So the meter would read 6.67 volts even though it should be reading 10 volts. This is a case where the meter is loading down the circuit. Sometimes this is difficult to avoid but at least being aware of the problem helps to understand why you are getting unexpected readings. This will happen when measuring voltages across high values of resistance.

A similar situation can come up when measuring current. We have been saying that we always talk about the current *through* a component so it make sense that in order to measure current we have to break into the circuit and route the current through the meter. Ideally, the meter, when measuring current, would have zero resistance, thereby contributing nothing to the total circuit resistance. Practically speaking, the resistance of an ammeter is very low, usually just a few ohms. This presents no problems in some cases, but, again, there are circumstances where it becomes a problem.

Figure 8 shows a simple series circuit with an ammeter connected in series. The resistance in the circuit is 1k ohm and the power source is a 20 volt supply. Without the ammeter in the circuit the current would be:

I_T=20v/1kohm=20mA

With the ammeter in the circuit we are adding 10ohms to the total resistance. Ten ohms is small compared to the 1k ohm resistor and will make practically no difference to the total current. On the other hand, if the circuit resistance was 20 ohms (which would give a current of 1 amp) adding the ammeter to the circuit would change the total resistance to 30 ohms instead of 20 ohms. This would limit the current to 667 mA instead of 1 amp so that meter would be affecting the operation of the circuit.

Again we must be aware that when connecting an ammeter in series with a circuit which has a low total resistance that the meter will affect the circuit noticeably.

Well, that about wraps it up for this month. Next time we'll talk about ideal and practical DC energy sources before we move on into some basics of Alternating voltage and current.

Hope I didn't load down your circuits. ■