Permanent Magnets

By "CATHODE RAY"

How They Fit Into the "Magnetic Ohm's Law"

ALTHOUGH electrically-energized magnetic circuits are discussed at length in most books on basic electrical engineering, permanent magnets are usually dismissed in a few descriptive paragraphs, leaving no very definite clues. Considering the scale on which permanent magnets are used in telecommunications, for loud speakers, gramophone pick-ups, microphones, telephone receivers, measuring instruments, and so forth, this seems strange.

The usual approach to magnetic circuits in general is by way of analogy from electrical circuits. Corresponding to electromotive force in the electrical circuit is magnetomotive force, expressed in ampere-turns. When one considers a permanent magnet circuit, however, where there are no ampere-turns (except for hypothetical ones in the molecular structure of the material), this concept does not get one very far. Yet the analogue of current—magnetic flux—is present; and one wants to be able to calculate how much of it one can get from a given magnet, or how much magnet and what shape will best give a required flux.

A second troublesome feature, which is common to all iron-cored magnetic circuits, permanent or energized, is the non-linear relationship between magnetomotive force and flux.

Now it happens that there is an electrical circuit which combines both of these features — non-linearity, and current without E.M.F. (apart from a rather nebulous electronic one). And as it is a simple and familiar circuit, it seems to be just the analogy that was wanted. The circuit in question is shown in Fig. 3a, and the graphical method of calculating the voltage and current in Fig. 3b.

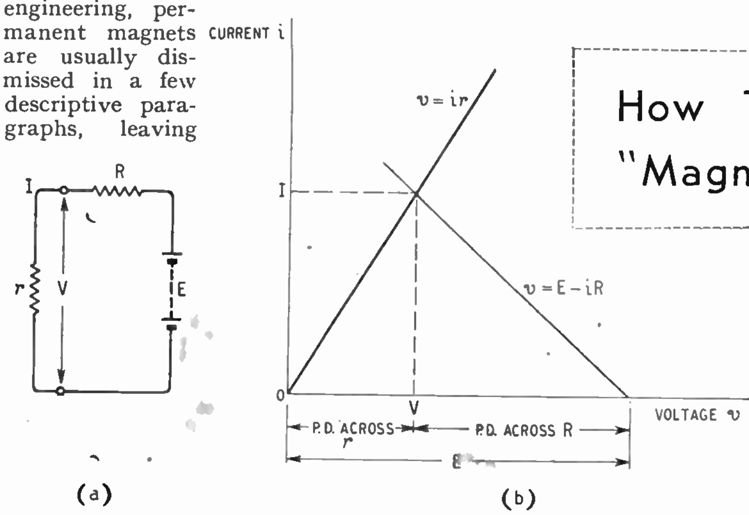


Fig. 1. Graphical method, based on Ohm's Law, of finding the current and voltage in a circuit with two resistances. When the resistances are linear, as here, it is quicker to do it mentally or by algebra, but in Fig. 2...

So that the relationship between energized and permanent magnetic circuits may be clear, let us approach Fig. 3 via the still simpler case of Fig. 1. Consider the left-hand side only, consisting of *r*. If this is an ohmic resistance, any potential difference across it is related to the current through it by Ohm's Law:

$$v = ir \quad \quad (1)$$

(The small letters v and i denote *any* values of voltage or current). Taking the right-hand side by itself, the voltage across the terminals is given by:

$$v = E - iR \quad \quad (2)$$

Here v and i are not necessarily the same as in (1). But when the

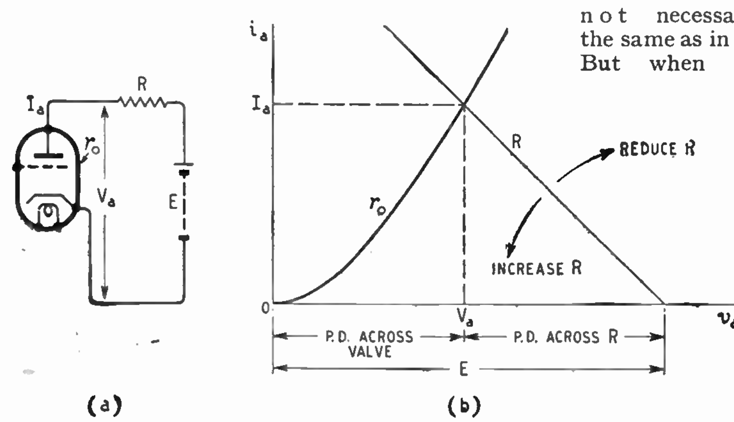


Fig. 2. The D.C. "resistance" of the valve, r_o, is known only as a graph, so the method of Fig. 1 must be used. This case is analogous to the simple magnetic circuit of Fig. 4. The effect on output, current and voltage of varying the load resistance can be seen by varying the slope of the load line.

two halves are put together, v and i must be the same in both. Call the particular values of v and i, which are the solutions of the above two simultaneous equations, V and I. Solving these equations:

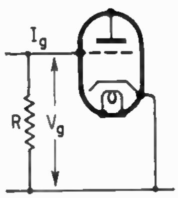
$$V = \frac{Er}{R + r} \quad \ldots \quad \ldots \quad (3)$$

$$I = \frac{E}{R + r} \quad \ldots \quad \ldots \quad (4)$$

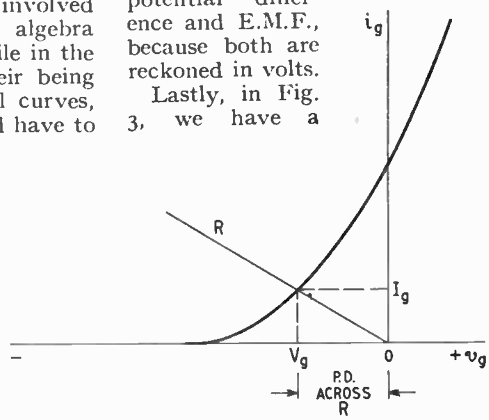
In case the solving of these equations is too great a strain on one's algebra, there is the alternative method of drawing the graphs representing the equations, as in Fig. 1b. The solution is given by their intersection, and as they are linear equations there is only one intersection.

Well, of course, that legendary creature "Any Schoolboy" could give the answers without setting pencil to paper in either method, because with ohmic or linear resistances it is so easy. If one or both of the resistances were expressible only by more involved equations, then some algebra might be necessary; while in the more likely event of their being known only as graphical curves, the second method would have to be used. For example, r might be r_o, the D.C. resistance of the anode-to-cathode path in a valve, and R its load resistance (Fig. 2a). The two graphs are then generally called the i_a/v_a characteristic of the valve and its load line respectively, and are very familiar in these guises (Fig. 2b). For brevity I have labelled them simply r_o and R, but, of course, the line "R" is really the graph of the linear equation $v_a = E - i_aR$. Incidentally, this graphical method would be even more necessary if r, too, were a non-linear resistance, such as a lamp. Note also that a single scale serves for both potential difference and E.M.F., because both are reckoned in volts.

Lastly, in Fig. 3, we have a case where r and E.M.F. are individually rather vague, but collectively can be expressed as an i_g/v_g curve. The actual voltage on the grid for any particular value of R can easily be found by the same general procedure as before (Fig. 3b). The externally-applied E.M.F., E, being zero, the load line starts from O; but, as the valve characteristic starts from a slightly negative v_g, some current flows notwithstanding.



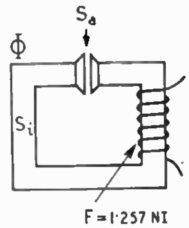
(a)



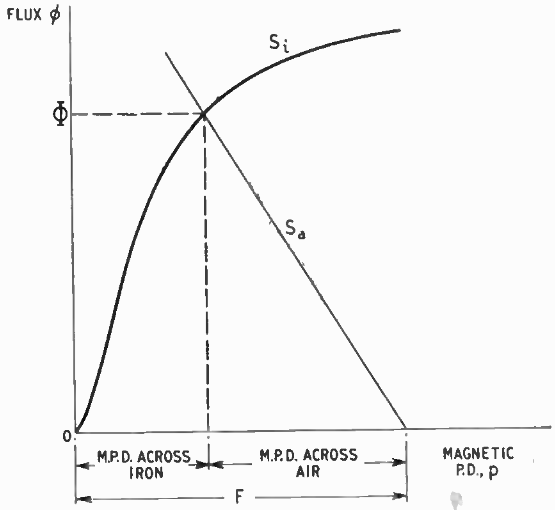
(b)

Fig. 3. This is similar in principle to Fig. 2, except that the source of E.M.F. is absent. It is analogous to a permanent magnet circuit; compare Fig. 7.

Fig. 4. Analysis of energized magnetic circuit for comparison with Fig. 2.



(a)



(b)

Fig. 2 is the analogue of a magnetic circuit consisting partly of air (linear) and partly of iron (non-linear), energized by the magnetomotive force of a current-carrying coil; and Fig. 3 a similar circuit but with a permanent magnet and zero externally-applied M.M.F. ("Iron" herein means any magnetic metal). Corresponding to current is flux Φ measured in lines (= maxwells); and corresponding to E.M.F. is M.M.F., F, which for practical convenience in the energized case is often reckoned in ampere-turns.

Supposing, then, one has an electromagnet operating in series with an air gap (Fig. 4a), it corresponds with the electrical circuit of Fig. 2a and can be calculated in the same way, merely changing the units (Fig. 4b). The "load line" is marked S_a, which stands for the reluctance of the air gap, and (in accordance with the magnetic "Ohm's Law") is equal to the magnetic difference of potential across it divided by Φ. So, corresponding exactly to (2), the equation of the line is:

$$p = F - \phi S_a \quad \ldots \quad (5)$$

And S_i is just the flux/magnetic-potential characteristic of the iron core.

I admit that this is not exactly the usual way in which the matter is put, and may be quite novel to some readers. But the analogy between the electric and magnetic circuits is sometimes confused

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because the two are not compared, as they are here, in their corresponding forms. There are very good practical reasons, which we shall come to in a minute, for dealing differently with magnetic circuits; but to get a good theoretical grasp of the matter beforehand it is advisable to compare Figs. 2 and 4 for a while. For instance, just as the E.M.F. developed by the battery in Fig. 2 is divided between the two resistive elements in the circuit, so the M.M.F. developed by the coil in Fig. 4 is divided between the two reluctances. If the M.M.F. available for the load be increased, by raising its reluctance (decreasing the slope of the S_a line) it is at the expense of flux. It is generally the aim to get as much flux as possible in the air gap; and for this purpose it is fortunate that the characteristic curves of most irons rise steeply at first, like that of a pentode. But in order to get a slight increase on the flux indicated in Fig. 4*b* it would be necessary to steepen the air-gap line considerably; that is to say, make its reluctance much less. The increase is only moderate even if one goes so far as to draw the line vertical, representing zero reluctance, obtainable only by closing the gap completely and thereby making it as useless as a short-circuited load would be in the electrical case.

Although the Fig. 4 form of diagram for the magnetic circuit is the most helpful for getting a clear picture of the theory by pondering on the similarities between it and the familiar valve loading diagrams, it is not the best for practical purposes. An i_a/v_a characteristic curve is useful only for the particular resistor or valve for which it has been drawn. With a valve, that is unfortunately unavoidable, because its "resistance" depends in a very complicated way on a large number of different variable such as the potentials of the various electrodes as well as all their dimensions. But the resistance of homogeneous solid materials is much simpler, being proportional to the length (l) and inversely as the cross-section area (a). The only other factor is the one that distinguishes materials from one another as regards resistiveness—the resistivity, ρ, which is the resistance of 1 cm. cube. So

$$r = \rho \frac{l}{a} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

For non-linear materials, a curve of current against voltage ($v = ir$) refers only to one particular size and shape of resistor. But by plotting current *density* (amps per sq. cm.) against voltage *gradient* (volts per cm.), the dimensions are incorporated into the co-ordinate scales, and we get a graph of ρ instead of r, one that applies to that material in general, irrespective of size. Thus, instead of plotting, as in Fig. 1b,

$$v = ir \left(= i\frac{l}{a}\rho \right)$$

we rearrange this to bring the l to the left-hand side:

$$\frac{v}{l} = \frac{i}{a}\rho \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

So the graph takes the form shown in Fig. 5*a*.

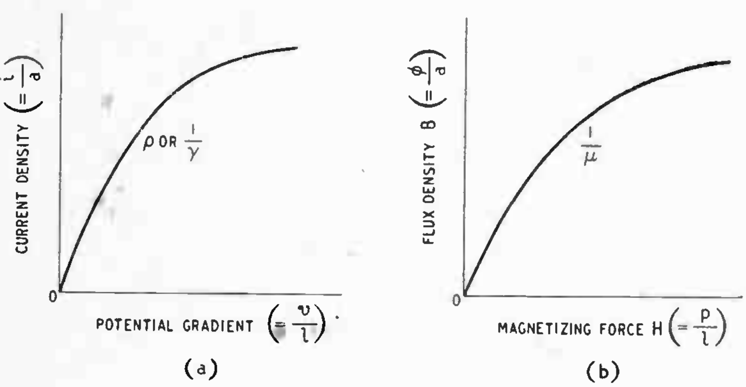

Fig. 5. When the non-linearity of a resistor is a peculiarity of the material of which it is made, it is more helpful to have a graph of the material, rather than of any particularly-sized piece of it, as would be shown in a Fig. 2*b* type of graph. This is done by altering the co-ordinate scales, as at *a*. The same argument applies to magnetic materials, which accounts for their graphs usually being plotted as at *b* (compare *a*) instead of as Fig. 4*b*.

There is every reason for adopting the same policy for magnetic materials, because, unlike resistive materials, they are all very non-linear; but, like resistance, reluctance varies in the same way with l and a. The magnetic quantity corresponding to ρ, called reluctivity, is not commonly used. But $\rho = \frac{1}{\gamma}$, γ being called the conductivity, and the magnetic equivalent of that is well known under the name of permeability, μ. So corresponding to (6) (modified by substituting $\frac{1}{\gamma}$ for ρ) is

$$S = \frac{1}{\mu}\frac{l}{a} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

B/H Curves

The permeability of all magnetic materials varies in a way that can only be recorded as a graph; and nobody wants to draw a separate graph like Fig. 4*b* for every possible size and shape of piece, when one plotted as in Fig. 5*b* gives the essential information from which the reluctance of any size can quickly be calculated. That is why the scale of flux (ϕ) becomes one of flux per unit cross-section area, or flux density, B; and the horizontal scale is in magnetomotive force (or magnetic potential difference) per unit length, which is rather confusingly called the magnetizing force, H. In other words, instead of plotting, as in the Fig. 4*b* type of curve:

$$p = \phi S \left(= \frac{l}{a}\frac{1}{\mu} \right)$$

the core dimensions are transferred from the curve to the co-ordinates:

$$\frac{p}{l} = \frac{\phi}{a}\frac{1}{\mu}$$

$$\text{or } H = B\frac{1}{\mu}$$

$$\text{or } B = \mu H \quad .. \quad .. \quad (9)$$

The information about any magnetic material, then, is usually given in the form of a B/H curve (as in Fig. 5*b*). Given the dimensions of any particular piece of it, the relationship ϕ/p is obtained by multiplying B by a and H by l. If a core consists of several sections in series, each with a different constant cross-section area, the magnetic potential drops for each (p_1, p_2, etc.) are calculated and added together to give the M.M.F. (ampere-turns) required to maintain the desired flux throughout the circuit.

For the sake of making the permeability of vacuum (and, for all practical purposes, air and other non-magnetic materials) = 1 and to avoid having to talk about ampere-turns when permanent magnets are in question, there is the alternative and slightly smaller unit of M.M.F. called (with no humorous intent) the gilbert. To convert a number of ampere-turns to gilberts, multiply by 1.257 (= 0.4π). If H is in gilberts per cm, then B for air = H.

Since all this is very thoroughly

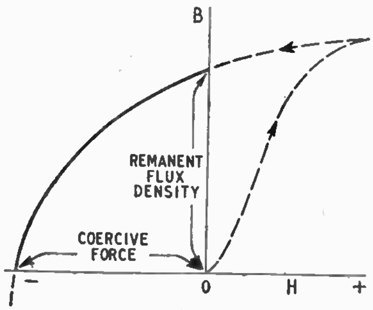

Fig. 6. Materials suitable for permanent magnets have a large part of their characteristic curve in the negative H quarter. (Compare Fig. 3*b*).

explained in textbooks we hurry on to the final goal—the permanent magnet circuit. Permanent magnets depend on the fact that when the magnetizing force applied to magnetic material is reduced, the characteristic curve doesn't follow the same path downward as it did upward. Materials most suitable for permanent magnets are those in which the departure is particularly large, as in Fig. 6, where the flux density is still near its maximum even when the magnetizing force has been completely removed; and to reduce it to zero necessitates a considerable negative magnetizing force. It is the section of curve in between these two points that gives us what we want, corresponding (except for the generalized type of scales) to Fig. 3*b*.

If Fig. 6 is converted to the ϕ/p co-ordinates of a particular magnet, by multiplying the B scale by a and the H scale by l, we get Fig. 7, which corresponds

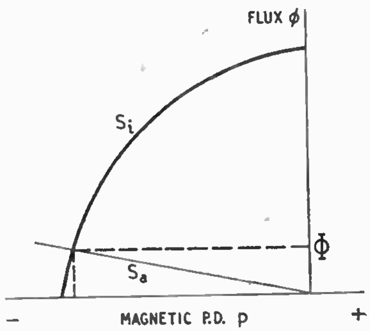

Fig. 7. This is the exact analogue of Fig. 3*b*, so the air-gap line, S_a, can be drawn in. But in practice the S_i line is not usually known, so the S_a line has to be transferred to a graph of the Fig. 6 type by multiplying the size factors, as in Fig. 8.

exactly to Fig. 3*b*, and can be used to investigate the magnet when in series with an air-gap, by drawing the S_a line, corresponding to the R line in Fig. 3*b*.

Neglecting magnetic leakage, the flux in iron and gap is the same, and is represented in Fig. 7 by the intersection of the two graphs, at value Φ. This diagram therefore shows the flux in the air gap when dimensions of gap and magnet are given. If these dimensions had been picked out of a hat or otherwise chosen at random, the chances would be that gap and magnet wouldn't match one another particularly well, any more than a valve

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picked at random would drive a particular loudspeaker with maximum efficiency. In Fig. 7, for example, the reluctance of the air gap is so large for the magnet that comparatively little flux is available in it. This is like a loudspeaker of excessively high impedance for the valve. Steepening the S_a line to represent a gap of lower reluctance, it is seen that the flux increases; until maximum flux is obtained by closing the gap completely—a magnetic short-circuit.

Generally one has an air gap of a particular size, and B/H curves of available magnet alloys, and one wants to know the dimensions of magnet that will give the required flux, or flux density; preferably using the smallest possible magnet. The dimensions of the magnet not being known, it is impossible to start by converting Fig. 6 (which one has) into Fig. 7.

There is a way, which for the sake of those sufficiently interested I will point out at the end, of getting over this by transferring the air-gap line to the B/H diagram; but in the meantime we can use Fig. 7 merely as an aid to grasping the basic principles. These are two in number, and both very simple. The first we have already assumed, that the flux is constant throughout the circuit. That is absolutely true, just as for current in an electric circuit. Neither current nor flux can quietly vanish anywhere around a circuit. But it can leak off so that not all of the total flux goes through a particular localized air gap. If the circuit is well designed, this leakage flux is relatively small, and we shall neglect it. (In practical engineering, it is taken care of by a factor based on experience of similar magnet systems).

So $\Phi = B_i a_i = B_a a_a$ .. (10)

$\therefore a_i = a_a \dfrac{B_a}{B_i}$ .. .. (11)

You are quite right if you guess that the subscripts *i* and *a* indicate iron and air respectively. And as we presumably know the air-gap area, a_a, and its flux density, B_a, all we need to get the magnet cross-section area is its flux density, B_i.

The second principle is that the total M.M.F. (F) in a magnetic circuit is equal to the sum of all the magnetic potential drops. (Compare the corresponding Kirchhof's Law for an electric circuit.) Applying it to our permanent magnet circuit; where there is zero F:

$0 = p_i + p_a = H_i l_i + H_a l_a$ .. .. (12)

$\therefore l_i = l_a \dfrac{H_a}{-H_i}$ .. .. (13)

We know that H_a is equal to B_a (because, if the right units are chosen, μ for air $= 1$), and the length of the gap, l_a, is given, so here we have the length of the magnet, l_i, in terms of H_i. (In practice, another factor is used to cover the fact that the effective length of the gap tends to increase near its edges.)

The B/H curve for the magnet metal gives H_i in terms of B_i, or vice versa; but how do we choose a particular combination of the two? Fig. 7 shows what seems to be a rather bad combination. The best, presumably, is that which requires the smallest magnet.

Combining (11) and (13).

Volume of iron $= l_i a_i$

$= \dfrac{B_a^2 l_a a_a}{-H_i B_i}$ .. .. (14)

Therefore, for given air-gap dimensions and flux density, the smallest volume of iron is needed when $-H_i B_i$ is a maximum. And, as you see, the volume of iron goes up in the same proportion as the volume of air gap, and as the square of the gap flux density.

The thing to do, then, is to plot $-H_i B_i$ against B (a convenient place is in the empty space to the right of the B scale as in Fig. 8), and connect a horizontal line from the peak of that curve to intersect the B/H curve, thus showing the most economical B_i and H_i—the last remaining data needed for finding the magnet dimensions. Instead of bothering to plot the $H_i B_i$ curve, a short cut that is near enough with most magnet curves is to complete the rectangle as shown dotted in Fig. 8, and use the B_i and H_i values given by the intersection of its diagonal with the B/H curve.

Suppose an air gap 0.25 cm long by 2 cm square (after using the factors that allow for flux "fringing") is to be given a flux density of 8,000 lines per sq. cm. (= 8,000 gauss), by means of a magnet made from alloy having the characteristics shown in Fig. 8. The most economical B_i is seen to be 5,000, corresponding to H =

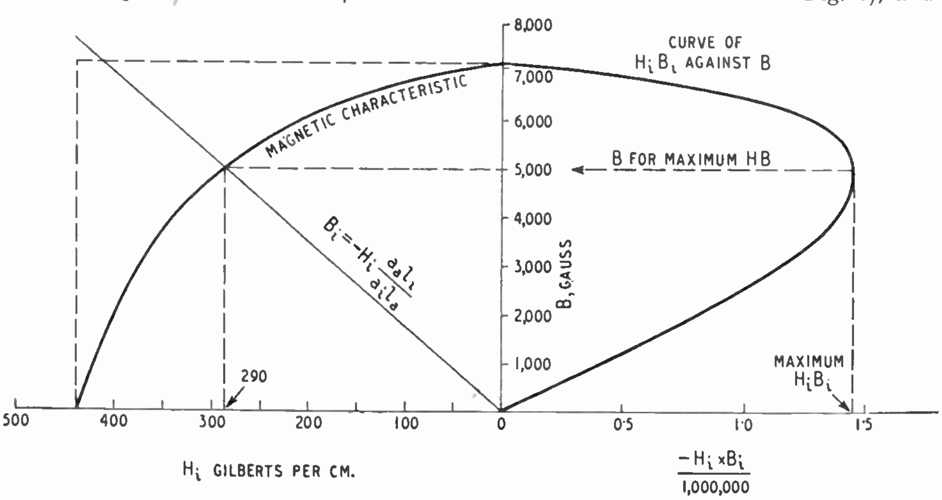

Fig. 8. Example of use of permanent-magnet B/H curve to find the smallest size of magnet necessary to excite a given gap.

-290. Then, applying (11) and (13), $a_i = \frac{B_a a_a}{B_i} = \frac{8,000 \times 4}{5,000}$ = 6.4 sq. cm., and $l_i = \frac{B_a l_a}{-H_i}$ $= \frac{8,000 \times 0.25}{290} = 6.9$ cm.

A magnet of these stubby dimensions may have to be connected to the given air gap by means of low-reluctance pole pieces; but that is as obvious as saying that a generator should be connected to its load by low-resistance leads. And in practice there are sometimes reasons for choosing magnet dimensions other than the most economical in material.

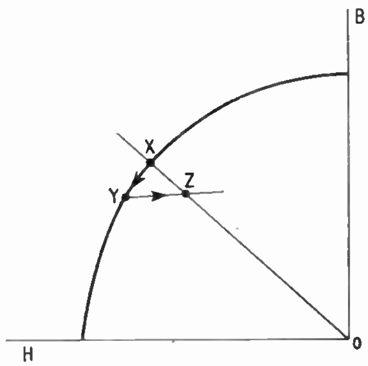

Fig. 9. If a magnetic circuit originally working at point X is demagnetized somewhat, say to Y; removing the demagnetizing influence causes it to return, not to X but to Z. To avoid the loss XZ taking place during service, magnets are previously brought by "aging" on to a flat curve like YZ.

To solve problems like the example above it is clear that no graph drawing is necessary so long as the best B of the magnet metal is given, and its corresponding H; only two easy formulæ.

There are cases, however, where it might at least be instructive to use the "load-line" technique of Fig. 7, if some means could be found of applying it to the B/H curve, instead of to a type of curve that cannot be drawn until we have the information we are trying to find. The S_a line of Fig. 7 mustn't be used in Fig. 6 (or Fig. 8) as it is, because the scales are wrong. To convert Fig. 6 to Fig. 7 we multiplied the B scale by a_i (giving ϕ), and the H scale by l_i (giving p). We could have done the same thing (less conveniently) by leaving the scale *numbering* alone—just changing the symbols—and fitting the curve to it by *dividing* its co-ordinates by the same factors. To convert a graph (to wit, the S_a line in Fig. 7) back to the original Fig. 6 scales of B and H, it is necessary to perform the reverse operation, viz., *multiply* the S_a line co-ordinates, so that what was a graph of

$$\phi = \frac{-p}{S_a} = \frac{-pa_a}{l_a}$$

becomes $Ba_i = \frac{-Ha_a l_i}{l_a}$

$$\text{or } B = \frac{-H a_a l_i}{a_i l_a} \quad \ldots \quad \ldots \quad (15)$$

Given a B/H curve for the magnetic metal, then, a transferred air-gap line can be drawn, as in Fig. 8. Either the slope of the line is given by already-known iron and gap dimensions, in which case the resulting flux-density is shown; or one can draw the line to suit the B/H curve, and then calculate the unknown dimensions from the slope. (Incidentally, $\frac{a_a l_i}{a_i l_a}$, which is the ratio B/-H, is called the unit permeance).

It should be fairly obvious how to apply the foregoing principles to more complicated situations. For example, if there were some M.M.F. coming from a coil, assisting or opposing the magnet, the starting point of the air-gap line in Fig. 7 would have to be to the right or left respectively of zero, by an amount equal to the M.M.F.

In these calculations it has been assumed that the whole of the permanent magnet is working at the same point on its B/H curve. In actual fact it would not be, but the difference is not enough to justify the appalling difficulties one would get into by departing from this simplifying assumption.

This article is only a framework of theory, which serves as a first approximation in design. In practice there are other considerations for which there is no room here.

There is one very important thing, however. Suppose a permanent magnet in service, say in a moving coil meter, comes under the influence of an external magmetic field that opposes its own field, so that the working point moves downwards, say from X to Y in Fig. 9. When this disturbing influence is removed, the *status quo* is not restored. Instead, the working point goes to some such place as Z, the flux being very little more than at Y, and decidedly lower than at X. The accuracy of the meter is permanently ruined. To avoid this, magnets are "aged" by demagnetizing them in advance to a greater extent than they are likely to experience after calibration. So variations take place along a nearly flat curve such as YZ.

This also shows the reason for warnings about it being a bad idea to take permanent magnet circuits to pieces. If the pole pieces are removed, the reluctance is increased, the "load line" moves anti-clockwise to some such position as in Fig. 7, and when put together again the system has lost a large part of its magnetism. One should also keep screwdrivers well away, because the horrid smack when such an implement is drawn against the magnet is also liable to de-gauss it appreciably.

Talking about gauss; magnetic units are a frequent source of confusion, which I shall try to dispel in a sequel. This inevitably brings in electric units as well, leading to consideration of the M.K.S. system of electro-magnetic units.