Feedback—Degenerative and Regenerative

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An analysis of the various forms of feedback, with particular attention to conditions which cause oscillation, and a mathematical presentation of the condition which must be present to cause oscillation.

ONE MIGHT SAY that feedback is as common as the measles. By the brief analysis presented here it is hoped that a clearer understanding is developed of the electronic phenomena.

In general, the block diagram for feedback is given in *Fig.* 1. If a voltage proportional to the output current or voltage of an amplifier is fed back in series with the input voltage so as to decrease the amplification the method is called negative, inverse, or degenerative feedback. The resulting amplifier has certain properties which may be desired despite a loss in amplification.

For simplicity, a sinusoidal input voltage E_1 and an absence of harmonics due to nonlinearity in the tube and related circuits are assumed. The input to the amplifier proper is the vectorial sum of E_1 and the feedback voltage, thus $E_i = E_1 + E_{fb}$. The complex voltage gain of

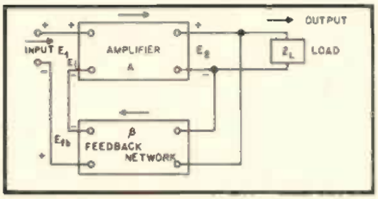

Fig. 1. General feedback configuration as applied to an amplifier.

the amplifier is given as $A = E_2/E_i$, and the feedback ratio is $\beta = E_{fb}/E_2$. It is seen, then, that $E_{fb} = \beta E_2 = \beta A E_i$, and $E_i = E_1/(1-\beta A)$, and $E_2 = AE_1/(1-\beta A)$. From the last expression the following is obtained:

$$\frac{E_2}{E_1} = \frac{A}{1-\beta A} = A'$$

which is the over-all complex voltage gain with feedback.

Now it follows that if βA is real and negative, then $A' < A$ and the amplifier is degenerative. If βA is real, positive, and less than unity, then $A' > A$ and the amplifier is regenerative. The voltage gain approaches infinity if βA is real and approaches *1*. If $\beta A = 1$ the amplifier is unstable and begins to oscillate. This latter condition is the Barkhausen criterion for sustained self-excited oscillation of a single tube circuit under the initial assumptions above. Generally speaking, the magnitude of the voltage gain depends on the absolute quantity $|1-\beta A|$. For very large values of negative feedback the expression $|1-\beta A|$ is very nearly $-\beta A$ and $E_2 \approx AE_1/-\beta A$ so that $E_2/E_1 = A' \approx -1/\beta$. To an arbitrary degree the voltage gain may be made independent of supply voltage changes, tube parameters, and characteristic curves by a high value of inverse feedback. Without considering the involvements of the theory, it is enough to state here that feedback reduces the nonlinear distortion produced in an amplifier for a given output by the ratio $1/(1-\beta A)$.

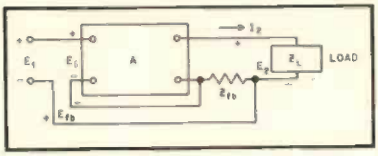

Fig. 2. Block schematic of connection for current feedback.

For any resistance-capacitance coupled amplifier the amplification remains constant for a certain large range of frequencies, but falls on either side of this range. This so-called mid-band range usually extends from below 100 cps to above 5000 cps. In the mid-band frequency range the output voltage is 180 deg. out of phase with the input voltage. The angle of phase shift of the output voltage relative to its mid-band phase is given as

$$\gamma = \text{arc tan } \omega r_h C_1$$

where C_1 is the parallel load capacitance and r_h is the effective output impedance of the stage (defined as the impedance that the plate circuit offers to an external voltage applied between the plate and the cathode).

If the voltage to be fed back is such as to make β independent of the value of the load impedance (which can be had by a simple voltage-divider across the load), then the condition is known as voltage feedback. If, however, the circuitry is as represented in *Fig.* 2, then the ratio of E_{fb} to the output *current*, I_2, is made constant by the feedback impedance Z_{fb}. This is known as current feedback. In the case of voltage inverse feedback, E_2 decreases as Z_L is decreased and E_{fb} also decreases; but the decrease in E_{fb} tends to increase the output voltage and to maintain it constant. With current inverse feedback I_2 increases as Z_L decreases, and E_{fb} increases; but the increase in E_{fb} tends to decrease the output current and to maintain it constant. Therefore, with voltage negative feedback the effective internal series impedance is made smaller by the feedback; whereas, with current inverse feedback the effective internal series impedance of the amplifier is made larger by the feedback. Thus it can be seen that at constant input voltage E_1 the former approaches the behavior of a constant-voltage source, while the latter approaches that of a constant-current source. In multistage amplifiers or in any situation where the amplifier must be terminated for power transfer it is necessary to know the effective internal impedance for optimum matching.

Cathode-Follower Analysis

The cathode-follower is a form of inverse feedback amplifier (see *Fig.* 3). This arrangement has two major advantages over the conventional ampli-

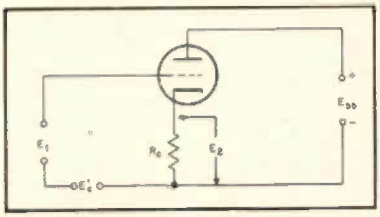

Fig. 3. Typical arrangement of a cathode follower.

fiers which place the load impedance in the plate circuit:

1. One of the load terminals may be grounded.
2. The impedance presented by the tube to the load is small enough that it can be easily matched to a transmission line.

The voltage developed across the load R_c opposes the applied signal voltage so that the circuit exhibits negative feedback characteristics. If the equivalent impedance and the transconductance are not too low, the voltage developed across the load will be almost equal to the applied signal voltage but never larger. The equivalent impedance is formed by the load in parallel with the term $r_p/(1+\mu)$, where μ and r_p have their customary significance.

Oscillators

It was shown earlier that the general requirement for oscillation is $\beta A = 1$. Consider the simple feedback circuit of *Fig.* 4. In the general case the following is true:

(Continued on page 58)

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FEEDBACK

(*from page 23*)

$$E_p = -\mu E_g \frac{Z}{r_p + Z}$$

$$A = -\mu \frac{Z}{r_p + Z}$$

$$\beta = \frac{1}{A} = -\left(\frac{1}{\mu} + \frac{r_p}{\mu Z}\right) = -\left(\frac{1}{\mu} + \frac{1}{g_m Z}\right)$$

which is simply applying the normal tube and circuit parameters to the Barkhausen criterion. In this particular case

$$E_{fb} = E_g$$

$$\beta = \frac{E_g}{E_p} = \frac{j\omega M I_L}{-(R + j\omega L) I_L} = \frac{-j\omega M}{R + j\omega L}$$

Also,

$$Z = \frac{(R + j\omega L)(1/j\omega C)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}.$$

Now substitute β and Z into the Barkhausen criterion:

$$\frac{-j\omega M}{R + j\omega L} = -\left[\frac{1}{\mu} + \frac{1}{g_m} \cdot \frac{R + \left(\omega L + \frac{1}{\omega C}\right) j}{(R + j\omega L)\frac{1}{j\omega C}}\right]$$

$$j\omega M = \frac{R}{\mu} + \frac{j\omega L}{\mu} + \frac{R + \left(\omega L - \frac{1}{\omega C}\right) j}{g_m (1/j\omega C)}$$

$$\frac{R}{\mu j\omega C} + \frac{L}{\mu C} + \frac{R}{g_m} + \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{g_m} - \frac{M}{C} = 0.$$

This is the circuit requirement for oscillation.

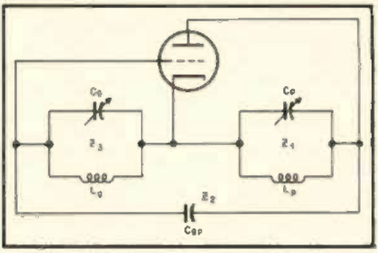

Fig. 5. A simple, tuned-grid, tuned-plate oscillator.

Similar analysis may be performed for any configuration. One other is here given as illustrated in *Fig.* 5. This becomes at once evident as a tuned-grid, tuned-plate oscillator.

$$Z_1 = j\left(\omega L_p - \frac{1}{\omega C_p}\right)$$

$$Z_2 = -\frac{j}{\omega C_{gp}}$$

$$Z_3 = j\left(\omega L_g - \frac{1}{\omega C_g}\right)$$

Completing the general Barkhausen criterion, we have:

$$\frac{1}{\mu} + \frac{1}{g_m} \cdot \frac{j\left(\omega L_p + \omega L_g - \frac{1}{\omega} \cdot \frac{1}{C_p + C_g + C_{gp}}\right)}{j^2\left(\omega L_p - \frac{1}{\omega C_p}\right)\left(\omega L_g - \frac{1}{\omega} \cdot \frac{1}{C_g + C_{gp}}\right)} - \frac{j\left(\omega L_g - \frac{1}{\omega C_g}\right)}{j\left(\omega L_g - \frac{1}{\omega} \cdot \frac{1}{C_g + C_{gp}}\right)} = 0$$

$$\frac{1}{\mu} - \frac{1}{g_m} \cdot \frac{j\left(\omega L_p + \omega L_g - \frac{1}{\omega C_o}\right)}{\left(\omega L_p - \frac{1}{\omega C_p}\right)\left(\omega L_g - \frac{1}{\omega} \cdot \frac{1}{C_g + C_{gp}}\right)} - \frac{\omega L_g - \frac{1}{\omega C_g}}{\omega L_g - \frac{1}{\omega} \cdot \frac{1}{C_g + C_{gp}}} = 0$$

where

$$C_o = C_p + C_g + C_{gp}$$

and:

$$\frac{1}{\mu} + \frac{1}{\omega L_g - \frac{1}{\omega} \cdot \frac{1}{C_g + C_{gp}}}\left[-\frac{1}{g_m} \cdot j \frac{\omega L_p + \omega L_g - \frac{1}{\omega C_o}}{\omega L_p - \frac{1}{\omega C_p}} - \omega L_g + \frac{1}{\omega C_g}\right] = 0$$

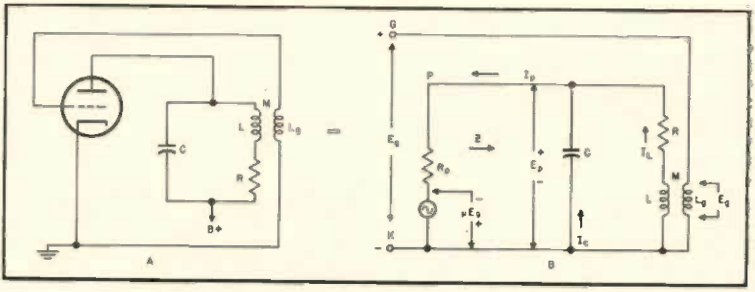

Fig. 4. (A) A simple feedback amplifier, and (B), its equivalent.

The generalized criterion used above depends on the assumption of circuit linearity and inductances of zero resistance for simplicity. It can be shown that

$$\frac{1}{\mu}+\frac{1}{g_m}\cdot\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)}-\frac{Z_3}{Z_2+Z_3}=0$$

is required for oscillation.