

CMBFS

Capacitance Measurement by Frequency Shift

Why, you might ask, should another article be written concerning capacitance measurement when dozens of articles have appeared on this subject in the past? I have reviewed some of them and find that specialized equipment is generally required that may not be available to the average amateur such as a precision calibrated variable capacitor, a grid dip meter, or a capacitance bridge. All techniques required calibrated standards of some sort which usually turn out to be the stumbling block for the average amateur.

What means of calibration is then available to all? The receiver, of course! Every ham has a receiver these days that is calibrated to within 5, 2 and frequently 1 kHz. Even most transceivers are accurate to the latter figure. The problem then is how to use this accurate frequency calibration to measure capacitance. The solution is readily



Finished unit.

evident — tune a simple self excited transistor oscillator to the high frequency edge of one of the amateur bands; connect the unknown capacitance across the oscillator circuit and measure the new lower frequency on the receiver. All that remains to be done is to derive the expression for relating the frequency shift to the unknown capacitance.

A basic Hartley oscillator circuit is shown in Fig. 1. According to R. F. Shea, *Transistor Circuit Engineering*, John Wiley & Sons, Inc., 1957, the oscillation frequency for a Hartley transistor oscillator is:

$$(1) f = \frac{1}{2\pi \sqrt{C(L_1 + L_2 + 2M) - \frac{(L_1 L_2 - M^2) h_{22b}}{h_{11b}}}}$$

which looks somewhat unmanageable. Fortunately, the expression can be greatly simplified. $L_1 + L_2 + 2M$ is nothing more than the total inductance L of the circuit. Let the second term:

$$\frac{(L_1 L_2 - M^2) h_{22b}}{h_{11b}} = A$$

then

$$(2) f = \frac{1}{2\pi \sqrt{LC + A}}$$

A is a constant involving the inductive terms L_1 , L_2 , & M as well as the transistor parameters h_{22b} and h_{11b} and Figs. 2 & 1. If the assumption is made that this term is negligible, then the familiar expression for the resonant frequency of a tuned circuit results.

$$(3) f = \frac{1}{2\pi \sqrt{LC}}$$

Suppose we assume that (3) determines the oscillator frequency for the moment. More will be said about the transistor loading factor "A" later.

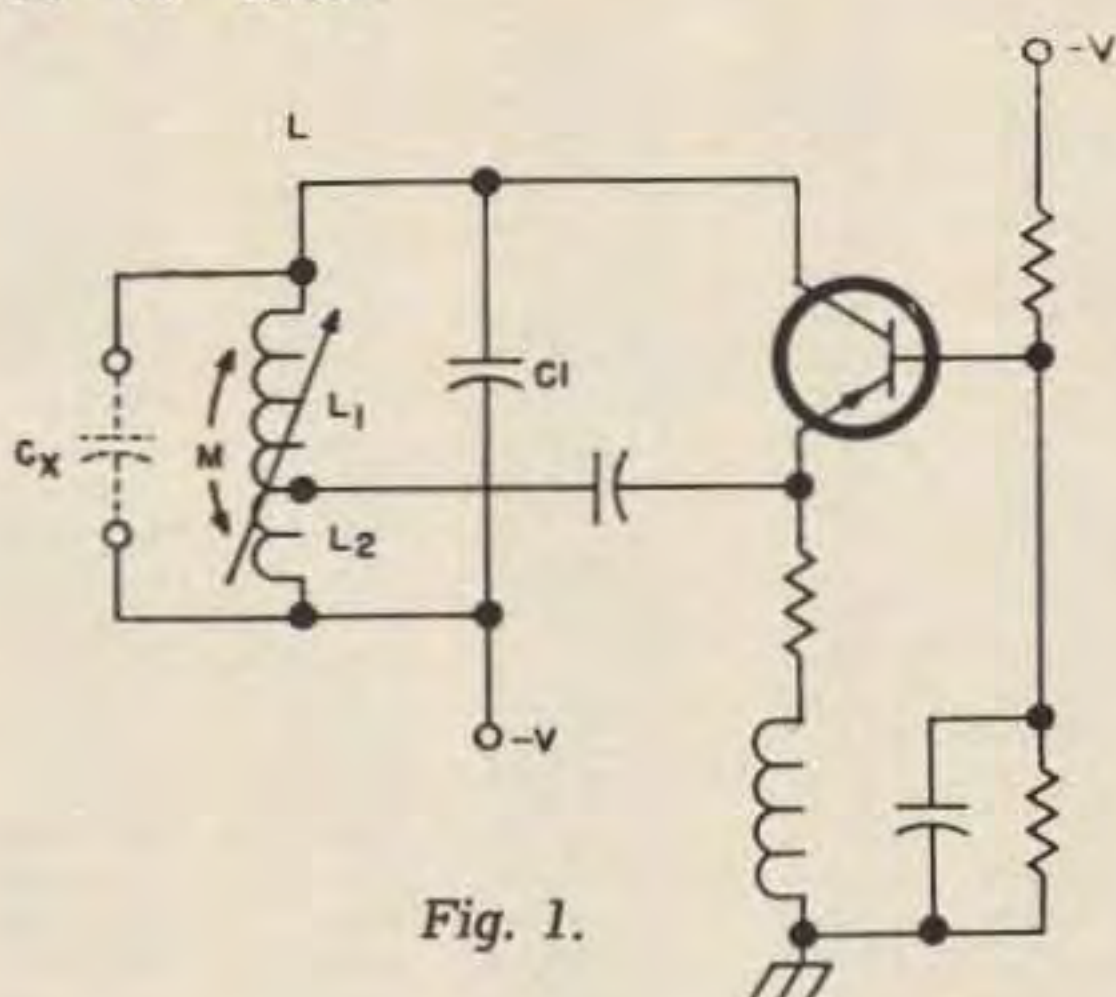


Fig. 1.

It can be shown (see Appendix I) that

$$(4) \quad C_x = \left(C_1 \frac{1}{1 - \frac{2\Delta f}{f_1} + \frac{\Delta f^2}{f_1^2}} - 1 \right)$$

where C_x = unknown capacitance

f_1 = basic oscillator frequency in kHz.

Δf = shift in frequency in kHz due to placing unknown capacitor C_x across tuned circuit

C_1 = fixed, known tank circuit capacitance

The $\Delta f^2/f_1^2$ term is very small compared to $2\Delta f/f_1$ and can be neglected for the moment. Equation (4) reduces to:

$$(5) \quad C_x = C_1 \left(\frac{1}{1 - \frac{2\Delta f}{f_1}} - 1 \right)$$

The significant facts that emerge from this equation are that the unknown capacitance C_x depends only on the shift in frequency Δf , the basic frequency f_1 and the tank capacitance C_1 . C_x is thus independent of L and other factors. To measure large values of C_x , C_1 must be large which dictates the use of the lowest frequency amateur band. 80 meters was selected since 160 isn't available on many receivers. Fortunately, 80 meters is also a wide band and contributes to the range of C_x .

Note that the shift in frequency (Δf) is much more important in determining C_x than the basic frequency f_1 . If f_1 is off by 1

Δf , kHz	C_x , pF	Δf , kHz	C_x , pF	Δf , kHz	C_x , pF	Δf , kHz	C_x , pF	Δf , kHz	C_x , pF
.0	.0000	1.0	.5140	2.0	1.0284	3.0	1.5432	4.0	2.0584
5.0	2.5740	6.0	3.0900	7.0	3.6063	8.0	4.1231	9.0	4.6402
10.0	5.1577	11.0	5.6756	12.0	6.1939	13.0	6.7126	14.0	7.2317
15.0	7.7512	16.0	8.2710	17.0	8.7913	18.0	9.3119	19.0	9.8330
20.0	10.3544	21.0	10.8762	22.0	11.3984	23.0	11.9210	24.0	12.4440
25.0	12.9674	26.0	13.4912	27.0	14.0154	28.0	14.5400	29.0	15.0650
30.0	15.5904	31.0	16.1161	32.0	16.6423	33.0	17.1689	34.0	17.6959
35.0	18.2232	36.0	18.7510	37.0	19.2791	38.0	19.8077	39.0	20.3367
40.0	20.8661	41.0	21.3958	42.0	21.9260	43.0	22.4566	44.0	22.9875
45.0	23.5189	46.0	24.0507	47.0	24.5829	48.0	25.1155	49.0	25.6485
50.0	26.1819	51.0	26.7157	52.0	27.2499	53.0	27.7845	54.0	28.3195
55.0	28.8549	56.0	29.3908	57.0	29.9270	58.0	30.4637	59.0	31.0007
60.0	31.5382	61.0	32.0761	62.0	32.6144	63.0	33.1531	64.0	33.6922
65.0	34.2317	66.0	34.7716	67.0	35.3120	68.0	35.8527	69.0	36.3939
70.0	36.9365	71.0	37.4775	72.0	38.0199	73.0	38.5627	74.0	39.1059
75.0	39.6496	76.0	40.1937	77.0	40.7382	78.0	41.2831	79.0	41.8284
80.0	42.3741	81.0	42.9203	82.0	43.4669	83.0	44.0139	84.0	44.5613
85.0	45.1091	86.0	45.6573	87.0	46.2060	88.0	46.7551	89.0	47.3046
90.0	47.8546	91.0	48.4049	92.0	48.9557	93.0	49.5069	94.0	50.0585
95.0	50.6106	96.0	51.1630	97.0	51.7159	98.0	52.2693	99.0	52.8230
100.0	53.3772	101.0	53.9318	102.0	54.4868	103.0	55.0423	104.0	55.5982
105.0	56.1545	106.0	56.7112	107.0	57.2684	108.0	57.8260	109.0	58.3840
110.0	58.9425	111.0	59.5014	112.0	60.0607	113.0	60.6204	114.0	61.1806
115.0	61.7412	116.0	62.3023	117.0	62.8638	118.0	63.4257	119.0	63.9880
120.0	64.5508	121.0	65.1141	122.0	65.6777	123.0	66.2418	124.0	66.8063
125.0	67.3713	126.0	67.9367	127.0	68.5025	128.0	69.0688	129.0	69.6356
130.0	70.2027	131.0	70.7703	132.0	71.3384	133.0	71.9068	134.0	72.4758
135.0	73.0451	136.0	73.6149	137.0	74.1852	138.0	74.7559	139.0	75.3270
140.0	75.8986	141.0	76.4706	142.0	77.0431	143.0	77.6160	144.0	78.1894
145.0	78.7632	146.0	79.3374	147.0	79.9121	148.0	80.4873	149.0	81.0629
150.0	81.6389	151.0	82.2154	152.0	82.7924	153.0	83.3697	154.0	83.9476
155.0	84.5259	156.0	85.1046	157.0	85.6838	158.0	86.2635	159.0	86.8436
160.0	87.4241	161.0	88.0052	162.0	88.5866	163.0	89.1685	164.0	89.7509
165.0	90.3338	166.0	90.9170	167.0	91.5008	168.0	92.0850	169.0	92.6696
170.0	93.2548	171.0	93.8403	172.0	94.4264	173.0	95.0129	174.0	95.5998
175.0	96.1872	176.0	96.7751	177.0	97.3634	178.0	97.9522	179.0	98.5415
180.0	99.1312	181.0	99.7214	182.0	100.3120	183.0	100.9031	184.0	101.4947
185.0	102.0868	186.0	102.6793	187.0	103.2723	188.0	103.8657	189.0	104.4596
190.0	105.0540	191.0	105.6488	192.0	106.2441	193.0	106.8399	194.0	107.4362
195.0	108.0329	196.0	108.6301	197.0	109.2278	198.0	109.8259	199.0	110.4245
200.0	111.0236	201.0	111.6231	202.0	112.2232	203.0	112.8237	204.0	113.4246
205.0	114.0261	206.0	114.6280	207.0	115.2304	208.0	115.8333	209.0	116.4367
210.0	117.0405	211.0	117.6448	212.0	118.2496	213.0	118.8549	214.0	119.4606
215.0	120.0669	216.0	120.6736	217.0	121.2808	218.0	121.8884	219.0	122.4966
220.0	123.1052	221.0	123.7144	222.0	124.3240	223.0	124.9341	224.0	125.5446
225.0	126.1557	226.0	126.7672	227.0	127.3793	228.0	127.9918	229.0	128.6048
230.0	129.2183	231.0	129.8323	232.0	130.4467	233.0	131.0617	234.0	131.6772
235.0	132.2931	236.0	132.9095	237.0	133.5265	238.0	134.1439	239.0	134.7618
240.0	135.3802	241.0	135.9991	242.0	136.6185	243.0	137.2384	244.0	137.8587
245.0	138.4796	246.0	139.1010	247.0	139.7229	248.0	140.3452	249.0	140.9681
250.0	141.5915	251.0	142.2153	252.0	142.8397	253.0	143.4645	254.0	144.0899
255.0	144.7157	256.0	145.3421	257.0	145.9690	258.0	146.5963	259.0	147.2242
260.0	147.8526	261.0	148.4815	262.0	149.1109	263.0	149.7407	264.0	150.3711
265.0	151.0020	266.0	151.6334	267.0	152.2654	268.0	152.8978	269.0	153.5307
270.0	154.1642	271.0	154.7981	272.0	155.4326	273.0	156.0676	274.0	156.7030
275.0	157.3390	276.0	157.9755	277.0	158.6126	278.0	159.2501	279.0	159.8881
280.0	160.5267	281.0	161.1658	282.0	161.8054	283.0	162.4455	284.0	163.0861
285.0	163.7273	286.0	164.3689	287.0	165.0111	288.0	165.6538	289.0	166.2970
290.0	166.9408	291.0	167.5850	292.0	168.2298	293.0	168.8751	294.0	169.5209
295.0	170.1673	296.0	170.8142	297.0	171.4615	298.0	172.1095	299.0	172.7579
300.0	173.4069	301.0	174.0564	302.0	174.7064	303.0	175.3570	304.0	176.0081
305.0	176.6597	306.0	177.3118	307.0	177.9645	308.0	178.6177	309.0	179.2714
310.0	179.9257	311.0	180.5805	312.0	181.2358	313.0	181.8917	314.0	182.5481
315.0	183.2050	316.0	183.8625	317.0	184.5205	318.0	185.1790	319.0	185.8381
320.0	186.4977	321.0	187.1578	322.0	187.8185	323.0	188.4797	324.0	189.1415
325.0	189.8038	326.0	190.4666	327.0	191.1300	328.0	191.7940	329.0	192.4584
330.0	193.1234	331.0	193.7890	332.0	194.4551	333.0	195.1218	334.0	195.7890
335.0	196.4567	336.0	197.1250	337.0	197.7938	338.0	198.4632	339.0	199.1331
340.0	199.8036	341.0	200.4747	342.0	201.1462	343.0	201.8184	344.0	202.4911
345.0	203.1643	346.0	203.8381	347.0	204.5124	348.0	205.1873	349.0	205.8628
350.0	206.5388	351.0	207.2154	352.0	207.8924	353.0	208.5701	354.0	209.2484
355.0	209.9272	356.0	210.6065	357.0	211.2864	358.0	211.9669	359.0	212.6479
360.0	213.3295	361.0	214.0117	362.0	214.6944	363.0	215.3777	364.0	216.0615
365.0	216.7459	366.0	217.4309	367.0	218.1164	368.0	218.8025	369.0	219.4892
370.0	220.1765	371.0	220.8643	372.0	221.5526	373.0	222.2416	374.0	222.9311
375.0	223.6212	376.0	224.3119	377.0	225.0031	378.0	225.6949	379.0	226.3873
380.0	227.0802	381.0	227.7737	382.0	228.4678	383.0	229.1625	384.0	229.8578
385.0	230.5536	386.0	231.2500	387.0	231.9470	388.0	232.6446	389.0	233.3427
390.0	234.0414	391.0	234.7407	392.0	235.4406	393.0	236.1411	394.0	236.8421
395.0	237.5438	396.0	238.2460	397.0	238.9488	398.0	239.6522	399.0	240.3561
400.0	241.0607	401.0	241.7659	402.0	242.4716	403.0	243.1779	404.0	243.8849
405.0	244.5924	406.0	245.3005	407.0	246.0091	408.0	246.7184	409.0	247.4283
410.0	248.1387	411.0	248.8498	412.0	249.5615	413.0	250.2737	414.0	250.9865
415.0	251.7000	416.0	252.4141	417.0	253.1287	418.0	253.8439	419.0	254.5597
420.0	255.2762	421.0	255.9932	422.0	256.7109	423.0	257.4291	424.0	258.1479
425.0	258.8674	426.0	259.5874	427.0	260.3081	428.0	261.0293	429.0	261.7512
430.0	262.4737	431.0	263.1968	432.0	263.9204	433.0	264.6448	434.0	265.3697
435.0	266.0962	436.0	266.8213	437.0	267.5480	438.0	268.2754	439.0	269.0033
440.0	269.7319	441.0	270.4611	442.0	271.1909	443.0	271.9213	444.0	272.6524
445.0	273.3840	446.0	274.1163	447.0	274.8492	448.0	275.5827	449.0	276.3168
450.0	277.0516	451.0	277.7869	452.0	278.5229	453.0	279.2595	454.0	279.9968
455.0	280.7346	456.0	281.4732	457.0	282.2122	458.0	282.9520	459.0	283.6924
460.0	284.4334	461.0	285.1750	462.0	285.9172	463.0	286.6601	464.0	287.4036
465.0	288.1478	466.0	288.8925	467.0	289.6379	468.0	290.3840	469.0	291.1306
470.0	291.8780	471.0	292.6259	472.0	293.3745	473.0	294.1237	474.0	294.8735
475.0	295.6240	476.0	296.3752	477.0	297.1269	478.0	297.8793	479.0	298.6324
480.0	299.3861	481.0	300.1404	482.0	300.8954	483.0	301.6510	484.0	302.4073
485.0	303.1642	486.0	303.9218	487.0	304.6800	488.0	305.4388	489.0	306.1984
490.0	306.9585	491.0	307.7193	492.0	308.4807	493.0	309.2429	494.0	310.0056
495.0	310.7690	496.0	311.5331	497.0	312.2978	498.0	313.0632	499.0	313.8292

Fig. 2. Frequency-shift capacitance equivalents.

kHz, the effect is only 1 kHz in 4000 kHz whereas a 1 kHz change in Δf has a much larger effect since Δf can vary between 0 to 500 kHz. What this means is that the linearity of your receiver dial calibration is more important than the absolute accuracy. Setting the basic oscillator frequency to 4001 instead of 4000 isn't much cause for concern. The shift in frequency is the important parameter.

The accuracy of C_x is dependent on the accuracy of C_1 . If C_1 is accurate to 5%, C_x will be accurate to 5%. If C_1 is accurate to 1/2%, C_x will be likewise. You can buy as much accuracy as you are willing to pay for. 5% is sufficient for most amateur applications but great accuracy can be achieved inexpensively in several ways, for example, padding up an undersized C_1 if there is precision capacitance measuring equipment available. If not, precision capacitances can be purchased from industrial electronic supply houses. It seems hardly worth buying a 5% unit for 60 cents when a 1% unit can be obtained for \$1.37. The Cornell Dubilier type CD19F102F500 capacitor can be obtained from major supply houses. Arco Electronics, Community Drive, Great Neck, N.Y. 11022 is the distributor for El Menco type DM20 capacitors which can be obtained at 1% or better tolerance on special order.

One nice feature of the CMBFS technique is that the oscillator is not critical. There is no precision or long term stability required. Temperature, voltage changes, etc. will have no appreciable effect. The only stability required is that long enough to last for 15 seconds — the length of time it takes to make a measurement. Inaccuracies are balanced out by adjusting the variable inductance L to produce a 4000 kHz oscillation frequency immediately prior to the capacitance measurement.

Returning to an earlier assumption, the next step is to verify the accuracy of equation (4) with regard to omission of the transistor loading factor "A". An oscillator, to be described in the next section, was constructed using a C_1 value of 1000 pF accurate to within 1/4 of 1% as measured on a precision laboratory bridge. A known C_x of 312.3 pF \pm 1/4% produced a frequency shift of 497 kHz. If these values are substituted into equation (4), C_x is calculated to be 303.9 pF or 8.4 pF less than it should be.

This amounts to an error of 2.7% and is attributed directly to transistor loading shifting the oscillator frequency. Another way of looking at it is that the transistor has added 27.7 pF of capacitance to the tuned circuit. The term "transistor loading factor" is used somewhat loosely. It also includes the circuit stray capacitances. To allow for the loading effect a constant K_1 is inserted into equation (4).

$$(6) \quad C_x = K_1 C_1 \left(\frac{1}{1 - \frac{2\Delta f}{f_1} + \frac{\Delta f^2}{f_1^2}} - 1 \right)$$

where $K_1 = 1.02768$

The term A could have been calculated directly from the inductance and transistor parameters but it would have been for an "average" transistor. The question is "What is the variability in this term with different transistors in the circuit?" To answer this question quantities of different transistors were plugged into the circuit and the change in Δf was noted. Intermediate frequency type 2N404's produced a variation of \pm 6 kHz shift out of 500 kHz. The high frequency type 2N964 produced a negligible variation in shift of \pm 1/4 kHz out of 500 kHz and were therefore judged the most satisfactory. The Motorola HEP 1 at 89 cents acts the same as the 2N964 and is recommended.

The average ham would hardly want to solve equation (6) every time he wanted to make a capacitance measurement and therefore a computerized solution was sought. With 1 KC increments in Δf being available and covering a 500 kHz range — 500 calculations have to be made. The computer is a natural tool for this job. The problem was programmed for a Univac 1107 and all 500 points were calculated in seven seconds. It would have taken me 41 hours with a desk calculator to get the same answers with many mistakes. The results are photographically reproduced in Fig. 2. Don't be fooled by all of the significant digits in the capacitance columns. Although the accuracy is inherently there, your answer is limited by the accuracy of C_1 and your receiver calibration.

Circuit and Construction Details

TR1, L1, C1 of Fig. 3 form the Hartley Oscillator circuit. TR2 is a buffer amplifier driven from the emitter tap on L1. Its function is to isolate any loading on the

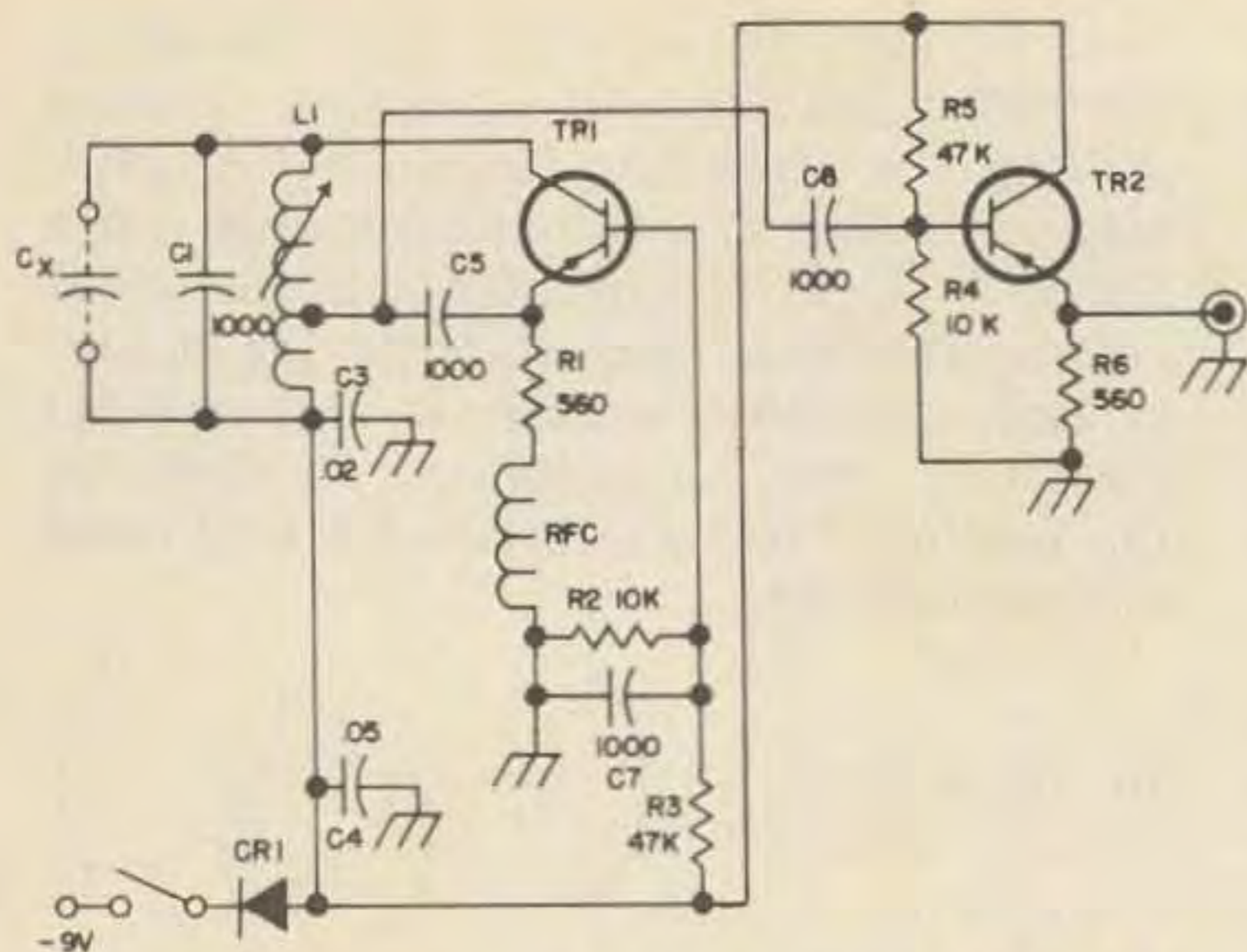
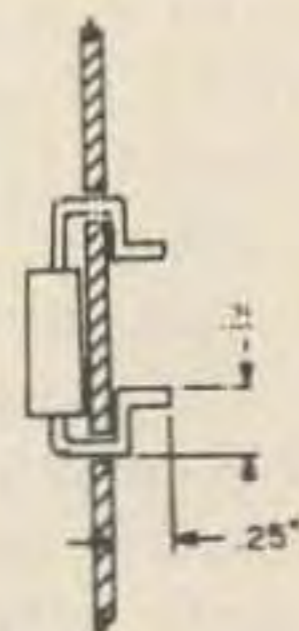
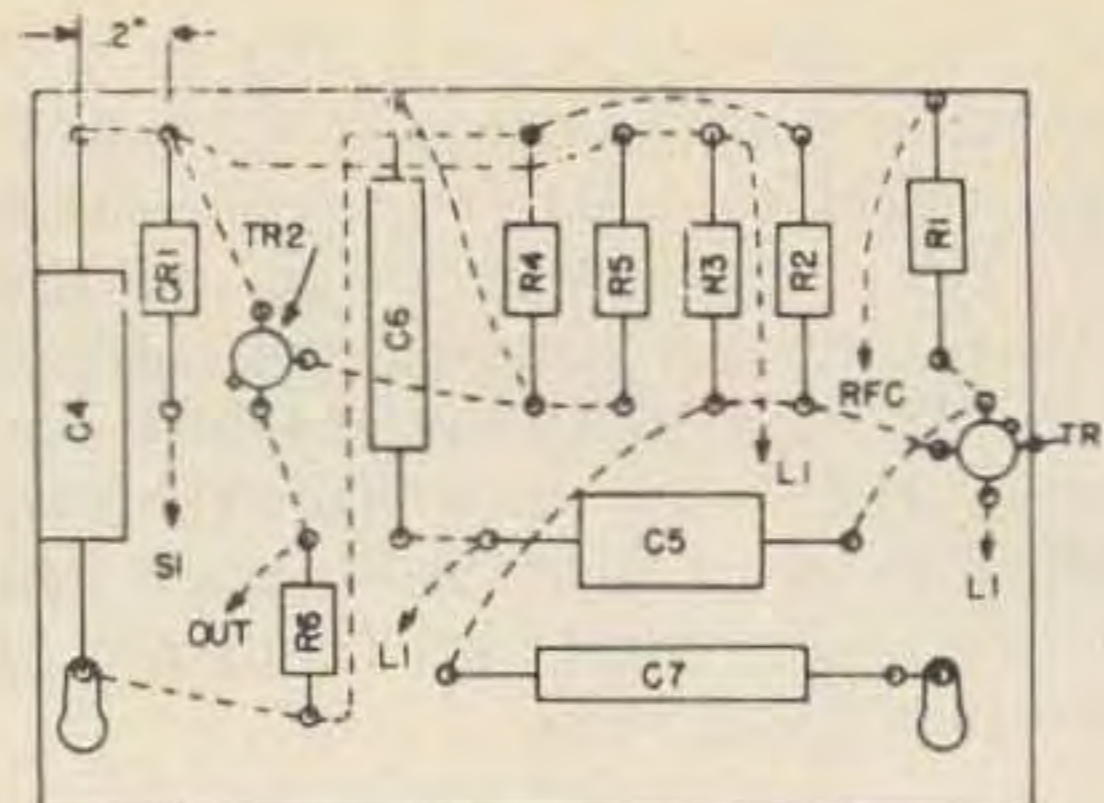


Fig. 3. Schematic. TR1, TR2: 2N964, HEP1; RFC: 2.5 MH National R100S; L1: 3/8" dia. slug tuned form, J.W. Miller 4400, 12 turns closewound No. 26E, tap 3 turns from the bottom; CR1: 1N34 type germanium diode; C1: 1000 pF precision capacitor (see text); C5: 1000 pF mica; other capacitors can be ceramic or paper. All resistors 1/2W, 10%.

output of TR2 from affecting the oscillator frequency. CR1 is a safety diode that prevents the application of reversed voltage from causing any damage. The circuit is conventional in all other aspects.

The complete unit is built into a 5" x 3" x 2" minibox with plenty of space to spare. All components except L1, RFC, C2, C1 and the on-off switch are mounted on a 2-1/4" x 1-7/8" piece of 85G24EP Vectorboard (holes on .1 inch centers staggered) and held onto the chassis by spade lugs. A layout of the board is shown in Fig. 4. The components are mounted by inserting the leads through the holes and crimping them as shown in Fig. 4. Although the components can wiggle when first installed, the board becomes one solid mass after wiring. Wiring is done on the back side of the board in



Method of crimping component lead.

Fig. 4. Component board. Component side solid, wiring side dotted lines.

conventional fashion. It is recommended that construction similar to that shown be followed in order to minimize the effect of stray capacitances on accuracy. 1% of 1000 pF is 10 pF which means that the rf wiring must be short, direct and kept away from ground. For this latter reason, the hot end of L1 and C1 must be isolated from ground. This is accomplished by mounting the binding posts on a 2-1/8" x 1-5/8" piece of phenolic and inserting behind a 1-13/16" x 1-1/16" cutout on the top front of the aluminum box as shown in the photograph. The terminals are mounted 13/16" apart to conveniently accept the leads of the capacitor to be measured.

C3, the .02 disc ceramic bypass capacitor, mounts between the bottom end of L1 and the spade lug ground. TR2 output feeds to a phono type jack. Although there is sufficient space to include a 9v battery inside the box, I decided to bring power in through a terminal strip instead. The chances are that the CMBFS unit will receive occasional use and that the battery will be dead when you do want to use it. I thought it best to use an external power supply or a battery borrowed from a transistor radio BC set when needed.

Operation

Apply voltage to the CMBFS unit. Tune your receiver to 4000 kHz. Connect the output lead to the receiver antenna terminal. Tune L1 until the signal from the oscillator is zero beat with the receiver. It should be a



Inside view of device.

well over S9 stable signal. Now connect the capacitor to be measured to the binding posts. You will note that the signal is no longer at 4000 kHz. Tune your receiver lower in frequency until the new signal is picked up and zero beated. If there is any question about it being the correct signal, bring your finger near the hot binding post and the frequency will shift slightly. Record the new frequency and subtract it from 4000 kHz to get the shift (Δf) in frequency. Now read the actual value of the capacitor corresponding to Δf directly from the chart (Fig. 2). That's about all there is to it.

It may not be necessary to actually connect the CMBFS unit output to the receiver antenna terminal. Radiation from a 2 or 3 foot piece of wire may be sufficient, depending upon the shielding of the receiver. More important is the elimination or reduction of 80 meter signals from other amateur stations that tend to confuse or lose the CMBFS signal. Disconnecting the 80M receiving antenna is desirable. I have found that switching to the 10 or 15 meter antenna or the dummy load is quite effective in reducing extraneous 80M signals.

The battery voltage isn't critical. A 1 volt shift from -9 to -8 volts causes a barely discernible several cycles shift in frequency. Although the oscillator will oscillate down to 2 volts, I don't recommend operating at this point because the loading factor K1 will noticeably increase and cause an inaccuracy in the measurement.

Summary

Equations — equations, the proof is in the performance! A number of capacitors were selected from the junk box, measured by this technique and compared to the 1/4% precision laboratory bridge. The receiver measurements were made with my old Collins 75A1 and repeated with a Drake TR3 transceiver. The results were as follows.

Face Value	CMBFS Value	Precision Bridge Value
10 pF	10.35 pF	10.45 pF
50 pF	50.8 pF	51.6 pF
100 pF	115.2 pF	115.0 pF
180 pF	175.0 pF	174.6 pF
270 pF	269.36 pF	268.4 pF

Not bad, considering that the CMBFS unit costs less than \$10 while the precision bridge costs over \$1,000.

Although this particular unit has a capacitance range up to 313 pF, there is no reason why a higher C1/L ratio can't be chosen to permit reading higher values of capacitance, that is, if you have a computer handy to give you a new set of computations. Another approach for extending the range is to keep C1 at 1000 pF, but split it into two parts with the unknown capacitor placed across one of the parts in a capacitive divider arrangement. This again requires recomputation. The present range satisfied the majority of my requirements in working with *rf* circuits and provided the excellent definition of .5 pF/kHz at the low end and .75 pF/kHz at the high end of the range. Thus this approach, coupled with the computer print-out rather than the usual plotted curve, permits measuring a fraction of a pF difference between 300 pF capacitors.

CMBFS isn't a technique for the production line testing of capacitors but it is well suited for the occasional amateur need and is capable of providing a high degree of accuracy at low cost.

APPENDIX I

$$f_1^2 = \frac{1}{4\pi^2 LC_1} \quad L = \frac{1}{4\pi^2 f_1^2 C_1}$$

with C_x in parallel with C_1

$$f_2^2 = \frac{1}{4\pi^2 L(C_1 + C_x)} \quad L = \frac{1}{4\pi^2 f_2^2 (C_1 + C_x)}$$

L is the same in both cases and can be equated

$$f_1^2 C_1 = f_2^2 (C_1 + C_x)$$

$$\frac{f_1^2 C_1}{f_2^2} = C_1 + C_x$$

$$\left(\frac{f_1^2 C_1}{f_2^2} - C_1 \right) = C_x$$

$$\left(\frac{f_1^2}{f_2^2} - 1 \right) C_1 = C_x$$

$f_2 = f_1 - \Delta f$ where Δf = difference in frequency

$$\left(\frac{f_1^2}{(f_1 - \Delta f)^2} - 1 \right) C_1 = C_x$$

$$\left(\frac{f_1^2}{f_1^2 - 2\Delta f f_1 + \Delta f^2} - 1 \right) C_1 = C_x$$

$$\left(\frac{1}{1 - 2\Delta f/f_1 + \Delta f^2/f_1^2} - 1 \right) C_1 = C_x$$

...W3HTF