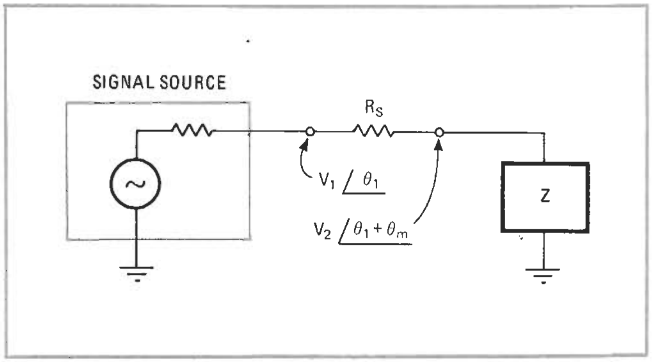
Measuring complex impedances at actual operating levels

by Jim Walworth
Honeywell Inc., Tampa Division, Tampa, Fla.

Complex impedance is usually measured with a vector impedance meter or a network analyzer. The vector impedance meter supplies its own signal source at a fixed level, which is sometimes lower than the normal operating level of the device under test. This approach can cause problems if the device involved is nonlinear. The network analyzer uses a dual directional coupler and measures the impedance relative to a 50-ohm system.

A simpler and equally effective means of measuring complex impedance is often overlooked as a useful data-gathering technique. By inserting a noninductive resistor in series with the unknown impedance, voltage and phase measurements can be made on each side of the resistor. This procedure allows in-circuit parameter measurements at the normal operating levels of the circuit. Additionally, the method requires less test equipment and is more versatile since data can be reduced graphically or mathematically.



1. Test setup. Unknown complex impedance Z can be determined by measuring the voltage drop and phase shift across noninductive resistor R_s. Complex impedance Z can then be found graphically with a modified Smith chart or mathematically with a calculator.

The circuit illustrated in Fig. 1 shows the voltage and phase relationships that must be determined. R_S is the noninductive resistor in series with the unknown impedance, Z. The signal source can be an external source or the circuitry that normally drives Z. The complex voltage at the input to R_S is $V_1 \underline{/\theta_1}$; and the complex voltage across the unknown impedance is $V_2 \underline{/\theta_1 + \theta_m}$, where θ_m is the phase shift across R_S.

Unknown impedance Z is calculated using vector algebra. The series combination of R_S and Z form a voltage divider, and $V_2 \underline{/\theta_1 + \theta_m}$ is given by:

$$V_2 \underline{/\theta_1 + \theta_m} = V_1 \underline{/\theta_1} \left[\frac{Z}{R_S + Z} \right]$$

Solving this equation for Z yields:

$$Z = R_S \left[\frac{V_2 \underline{/\theta + \theta_m}}{V_1 \underline{/\theta_1} - V_2 \underline{/\theta_1 + \theta_m}} \right]$$

or:

$$Z = R_S \left[\frac{V_2 \underline{/\theta_m}}{V_1 - V_2 \underline{/\theta_m}} \right] \quad (1)$$

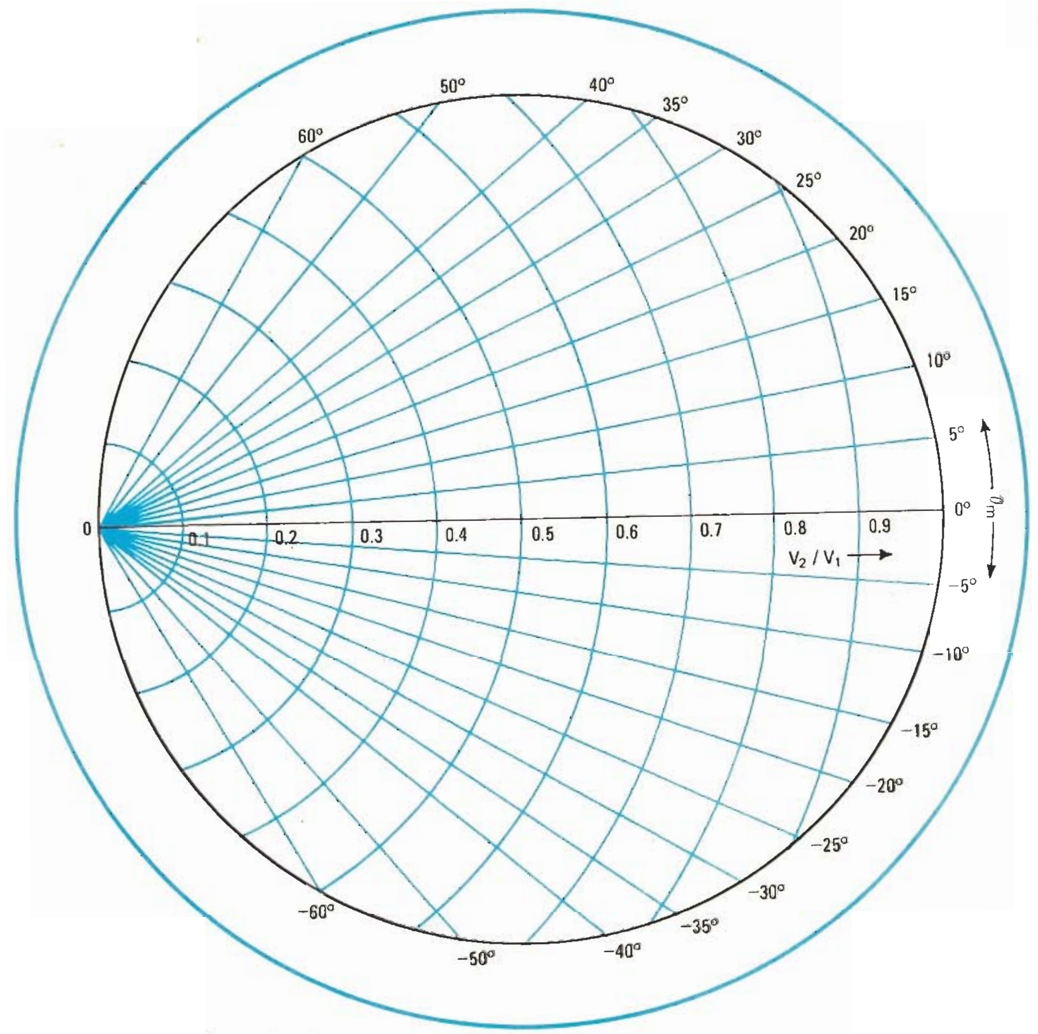
The term θ_m is the relative phase across the resistor and, therefore, the desired phase parameter to measure.

The last equation for Z can easily be solved with some of the scientific calculators now available, or it can be solved graphically. Normalizing Eq. 1 in terms of resistance R_S produces:

$$\frac{Z}{R_S} = \frac{(V_2/V_1) \underline{/\theta_m}}{1 - (V_2/V_1) \underline{/\theta_m}} \quad (2)$$

If this equation is plotted in polar coordinates, it forms a circle having its center point at −1. If it is plotted on a Smith chart, the circle's center lies at the far left side of the chart with a radius of V_2/V_1 and an angle of θ_m.

Figure 2 is a Smith chart showing the contours of Eq. 2. It now becomes a simple matter to determine an unknown impedance quickly by measuring the voltage



2. Graphical solution. The value of complex impedance Z, normalized with respect to noninductive resistor R_S, can be found graphically from this modified Smith chart. The chart establishes the coordinates for the voltage ratio of V_2/V_1 and the relative phase shift of θ_m.

ratio of V_2/V_1 and its relative phase, θ_m.

Suppose an unknown complex impedance is to be measured. The first step is to estimate the relative impedance magnitude and choose a noninductive resistor having such a value—511 ohms, for this example. (Optimum accuracy is obtained when R_S approximately equals the absolute value of Z.) By using the test setup of Fig. 1, the following data is then taken:

$$V_1 = 1.0\ V$$
$$V_2 = 0.6\ V$$
$$\theta_m = -10^\circ$$

and:

$$(V_2/V_1)\underline{/\theta_m} = 0.6\underline{/-10^\circ}$$

The point, $0.6\underline{/-10^\circ}$, is next plotted on the modified Smith chart of Fig. 2, and the impedance, Z_n, which is normalized to 511 ohms, can be read off the chart in the conventional manner:

$$Z_n = 1.28 - j0.58$$
$$Z = (511\ ohms) \times (1.28 - j0.58)$$
$$Z = 654 - j296 = 718\underline{/-24.5^\circ}$$

This procedure should be repeated until the computed magnitude of impedance Z is the same order of magnitude as the estimated value for resistor R_S.

The same equation—Eq. 2—can be solved mathematically on a scientific calculator. For this example, the measured data can be reduced to:

$$Z = 726\underline{/-24.29^\circ} = 661.7 - j298.6$$

The mathematical solution is more accurate than the graphical one, but the graphical technique is quicker. The accuracy of the results depends on the tolerance and quality of the resistor used, the accuracy of the test equipment, and the accuracy of the data-reduction technique.

A modified Smith chart can also be a powerful analysis aid when VSWR measurements are to be made at low frequencies. A series resistor is chosen equal to the characteristic impedance, Z_o, of the system, and voltages V_1 and V_2, as well as phase θ_m, are measured. The maximum acceptable VSWR circle is drawn on the chart, and $(V_2/V_1)\underline{/\theta_m}$ is plotted at each frequency of interest. Since the chart is normalized to Z_o, all points of $(V_2/V_1)\underline{/\theta_m}$ falling outside of the circle are out of specification. □

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