Designer's casebook

Digital pulses synthesize audio sine waves

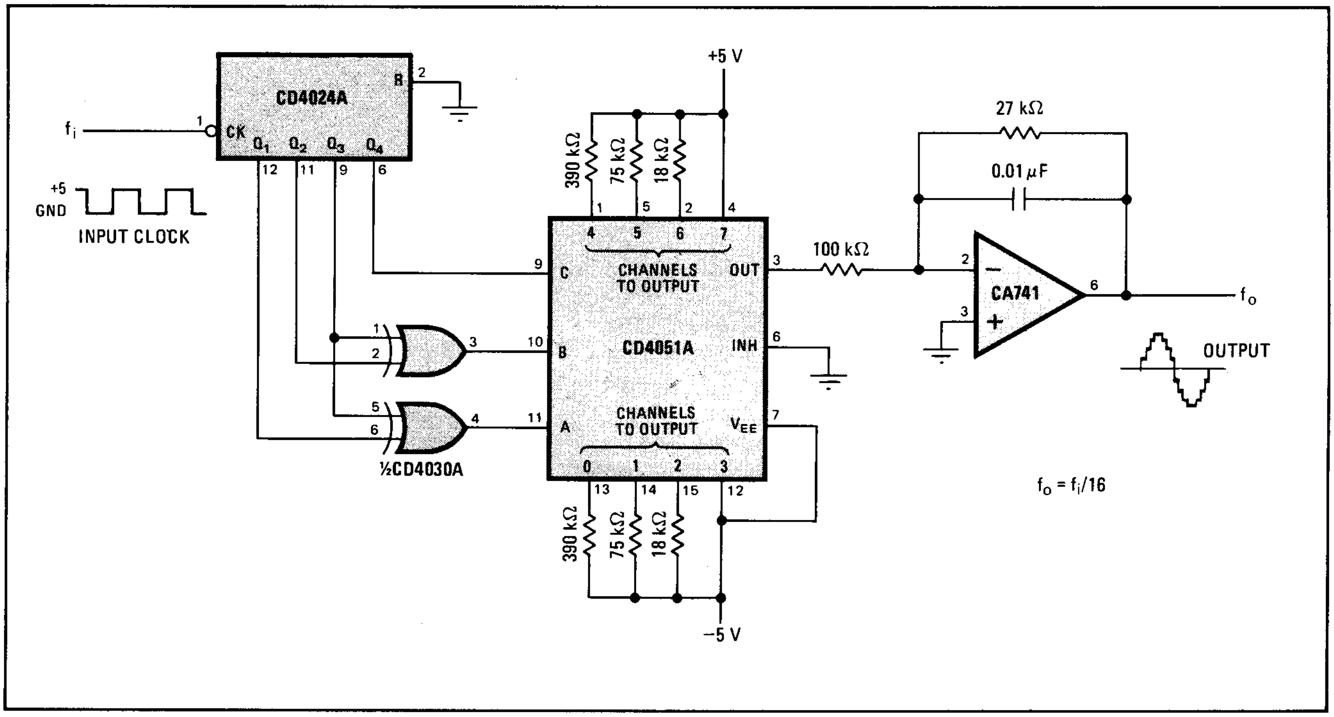
by Patrick L. McGuire
General Dynamics, Pomona, Calif.

Audio tones are often employed for signaling and control in digital systems. To minimize harmonic distortion and channel cross talk, the waveforms of these tones must resemble sine waves as nearly as possible. If the square-wave signal typical of a digital system were used directly, it would have to be thoroughly filtered. Integrated circuits that produce sinusoids from digital inputs, now available commercially, are powerful for their specific functions. However, the technique shown here offers a more general approach to the generation of sine waves or other repetitive waveforms. It requires only a few inexpensive components and dissipates very little power because it is implemented with complementary-MOS circuits.

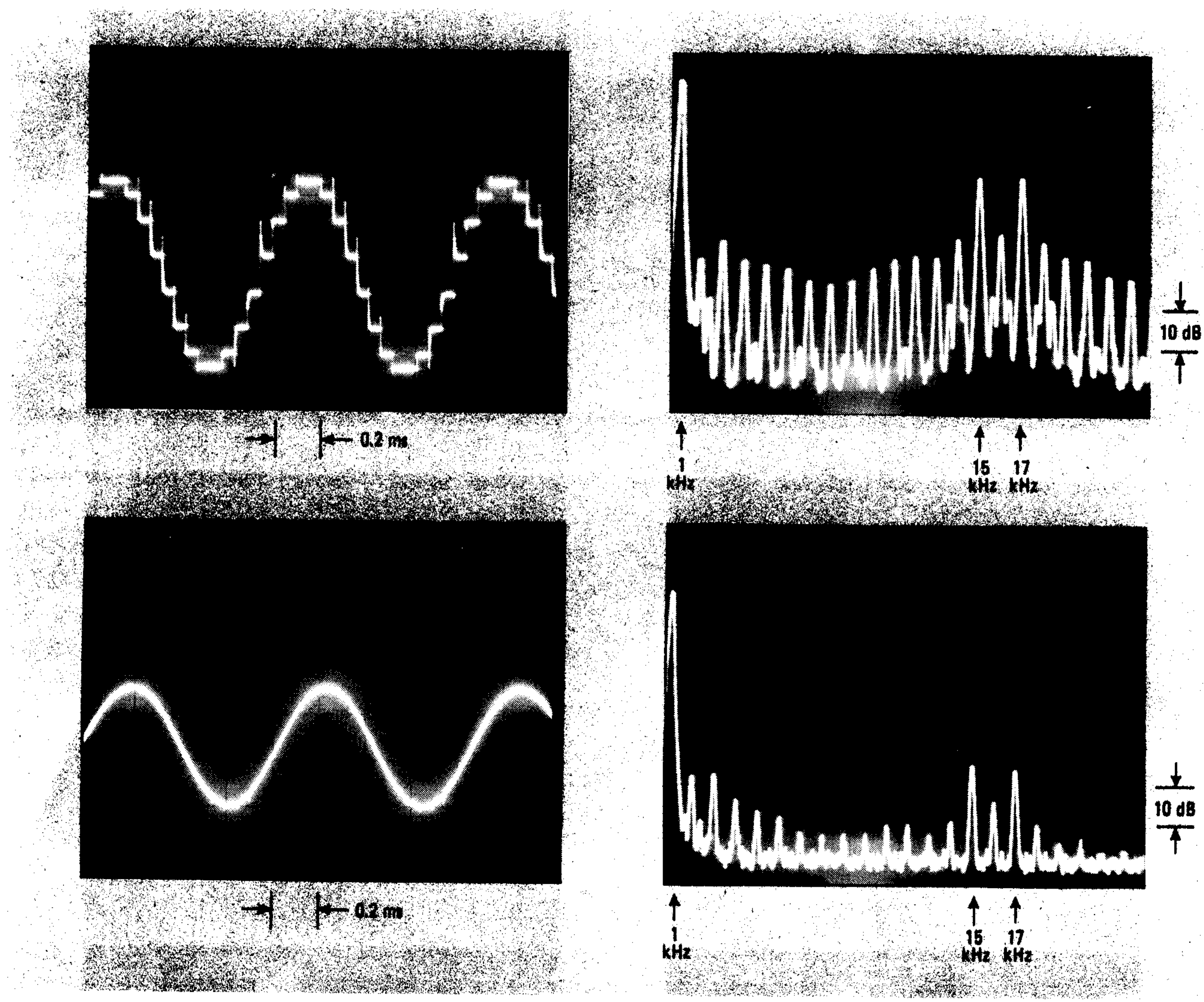
The circuit shown in Fig. 1 generates a 1-kilohertz sine wave from a 16-kHz clock input. The input clock drives a CD4024 counter, which, through some count modification in exclusive-OR gates, drives a CD4051 multiplexer. The eight channels of the multiplexer can deliver any of four different positive currents or four

GENERATING SINE WAVES

Pulse No.	Outputs from CD4024 Q_4	Q_3	Q_2	Q_1	Inputs to CD4051 C	B	A	ON channel of CD4051	Output voltage from 741
⋮		⋮				⋮		⋮	⋮
14	1	1	1	0	1	0	1	5	−0.77
15	1	1	1	1	1	0	0	4	−0.28
0	0	0	0	0	0	0	0	0	0.28
1	0	0	0	1	0	0	1	1	0.77
2	0	0	1	0	0	1	0	2	1.14
3	0	0	1	1	0	1	1	3	1.35
4	0	1	0	0	0	1	1	3	1.35
5	0	1	0	1	0	1	0	2	1.14
6	0	1	1	0	0	0	1	1	0.77
7	0	1	1	1	0	0	0	0	0.28
8	1	0	0	0	1	0	0	4	−0.28
9	1	0	0	1	1	0	1	5	−0.77
10	1	0	1	0	1	1	0	6	−1.14
11	1	0	1	1	1	1	1	7	−1.35
12	1	1	0	0	1	1	1	7	−1.35
13	1	1	0	1	1	1	0	6	−1.14
14	1	1	1	0	1	0	1	5	−0.77
15	1	1	1	1	1	0	0	4	−0.28
0	0	0	0	0	0	0	0	0	0.28
1	0	0	0	1	0	0	1	1	0.77
⋮		⋮				⋮		⋮	⋮



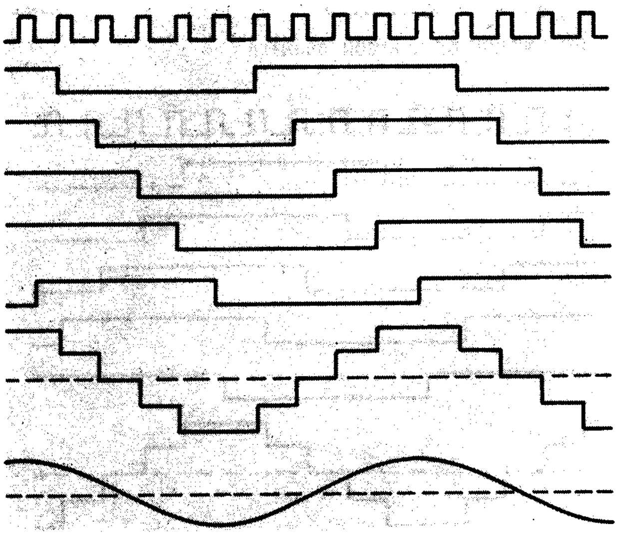
1. Sine-wave generator. The 16-kHz input clock drives a counter that, through X-OR gates, drives a multiplexer. The multiplexer simply routes currents in amplitudes proprotional to a sampled sine wave into the summing junction of the op amp. The feedback capacitor rolls off the frequency response of the amplifier at about 600 Hz. This technique of waveform generation can also be used for other repetitive waves.



2. Output. Wave shape and spectral composition of output from circuit in Fig. 1 are shown with and without filter capacitor in feedback circuit of output operational amplifier. Without filtering, waveform (a) clearly shows the discrete steps of synthesis, and spectrum (b) contains harmonics at 15 kHz and 17 kHz that are only 25 dB below the fundamental 1-kHz output. With frequency response rolled off at about 600 Hz by addition of filter capacitor to circuit, waveform (c) is smoothed, and all harmonics in spectrum (d) are down by more than 45 dB.

different negative currents to the summing junction of a 741 operational amplifier. The table indicates how the counter and X-OR gates ensure that these currents are delivered in the proper sequence to produce the output waveform shown in Fig. 2. The wave is smoothed by the feedback capacitor in the op-amp circuit.

The photographs in Fig. 2 demonstrate the step-by-step generation of the sine wave and the smoothing effect of the filter capacitor, as well as the harmonic content of the output. Without the filtering, the 15th and 17th harmonics are only 25 decibels weaker than the fundamental output signal, but the capacitor adds a corner at about 600 Hz so that these harmonics are reduced to 45 dB below the signal. □



Create Sinewaves Using Digital IC's

Digital techniques can be used to synthesize sinewaves whose amplitude and frequency can be precisely and rapidly controlled and whose distortion is low

DON LANCASTER

EVERY ONCE IN A WHILE A REALLY GREAT IDEA gets buried deep in the technical literature. For instance, back in 1969, a very elegant and ultra-simple way to generate sinewaves digitally appeared. Then, apparently, it was nearly forgotten. Today we can use a $1.00 CMOS integrated circuit, three or four 5% resistors, and this "lost" method to build ultra-simple digital sinewave sources—sinewave sources whose amplitude and frequency we can precisely and rapidly control, and whose distortion is very low. We can use sinewaves like these in electronic music, lab function generators, sweepers, microprocessor and minicomputer analog I/O, digital cassette recorders and MODEMS. But, you're sure to find lots more places where you can use these simple, quick and sophisticated techniques.

The basic idea

Any method of generating sinewaves digitally is usually a two-step process. First, you generate a convenient waveform that consists of a fundamental and some harmonics. Then you get rid of the harmonics by filtering them out. The trick is to pick a convenient waveform that has as few harmonics as possible to start with. We also want the harmonics to be as small as possible, and we want them to be as high an order as possible. All these requirements simplify our filtering and let us change frequency over a reasonable range without necessarily changing our filtering.

Our search for a convenient waveform starts with a *symmetrical* one. This automatically gets rid of all the even harmonics. From here, we want to pick some waveform that inherently doesn't have as many of the odd harmonics as is possible. Ideally, we'd like to get rid of all the low-order odd harmonics. Directly using squarewaves doesn't look too promising because of a third harmonic only 10 decibels (1/3 amplitude) down from the fundamental that's staring you in the face. Similarly, most any relatively simple system based on binary counters will probably also have lots of strong, low-order odd harmonics.

The secret of digital sinewave generation is shown in Fig. 1. You use a circuit called a *walking-ring* or a *Johnson* counter to ultimately generate your sinewave. You make the counter as long as you have to. The longer the counter, the more parts, but the higher the odd harmonics you end up with and the weaker they are. A second part of the secret is that you combine outputs of the counter with resistors, into either a single small-value summing resistor, or into the summing input on an operational amplifier. *But you skip one counter stage in your summing.* This makes our sinewave waveform take twice as long automatically on the peaks and valleys. As we'll shortly see, the waveforms look rather strange and choppy before filtering, but *they have no low-order harmonics!*

You can make a walking-ring counter out of type-D flip-flops or out of many different types of shift registers. The CMOS 4018 is ideal for 6, 8, and 10-step sinewave synthesis, as we'll shortly see, and several 4018's can be cascaded for longer sequences. Our ten-step system takes only one 4018 (or five type-D flip-flops). Pick the resistors just right, and the first harmonic after the fundamental is the *ninth*, and it's almost 20 dB (one-tenth amplitude) down from the fundamental *before* you do any filtering. The only other low-order harmonics are the 11th, the 19th, 21st, 29th, 31st, and so on. All these are so low in amplitude and so high in frequency that if you get rid of the ninth by low-pass filtering, the rest will utterly disappear.

A type-D flip-flop or a register stage is a clocked logic block. When an input clock arrives, information on the D input is passed onto the Q output and its complement is passed onto the $\bar{Q}$ output. (If D is a "1", clocking puts a "1" on Q and a "0" on $\bar{Q}$. If D is a "0", clocking puts a "0" on Q and a "1" on $\bar{Q}$.)

To build a walking-ring counter, connect the Q output of one stage to the D input of the next stage and so on down the line. At the last stage use the complementary $\bar{Q}$ output to feed back to the D input of the first stage. If we use a five-stage register and start with

00000, one clocking gives us 10000 since the $\bar{Q}$ output of the last stage was a "1" and gets passed on to the first stage. More clockings give us 11000, 11100, 11110, and 11111. The $\bar{Q}$ output is now a 0, so the next clocking gives us 01111, 00111, 00011, 00001, and finally 00000, repeating the ten-step sequence as we close the series. The length of our sequence is ten or twice the number of stages.

The sequence length usually equals twice the number of stages in use. If we look at the five outputs A through E in Fig. 1, we see that we get a group of five *phase-shifted* squarewaves. We now sum *four of these five waveforms* with just the right "magic" resistor values, and we get a composite waveform that is a fundamental sinewave along with low-amplitude ninth, eleventh, and a few very small and very high-order remaining odd harmonics. For many uses you can use this sinewave pretty much as is, but it's a simple matter of filtering to get a sinewave of good purity.

Our clock frequency sets the output frequency. With a five-stage, ten-step system, the clock input is ten times the output frequency. As the clock frequency changes, so does the output on a nearly instantaneous basis, since there are no time constants or inductors in the circuit. Note also that a sudden change in clock frequency *coherently* changes the sinewave without any transients or jumps.

With CMOS and relatively light loading (20K or more) the output logic swing is equal to the supply voltage, so we can change the output amplitude either by changing the supply voltage or by changing the gain (digitally or otherwise) of any op-amp that's summing our phase-shifted squarewaves into the composite sinewave output.

Circuits

Figs. 2 and 3 show us two circuits using a single 4018 CMOS register that you can use for digital sinewave generators.

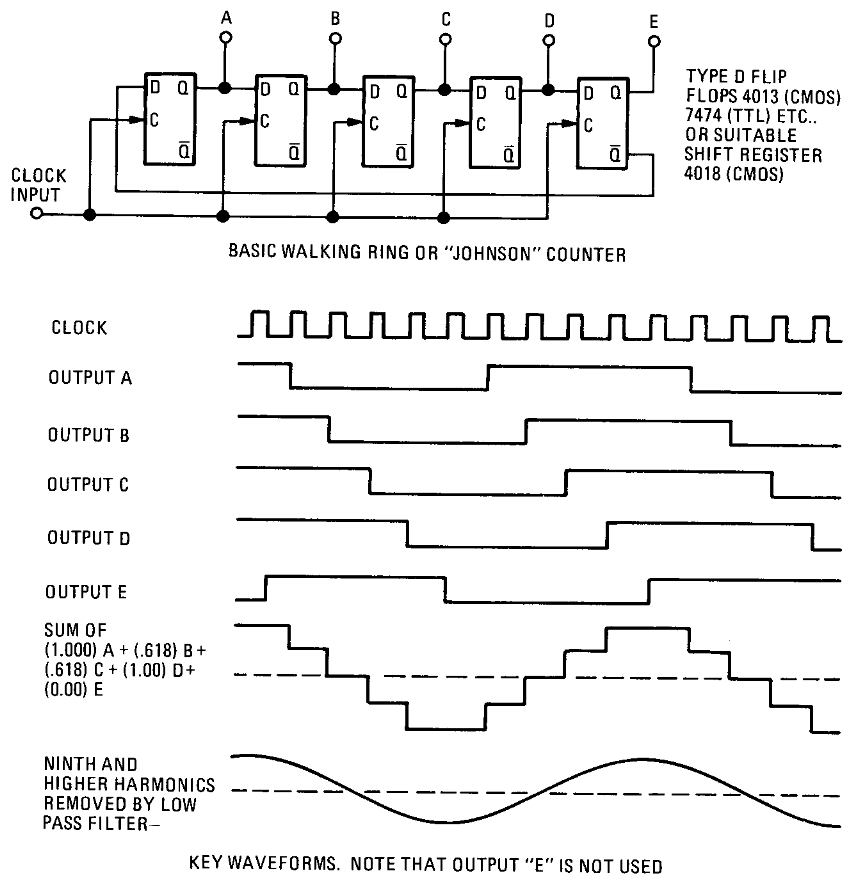
In Fig. 2, we've built a four-stage counter that gives us an eight-step output using one IC and three resistors. The output is summed across the 4.7K resistor and then actively filtered by the PNP emitter follower and the third-order Bessell active filter. This particular circuit is used in a digital cassette recording system, where a digital "1" is a 2400 hertz and a digital "0" is a 1200 hertz sinewave recorded on the tape. You can shift the cutoff frequency of the active filter by proportionately changing capacitors C1, C2, and C3. Doubling these capacitors reduces the cutoff frequency in half and so on. The input clock of this circuit is eight times the output frequency.

In Fig. 3, we have a five-stage counter and four resistors that sum into a type-741 operational amplifier. The op amp is filtered with a single capacitor. This particular circuit drives the transmitter speaker of a "103" style MODEM, outputting a 1070 hertz sinewave for a digital logic "0" and a 1270 hertz sinewave for a digital logic "1". This time the clock frequency is ten times these output values.

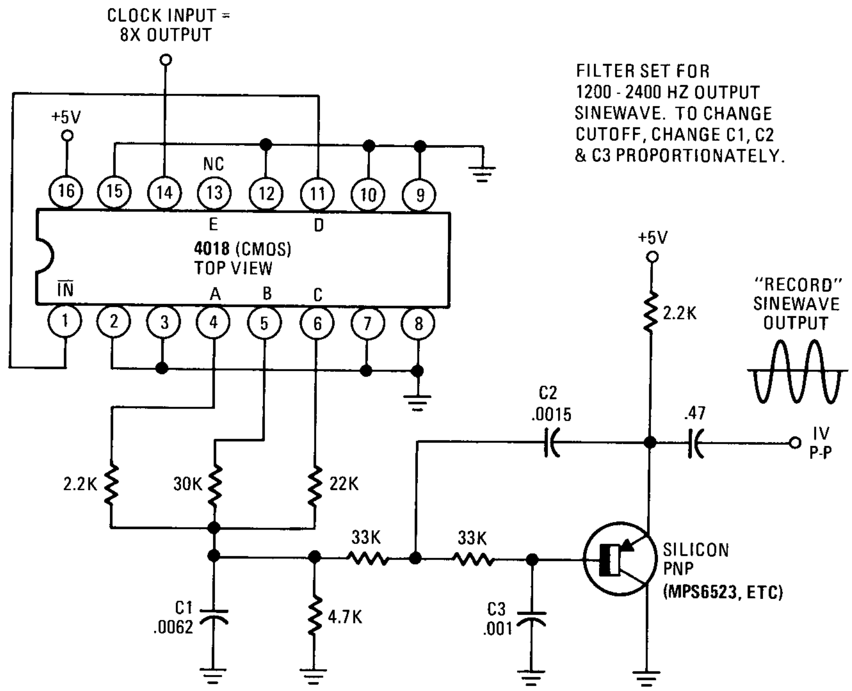
Magic numbers

Figures 4, 5 and 6 show us the magic resistor values, the circuits, and the waveforms for three, four and five-stage registers of length 6, 8, and 10. The value in parenthesis is the resistor ratio we want, while the

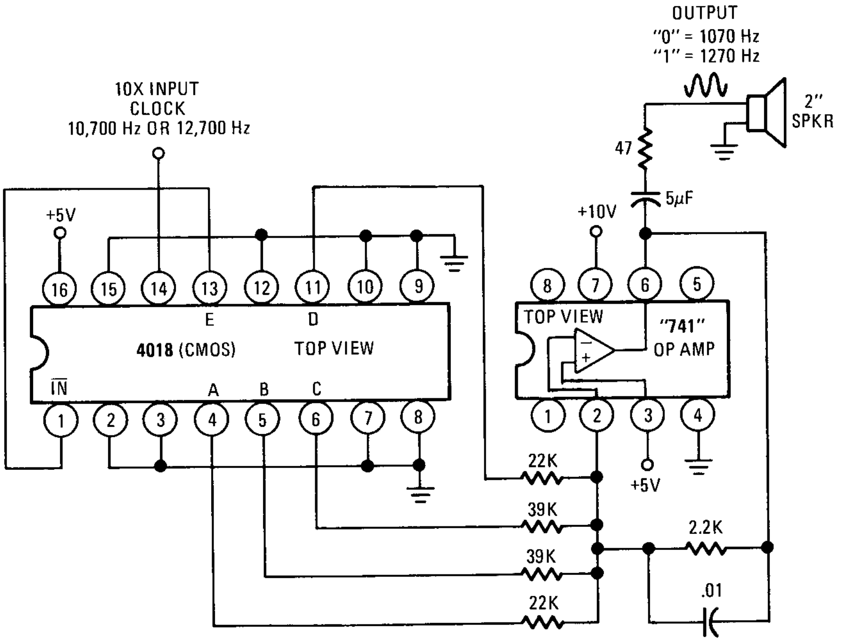
1. THE WALKING RING COUNTER and its waveforms, which combine to produce a good sinewave (after filtering).



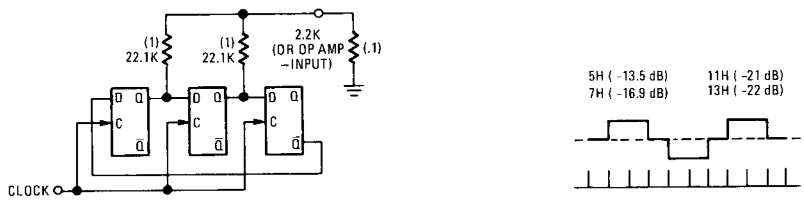
2. AN EIGHT-STEP GENERATOR, as used in a digital cassette recording system.



3. **10-STEP SINEWAVE GENERATOR drives the transmitter speaker of a MODEM.**

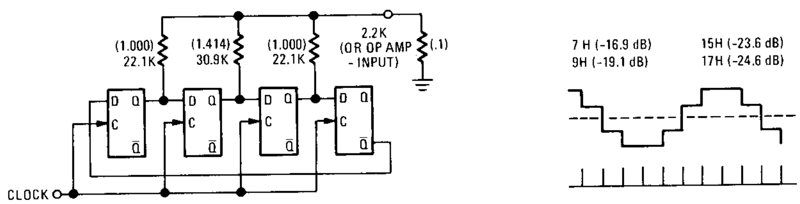


4. **THREE-STAGE DIGITAL SINEWAVE GENERATOR. Resistance values are shown in ratios (parentheses) and ohms. The amplitude of the harmonics are also shown.**



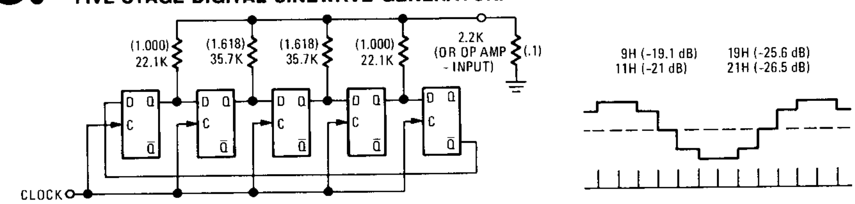
THREE-STAGE. CLOCK IS SIX TIMES OUTPUT FREQUENCY

5. **FOUR-STAGE DIGITAL SINEWAVE GENERATOR.**



FOUR-STAGE. CLOCK IS EIGHT TIMES OUTPUT FREQUENCY

6. **FIVE-STAGE DIGITAL SINEWAVE GENERATOR.**



FIVE-STAGE. CLOCK IS TEN TIMES OUTPUT FREQUENCY

resistor value has been rounded off to a stock 1 percent value. Actually, 5 percent resistors are more than adequate for practically all sinewave generators, particularly for those of ten stages or less.

You'll get more performance by lengthening the registers and using more resistors. Values for any length are shown in Table I, along with the harmonics and their strengths that you can expect. Once again, the parenthesis values are exact ratios, while the resistor values are 1 percent based on 22.1 K being the smallest value used.

These longer lengths make filtering more easy since the odd harmonics you do get are higher in frequency and lower in amplitude as you add stages. Note that the input clock frequency goes up as you add stages. Two or more 4018's can be cascaded as needed for these longer lengths. Usually binary lengths of 8 and 16 or decimal lengths of 10 or 20 make for the easiest interface with the system timing in the rest of your circuit.

Note that with a longer register, you can have a fixed filter and still operate over a wide frequency range. For instance, with a 10-stage register and a 10:1 frequency change, the lowest harmonic of the lowest output frequency will still be 1.9 times the frequency of the highest output frequency and reasonably easy to get rid of with a sharp-cutoff filter.

Some loose ends

You may have to look into several details when generating your own digital sinewaves. These include the counter sequences, resistor tolerances, offsets, and the choice of filtering.

Walking-ring counters longer than two stages have *disallowed sequences* that make up the difference between the total possible counter states and the states you are actually using. For instance, a three-stage counter has the valid 000, 100, 110, 111, 011, 001 and back to 000 six-count state sequence. It also has a disallowed 101, 010, 101, 010 . . . . two-count rut it can get into. All walking-ring counters must be set up to eliminate the disallowed states.

This is done internally in the 4018 counters, and cascaded 4018's will probably eliminate most if not all the possible disallowed sequences. You can also use a reset button or signal to get your sequence off on the 000 state before you begin. Or you can add gating to force the internal stages to zero when the end stages are zero; or to all ones when the end stages are ones. (If you have only a common reset line for all stages, be sure to shorten the reset pulse so it doesn't permanently hang up the register.)

How accurate do the resistors have to be? For registers of ten stages or less, a tolerance of 5 percent is good enough, even though we've shown you 1 percent values. Resistors out of tolerance introduce lower-order odd harmonics, but for most 5 percent variations, these should be 40 decibels below the fundamental or lower.

Note that there will be a DC offset in the output sinewave that usually must be eliminated somehow. The simplest way is with a blocking capacitor as we did with the output capacitor in Figs. 2 and 3. With our CMOS outputs, we have a choice of summing to the positive supply voltage, to ground, or to an op amp's inverting input biased halfway between positive supply and ground. In Fig.

TABLE I—DESIGN INFORMATION FOR LONGER GENERATORS

STAGES	CLOCK	RESISTORS*	HARMONICS
6	12x	22.1K; 38.3K; 44.2K; 38.3K; 22.1K (1.000) (1.732) (2.000) (1.732) (1.000)	11H (−21 dB) 23H (−27 dB) 13H (−23 dB) 25H (−28 dB)
7	14x	22.1K; 40.2K; 49.9K; 49.9K; 40.2K; 22.1K (1.000) (1.803) (2.248) (2.248) (1.803) (1.000)	13H (−23 dB) 27H (−29 dB) 15H (−24 dB) 29H (−29 dB)
8	16x	22.1K; 41.2K; 53.6K; 57.6K; 53.6K; 41.2K; 22.1K (1.000) (1.849) (2.412) (2.613) (2.413) (1.849) (1.000)	15H (−24 dB) 29H (−29 dB) 17H (−25 dB) 31H (−30 dB)
9	18x	22.1K; 41.2K; 56.2K; 63.4K; 63.4K; 56.2K; 41.2K; 22.1K (1.000) (1.877) (2.532) (2.879) (2.879) (2.532) (1.877) (1.000)	17H (−25 dB) 35H (−31 dB) 19H (−26 dB) 37H (−31 dB)
10	20x	22.1K; 42.2K, 57.6K; 68.1K; 71.5K; 68.1K; 57.6K; 42.2K; 22.1K (1.000) (1.896) (2.618) (3.077) (3.236) (3.077) (2.618) (1.896) (1.000)	19H (−26 dB) 39H (−32 dB) 21H (−27 dB) 41H (−33 dB)
16	32x	22.1K; 43.2K; 63.4K; 80.6K; 93.1K; 105K; 110K; 113K; (1.000) (1.961) (2.847) (3.624) (4.262) (4.736) (5.027) (5.125) 110K; 105K; 93.1K; 80.6K; 63.4K; 43.2K; 22.1K (5.027) (4.736) (4.262) (3.624) (2.847) (1.96) (1.000)	31H (−30 dB) 63H (−36 dB) 33H (−31 dB) 65H (−36 dB)
n	2nx	$1;\ \frac{\sin\frac{2\pi}{n}}{\sin\frac{\pi}{n}};\ \frac{\sin\frac{3\pi}{n}}{\sin\frac{\pi}{n}};\ \frac{\sin\frac{4\pi}{n}}{\sin\frac{\pi}{n}};\ \ldots\ \frac{\sin\frac{(n-1)\pi}{n}}{\sin\frac{\pi}{n}}$	$(2n-1)H\ ({}^{1}/_{2}n-1)$ $(4n-1)H\ ({}^{1}/_{4}n-1)$ $(2n+1)H\ ({}^{1}/_{2}n+1)$ $(4n+1)H\ ({}^{1}/_{4}n+1)$

*** The resistor values in parentheses are exact ratios; values in ohms are rounded off to stock 1 percent values.**

2, we've summed to ground so the emitter follower will have a reasonably constant emitter current and thus not distort the waveform. In Fig. 3 we sum to one-half the supply voltage to minimize the offset at the output even before capacitor coupling.

A final detail is filtering. Complete information on active filters appears in the *Active Filter Cookbook*. For some uses, the harmonics are high enough in frequency that they can simply be ignored. For others, a simple capacitor or two to introduce rolloff is all you need. For more critical uses, a better quality active filter is called for.

Sharp-cutoff low-pass filters using Butterworth or Chebyshev response curves have the advantages of producing very clean sinewaves with a minimum of circuitry. But these sharp filters have one possible drawback—if the input sinewave is changing or jumping between several frequencies, the filters will introduce *group-delay distortion*, or simple smearing that will generally mess up any sudden input frequency changes. Where you are suddenly or often changing input frequencies, use the higher-order "more gentle" Bessell active filters since Bessell filters are designed to absolutely minimize this form of distortion.

These circuits actually make digital sinewaves easier and simpler and cheaper than analog ones, so there should be all sorts of things you can do with them. Let us know what uses you come up with. **R-E**

FOR MORE READING:

Generating digital sinewaves—
"Digital Generation of Low Frequency Sinewaves," Anthony C. Davies, *IEEE Transactions on Instrumentation and Measurement,* IM18, No. 2, June 1969 PP 97-105.

Active filters—
The Active Filter Cookbook No. 21168, Howard W Sams, Indianapolis, IN, 46206.

letters

DIGITAL SINEWAVES

Don Lancaster's article entitled "Create Sinewaves Using Digital IC's" that appeared in the November, 1976, issue was both interesting and useful. Using a 4018 IC, we built the five-stage synthesizer shown in Fig. 6 of the article. With 1% summing resistors, the output waveform shown in Fig. 1 was obtained. The scope was set to 1 ms-per-division and 0.5 volts-per-division. The harmonic content of the output signal was analyzed and is listed in Table 1.

TABLE 1—HARMONIC CONTENT of five-stage synthesizer.

Harmonic	Frequency (Hz)	Amplitude (dB)
1	200	0
3	600	–17
5	1000	<–60
7	1400	–14
9	1800	–20
11	2200	–21
13	2600	–27
15	3000	<–60
17	3400	–21
19	3800	–16
21	4200	–17
23	4600	–33
25	5000	<–60

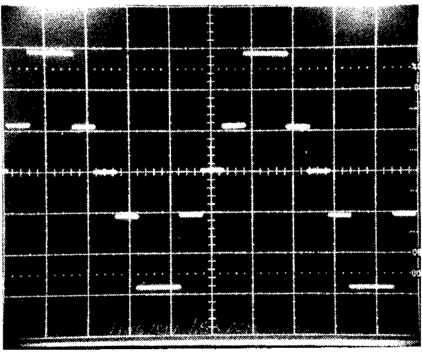


FIG. 1

TABLE 2—HARMONIC CONTENT of modified synthesizer.

Harmonic	Frequency (Hz)	Amplitude (dB)
1	200	0
3	600	–57
5	1000	<–60
7	1400	–54
9	1800	–19
11	2200	–20
13	2600	–60
15	3000	<–60
17	3400	–60
19	3800	–25
21	4200	–26
23	4600	<–60
25	5000	<–60

In the waveform of Fig. 1, the distance between the bottom level and the first step is larger than the distance between the first step and the second step. A better sinewave approximation would have a larger distance between the 1st and 2nd steps, than between the bottom level and the first step. If the 22.1K and 35.7K resistors are switched, the output waveform shown in Fig. 2 is obtained. The scope settings are the same as before. The harmonic content of this output signal was analyzed and is listed in Table 2.

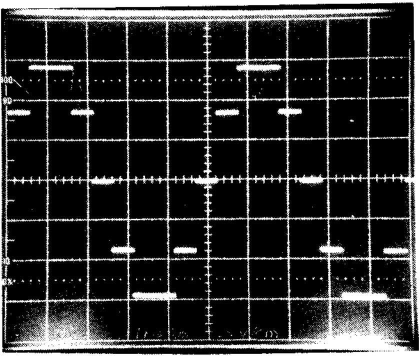


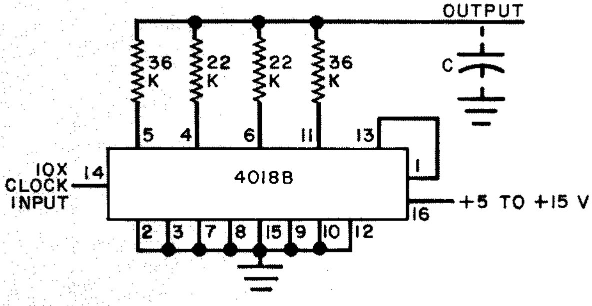
FIG. 2

The modification produces a better approximation that follows the harmonic content specified in the article.

The resistor values obtained in Table 1 of the article are also incorrect. The proper values can be determined from the reciprocal of the numbers shown in the parenthesis in Table 1 of the article. For a six stage counter, the resistors should be: 44.2K (1.000), 25.5K (.577), 22.1K (.500), 25.5K (5.77), and 44.2K (1.000).

Thanks again for a useful and interesting article, keep up the good work.

JOHN PEASE and
GIL JOHNSON
Marquette Electronics
Milwaukee, WI



Digital Sine-Wave Generator

5

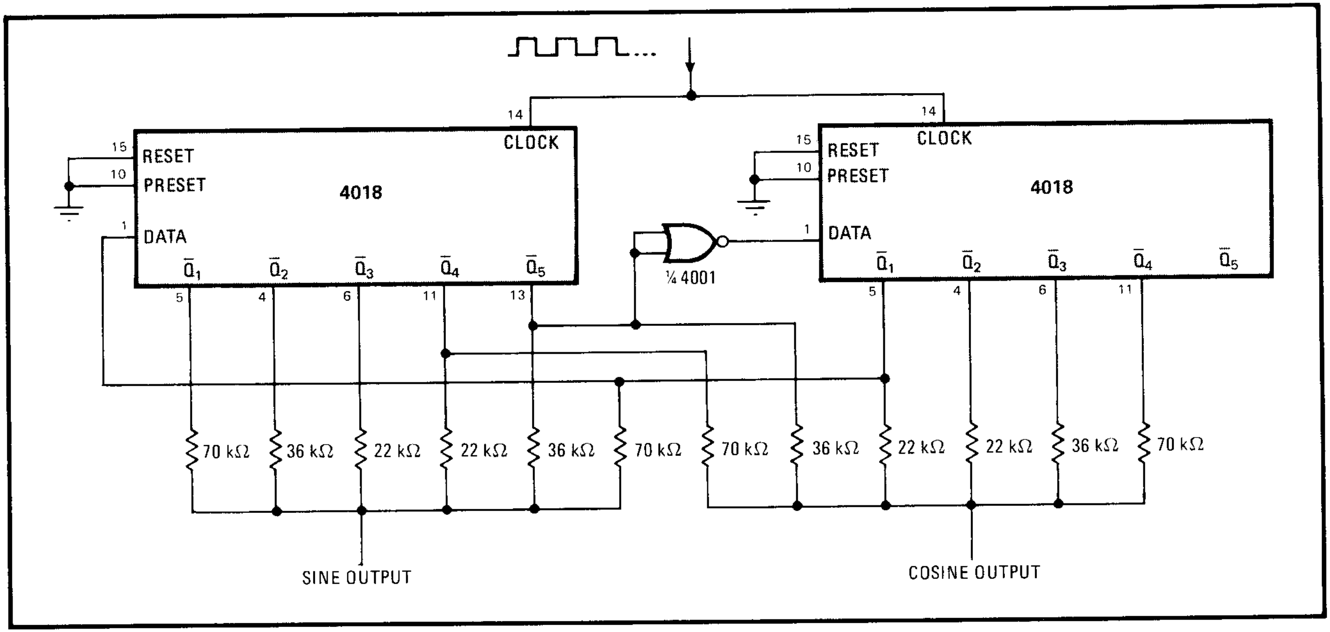
This circuit uses a clock frequency 10 times the required output frequency. The walking-ring counter and resistor summing network will produce a "chunky" waveform at the output. However, you can filter this waveform since it is basically a sine wave with a little of the 9th and 11th harmonics present. You can either ignore the harmonics or use a capacitor (shown dotted on the schematic) as a filter. If desired, an active filter can be used. The unfiltered output swings the full supply voltage which can range from 3 to 15 volts.

Ring counter synthesizes sinusoidal waveforms

by Timothy D. Jordan
Texas A & M University, College Station, Texas

A digital circuit composed of only two counters and a weighted resistor network is as good at producing sine and cosine waveforms as many quadrature oscillator networks. Because matched components are not used, design considerations are radically simplified.

Use of the digital technique eliminates many components. The upper frequency limit of the oscillator is 250 kilohertz, and it is not affected by the frequency limitations of operational amplifiers, because no op amps are used. Tweaking the oscillator is not necessary, because no special circuitry is needed. And the sine and cosine waveforms are equal in magnitude at every frequency, because no integrating or differentiating circuits are used. It is even possible to transform the circuit into a digital-to-sine-wave converter with little modification, if



Digitized waves. Frequency dividers and weighted resistor network generate sine and cosine waveforms. The 4018 counters divide input frequency by 12. First counter provides 12-bit approximation of sine function. Second device lags by 90°, producing cosine waveform.

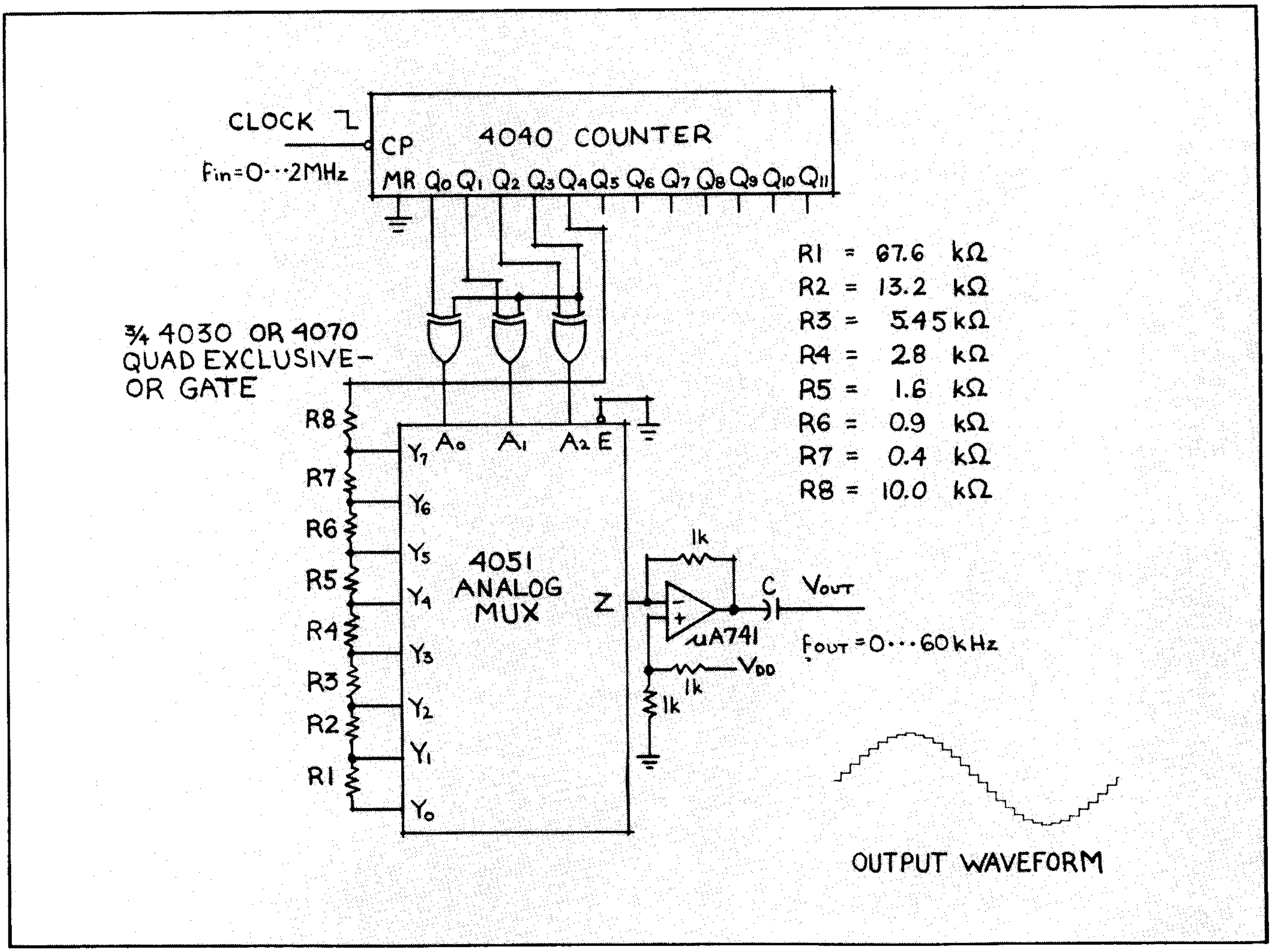
the counters' parallel input ports are used to accept binary signals.

As shown in the figure, two cascaded 4018 complementary-metal-oxide-semiconductor integrated circuits wired as a single ring counter are driven by the master clock. The 4018s divide the input frequency by 12. The digital clock advances the ring counter by one count on the positive clock transition, and each output port moves from the high to low state sequentially.

The resulting current through the weighted resistor network at the counter's output produces a 12-step approximation of a sine wave. The output stages of the second 4018 produce a cosine wave, since it is delayed three clock periods, or one quarter of a cycle, with respect to the first counter.

The first appreciable harmonics to appear at the output are the 11th and 13th, and they may be filtered out with a passive resistance-capacitance filter. Identical filters should be used for each counter so that the phase shift introduced is equal for both output waveforms. The input frequency may be as high as 3 megahertz; above 1 MHz, no filtering is necessary. □

CIRCUIT IDEAS



DIGITALLY CONTROLLED SINE WAVE GENERATOR

PETER ALFKE

Digitally derived sine waves are ideal in instruments that cover a wide frequency range, and in frequency shift keying, e.g., for data storage on audio cassette. Conventionally, these sine waves are created by filtering symmetrical square waves through low-pass filters. Such filters require several precision components (capacitors, resistors, or even inductors) and work properly only over a limited frequency range.

The approach used here eliminates the filtering and the frequency-sensitive components by generating a staircase approximation of a sine wave (of arbitrary frequency) through a specialized digital-to-analog converter.

The counter, with outputs Q_0 through Q_4, counts clock pulses at 32 times the rate of the sine wave frequency to be generated (a 1-kHz sine wave requires a 32-kHz clock rate).

The three least significant counter outputs are fed through three exclusive-OR gates into the address inputs of an 8-input analog multiplexer. The next counter output (Q_3) controls the exclusive-OR gates so that the address inputs to the multiplexer first count up (0...7), then count down (7...0), then count up again.

The inputs of the analog multiplexer are connected to a resistor chain, and the multiplexer output feeds into the inverting input of an operational amplifier. The non-inverting op amp input is connected to 50% of the supply voltage, e.g., 2.5 V in a 5 V system.

Address 0 connects the highest, and address 7 the lowest, resistor value to the op amp input. The other end of the resistor chain is tied to the most significant counter output, so that it alternates between sourcing current into the amplifier and sinking current from the amplifier. By proper choice of the eight resistor values, the amplifier output is a 32-step approximation of a sine wave with a dc component equal to half the CMOS supply voltage. **R-E**

Reprinted from "Fairchild Journal of Semiconductor Progress, Vol. 6 No. 2"