Technical Article

Hysteresis Loss: Estimation, Modeling, and the Steinmetz Equation

5 days ago by Dr. Steve Arar

The hysteresis effect is one of the main sources of loss in ferromagnetic materials. In this article, we learn to calculate the hysteresis loss of a magnetic core and work through some example problems.

The previous article in this series discussed the relationship between a magnetic core's hysteresis loss and its B-H curve. We'll start off this article by demonstrating how the B-H curve area can be used to estimate hysteresis loss. We'll then learn about the Steinmetz equation, which is an empirical method for estimating the loss. Finally, we'll wrap up with a brief discussion of how hysteresis loss is modeled in inductors and transformers.

Hysteresis Loss and the B-H Curve Area

The hysteresis effect causes a ferromagnetic material's B-H curve to be multivalued, producing a distinctive hysteresis curve. To take a material through one cycle of magnetization along its hysteresis curve requires an amount of work proportional to the area inside the curve. In mathematical language, the hysteresis loss per unit volume of the material is given by:

$$w_h = \oint H \, dB$$

Equation 1.

where the integral is taken over one cycle of the hysteresis loop. To reduce the hysteresis loss, we use cores made of soft ferromagnetic materials that have a small hysteresis loop, indicating that they have low energy loss per cycle.

Equation 1 gives us the dissipated energy for one cycle. With an AC excitation, the frequency of AC current determines how many times the core material cycles through the hysteresis loop per second. Since power is the rate of energy transfer or conversion per unit time, the total power dissipated due to hysteresis over the course of one cycle is:

$$P_h = fV_c \times \oint H \, dB$$

Equation 2.

where:

f is the frequency of operation

V_c is the volume of the core.

Let's look at an example.

Example 1: Estimating a Hysteresis Loop with a Parallelogram

Figure 1 shows the hysteresis curve of a hypothetical material. Noting that the curve resembles a parallelogram, let's estimate:

- The hysteresis loss per m^3 per cycle.
- The hysteresis loss per m^3 at a frequency of 50 Hz.

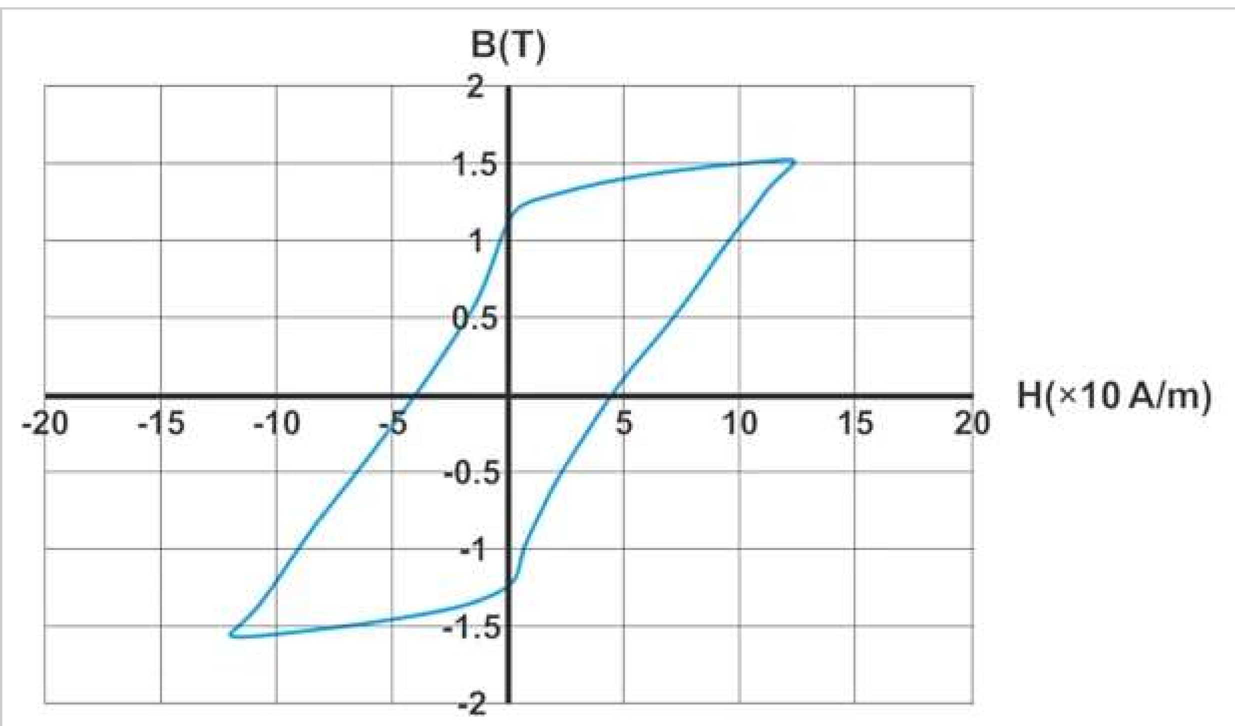

Figure 1. Hysteresis curve of a magnetic material.

The energy density lost in one cycle is equal to the area of the B-H curve. Instead of finding the exact value of the area, we'll estimate it by drawing a parallelogram that matches the size of the B-H curve (Figure 2).

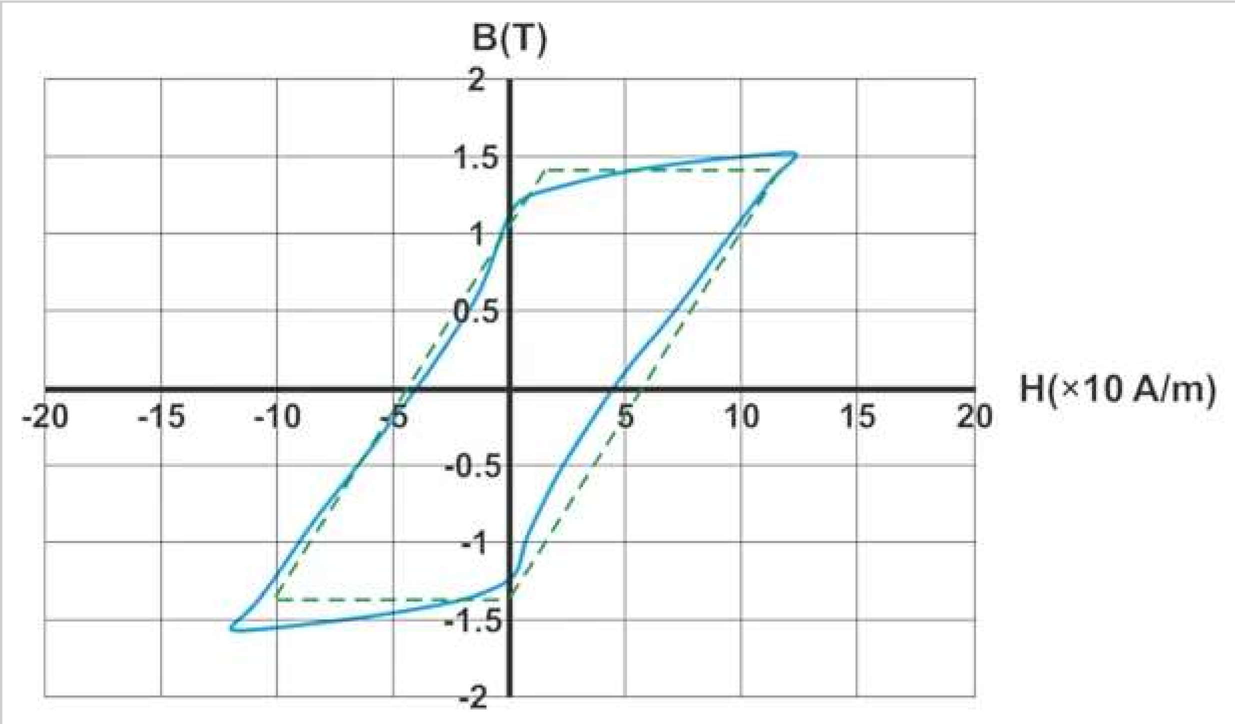

Figure 2. The green parallelogram estimates the B-H curve area.

The base of the parallelogram is 10 × 10 = 100 A/m. Its height is 2.75 T, making the parallelogram's area 275 J/m^3. At 50 Hz, the power loss density is:

$$p_h = A_{curve} \times f = 275 \frac{\mathrm{J}}{\mathrm{m}^3} \times 50 \frac{1}{\mathrm{s}}$$
$$= 13750 \frac{\mathrm{W}}{\mathrm{m}^3}$$

Equation 3.

where A_{curve} is the estimated area of the curve.

Hysteresis Loss Analysis for Rectangular B-H Loops

To better understand how different parameters impact the hysteresis loss, let's assume the following:

- The B-H curve is rectangular.
- Its operating point is at the origin.

Because the operating point is at the origin, the sides of this rectangle extend equally far in the positive and negative directions. The rectangle's height is therefore equal to $2B_m$ and its base to $2H_m$, where B_m and H_m are the peak values of the flux density and the magnetic field intensity, respectively. That being the case, the hysteresis loop area (A_{curve}) is equal to $4B_mH_m$.

From Equation 2, the total power loss due to the hysteresis is:

$$P_h = fV_c \times A_{curve} = fV_c \times (4B_mH_m) = 4fV_c\frac{B_m^2}{\mu_0\mu_r}$$

Equation 4.

where μ_0 is the permeability of free space and μ_r is the material's relative permeability.

Assuming that the hysteresis curve remains rectangular for a range of flux densities, we observe that the volume density of hysteresis loss has the following general form:

$$p_h = k_h f {B_m}^2$$

Equation 5.

where k_h is the hysteresis loss coefficient, a material property that may be found from the manufacturer's data. For 2.5% silicon steel, k_h is 93.89 Ws/(T^2m^3).

In the above equation, B_m is the peak value of the flux density with sinusoidal excitation at the frequency f. Equation 5 shows two important properties of hysteresis loss:

- The hysteresis loss increases as the frequency of the signal rises because the magnetic domains of the core switch faster at higher frequencies.
- The hysteresis loss increases with the applied signal level.

We can reveal a further property of hysteresis loss by rewriting Equation 4. Assuming that the inductor is a solenoid having N turns and length l, a current of i flowing through the inductor produces a magnetic field intensity of $H = Ni/l$. Therefore, Equation 4 can be rewritten as:

$$\begin{aligned} P_h &= 4fV_cB_mH_m = 4fV_c\mu_0\mu_r{H_m}^2 \\ &= 4fV_c\mu_0\mu_r \times (\frac{N}{l})^2 \times {I_m}^2 \end{aligned}$$

Equation 6.

This version of the equation makes the relationship between relative permeability and hysteresis loss clearer.

Because increasing the temperature increases the random thermal motion of atoms, which tends to randomize the magnetic domains, the relative permeability of many materials reduces with temperature. As Equation 6 shows, the hysteresis loss reduces with temperature for these material types.

Example 2: Calculating the Hysteresis Loss of a Solenoid

Consider a solenoid with the following characteristics:

- 10 total turns ($N = 10$).
- A cross-sectional area of 100 mm^2 ($A_c = 100\ mm^2$).
- A length of 10 cm ($l_c = 10$ cm).

Assume that the core of this solenoid has $\mu_r = 2000$ and exhibits a rectangular hysteresis curve. If the applied current is $I = 0.5\sin(2\pi \times 10^5 \times t)$, let's calculate the hysteresis loss and the loss's equivalent series resistance.

Since the hysteresis curve is rectangular, we can apply Equation 6. First, let's find the volume of the core:

$$V_c = A_c l_c = (100 \times 10^{-6}) \times (10 \times 10^{-2}) = 10^{-5}\ \mathrm{m}^3$$

Equation 7.

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We then plug the values into Equation 6, leading to:

$$\begin{aligned} P_h &= 4fV_c\mu_0\mu_r \times (\frac{N}{l})^2 \times I_m^2 \\ &= 4 \times 10^5 \times 10^{-5} \times (4\pi \times 10^{-7}) \times 2000 \times \left(\frac{10}{10 \times 10^{-2}}\right)^2 \times 0.5^2 \\ &= 25.13 \text{ W} \end{aligned}$$

Equation 8.

To find the equivalent series resistance that produces the same power loss as the hysteresis effect, we find the resistance that dissipates 25.13 W when a sinusoidal current with amplitude 0.5 A passes through it:

$$\frac{1}{2}R_s I_m^2 = 25.13 \Rightarrow R_s = \frac{2 \times 25.13}{0.5^2} = 201\ \Omega$$

Equation 9.

This series resistance burns the same power as the hysteresis effect of the core. However, as we'll discuss later in this article, the core loss is commonly modeled as a resistance in *parallel* with the inductance of the structure.

The Steinmetz Equation

If we assume that the B-H curve is a parallelogram or a rectangle, we find that the hysteresis loss follows the general form shown in Equation 5. Empirical data, however, shows that the hysteresis loss density is actually given by:

$$p_h = k_h f(B_m)^n$$

Equation 10.

where n is the Steinmetz index, so called after the American mathematician and electrical engineer Charles Proteus Steinmetz. The value of n, which is found empirically, is in the range of about 1.6 to 3. It depends on the material properties and is usually given for a specific range of flux density values.

But why does the Steinmetz equation suggest that the energy loss is proportional to $(B_m)^n$ instead of $(B_m)^2$? When deriving Equation 4, we assumed that the shape of the hysteresis curve doesn't change with the applied signal level. In practice, this generally isn't the case. Figure 3 shows the magnetization of a ferromagnetic material as we gradually increase the amplitude of the excitation signal.

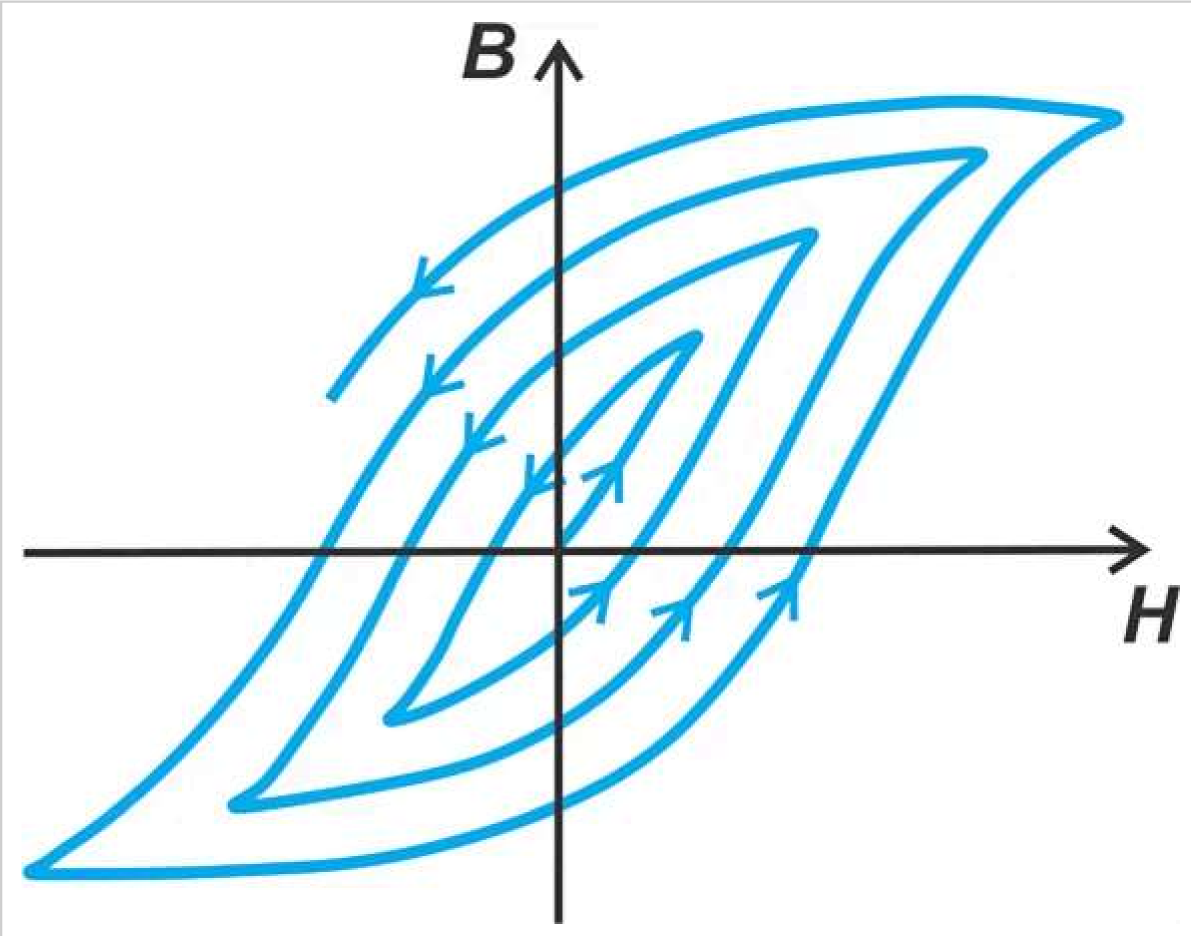

Figure 3. Hysteresis loop for a growing excitation signal.

We can see that both the size and shape of the hysteresis loop can change with the magnitude of the applied field. Also, the peak value of the flux density B_m doesn't increase linearly with H_m. The Steinmetz equation uses an empirically determined power exponent to account for these effects.

Example 3: Using the Steinmetz Hysteresis Loss Equation

When the maximum flux density is B_{m1} = 1.5 T and the frequency is f_1 = 50 Hz, the hysteresis power loss density of a magnetic material is p_{h1} = 13750 W/m^3. What would the power loss density be for B_{m2} = 1 T at f_2 = 40 Hz, if the power exponent in the Steinmetz equation is n = 1.6?

Let the new power loss density be p_{h2}. From Equation 10, we have the following relationship for the two experiments:

$$\frac{p_{h1}}{p_{h2}} = \frac{f_1}{f_2} \times \left(\frac{B_{m1}}{B_{m2}}\right)^n$$

Equation 11.

Plugging in the values given above, we obtain:

$$\frac{13750}{p_{h2}} = \frac{50}{40} \times \left(\frac{1.5}{1}\right)^{1.6} \Rightarrow p_{h2} = 5750 \text{ W/m}^3$$

Equation 12.

There are a couple of things I'd like to mention before we leave this topic. First, the area of the B-H loop—and, by extension, the hysteresis loss—may increase with frequency. To take this effect into account, the frequency (f) in the above equations may be replaced by f^a, where a is greater than unity.

Second, the parameters of the Steinmetz equation may be specified to account for the total core loss, which would include the losses from eddy currents as well as from hysteresis. When that's the case, the power loss density equation is:

$$p_c = p_h + p_{eddy} = kf^a(B_m)^b$$

Equation 13.

We'll discuss the eddy current loss in greater detail in the next article.

Core Losses Modeled as a Parallel Resistance

Noting that $\phi = BA$ and $i = Hl/N$, we can rescale the B-H curve to obtain the ϕ-i curve for the core. For a B-H characteristic with no hysteresis, a sinusoidal flux in the core is produced by a non-sinusoidal current because of the B-H curve nonlinearity. This is shown in Figure 4.

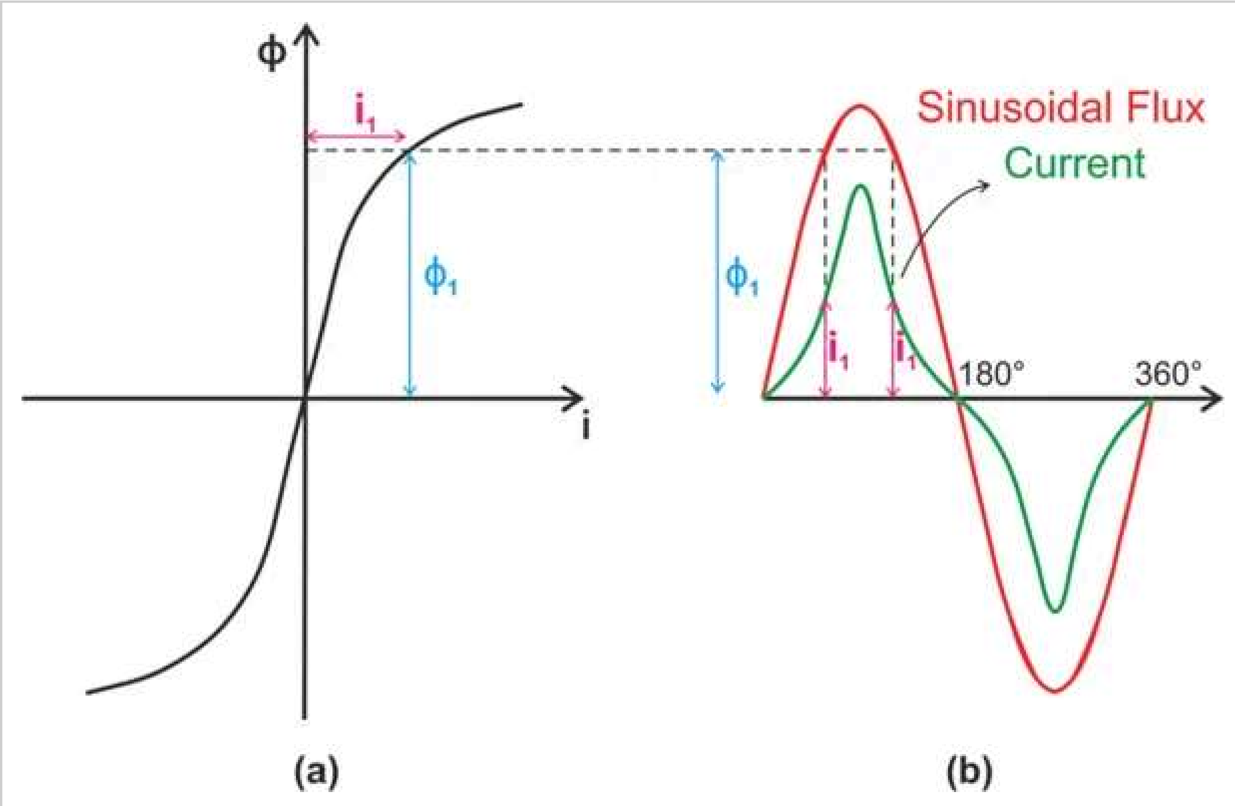

Figure 4. (a) Magnetization curve and (b) flux and magnetization current waveforms in the absence of the hysteresis effect.

The non-sinusoidal current is in phase with the flux and has a symmetrical ascent and descent. The fundamental component of the current waveform lags the voltage across the inductor by 90 degrees, which corresponds to an ideal lossless inductor. Since we started by assuming that the core isn't hysteretic, this is no big surprise—there's no hysteresis loss, and the winding and core together act like an inductor.

As we see in Figure 5, taking the hysteresis effect into account results in a current waveform with asymmetrical ascent and descent.

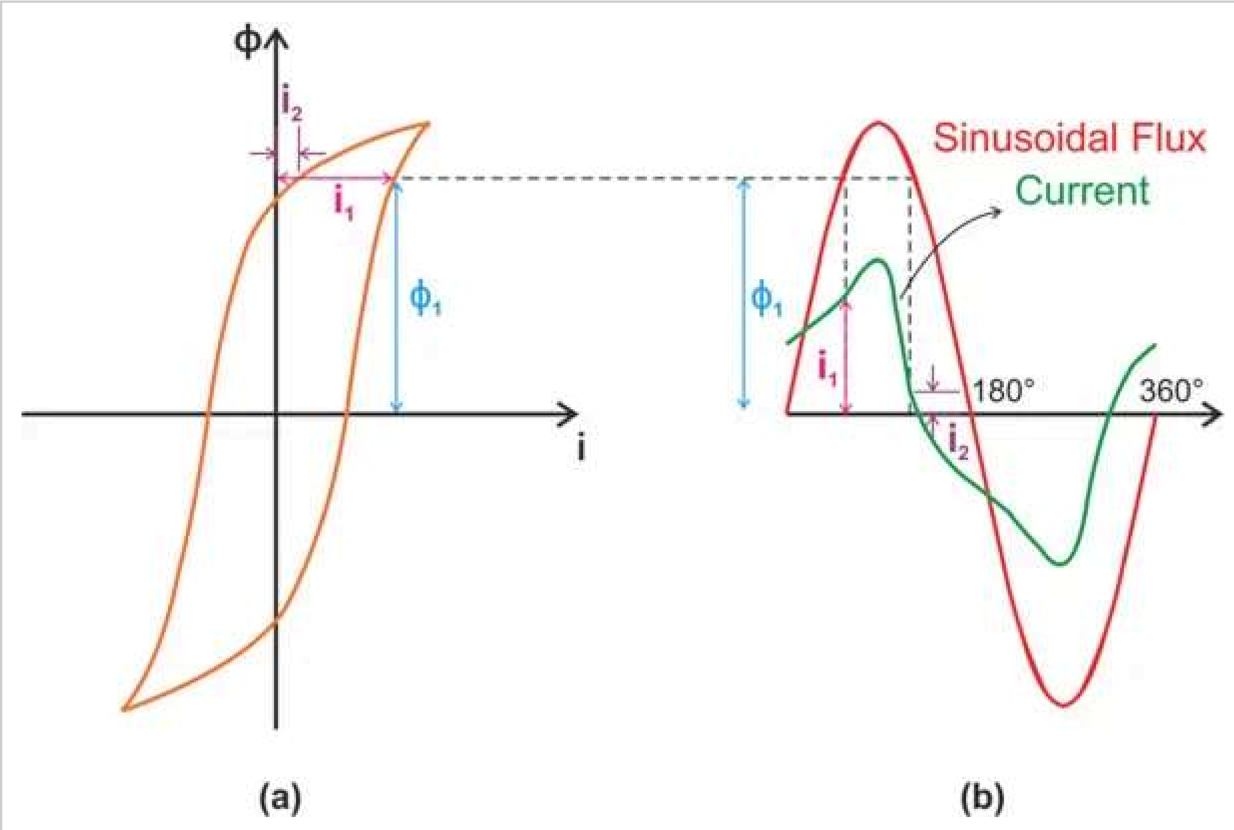

Figure 5. (a) Magnetization curve and (b) flux and magnetization current waveforms when the hysteresis effect is present.

As you can see, maintaining a sinusoidal flux in a core with hysteresis requires a non-sinusoidal, asymmetrical current. This current can be split into two different components:

- I_m, which—as in the ideal, lossless case—is in phase with the flux.
- I_c, which is in phase with the voltage across the inductor.

I_m lags the voltage across the inductor by 90 degrees, producing an inductive term. I_c, however, produces a resistive term. Since the total current (I_ϕ) is the sum of these two components, the equivalent circuit of the winding and core combination is an inductor in parallel with a resistance, where the resistance models the hysteresis loss (Figure 6).

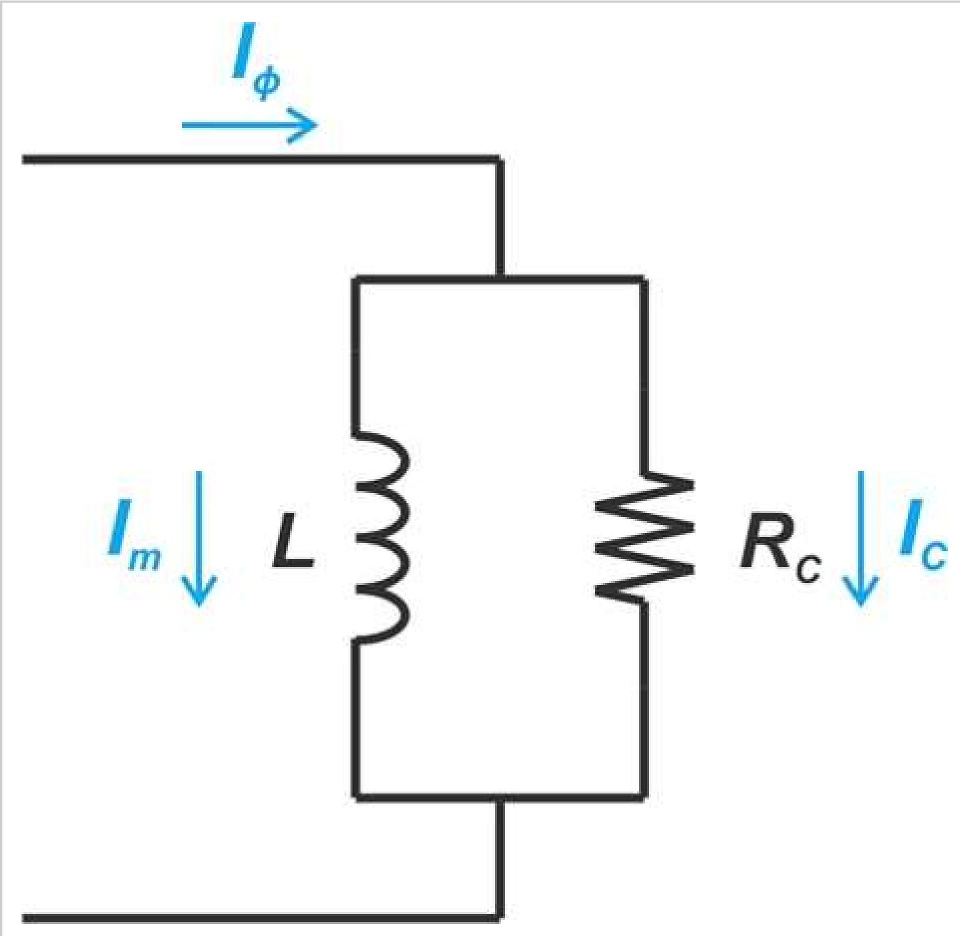

Figure 6. A parallel resistance can be used to model core losses.

In general, the parallel resistance (R_C) can account for both the hysteresis loss and the eddy current loss. Similarly, we commonly model the core losses of a transformer by a frequency-dependent resistance in parallel with the primary winding.

Wrapping Up

In this article and the preceding one, we discussed hysteresis loss in magnetic cores. Next time, we'll take a close look at eddy current loss. Whereas an earlier article in this series addressed the effects of eddy currents on magnetic cores, we'll now turn our attention to causes, analysis, and mitigation strategies.

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