What designers should know about THERMAL ANALYSIS

What you know about structural analysis will help run an accurate thermal analysis. Of course, there are a few variations.

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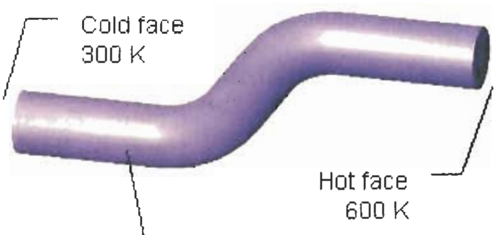
Thermal and structural analyses are the most frequently performed finite-element analyses. Both use the same type of partial-differential equations and are analogous in other ways as well.

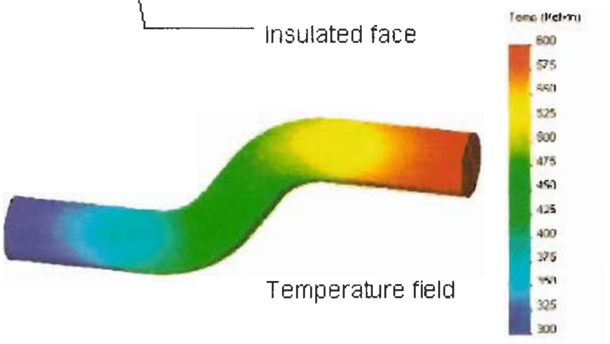
SIMILARITIES

Let's start with thermal analyses. The primary unknowns are temperatures at nodes of the finite-element mesh. Temperature is scalar so it needs only one degree of freedom at each node, whether the model is 2D or 3D and regardless what elements mesh the model. Once temperatures are found at the nodes, they are interpolated over elements to calculate temperatures in the entire model. Temperatures are then differentiated to find the temperature gradients. Finally, the software calculates heat flux based on the temperature gradient and thermal conductivity of the material.

This is much like structural analysis in which primary unknowns, displacements, are first found for nodes then are interpolated over elements. A displacement field is then differentiated to find strains, and stresses are calculated based on strain and material elasticity.

However, displacements are vectors and so are defined by more than one component. It takes two components (two degrees of freedom) to describe nodal displacements in 2D models, three components (three degrees of freedom) in 3D models meshed with solid elements, and six components (six degrees of freedom) in 3D models with shell elements. Since each nodal degree of freedom is an unknown it takes more computing power for structural analysis because it has more unknowns.





Temperature field

Heat flows through a solid rod from hot face to cold in the absence of convection (no heat escapes "along the way").

Heat flux

Results from static-structural analysis describe a deformed structure in equilibrium between applied loads and reactions provided by supports. A steady-state thermal analysis provides an analogy in which heat flows at a constant rate. It's also in equilibrium.

Thermal analysis deals with

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heat *flow* in a solid body, so there must be a way for heat to enter and to exit that solid body. If heat enters a body with no way to exit, the body's temperature would theoretically rise to infinity. This is analogous to applying a structural load to an unsupported model. The result: infinite displacements.

Heat flowing inside a solid, and heat entering and leaving the solid, are governed by different mechanisms. Heat inside moves by conduction, while heat entering or escaping the solid moves by convection, or radiation, or both.

Heat transfer by conduction, for example, is heat flowing through a wall. It is described by:

$$Q_{conduction} = KA(T_{hot} - T_{cold})/L$$

where $Q_{conduction}$ = rate of heat flow, K = thermal conductivity, A = area of the wall, T_{hot} = temperature on the hot side, T_{cold} = temperature on the cold side, and L = wall thickness.

Heat transfer by convection moves heat to or from an external face of a solid body and surrounding fluid, such as air, water, or oil. The heat moved by convection is described by:

$$Q_{convection} = hA(T_s - T_f)$$

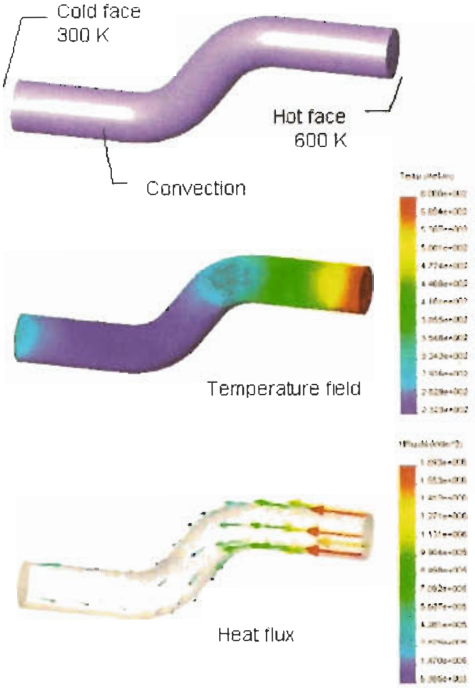
where $Q_{convection}$ = flow rate by convection, h = convection coefficient, T_s = surface temperature, and T_f = fluid temperature.

The convection coefficient, *h*, strongly depends on whether convection is natural or forced. Natural convection takes place only in the presence of gravity because fluid movement depends on the difference in the specific gravity between cold and hot fluid. Forced convection, such as forcing air around a warm solid with a fan, does not need gravity. Convection coefficients also strongly depend on the type of medium such as air, steam, water, or oil, which surrounds the solid body.

Now consider a solid bar in which one end is kept at 300°K and the other at 600°K. This is analogous to prescribed displacements in structural analysis. The temperature difference between the ends creates heat flow. The heat flow stabilizes after an initial transient period.

Assume the bar is insulated so no heat escapes through the sides. All heat entering one face must exit through the other. Heat flux is a vector and can be plotted using arrows (beside commonly used fringe plots). All heat flux

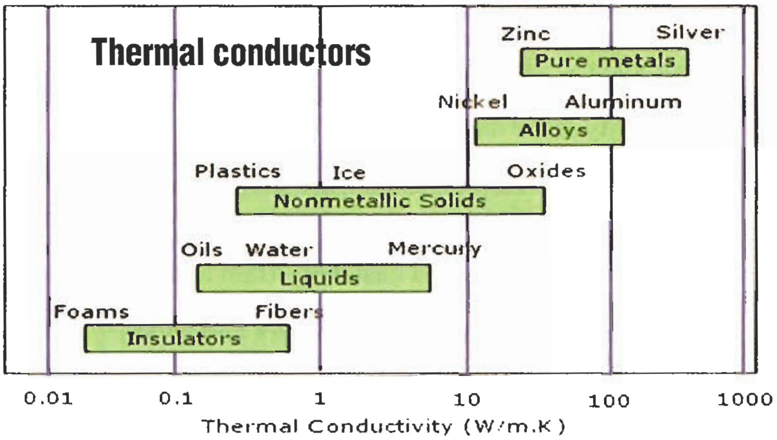
The solid rod with convection



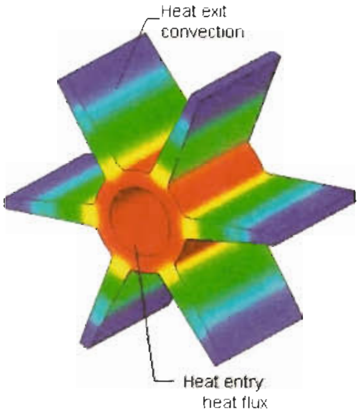
Vectors (bottom) show heat flow in a bar after adding convection.

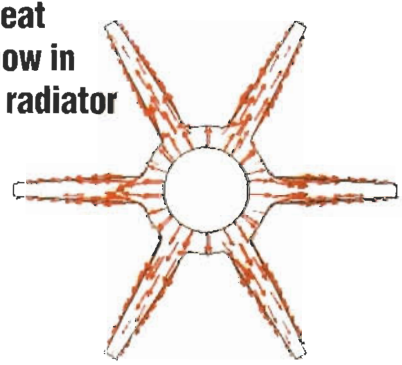
Comparing structural to thermal analysis

STRUCTURAL	THERMAL
displacement $[m]$	temperature $[K]$
strain $[1]$	temperature gradient $[K/m]$
stress $[N/m^2]$	heat flux $[W/m^2]$
pressure $[N/m^2]$	heat flux $[W/m^2]$
volume load $[N/m^3]$	heat power $[W/m^3]$
prescribed displacement	prescribed temperature
elastic support	convection coefficient
static analysis	steady state analysis
dynamic analysis	transient analysis


Thermal conductors

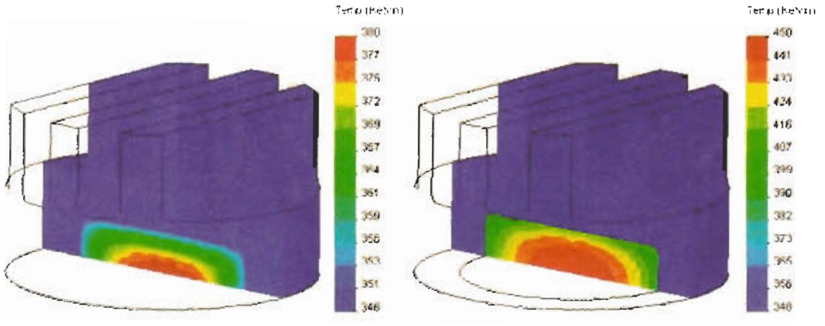
Conduction coefficients show that thermal conductivities vary widely among materials.





Heat flow in a radiator

How resistance layers change temperatures



Plots compare temperature distributions in the microchip assembly without (left) and with (right) a thermal-resistance layer separating the chip from the radiator. Outside faces have the same temperature but the chip surrounded by a thermal-resistance layer runs hotter.

vectors are tangent to the outside faces of the bar, because in the absence of convection, no heat moves across the boundaries.

Adding convection significantly changes heat transfer. For example, make the surrounding fluid a water temperature below 300°K. The two ends maintain their prescribed temperatures, but heat entering the model through the hot face now does not make it to the other end. It all gets dissipated by convection. In fact, the direction of heat flow near the cold face reverses. Defining a lower convection coefficient would eliminate this back flow of heat.

Consider a heat sink in which a ceramic microchip generates heat throughout its entire volume. Heat travels to the aluminum radiator by conduction, then dissipates to ambient air by convection.

Adding a cooling fan or immersing the radiator in water does not change the nature of hear transfer in the heat sink. The radiator still removes heat by convection. From an FEA point of view, the only difference between air, water, or oil as cooling medium and between natural and forced convections are different convection coefficients.

Convection rate on the outside faces of the radiator determines the temperature of those faces, even though the microchip produces the same amount of heat. In case of *natural convection* and a low convection coefficient, the outside fins get hotter because of a steeper temperature gradient between the radiator and ambient air ($T_s - T_a$) is required to move the given amount of heat. In case of *forced convection,* (a higher value of convection coefficient) the radiator stays cooler because a smaller temperature gradient moves the same amount of heat from solid body to surrounding fluid. Plotting heat flux as vectors lets analysts see heat movement from radiator to ambient air.

There's one more important issue: thermal resistance at the boundary between the microchip and radiator. No two solid surfaces are ever in perfect contact. There are always tiny air gaps between them.

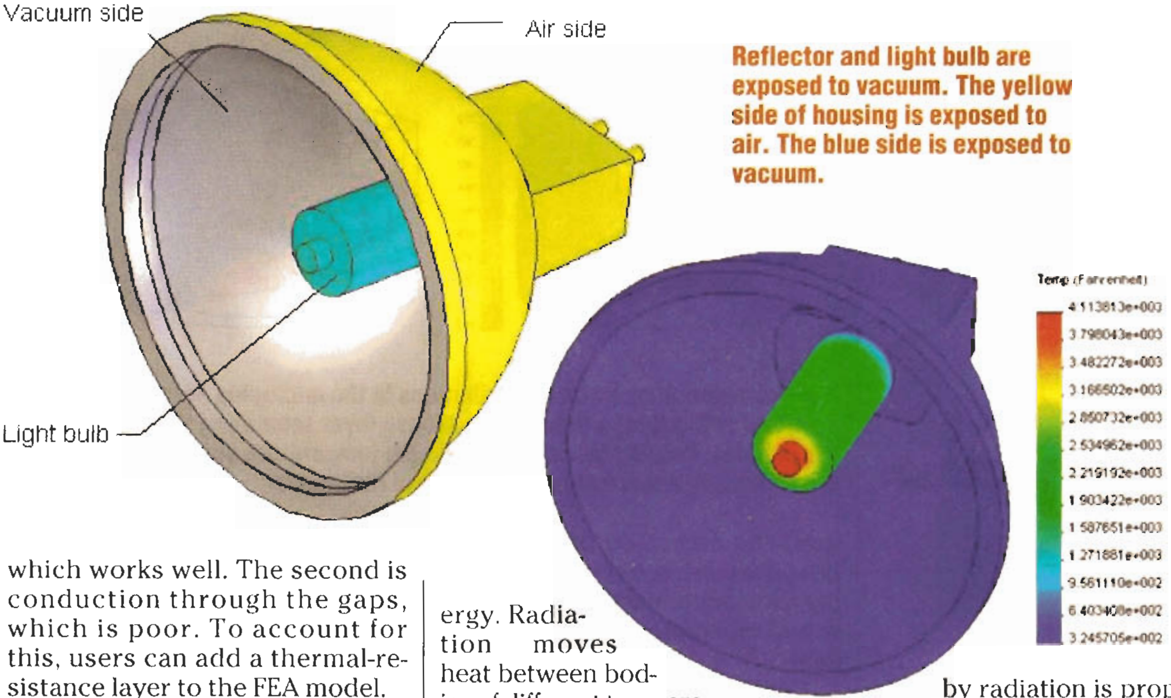
There are two modes of heat transfer through the boundary between two contacting faces. The first is conduction through points of solid-to-solid contact,

A few heat-transfer coefficients

Medium	Heat Transfer Coefficient h (W/m^2 . K)
Air (natural convection)	5-25
Air/superheated steam (forced convection)	20-300
Oil (forced convection)	60-1800
Water (forced convection)	300-6000
Water (boiling)	3000-60,000
Steam (condensing)	6000-120,000

Heat-convection coefficients vary among different media.

Radiation and spot lights



Reflector and light bulb are exposed to vacuum. The yellow side of housing is exposed to air. The blue side is exposed to vacuum.

which works well. The second is conduction through the gaps, which is poor. To account for this, users can add a thermal-resistance layer to the FEA model.

Thermal-resistance layers do not change how much heat is generated or change the outside temperature of the radiator. However, in the presence of a thermal-resistance layer, the microchip must run hotter because a higher temperature gradient across the thermal-resistance layer is required to push heat across it.

Heat transfer by radiation can be ignored because at the working temperatures of the heat sink, it's negligible, although there are times when it is not.

HEAT TRANSFER BY RADIATION

Radiation moves heat between a solid body and its surroundings by carrying it off as electromagnetic radiation.

Heat exchange by radiation always takes place, regardless of temperature and whether bodies are in a fluid or vacuum. At room temperatures, however, radiation is low compared to convective heat transfer. Hence, it's often ignored.

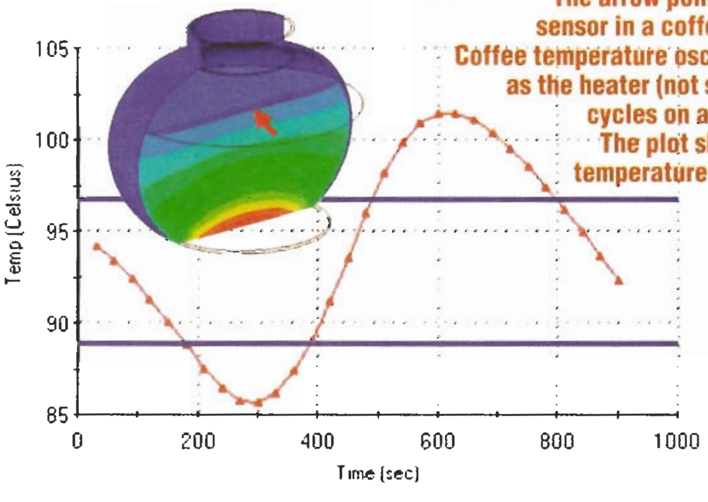
All bodies radiate thermal energy. Radiation moves heat between bodies of different temperatures and can send heat away into space. The heat exchanged by radiation between faces of two solid bodies with temperatures T_1 and T_2 is described by:

$$Q_{\text{radiation}} = s\epsilon\,(T_1^4 - T_2^4)$$

where $Q_{\text{radiation}}$ = heat flow by radiation, s = Stefan-Boltzman constant, and ϵ = emissivity of the radiating face. Because heat transfer by radiation is proportional to the fourth power of the absolute temperature, the amount transferred becomes significant at higher temperatures.

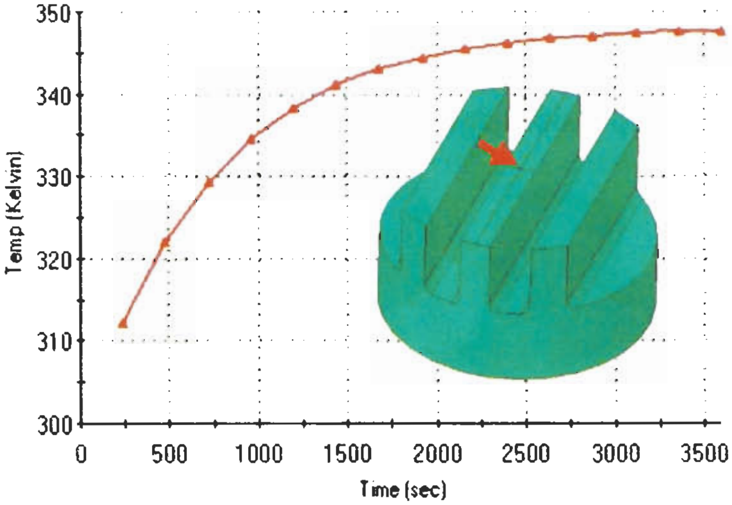
For example, consider a spotlight illuminating a large vacuum chamber. The chamber is so large we can ignore heat reflected from chamber walls back to the light. The light bulb and reflector are in a vacuum, while the back of the

Thermal inertia and power cycling



The arrow points to a sensor in a coffee pot. Coffee temperature oscillates as the heater (not shown) cycles on and off. The plot shows a temperature cycle.

Warming up to steady state



The plot shows the temperature at the top of the cooling fin (arrow) after the power is turned on at time *t* = 0. Every transient thermal analysis requires an initial temperature and time duration.

aluminum housing is surrounded by room-temperature air.

Heat radiates from the light bulb to the outside space and the parabolic reflector. Only a small portion of heat enters the housing directly by conduction where the bulb contacts the housing. Conduction moves heat inside the housing from vacuum side to air side. Convection dissipates heat from faces exposed to air.

Because radiation becomes effective only at high temperatures, the light bulb must get hot to dissipate the generated heat. The temperature of the aluminum housing is practically uniform because heat conducts easily in aluminum.

This example illustrates that cooling in vacuum is a major challenge for spacecraft designers. Even though space vehicles operate in near 0°K, electronic components can easily overheat because radiation, the only cooling mechanism, is effective only at high temperatures. To reach steady state, where the heat generated equals the heat dissipated by radiation, an electronic device must reach higher temperatures than the same device cooled by convection, which is effective at lower temperatures.

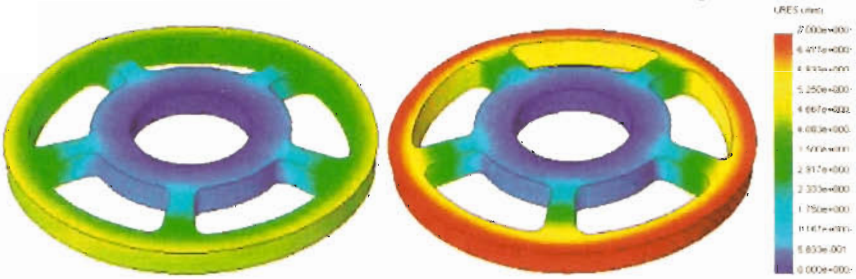
The heat sink and spotlight dealt with heat transfer in steady state and with enough time to stabilize heat flow. An analysis of steady-state heat transfer is not based on the initial temperatures or how long a system took to reach a steady state. That could take seconds or days.

TRANSIENT THERMAL

A coffee pot and the heater it sits on provide an example of transient-thermal analysis. The heater turns on when coffee temperature drops below 88°C and turns off when it reaches 93°C. Because of thermal inertia, coffee temperatures oscillate outside of this range.

Two heat-transfer mechanisms are at work in the pot. A combination of convection and conduction (the pot's bottom does not completely contact the heater) moves heat from heater to glass. Conduction moves heat across the glass to surrounding air and to the coffee. Convection moves heat from the coffee's top surface and from external surfaces of the pot. An important consideration in the coffee-pot model is the definition of the conduction coefficient for coffee. In reality, heat transfer in coffee takes place predominantly by convection due to fluid movement induced by heating. In an FEA model, however, coffee must be modeled as a solid body because the software cannot simulate fluid flow. All heat transfer in coffee must be modeled by conduction, so an artificially high conductivity must be assigned to coffee to compensate for the lack of convective heat

When modulus of elasticity changes with temperature



Rings are supported on the inner circumference and loaded downward on their outer edges. Displacements for the model with a constant modulus of elasticity (left) significantly differ from the model with a temperature dependent modulus (right).

transfer. Also, no phase change effects (boiling) can be modeled either. In fact, FEA is not recommended to solve this problem, which would be better handled using Computational Fluid-Dynamics (CFD) tools. Showing this problem provides an opportunity to emphasize the fact that FEA is a tool for analysis of heat transfer in solids, not in fluids.

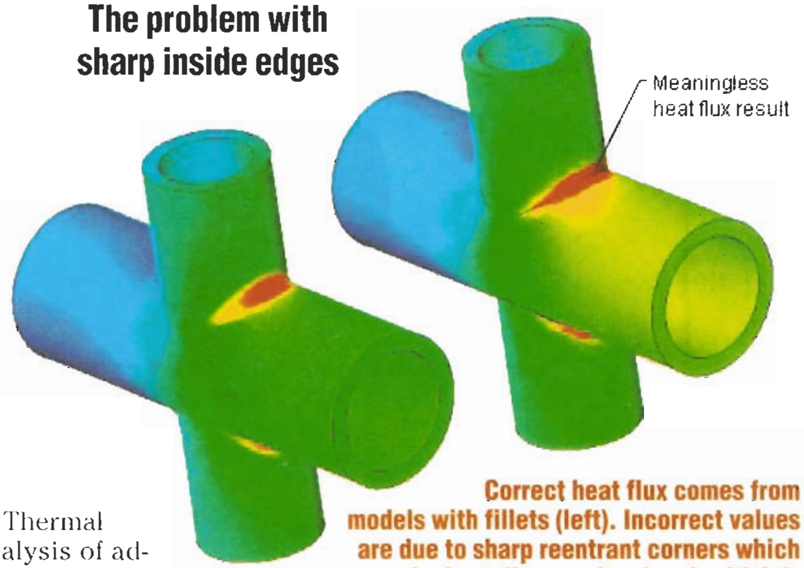
A solid body with heat flowing through it will not all be at one temperature. Consequently, different portions of the body expand and shrink at different rates. Stresses caused by this expansion or shrinkage are called thermal stresses.

Suppose hot coffee (93°C) is poured into a porcelain mug. What are the thermal stresses in the mug? To find out, run a thermal analysis that reveals the temperature distribution. Apply a temperature of 93°C to the inside faces of the mug and define convection coefficients for the outside faces. Because cooling is relatively slow, use a steady-state thermal analysis to calculate the temperature distribution in the coffee mug. It won't be uniform, so there will be thermal stresses. Having found a temperature field in the mug, calculate thermal stresses by exporting the temperature field from thermal analysis to structural static analysis.

NONLINEAR THERMAL ANALYSIS

All examples so far involved properties characterizing heat transfer — material conductivity, convection coefficients, emissivity, and heat power — that remain constant and independent of temperature. These assumptions are often acceptable for traditional engineering materials such as steel and aluminum, which are often subjected to a narrow range of operating temperatures.

The problem with sharp inside edges



Correct heat flux comes from models with fillets (left). Incorrect values are due to sharp reentrant corners which make heat-flux results singular (right).

Thermal analysis of advanced materials such as composites, may require a nonlinear approach when conductivity is a function of temperature, while heat load and convection coefficients are functions of temperature and time. Furthermore, properties that determine structural response, such as the modulus of elasticity, may also be functions of temperature.

To illustrate, consider a component heated so as to cause a nonuniform temperature distribution. Also make the material's modulus of elasticity temperature dependent. Results from such a structural static analysis under a bending load show that ignoring the effect of temperature on the modulus of elasticity produces a 40% displacement error.

MORE MODELING CONSIDERATIONS

Similarities between thermal and structural analysis mean most modeling techniques are identical and that skills acquired in one type of analysis are useful in the other. The difference in modeling techniques, however, begins with thermal-boundary conditions. The definition of symmetric boundary conditions in thermal analysis is based on the observation that if geometry and boundary conditions are symmetrical, no heat flows through the plane of symmetry. Therefore, after simplifying the model to ½ (or ¼ in case of a double symmetry), nothing must be done to the faces exposed by cuts. A convection coefficient of zero for those faces means no heat flows across them.

Analysis of heat flux requires a detailed edge model. Sharp reentrant corners — those not blended with joining surfaces — are not suitable for heat-flux analyses near corners. Heat flux is high and meaningless, a singularity, in sharp inside corners. This is directly analogous to sharp inside corners that generate stress singularities in structural models. **MD**

All images are created using CosmosWorks FEA software.

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