# Experimenter's Corner

By Forrest M. Mims

## ANALOG COMPUTER CIRCUITS, PART 2

IN PART 1 we looked at ways to use resistors to add and multiply. We also learned how to use op amps to multiply, divide, add, average and subtract. Although an op amp and a few resistors can multiply and divide, manual adjustment of at least one potentiometer is necessary. We noted in Part 1, however, that there are many analog computer circuits that respond to incoming voltages rather than manually adjusted potentiometers. The op amp adder, averager and subtracter circuits described in Part 1 have this ability.

One way to multiply or divide two voltages is to convert both to their logarithms. Multiplication is accomplished by adding the two logs with a summing amplifier. Division can be performed by subtracting the log of the divisor from the log of the dividend with a difference amplifier. The antilog of the result is the product or quotient, as the case may be.

Now that the pocket calculator has replaced the slide rule, logarithms are not used nearly as often as they once were. So let's take time out for a brief refresher course before moving on.

**Logarithms.** Any decimal number can be expressed as a power of ten. For example, 1,000 is $10^3$ and 736 is $10^{2.8669}$. In both cases, the exponent of the base 10 is referred to as the number's logarithm. One important aspect of logarithms is revealed by the following table.

| Number | Power of Ten | Logarithm |
|---|---|---|
| 1 | $10^0$ | 0 |
| 10 | $10^1$ | 1 |
| 100 | $10^2$ | 2 |
| 1,000 | $10^3$ | 3 |
| 10,000 | $10^4$ | 4 |
| 100,000 | $10^5$ | 5 |
| 1,000,000 | $10^6$ | 6 |

As you can see, a very wide range of decimal numbers occupies a very small range of logarithms. The resulting compression provides a handy shorthand method for processing very large numerical variations.

We noted earlier that two numbers can be multiplied by adding their logs or divided by subtracting their logs. That's how a slide rule works. It's also possible to add and subtract numbers using ordinary rulers. Place one ruler atop the other. Then align the 0 on the top ruler with one of the numbers being added on the bottom ruler. Next, find the second number being added on the top ruler. This number will point to the sum on the bottom ruler.

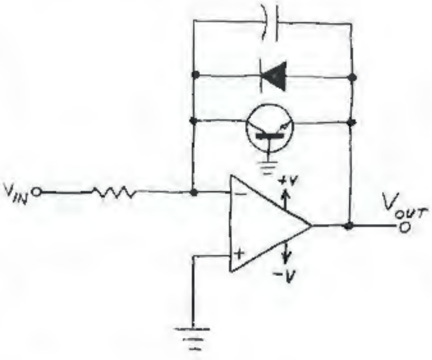


*Fig. 1. Schematic of a basic logarithmic amplifier.*

Rulers have a linear scale—their divisions are equally spaced. A slide rule, on the other hand, has a logarithmic or compressed scale. When you multiply two numbers with a slide rule, you are actually adding their logs.

Look back at the table and multiply 1,000 × 100 to see how this works. The log of 1,000 is 3 and the log of 100 is 2. $3 + 2 = 5$ so the log of $1,000 \times 100$ is 5. From the table, 5 is the log of 100,000 (or 100,000 is the antilog of 5) so $1,000 \times 100 = 100,000$. Try dividing a few numbers in the table by subtracting the log of the divisor from the log of the dividend and taking the antilog of the remainder to obtain the quotient.

Before the advent of the pocket calculator, the use of logarithms was standard procedure when multiplying and dividing very large or very small numbers. Logs are also handy for extracting roots. The cube root of 27, for example, is found by dividing the log of 27 by 3 and extracting the antilog of the result. (The log of 27, 1.4314, divided by 3 is 0.4771; the antilog of 0.4771 is 3, the cube root of 27.)

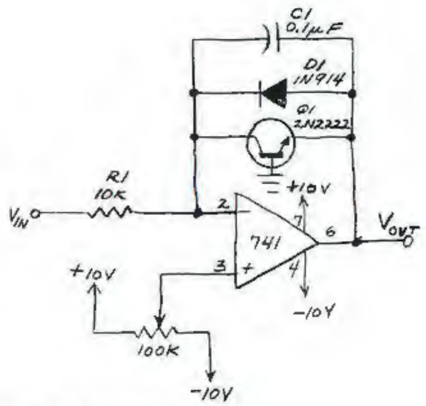


*Fig. 2. A practical logarithmic amplifier circuit.*

Incidentally, numbers in any number system can be expressed as logarithms. Can you figure out the logarithms of the binary sequence 1, 10, 100, 1000 . . . 10000000? (Hint: 1000 is $2^3$.)

**The Logarithmic Amplifier.** The voltage drop across a diode is related logarithmically to the current flowing through it. This makes possible the conversion of a voltage into its log.

Practical log conversion is best achieved by using a transistor in a common- or grounded-base configuration instead of a diode. Figure 1 shows how the transistor is connected in place of an op-amp's feedback resistor to give what is called a *transdiode logarithmic amplifier*. Although the circuit is an amplifier, you can think of it as a log generator to avoid confusion.

Not all transistors exhibit logarithmic properties over as wide a range as might be required. Many, however, do and one readily available type is the 2N2222 (equivalent to Radio Shack type RS2009).

You can easily assemble a breadboard log amplifier with the help of a 741 or any other frequency-compensated op amp. Figure 2 shows the details of a practical version of the circuit in Figure 1. Capacitor *C1* does not assist in the log conversion process. Instead, it reduces the ac gain of the op amp and helps eliminate high-frequency oscillation which might otherwise occur. Diode *D1* protects the transistor from excessive reverse base-emitter bias from the op amp's output.

On the following page are the results I obtained from a breadboard version of the circuit in Fig. 2.

*Fig. 3. Operation of a log amplifier plotted on semi-log graph paper.*

| **Input** (mV) | **Output** (mV) |
|---|---|
| 1 | −322 |
| 10 | −371 |
| 100 | −432 |
| 1,000 | −494 |
| 10,000 | −557 |

In all cases, the output voltage was inverted (negative), but this is of no major consequence as we can either ignore the polarity or, if desired, change it with an inverting buffer.

Figure 3 shows the data in the table plotted on a semi-log graph. The graph is called semi-log because one axis is linear (the output voltage) and the other is logarithmic (the input voltage). A plot of the data produces a straight line on the semi-log graph, so we know the log amplifier is reasonably accurate over the given range.

Now that we've seen how a real log amplifier works, let's look at a few of its characteristics. First, notice the very small range in output voltage (a few hundred millivolts) that results from the huge swing in input voltage (10,000 millivolts). This characteristic of log amplifiers is ideal for compressing very large voltage excursions into more manageable form.

A second characteristic is that the transfer function of our log amplifier is *not* $-V_{OUT} = \log(V_{in})$. Rather, it's approximately $-V_{out} = 0.06 \log V_{in} + K$ where $K$ is a constant. For the log amplifier I built, $K$ is 0.495. Your amplifier might yield a slightly different $K$. You can use a programmable calculator to compute the exact transfer function.

A third characteristic of our log amplifier is that it is temperature sensitive. That's not good because the current flowing through the 2N2222 causes heating which can alter the accuracy of the circuit. The error this introduces can be substantial, easily several percent.

Yet another characteristic of the amplifier is that the input offset voltage of the op amp can cause a substantial but predictable error when the input voltage is small. This problem can be alleviated by connecting a 10,000-ohm potentiometer to the 741 as shown in Fig. 4. Pin 2 of the 741 is then temporarily shorted to ground and the offset potentiometer is adjusted until VOUT is exactly zero volt.

A more significant error is introduced by the op amp's bias current. This ranges from 80 to 500 nonoamperes for the 741. Figure 4 also shows how to compensate for this problem by temporarily replacing the components in the feedback loop with a 100,000-ohm resistor and adding a bias current potentiometer. The pot is then adjusted until –VOUT exactly equals VIN over as wide a voltage range as you expect the amplifier to receive.

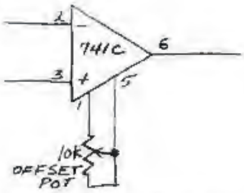


*Fig. 4. Adding an offset pot to the log amplifier.*

You don't have to make all these calibration adjustments when building a simple log amplifier for experimental purposes. But if you decide to build your own analog computer, for best results you'll need to calibrate or *trim* each op amp using the methods described.

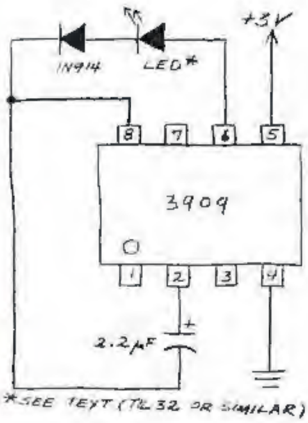
**The Antilogarithmic Amplifier.** Analog computing circuits that use log amplifiers require one or more antilog amplifiers to convert results back into linear form. Antilog amplifiers can also be used to expand narrow ranging input voltages into much wider and therefore more easily resolved form.

## PROJECT OF THE MONTH

### LED TRANSMITTER MODULE

Here's a miniature LED transmitter module you can assemble in half an hour or so. It makes an ideal mate for the miniature phototransistor receiver module described in the last Project of the Month. Alternatively, it can be used on its own as a miniature infrared beacon.

Figure A is a complete circuit diagram of the transmitter. The circuit uses a 3909 LED flasher IC for the utmost in simplicity. This IC is designed to drive red LEDs and *not* IR (infrared) LEDs. IR LEDs have a lower forward voltage drop (about 1.2 volts) than red LEDs (about 1.7 volts). This means you can fool the 3909 into driving an IR LED by adding an ordinary silicon diode in series with the LED. A diode like the 1N914 has a drop of 0.6 volt and this gives a total drop of about 1.8 volts when connected in series with an IR LED.



*SEE TEXT (TIL 32 OR SIMILAR)

*Fig. A. An infrared light emitting diode transmitter.*

For best results, use a GaAs:Si LED instead of a GaAs LED. Both types are available from companies that advertise in the Electronics Marketplace section of this magazine. GaAs LEDs emit at a peak wavelength of 900 nanometers (nm) while GaAs:Si LEDs have a peak wavelength of about 935 nm. Visible light ranges from about 400 nm to 700 nm.

*Fig. B. LED transmitter module.*

Most GaAs:Si LEDs are at least twice as efficient as GaAs units, and that's why they will work better in this project. GaAs units have a much faster rise time, but this is irrelevant because the rise time of the transmitter is not fast enough to tax a GaAs:Si diode.

Figure B is a photo of the interior view of the transmitter module and Fig. C shows its assembly details. Begin assembly by installing the capacitor and diode in the bottom of the module header and inserting their leads deep in the indicated pin slots. Then install the LED as shown in Fig. C, making sure its leads are oriented properly and that it doesn't protrude too far over the edge of the header. Secure all the leads in place with a small amount of solder and clip off the excess lead lengths close to the header pins.

Next, place the pins of the IC adjacent to or inside the slots in the appropriate header pins. Make sure they don't protrude too far or the module cover will not fit. Then carefully solder the pins in place. Use a small file to remove excess solder from the outside edges of the header pins so that the module cover will fit. Then bore a hole (⅛-inch if you use a TIL32 LED) in the module cover and snap the cover in place.

Unless you use a red LED, you'll need a receiver such as the phototransistor receiver module described in the previous Project of the Month to test the transmitter. Insert the module in a solderless breadboard and connect a 3-to-6-volt supply to the power connections. With a 3-volt supply, the transmitter LED will flash on and off at 360 Hz. If you connect an earphone to the receiver module and point the LED toward the phototransistor, you will hear a loud tone. Block the path between the two modules and the tone will stop.

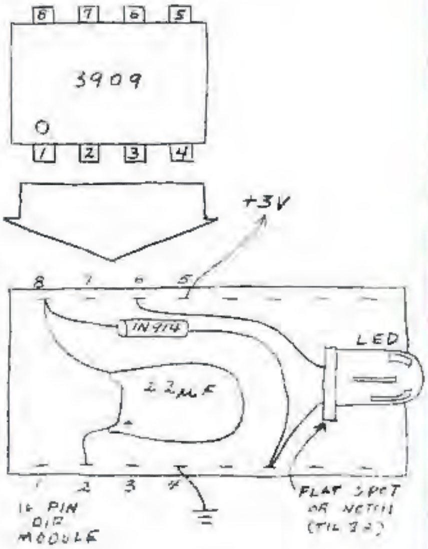


*Fig. C. Assembly details of LED transmitter module.*

Remember to be careful if you connect an earphone to the output of the receiver module instead of a small speaker. The sound generated by the earphone can be *very* loud.

Try experimenting with the two modules to see how far you can separate the receiver module from the transmitter module and still recover a usable signal. Also, try using the two modules as an object detector by pointing both units at a white card and seeing how far away the card can be placed without losing the signal. ◇

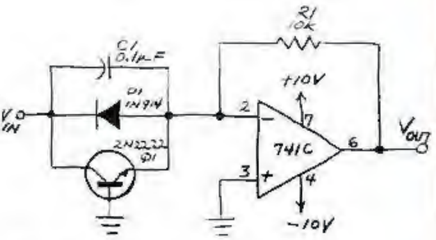


*Fig. 5. Schematic of an anti-logarithmic amplifier.*

If the transfer function of an ideal log amplifier is $V_{OUT} = \log (V_{IN})$ then the transfer function of an ideal antilog amplifier is $V_{OUT} = 10^{V_{IN}}$. In an actual circuit, however, the transfer function is the inverse of the log amplifier's. The differneces between ideal and actual transfer functions are therefore compensated.

Figure 5 shows the circuit for a working antilog amplifier you can make. An interesting experiment is to connect the input of the antilog amplifier to the output of the log amplifier in Figure 2. If both amplifiers are perfectly accurate, the transfer function for the combination will be $V_{OUT} = V_{IN}$.

Here are the results I obtained from a log-antilog combination with *no* calibration adjustments:

| $V_{IN}$ (mV) | $V_{OUT}$ (mV) |
|---:|---:|
| 1 | 1 |
| 10 | −6 |
| 100 | −111 |
| 1,000 | −1,205 |
| 10,000 | −11,490 |

As you can see, the error is fairly high. Calibrating both amplifiers using the methods previously outlined will provide much better results.

**The Analog Multiplier.** Now that we've built log, antilog and summing amplifiers, we can build an analog multiplier. A block diagram for the multiplier is shown in Fig. 6 and a complete circuit in Fig. 7.

The maximum error of the multiplier is easily in excess of 10 percent. Can you improve this figure over several decades of input voltage? (Hint: Use careful calibration procedures and try to keep all feedback transistors at the same temperature by, say, bonding them together with epoxy cement.)

You can convert the multiplier into an analog divider simply by changing the summing amplifier into a difference amplifier. See last month for details.

**Single-Chip Multipliers.** Contrary to

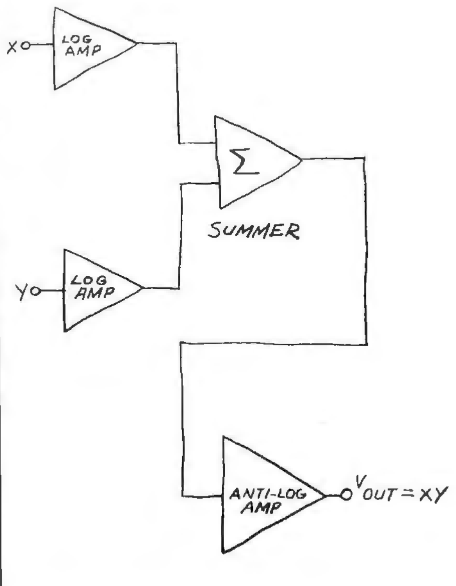


*Fig. 6. Block diagram of a logarithmic multiplier.*

my usual practice of breadboarding every circuit that appears in this column, I must confess to not having assembled the multiplier in Fig. 7. Several single IC multipliers that include all necessary amplifiers and transistors on the same silicon chip are now available, and they're much easier to use and more accurate because all circuit elements on the chip are at the same temperature. One such multiplier is Motorola's MC1595.

Single-chip multipliers like the MC1595 require many external calibration resistors, but recently Raytheon and Analog Devices introduced single-chip multipliers that include built-in error correcting features. Raytheon's chip is the 4200 and Analog Devices' is the AD534.

The 4200 is much less expensive than the AD534, but the latter is far superior to any previous single-chip multiplier because it includes 12 calibration resistors that have been factory-trimmed to a high degree of accuracy by a pulsed laser. The laser zaps away bits of thin-film calibration resistors that have been previously deposited directly on the silicon chip until a specified accuracy is reached.

The AD534 is being billed as the first single-chip analog computer. Having experimented with both overly demanding, temperature-sensitive log amps and now the AD534, I'm more than willing to accept this enthusiastic claim. Figure 8 shows why. All the circuits shown are complete—*no* calibration resistors are required.

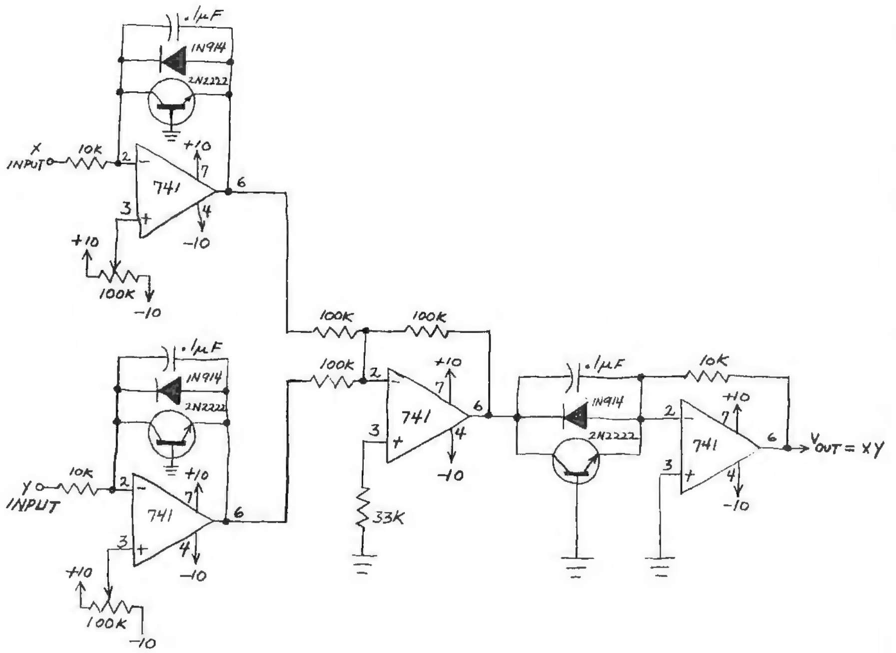


*Fig. 7. Analog logarithmic multiplier circuit.*

Here's an example of the results I obtained for an AD534 connected as a multiplier and a square rooter:

| X | $X^2$ | AD534 | $\sqrt{10X}$ | AD534 |
|---|---|---|---|---|
| 1 | 1 | .95 | 3.16 | 3.09 |
| 2 | 4 | 4.08 | 4.47 | 4.51 |
| 3 | 9 | 9.20 | 5.48 | 5.52 |
| 4 | 16 | 16.24 | 6.32 | 6.36 |
| 5 | 25 | 24.40 | 7.07 | 7.11 |
| 6 | 36 | 35.20 | 7.75 | 7.72 |
| 7 | 49 | 48.20 | 8.37 | 8.42 |
| 8 | 64 | 63.20 | 8.94 | 8.90 |
| 9 | 81 | 79.80 | 9.49 | 9.50 |
| 10 | 100 | 98.70 | 10.00 | 10.05 |

As you can readily see, the AD534 is exceptionally accurate. If you want to experiment with the AD534, you'll have to order one from an Analog Devices representative. Write the company for a list of reps and a specification sheet (Route One, Industrial Park, Box 280, Norwood, MA 02062). The single-quantity price is $26.00 for the lowest accuracy version (AD534J; ±1% total error). If the price seems high, look at it again after you've spent a frustrating evening trying to calibrate a homebrew multiplier. If you prefer digital circuits, consider the cost of the hardware and the time to develop software for a microprocessor that will perform the same functions. ◇

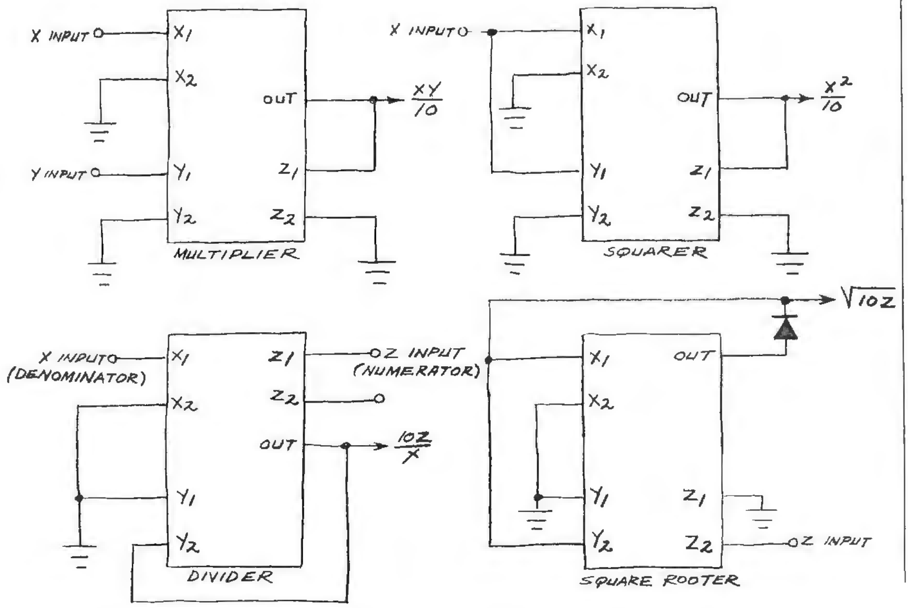


*Fig. 8. Applications for the AD534 multiplier.*