Automatic flight control and automatic landing system using analogue computing units by Elliott Automation Ltd.

COMPUTER EVOLUTION

PART TWO

By S. A. HODSON B.Sc.

BASIC ANALOGUE CIRCUITS

In the opening article of this series, the differences between analogue and digital computers were explained, and their early development from simple calculating machines was traced. Now we shall come up to date and look at modern analogue computers and some of their uses.

Two important concepts were described in the previous article. The first of these concepts was that of the "model". It should be obvious that a scale model of a ship or an aeroplane will be a great help in predicting its behaviour when it is built full size.

What may not be so obvious is that, especially in these days of electronics, the model need not be a physical scale model at all, but can be a model made in any medium the modeller likes to use. Of course some media are more direct than others, and it is this that directs the attention to the first means of classification of analogue computers: "Direct" and "Indirect". A scale model of an aeroplane in a wind tunnel is a "direct" computer, whereas an electronic analogue computer, programmed to represent the aeroplane in the air flow, would be an "indirect" computer.

In this case, the electronic computer works by representing the air flow over the aircraft; and it does this by means of mathematical equations that can take into account every bump or curve on its skin. What is perhaps more important is that these equations can be altered to take account of a design change without the aircraft having to be hacked about by a welding torch, in fact the performance of the whole plane can be predicted before hacksaw has touched metal.

These mathematical equations or "functions" as they are more properly called, are the second of the two concepts mentioned above: that of being able to represent any physical occurrence by means of an equation.

COMPUTING MEDIA

There are three main computing media in the field of analogue computing. The first of these is that of mechanics. Scale models are a good example of the direct application of a computing device in this field, while a slide rule serves to illustrate the indirect use of a mechanical device.

The use of the term "computer" about a scale model is not as loose as it may seem. Take for example the case of the model in the wind tunnel. Physical distances and hence velocities have obviously to be scaled, but what about the pressure, the density, and even the composition of the air flowing in the tunnel? These must be scaled too.

A whole science known as "dimensional analysis" has grown up around these scale models, and a lot of time is devoted to the calculation of the correct scaling factors for all the parameters involved in a scale model.

The second medium used in analogue computing is the fluid; here the term fluid includes both liquids and gases, thus embracing the sciences of hydraulics and pneumatics in one term. The fluid medium is mostly commonly used in the direct fashion in scale models of dams and hydroelectric schemes.

It can, however, be used indirectly, and a good example of this is the "electrolytic tank". This device is used mainly in computations involving field theory of one form or another. The details are not important here but the general idea is to have a tank full of an electrolyte and to immerse electrodes in this tank. The arrangement of the electrodes represents the system being investigated, and when they are charged up, the value of the electric field at any point in the electrolyte can be used to calculate the behaviour of the system.

The third, and certainly the most widely used, of the three media that have been mentioned is, of course, that of electronics; and it is the application of this medium that is of interest here.

It should be noted that nearly all the devices described so far can only be used for one purpose. For instance, the model aircraft can only represent one full size machine, any other design will have to have a different model. A computer of this type is known as a "fixed purpose" machine, and as such is limited in its field of operations.

MATHEMATICAL FUNCTION

The great advantage of an electronic computer is that it is a general purpose machine and can be programmed for one job then, when that job is finished, programmed for something entirely different. To achieve this flexibility of operation, the electronic machine works in the realm of the mathematical function, and it is to the explanation of these that the next few paragraphs must be devoted.

Suppose that a capacitor is being charged from a battery, through a resistor. The voltage and current wave-forms will look like Fig. 2.1.

If the graphs of V and I are examined more closely, it will be seen that the actual value of I is directly proportional to the slope of V. That is, near the origin of the graphs, V is sloping upwards quite sharply, and I has a high positive value. As time progresses, V slopes less sharply and the value of I drops away. In mathematical terms this can be expressed

$$I = C\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) \qquad (1)$$

where the term $\mathrm{d}V/\mathrm{d}t$ is used to represent the rate of change of voltage V with time. The operation performed on V to get $\mathrm{d}V/\mathrm{d}t$ is known as "differentiation". The letter d is an arbitrary symbol of differentiation. Similarly, to get back to V from $\mathrm{d}V/\mathrm{d}t$ the process used is known as "integration", and may be written thus:

$$V = \frac{1}{C}\int I\mathrm{d}t \qquad (2)$$

Two very similar equations can be written to represent the behaviour of an inductor namely:

$$V = L\left(\frac{\mathrm{d}I}{\mathrm{d}t}\right) \qquad (3)$$

and

$$I = \frac{1}{L}\int V\mathrm{d}t \qquad (4)$$

The elongated S sign denotes integration.

No apology is made for starting at such an elementary point in the theory of functions, since these equations are by far the most important in the realm of analogue computing. It is in fact possible, with these four equations to set up solutions to the most complex differential equations imaginable.

Just as it is possible to differentiate V once and obtain $\mathrm{d}V/\mathrm{d}t$, it is equally possible to do it again and end up with $\mathrm{d}^2V/\mathrm{d}t^2$.

An easy way of understanding this is to consider a car travelling along a road, and to let the distance it has covered be x miles. Then if x were to be differentiated $\mathrm{d}x/\mathrm{d}t$ would be obtained which is the velocity of the car in miles per hour. A further differentiation would give $\mathrm{d}^2x/\mathrm{d}t^2$ which is its acceleration, in miles per hour per hour, and so on. In just the same manner integration may be performed again and again.

In all these examples the function "time" t has been involved and the differentiations and integrations that have been performed have been done with respect to time. Any computations done with respect to time in this manner would be known as "real time" computations.

A great deal of analogue computing is done with respect to time, although, as will be seen, it need not always be real time that is used. In some cases it is very convenient to use "half time" or "quarter time". This gives a very powerful method of speeding up what may be a tedious calculation.

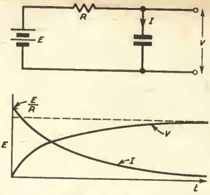


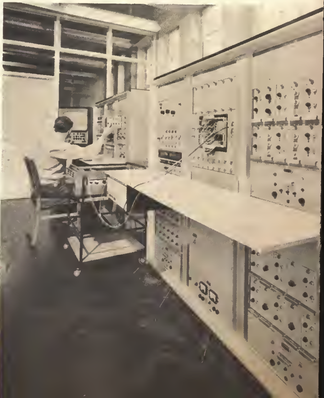
Fig. 2.1. Capacitor charging circuit with voltage and current waveforms

Having described the basic formulae involved in calculus (this is the term used to describe integration and differentiation), it is possible now to turn to the differential equation, which forms the basis of all computations performed on an analogue computer. The general form of such an equation is:

$$a + bx + c\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) + d\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right) + e\left(\frac{\mathrm{d}^3x}{\mathrm{d}t^3}\right) + \ldots = 0 \qquad (5)$$

This looks positively frightening, and as it stands, has no solution. However, if it is broken up into its separate terms, it will be seen that each term is no more than one differentiation of the previous term with a different constant attached. When all the terms are added together they might, for instance, represent the flow of air across an aircraft's wing surfaces, or, in a simplified form, they might, as has already been seen in previous equations, represent the behaviour of a capacitor or an inductor.

Solartron analogue computer in use in the electrical and mechanical research laboratories at the University of Sheffield



OPERATIONAL AMPLIFIER

To turn now to the actual hardware involved, the basic linear computing unit is the "operational amplifier" (see Fig. 2.2). The amplifier has a very high gain, and its input current is assumed to be zero.

If this is the case, then $I_1 = I_2$, putting this in another form gives

$$\frac{V_1 - V_g}{Z_1} = \frac{V_0 - V_g}{Z_2} \qquad (6)$$

Now if the gain of the amplifier is in the thousands or even millions, then V_g can be neglected in comparison with V_1 and V_0, and this equation becomes

$$\frac{V_1}{Z_1} = \frac{V_0}{Z_2}$$

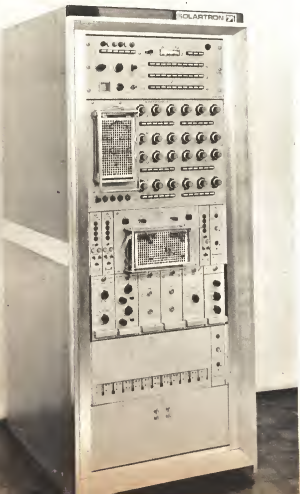
or

$$\frac{V_0}{V_1} = \frac{Z_2}{Z_1} = G$$

where G is the "closed loop" gain of the unit as a whole. Thus the gain of this device can be controlled at will by the user simply by juggling with the two impedances Z_1 and Z_2.

Suppose now that Z_1 was a resistance of 10kΩ and Z_2 a resistance of 100kΩ, then the gain G would be 10 and the output voltage V_0 would be 10 times the input voltage V_1. This is a simple way of multiplying a variable voltage by a constant. In fact it has performed one of the operations required to form equation 5.

The patch board and analogue control panel on the Solartron basic 24 amplifier equipment



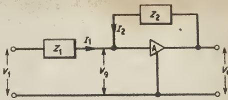


Fig. 2.2. Basic operational amplifier

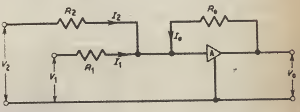


Fig. 2.3. Two inputs fed into a basic amplifier

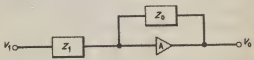


Fig. 2.4. Simplified diagram of an operational amplifier

In the above diagrams A is normally prefixed with a minus sign to denote 180 degrees phase shift

If the input voltage were to represent dx/dt, and $Z_2/Z_1 = c$, then the output voltage will be $C(dx/dt)$; one of the terms in equation 5.

Consider now what would happen if two inputs were added on to a basic amplifier, as in Fig. 2.3.

Now, using the same assumptions as before,

$$I_0 = I_1 + I_2$$

then

$$\frac{V_0}{R_0} = \frac{V_2}{R_2} + \frac{V_1}{R_1}$$

therefore

$$V_0 = \left(\frac{R_0}{R_2}\right) V_2 + \left(\frac{R_0}{R_1}\right) V_1$$

but R_0/R_2 and R_0/R_1 can be varied independently of each other, and hence it is possible to add two variables together. For instance:

Let

$$V_1 = 1 \quad \text{and} \quad R_0/R_1 = a$$

and

$$V_2 = x \quad \text{while} \quad R_0/R_2 = b$$

then $V_0 = a + bx$, which are the first two terms of equation 5. It is easy to see how this technique can be extended to accomodate any number of inputs with different multipliers for each one. The only thing that remains now is to be able to differentiate and integrate electronically. Once this is possible, the whole of equation 5 will be constructed from just one input.

The only type of amplifier that has been dealt with so far is that in which the two impedances, Z_0 and Z_1 (see Fig. 2.4) were both resistances.

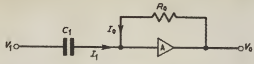


Fig. 2.5. Z_1 is represented by a capacitor

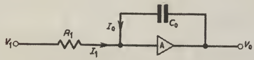


Fig. 2.6. Z_1 is represented by a resistor and Z_2 by a capacitor

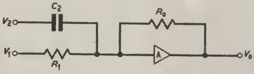
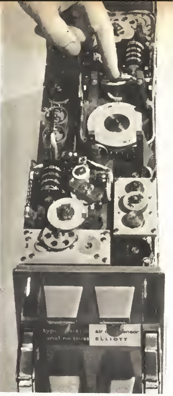


Fig. 2.7. A differentiator C_2 and scaler R_1 are combined

In the above diagrams A is normally prefixed with a minus sign to denote 180 degrees phase shift



Elliott air data analogue computer for aircraft. Signals from the aircraft's sensors are converted for use in flight control

It may have been noted that, in Fig. 2.4, no earth or zero voltage line has been drawn. This is a matter of convenience, and provided that all voltages given on a diagram are given with respect to earth, then no confusion should arise. This makes the drafting of large, more complex circuits, a very much less tedious task.

Having described the results of calling Z_0 and Z_1 resistances, consider now what would happen if one of them, say Z_1, were to be a capacitance, leaving Z_0 as a resistance, as in Fig. 2.5.

Assuming, as before, that the amplifier draws no current at its input, then $I_0 = I_1$
hence, using equation **1**

$$\frac{V_0}{R_0} = I_1 = C_1\left(\frac{dV_1}{dt}\right)$$

or

$$V_0 = R_0C_1\left(\frac{dV_1}{dt}\right)$$

This means that the output of this type of operational amplifier is directly proportional to the differential of the input. It is now that the possibilities of such an amplifier begin to make themselves felt. Given, say, x in equation **5**, and this may be the distance that a car has travelled as read from its trip-meter, then solely by using a train of differentiators, as in Fig. 2.5, dx/dt, and all the further derivatives of x, may be found. This gives the speed of the car at any one point; also, its acceleration, its rate of change of acceleration, and so on.

Supposing Z_0 and Z_1 were to be interchanged, making Z_0 a capacitance, and Z_1 a resistance, as in Fig. 2.6.

In this case

$$I_0 = I_1 = \frac{V_1}{R_1} = C_0\left(\frac{dV_0}{dt}\right) \qquad \textbf{(7)}$$

Now, remembering that, to get from dV_0/dt to V_0, it is necessary to integrate, it is possible to integrate both sides of equation **7**, and get

$$C_0V_0 = \int\left(\frac{V_1}{R_1}\right)dt \quad \text{or} \quad V_0 = \frac{1}{C_0R_1}\int V_1 dt$$

The dt is included to show that the integration has been performed with respect to time.

Not only is it possible to differentiate and multiply with an operational amplifier, it is also possible to integrate.

The flexibility of these units is such that they can be mixed up together to give more complex results without having to use large quantities of hardware. For instance, Fig. 2.7 shows how a differentiator and a scaler can be mixed together. This dodge can save two operational amplifiers straight away.

It may have been noticed that nowhere in the preceding paragraphs is an inductor mentioned.

The reason for leaving out the inductor is that in practical circuits for this purpose they are never used. Capacitors are cheaper, smaller, easier to obtain, and more stable than inductors. Furthermore, there just isn't any need for them, since all the functions that are needed can be performed using capacitors alone.

In the next article it is intended to describe how these methods are put to use in practical analogue computers; types of d.c. amplifier that can be used; setting up for computation.