E-Z MATH

NETWORK ANALYSIS & THEVENIN'S THEOREM

By Louis E. Frenzel

When you've got a circuit schematic with enough interconnections to make a traffic planner ill, how do you analyze it without getting ill? Network analysis!

MOST OF THE MATHEMATICAL OPERATIONS USED IN electronics are performed for one of two purposes. First, the math helps to analyze a circuit. In analyzing a circuit, you find out not only how it works, but what the various current, voltage, and impedance levels are. Circuit analysis allows you to experiment with the circuit mathematically to determine how conditions change if one or more parameters are varied.

The second reason for this type of math is circuit design. Whenever you wish to create a new circuit to perform a specific function, you must be able to specify what you want, then work backwards to synthesize the circuit that will give it to you. The math lets you do that on paper.

Circuit analysis in electronics is really not all that difficult. Typically, a great deal of it can be done with the basic electrical laws you no-doubt learned somewhere along the line. Specifically, I am speaking of Ohm's law and Kirchhoff's laws. Both are simple mathematically and very easy to understand and apply. (See the sidebar entitled "Ohm's and Kirchhoff's Laws" for a quickie review of those important electronics laws.)

When the circuits are essentially simple series and parallel circuits, all you need is Ohm's and Kirchhoff's laws to analyze and solve them completely. But as the circuits get more complex, the laws, while they still apply, are more difficult to use. In some cases complex circuits defy analysis or design. That's where special circuit theorems come into play. Over the years, many different techniques have been developed for analyzing and designing electronic circuits. The theorems are really not any more difficult to understand or apply than Ohm's or Kirchhoff's laws, but they have *high falutin'* sounding names and often intimidate people. In reality, they are nothing more than tricks and shortcuts that help you speed up and simplify the analysis and design of circuits.

In this article, we'll cover the popular and widely used Thevenin's theorem.

Voltage Sources

Almost any electronic circuit can be represented by the simple equivalent circuits shown in Fig. 1. The circuits consist of a voltage source with an internal impedance and some type of load. In Fig. 1A, the source is a battery with a voltage of V_S and its internal resistance R_S connected to a resistive load R_L. A simple example might be a flashlight where the battery represents some 1.5-volt D cells, and the load represents the light bulb. Another example might be where the battery in Fig. 1A represents a DC electronic power supply with its internal resistance R_S while the load represents one or more electronic circuits.

In Fig. 1B, the voltage source is an AC generator with its internal impedance connected to a load. That might represent any number of circuits. For example, the AC generator might represent the output of an amplifier with its internal impedance R_S, while the load R_L may represent a speaker. Another example is that the voltage source might represent the class-C output amplifier of a CB transmitter with its output impedance connected to a 50-ohm load which represents an antenna. Those are only a few examples, but you get the idea. Almost any electronic circuit can be simplified until it is represented by an equivalent circuit similar to those shown in Fig. 1. In some cases, you will hear the internal resistance

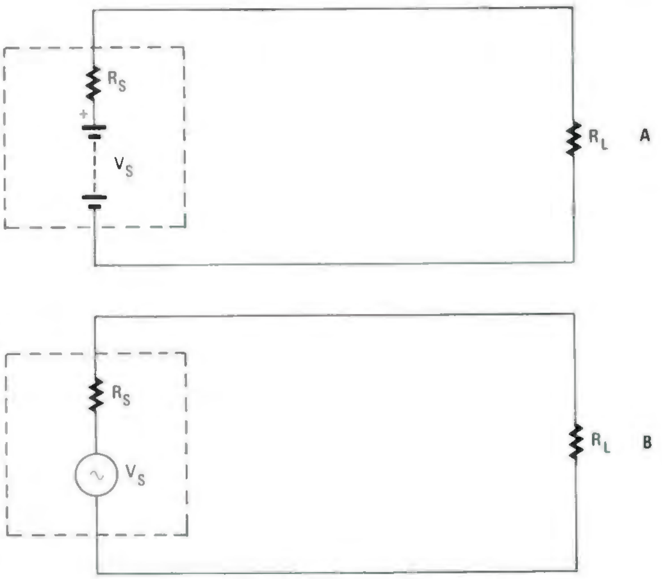

Fig. 1—These are the simplest ways of representing a current or voltage source. They are known as Thevenin equivalents and they greatly simplify circuit analysis.

Ohm's and Kirchhoff's Laws

Ohm's law is a mathematical statement of the relationship between the current, voltage and resistance (or impedance) in a circuit. It states:

"The current is directly proportional to the applied voltage and inversely proportional to the resistance (or impedance)."

In mathematical form, Ohm's law is expressed as:

$$I = V/R$$

where I is the current in amperes, V is the voltage in volts, and R is the resistance in ohms. For example, if a 20 ohm resistor is connected to a 12-volt battery, the current is:

$$I = 12/20 = .6 \text{ ampere}$$

Using algebra, you can rearrange the above formula to calculate the voltage or resistance:

$$V = IR$$
$$R = V/I$$

Kirchhoff's Laws

There are two basic Kirchhoff's laws, one for series circuits, the other for parallel circuits. Kirchhoff's voltage law for series circuits states:

"The sum of the voltage drops across components in a series circuit is equal to the source voltage." In Fig. A, that can be written mathematically as:

$$V_S = V_1 + V_2 + V_3$$

where V_S is the source or applied voltage and V_1, V_2 and V_3

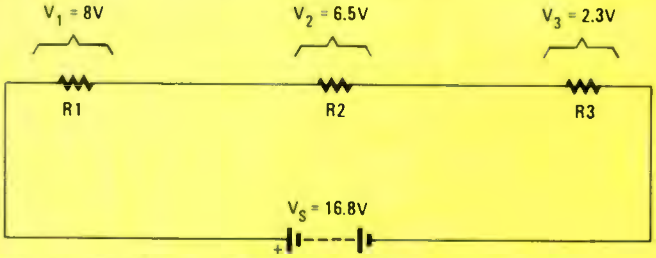

Fig. A—Kirchhoff's voltage law for series circuits simply states that the input voltage must equal the sum of the voltage drops. That means no voltage "leaks" out.

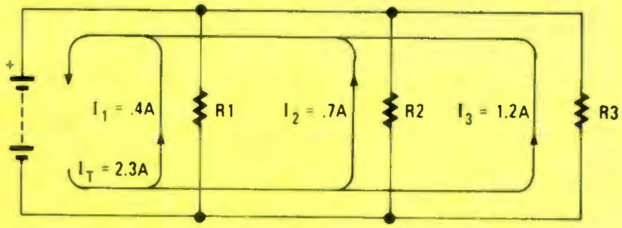

Fig. B—Kirchhoff's current law for parallel circuits simply states that the source current must equal the sum of the current through all branches. The electrons must remain in the circuit and return to the current source.

are the voltages across R1, R2, and R3. In Fig. A, the source voltage is:

$$8 + 6.5 + 2.3 = 8 \text{ volts}$$

You can also rearrange the formula to solve for one of the resistor voltages if the source voltage and the other component voltages are known. For example:

$$V_1 = V_S - V_2 - V_3$$

Kirchhoff's current law for parallel circuits states:

"The sum of the currents in the branches of a parallel circuit is equal to the total line current drawn from the source."

In Fig. B, that is expressed as:

$$I_T = I_1 + I_2 + I_3$$

where I_T is the total line current supplied by the battery and I_1, I_2 and I_3 are the individual branch currents in the resistors. In that case, the total line current is:

$$I_T = .4 + .7 + 1.2 = 2.3 \text{ A}$$

You can also rearrange the formula to solve for any branch current. To find I_2 for example:

$$I_2 = I_T - I_1 - I_3$$
$$U_2 = 2.3 - .4 - 1.2 = .7 \text{ A.}$$

The three laws are used in the solution to almost any circuit problem. Commit them to memory and apply them as needed. ■

referred to as the *output impedance* of the voltage source.

The whole idea of most circuit theorems is to attempt to simplify the larger, more-complex electronic circuits and convert them so they can be represented by a simple circuit like those given in Fig. 1. In most cases, all circuits can be reduced to the equivalent of a voltage source in series with its internal resistance and the load. That is, in fact, the whole purpose of Thevenin's theorem. By using Thevenin's theorem along with the usual Ohm's and Kirchhoff's laws, you can easily reduce most circuits into a Thevenin's equivalent which is essentially a voltage source in series with its internal resistance and a load. But before we talk in detail about the Thevenin's theorem process, let's get a little-more familiar with voltage sources.

In electronic-circuit design, engineers strive to achieve what is known as an ideal voltage source. An ideal voltage source is some component or circuit that supplies a fixed output voltage to a load. That voltage, of course, will cause current to flow through the load and in that way produce some useful end effect. The problem is most voltage sources are not ideal. They will not provide a constant output voltage for all load values. Changing the load resistance invariably will change the amount of voltage supplied by the voltage source. The reason for the change is that all practical voltage sources have a built-in internal resistance or output impedance. Take a battery for example. It is about as close to a perfect voltage source as you can find in practice. You can almost connect any value of load resistance to a typical D cell and its output voltage will remain at approximately 1.5 volts. The reason for that is that the battery has an extremely low internal resistance (output impedance). It is usually so low, compared to the load resistance, that it is negligible, and so it is considered to be zero in most cases. But as you make the load resistance smaller, the internal resistance becomes a larger percentage of the total circuit resistance. It is at that point that decreasing the load resistance causes the output voltage to drop considerably. Let's take a look at that phenomenon in a little greater detail.

Figure 2A shows a voltage source V_S with its internal resistance R_S. With no load connected to the battery, no current will flow. With no current flowing through the internal resistance, no voltage drop will be produced across it. For that reason, the voltage at terminals A and B will simply be the natural voltage produced by the battery. That is known as the open-circuit voltage, which is just V_S here.

Now, take a look at Fig. 2B. A load, R_L, is now connected to the voltage source. That, of course, causes current to flow through the load and the internal resistance. The current through the internal battery resistance will produce a voltage drop across it, denoted V_i. The voltage drop, when added to the load voltage, will equal V_S just as Kirchhoff's law says. The voltage across the internal resistance will, of course, take away from the voltage that appears across the load. The best way to see that is to take an example.

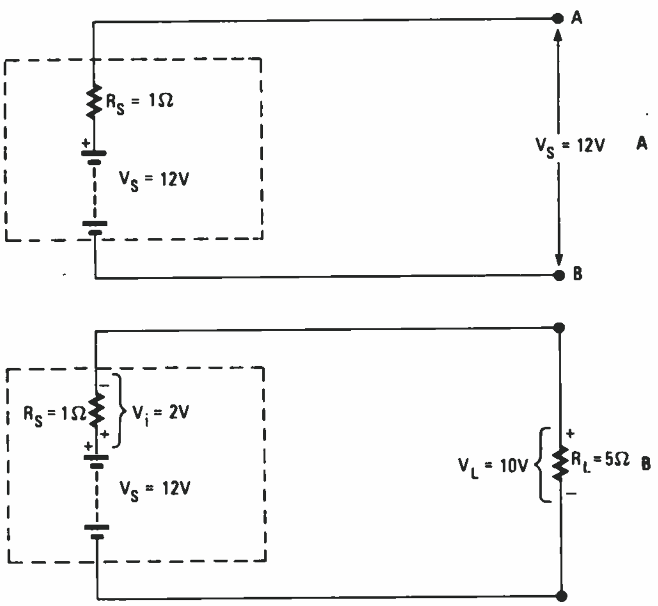

Fig. 2—In an unloaded state (A), the voltage source can be treated as though its internal resistance does not exist. When a load is applied (B), the resistance becomes noticeable.

Suppose that we have a 12-volt battery with an internal impedance of 1 ohm. Then suppose we connect a 5-ohm load to the circuit. We know that the total circuit resistance R_T is the sum of the individual resistors, in this case, the 5-ohm load resistance plus the 1-ohm internal resistance for a total of 6 ohms:

$$R_T = R_S + R_L = 1 + 5 = 6 \text{ ohms}$$

We can now use Ohm's law to find out what the current is:

$$I = V/R = 12/6 = 2 \text{ A}.$$

We know that the total circuit current now is 2 amperes. To find out what the voltage across the load is, we again use Ohm's law. Remember that the voltage is simply the current multiplied by the resistance. The load voltage is:

$$V_L = I \times R_L = 2(5) = 10 \text{ volts}$$

Well, you can see that the voltage across the load is not the entire 12 volts generated by the battery. All we are getting is 10 volts across the load. Where is the other 2 volts? The answer is, it is being dropped across the battery's internal resistance. Looking at Fig. 2B, you can see that as current flows through the internal resistance, it produces a voltage drop of:

$$1 \text{ volt} \times 2 \text{ amperes} = 2 \text{ volts}.$$

The polarity of that voltage drop is as shown in the figure. The polarity is opposite to that of the battery polarity and so the two voltages oppose one another. That's why we say that the 2 volts across the internal resistance takes away from the battery voltage leaving only 10 volts across terminals A and B, which are the actual battery terminals. That is why you see only 10 volts across the load. All that makes sense because Kirchhoff's law tells us that the sum of the voltage drops is:

$$V_S = V_i + V_L = 2 + 10 = 12 \text{ volts}$$

Now you can see why an ideal voltage source is so desirable. With zero internal resistance, all of the voltage generated is applied across the load. Since perfect voltage sources can't be obtained in practice, then the objective is simply to keep the internal resistance as low as possible. That is always the basic design objective of most engineers. The lower the internal resistance compared to the load resistance, the greater the amount of voltage that appears across the load.

One way to analyze a voltage source with its load is to look at the internal resistance R_i and the load resistance R_L as a voltage divider. It can be more-easily seen if we redraw the circuit as shown in Fig. 3. A voltage divider is simply two or more resistors that are used to develop an output voltage somewhat lower than the input voltage. Normally, we design voltage dividers by choosing two resistor values that will give a desired output voltage for a given input voltage. The basic formula used for figuring the output voltage of a voltage divider given the resistor values and the source voltage is:

$$V_O = V_S R_L/(R_S + R_L)$$

Here, V_O is the output voltage across the load, V_S is the open circuit source voltage, R_L is the load resistance, and R_S is the internal resistance of the voltage source. As an example of the use of the formula, let's use the 12-volt battery we assumed before, but this time assume it has an internal impedance of 0.1 ohm instead of 1 ohm. The load resistance is still 5 ohms. The voltage across the load in that case then would be:

$$V_O = 12(5/(5 + 0.1)) = 12(.98) = 11.76 \text{ volts}$$

Here you can see that when the internal resistance is only a tenth of an ohm instead of 1 ohm, the output voltage is 11.76 volts, or very much closer to the source voltage than the 10 volts produced with an internal impedance of 1 ohm. Now you can see why it is desirable to keep the internal impedance low.

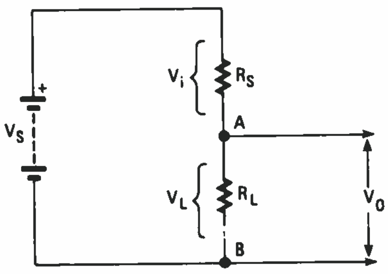

Fig. 3—The source's internal resistance and the external load form a voltage divider making the output voltage of the source easy to determine with voltage divider relationships.

One of the main problems in dealing with voltage sources is that it is often very difficult to determine the internal impedance of a source. For example, how do you know what the internal resistance of a battery is? We know that it is extremely low when the battery is good and, in most cases, it can be ignored or assumed to be zero. But as the battery discharges, the internal resistance increases. That's why when a battery goes bad, its output voltage drops to a very low level. When you connect a load to it, the current that flows produces a large internal voltage drop, leaving less voltage at the battery terminals for the external circuit.

Unfortunately, you just can't get an ohmmeter and connect it to a battery and expect to measure the internal resistance. Neither can you do that on any other kind of voltage source nor AC generator. Yet it is important to be able to determine what that internal impedance is so that you can perform the circuit analysis. Well, there is a way to determine the internal impedance, although not by direct measurement. Let's take an example, but this time let's use an AC-voltage source.

Refer to Fig. 4. There is an AC generator with an internal impedance of R_S. It might be a signal generator or function generator used for bench testing. Let's assume for a moment that we don't know what the internal impedance is. Through a series of measurements and calculations though, we can determine the output-impedance value. To do that, use the following procedure:

1. Measure the open-circuit generator voltage. With no load on the circuit, connect a multimeter or oscilloscope across the generator output terminals A and B. Measure the output voltage. That is the real generator source voltage V_S as no current is flowing through the internal resistance R_S. Assume you measure 6 volts.

2. Next, connect a load R_L to generator terminals A and B. In general, it is best to connect a load resistance that is close to, but somewhat higher than, the internal impedance. Since you don't know what the internal impedance is, it is difficult to estimate what kind of load resistance to connect. Although in some cases you may have a feel for the general range of the output impedance. If you do, then assume a load resistance that is somewhat higher. In that case, let's just say that you guessed the output impedance is about 50 ohms, but you don't really know for sure. Say you picked a 100 ohm load resistance.

3. With the load connected, measure the load voltage. Let's say you measured 4 volts.

4. You now have enough information to calculate the circuit current. You can determine the current with Ohm's law by simply dividing the load voltage by the load resistance:

$$I = V_L/R_L = 4/100 = .04A.$$

5. Now you are ready to actually calculate the internal generator impedance. You do that with a combination of both Kirchhoff's and Ohm's laws. You know that the generator produces 6 volts output under open-circuit condition. With the load connected, you get 4 volts at the output terminals. From Kirchhoff's law then, you know that the voltage dropped across the internal impedance V_i is the difference between those two voltages:

$$V_S + V_i + V_L$$
$$V_i = V_S - V_L$$
$$V_i = 6 - 4 = 2 \text{ volts}$$

Knowing that 2 volts is dropped across the internal resistance and knowing that the current flowing through it is .04

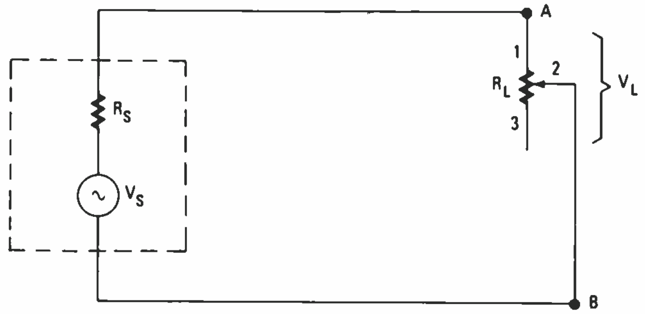

Fig. 4—The internal impedance of a source can be determined by making a few measurements and using Ohms law.

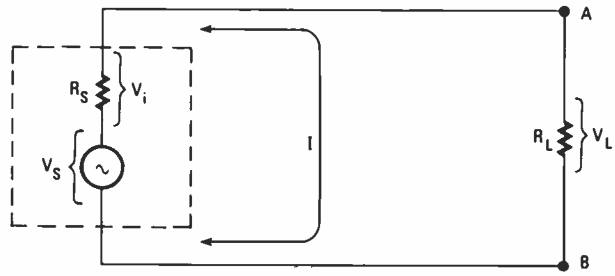

Fig. 5—A quick technique for measuring the output impedance of any device can be performed using a potentiometer, and by taking two measurements.

amperes, the source impedance can be calculated as follows:

$$R_S = 2/.04 = 50 \text{ ohms}$$

Another method of measuring the internal resistance or output impedance of a voltage source is to use the technique illustrated in Fig. 5. It is called the half-voltage method. Use the procedure described below:

1. Estimate the value of the output impedance, R_S. Let's assume that you guess that R_S is near 600 ohms.

2. Select a variable resistor whose value is larger than the estimated value. For example, you may choose a potentiometer with a value of 1000 ohms. In general, you may want to make the resistance larger just in case the actual output impedance is larger. You might choose a 2.5K- or 5,000-ohm potentiometer for the application. That is the load resistance, R_L.

3. Measure the open circuit or no-load output voltage of the generator at terminals A and B as you did before and record the value as V_S. Assume that it is 8 volts.

4. Connect the potentiometer across the voltage source as a variable load resistor R_L as Fig. 5 shows. Connect a voltmeter or oscilloscope across the load resistance. Vary the load resistance until the output voltage V_L drops to one-half of the open-circuit value. In that case, you would adjust the potentiometer until the load voltage was 4 volts.

5. Without disturbing the potentiometer, disconnect it from the voltage source. Then, using an ohmmeter, measure the resistance between the arm of the pot and the end you connected to the circuit, in this case, terminals 1 and 2. The resistance value will be equal to the output resistance of the circuit, or:

$$R_S = R_L.$$

Assume that you measured a value of 650 ohms across the potentiometer. You would know then that the output impedance was 650 ohms.

Why does the technique work? It works because you are adjusting the load resistance to half the output voltage, and so you are dividing the output voltage into two equal parts, one part of which is dropped across the load itself, which you are measuring, while the other half is dropped across the internal resistance. If the load voltage and internal-resistance voltages are equal, then their resistance values should also be equal because the current flowing in both is the same. That is why when the output voltage is dropped to one half, the load resistance equals the internal resistance.

Remember those important techniques when working with voltage sources. They will help you determine internal impedances and help you to calculate them.

Now we are ready to take a look at Thevenin's theorem itself.

Thevenin's Theorem

Thevenin's theorem describes a process of converting any complex electrical or electronic circuit into a simple equivalent circuit consisting of a single voltage source in series with an internal impedance. In other words, a complex electronic circuit represented by the box in Fig. 6A would be converted into a DC or AC voltage source as illustrated in Fig. 6B or 6C. The equivalent circuits consist of an equivalent or Thevenin's voltage source designated V_{th} in series with an equivalent internal resistance designated the Thevenin's resistance or R_{th}. The mathematical process of converting the larger, more complex circuit into its Thevenin's equivalent is called "Theveninizing" a circuit. The mathematical process of Thevenin's theorem takes the voltages and resistances of the complex electronic circuit and uses them to calculate V_{th} and R_{th}. That's what we want to show you how to do here.

Let's start with a relatively simple circuit, but one that is widely used and that Thevenin's theorem makes much easier to analyze. Figure 7 shows a DC-voltage source connected to a simple two-resistor voltage divider. A load resistance R_L is connected across the voltage-divider output. Typically the problem is to determine the output voltage across terminals A and B, or V_{AB}, for different values of load resistance. Of course, you can use standard circuit-analysis techniques for computing parallel- and series-resistance values, then use Ohm's and Kirchhoff's laws to compute the value for the output voltage for each load resistance. That is a rather time-consuming process. A much easier approach is to simply Theveninize the circuit, coming up with a simple voltage-source equivalent to which the load is connected. With such an arrangement, the output voltage for various values of load resistance can be more quickly and easily obtained.

To Theveninize a circuit, the first step is to remove the load resistance as shown in Fig. 8A. Then, use standard Ohm's- and Kirchhoff's-law procedures to compute the voltage that appears across A and B without the load. That is called the

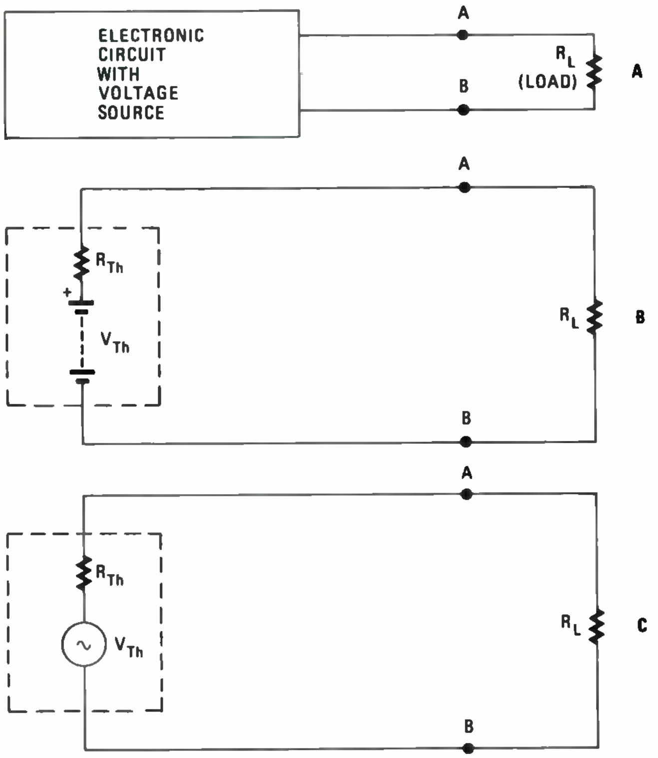

Fig. 6—Any complete circuit (A) can be replaced by its DC Thevenin (B) or AC Thevenin equivalent for analysis.

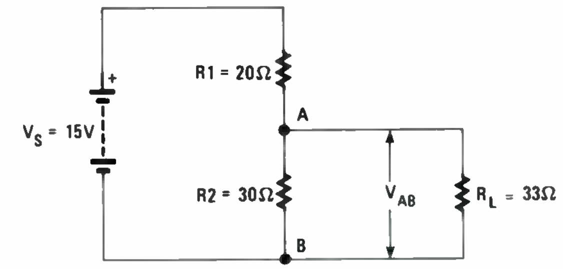

Fig. 7—Of course, you can use techniques for computing parallel and series resistance values, and use Ohm's law to compute the output voltage for each load resistance. A much easier approach is to simply Theveninize the circuit.

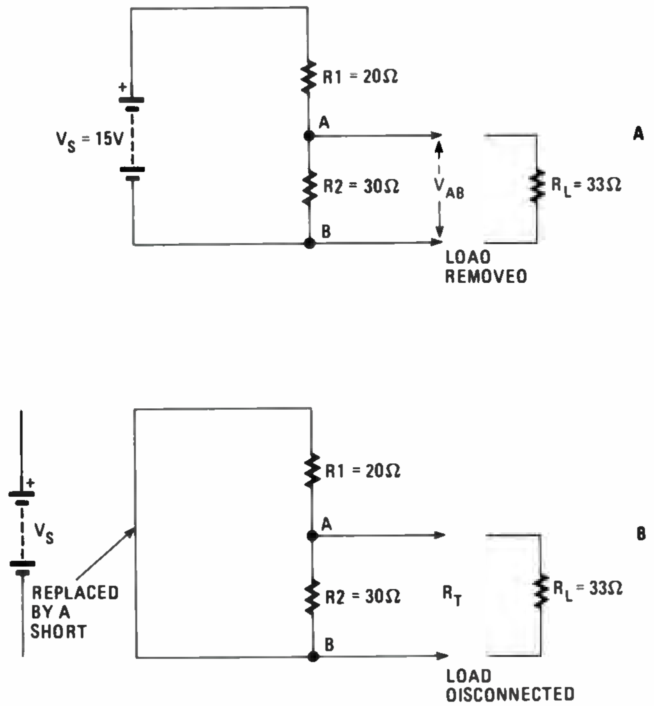

Fig. 8—The first step in Theveninizing a circuit is to analyze the circuit without the load (A) to find the voltage. Next you must short the voltage source(s) to determine the thevenin equivalent impedance. Finally you replace the load, but now connected to the equivalent.

Thevenin's-equivalent voltage, V_{th}:

$$V_{th} = V_{AB}$$

To compute the Thevenin's-equivalent voltage between points A and B, you can use the simple voltage-divider formula given earlier. We can express that as follows:

$$V_{AB} = V_S R1/(R1 + R2)$$

Using the values in Fig. 8, the Thevenin's-equivalent voltage then is:

$$V_{th} = 15(30)/(20 + 30) = 15(30/50) = 15(.6) = 9 \text{ volts}$$

The equivalent voltage source for the circuit then is a 9-volt battery.

Next, we need to compute the Thevenin's-equivalent resistance. Again, that is done by assuming that the load resistance is disconnected. It also assumes that the voltage source is replaced by a short circuit. That condition is illustrated in Fig. 8B. With the voltage source shorted, you simply calculate the total equivalent resistance between terminals A and B. In this case, the 20- and 30-ohm resistors are simply in parallel with one another, and so the equivalent resistance can be computed by using the standard parallel-resistance formula. The total resistance is:

$$R_T = R1R2/(R1 + R2) =$$
$$20(30)/(20 + 30) = 600/50 = 12 \text{ ohms}$$

The total resistance therefore, is the Thevenin's-equivalent resistance R_{th}:

$$R_{th} = R_T$$

We can now draw the complete Thevenin's equivalent, which is illustrated in Fig. 9. It is simply a 9-volt battery in series with a 12-ohm resistor. The load is reconnected to terminals A and B. Now using Ohm's and Kirchhoff's laws for additional calculations, the equivalent circuit, which is easier to work with, will produce exactly the same output voltage for varying load values as the original circuit. The load voltage, V_L, between A and B is found as follows:

First, calculate the total circuit resistance, R_T:

$$R_T = R_{th} + R_L$$
$$R_T = 12 + 33 = 45 \text{ ohms}$$

Next, find the current, I:

$$I = V_{th}/R_T = 9/45 = .2 \text{ A.}$$

The load voltage, V_L, then is:

$$V_L = I \times R_L = .2(33) = 6.6 \text{ V.}$$

Now, let's take a somewhat more complex circuit. Refer to Fig. 10A. Again we wish to compute the Thevenin's-equivalent voltage and resistance. As usual, we begin by removing the load resistor. See Fig. 10B. Then we calculate the output voltage between terminals A and B. Note that we are using an AC generator. That is just to remind you that Thevenin's theorem works for both AC and DC circuits. The generator in the circuit supplies 6 volts to the network made up of resistors R1, R2, and R3. Note that R1 and R2 form a voltage divider across the generator. Since the two values of resistance are equal, then the source voltage will simply divide equally across them. That means that there will be 3 volts dropped across R1 and 3 volts dropped across R2. The voltage across R2 is the output voltage applied to terminals A and B. The voltage is, of course, applied through resistor R3. Since there

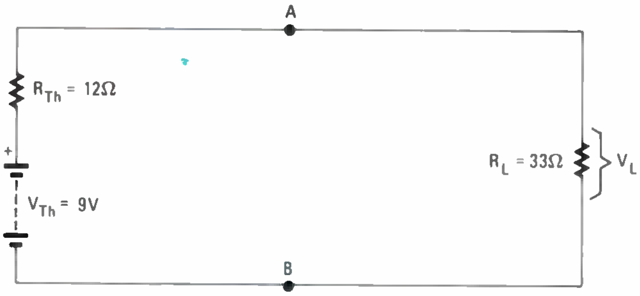

Fig. 9—This is the complete Thevenin's equivalent of Fig. 7. The load is reconnected to terminals A and B. Using Ohm's and Kirchhoff's laws the equivalent circuit will produce exactly the same output voltage values.

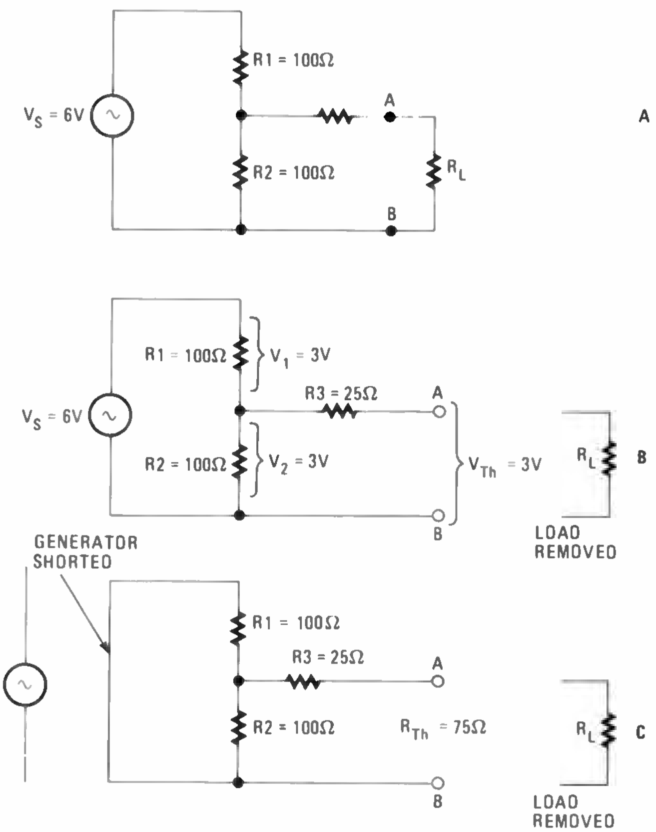

Fig. 10—We wish to compute the Thevenin's equivalent voltage and resistance of the circuit in A. Begin by removing the load (B) and calculating the output voltage. Note that we are using an AC generator.

is no load connected to terminals A and B, then no current flows through R3, and so there is no voltage across it. So the voltage across terminals A and B is simply the voltage across R2, or 3 volts. That is the Thevenin's-equivalent voltage, V_{th}.

Next, compute the Thevenin's-equivalent resistance. That is done by shorting the generator and computing the total value of resistance between terminals A and B. The equivalent circuit is shown in Fig. 10C. First note that R1 and R2 are simply put in parallel. Putting two equal values of resistance in parallel gives a total resistance value of one-half the value of one of the resistors. The total equivalent resistance of R1 and R2 in parallel then is 100/2 = 50 ohms. The total resistance between A and B then is simply 50 ohms added to the value of R3 of 25 ohms, for a total of 75 ohms. That is the Thevenin's-equivalent resistance, R_{th}. Now you can connect various values of load resistance and compute the output voltage for each using Ohm's and Kirchhoff's laws.

Practice Problem

1. Now it's time for you to try it yourself. Refer to Fig. 11. Calculate the Thevenin's equivalent voltage and resistance.

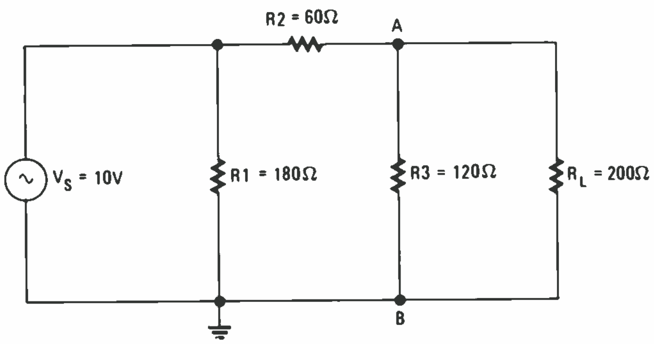

Fig. 11—When trying out this problem, remember these steps: remove the load; compute the Thevenin voltage; compute the Thevenin resistance; connect the load to the equivalent circuit to observe its behavior.

You really appreciate the power and convenience of a procedure such as Thevenin's theorem when you encounter a rather complex circuit to analyze. A good example is the bridge circuit shown in Fig. 12. All bridge circuits, whether they are made with resistors or a combination of resistors, capacitors, inductors, or transistors, have four arms. In general, most bridge circuits are balanced. In other words, the relationship between the resistances and/or impedances of the various arms is as follows:

$$R1/R2 = R3/R4$$

So, when the ratios of the resistances are equal, the bridge is said to be balanced. If that is true, the voltage at A is equal to the voltage at B with respect to ground and no current flows through the load resistance.

Changing one of the arm values will cause the bridge to become unbalanced. Depending upon the nature of the change, that will cause current to flow through the load resistance from A to B or from B to A, depending on the conditions. Our job is to calculate the current in R_L given the values in Fig. 12.

Bridge circuits in general are a pain in the neck to analyze. Particularly if you wish to determine the amount of current through the load for different values of load resistance. The computations are complex, messy, and time consuming. The chances for making an error are also high. But then, along comes Thevenin's theorem to the rescue. There is no reason why you can't convert the more complex circuit into a simple Thevenin's equivalent. Let's see how to do that.

Begin as before by removing the load resistance from the circuit. To find the Thevenin's-equivalent voltage, all we have to do is to compute the amount of voltage between terminals A and B. That shouldn't be too difficult.

In examining the bridge circuit without the load, you may recognize the fact that it actually consists of two voltage dividers connected across the source voltage. If we redraw the circuit as shown in Fig. 13, you will see that more clearly. One voltage divider is made up of R1 and R2, while the other is

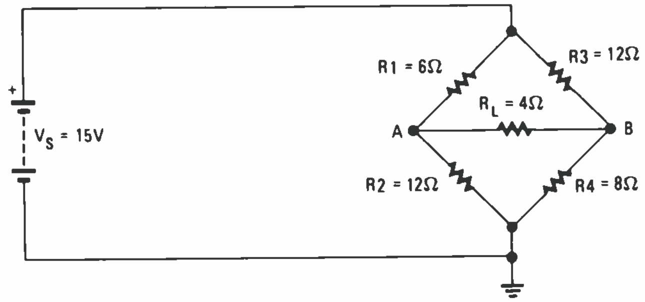

Fig. 12—Bridge circuits are difficult to solve with more basic methods. However, they can easily be analyzed using Thevenin's Theorem and a little math.

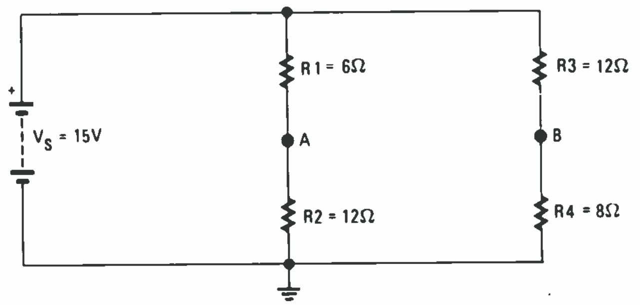

Fig. 13—Even with a bridge circuit, the first step is to remove the load from the circuit before analysis.

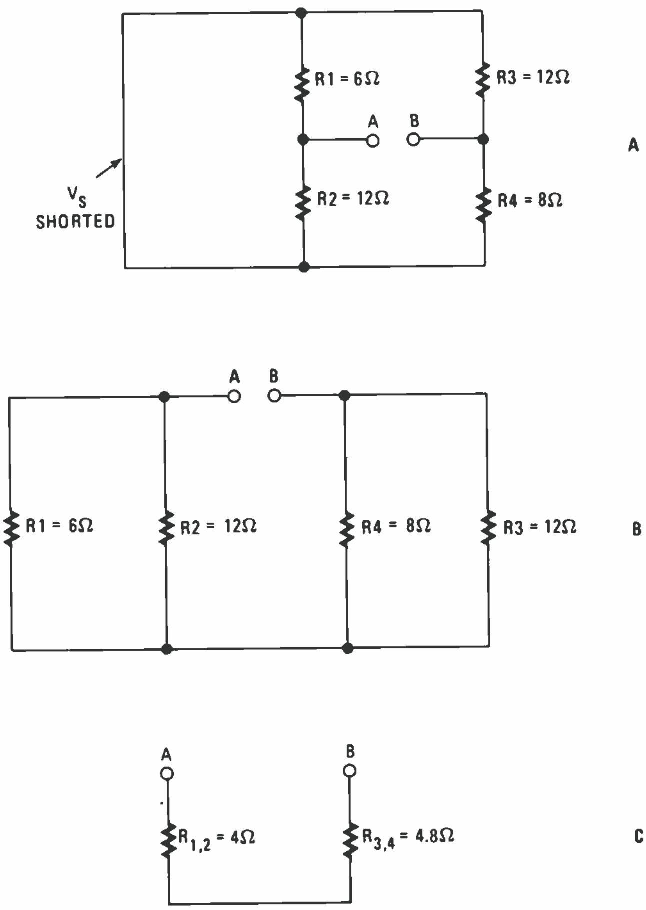

Fig. 14—After shorting the voltage source, the circuit looks a little strange (A), but that can be resolved by rearranging (B). Then we break the resistances down into simpler equivalent forms before final analysis.

made up of R3 and R4. What we wish to know is the voltage between terminals A and B. But to find it, we must first determine the voltage at A with respect to ground and the voltage at B with respect to ground. To do that we can simply apply the voltage divider equation given earlier.

The voltage at point A with respect to ground (V_A) is computed as follows:

$$V_A = V_S R2/(R1 + R2) = 15 \times 12/(6 + 12)$$
$$= 15(12/18) = 15(0.6666) = 10 \text{ volts}$$

The voltage at point B with respect to ground (V_B) is computed in exactly the same way:

$$V_B = V_S R4/(R3 + R4) = 15 \times 8/(8 + 12)$$
$$= 15(8/20) = 15(.4) = 6 \text{ volts}$$

We now know the voltages at A and B with respect to ground. The voltage directly between A and B is simply the difference between those two voltages. From that, V_{AB}, which is also the Thevenin's-equivalent voltage, V_{th}, is:

$$V_{AB} = V_{th} = V_A - V_B = 10 - 6 = 4 \text{ volts}$$

Since V_A is greater than V_B, A is positive with respect to B.

Now let's find the Thevenin's-equivalent resistance. As before, the load is assumed to be disconnected. Next, we short out the source voltage as we did before. The resulting circuit is shown in Fig. 14A. That is a good example of how not to draw a schematic diagram. To understand the circuit better, we can redraw it as shown in Fig. 14B. As you can see, we have a simple series-parallel combination circuit made up of R1 and R2 in parallel, and R3 and R4 in parallel, and the two combinations in series. We can apply the standard resistor formulas to the problem.

First, we find the parallel resistance of R1 and R2, which we call $R_{1,2}$. That is done as follows:

$$R_{1,2} = R1(R2)/(R1 + R2) = 6(12)/(6 + 12)$$
$$= 72/18 = 4 \text{ ohms}$$

The parallel combination of R3 and R4 we can designate as R_{34} and it is computed as follows:

$$R_{3,4} = R3(R4)/(R3 + R4) = 12(8)/(12 + 8)$$
$$= 96/20 = 4.8 \text{ ohms}$$

The total resistance then is simply the sum of the two equivalent resistances shown in Fig. 14C. To find the total all we do is add the two equivalent resistances:

$$R_T = 4 + 4.8 = 8.8 \text{ ohms}$$

Of course, the total resistance is the Thevenin's-equivalent resistance, R_{th}.

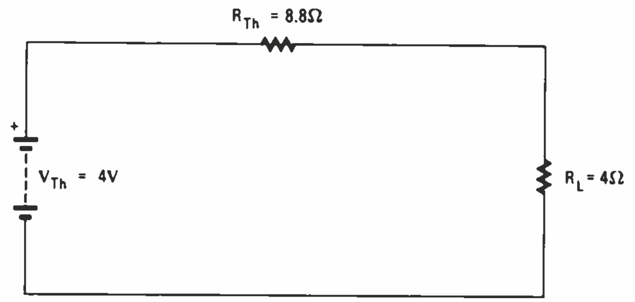

Fig. 15—The equivalent of the bridge circuit looks like all the other equivalent circuits, indicating the usefulness of Thevenin's Theorem in circuit analysis.

At last we can draw the full Thevenin's equivalent, which is illustrated in Fig. 15. A DC voltage of 4 volts is connected in series with its resistance of 8.8 ohms. Our load resistance is 4 ohms. Calculating the voltage across the load resistance is now easy.

To do that, we first find the total circuit resistance, which in this case is the Thevenin's-equivalent resistance plus the load resistance, or:

$$R_T = R_L + R_{th} = 4 + 8.8 = 12.8 \text{ ohms}$$

Next, we find the total circuit current with Ohm's law:

$$I = V_{th}/R_T = 4/12.8 = .3125 \text{ amperes}$$

Now the voltage across R_L is simply that current multiplied by the load resistance, or:

$$V_L = IR_L = .3125 \times 4 = 1.25 \text{ volts}$$

Exercise Problem

2. Now have one more go at using Thevenin's theorem yourself. Refer to Fig. 16. Calculate the Thevenin's equivalent voltage and resistance.

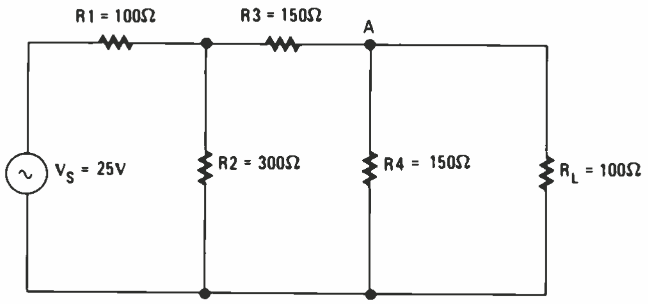

Fig. 16—Follow the basic steps for using Thevenin's Theorem when working the practice problem for this circuit.

Next month we'll cover some of the more interesting techniques for circuit analysis. Till then, practice what you've learned and Theveninize! ■

Answers

1. Refer to Fig. 17A.

a. Remove the load resistance R_L.

b. Calculate the Thevenin's equivalent voltage V_{th} across R3. $V_{th} = V_3$.

Calculate the total circuit resistance R $1_{2,3}$. R2 and R3 are in series to produce:

$R_{2,3} = R2 + R3$

$= 60 + 120 = 180$ ohms.

R_{23} is in parallel with R1. Since R1 and R_{23} are equal, the equivalent is:

$R_{1,2,3} = 180/2 = 90$ ohms.

Find the total current:

$I = V_S//R_{1,2,3} =$

$10/90 = .111A$.

Compute the current in R3(I_3):

$I_3 =$

$V_S/(R2 + R3) = 10/180 = .0555$ A

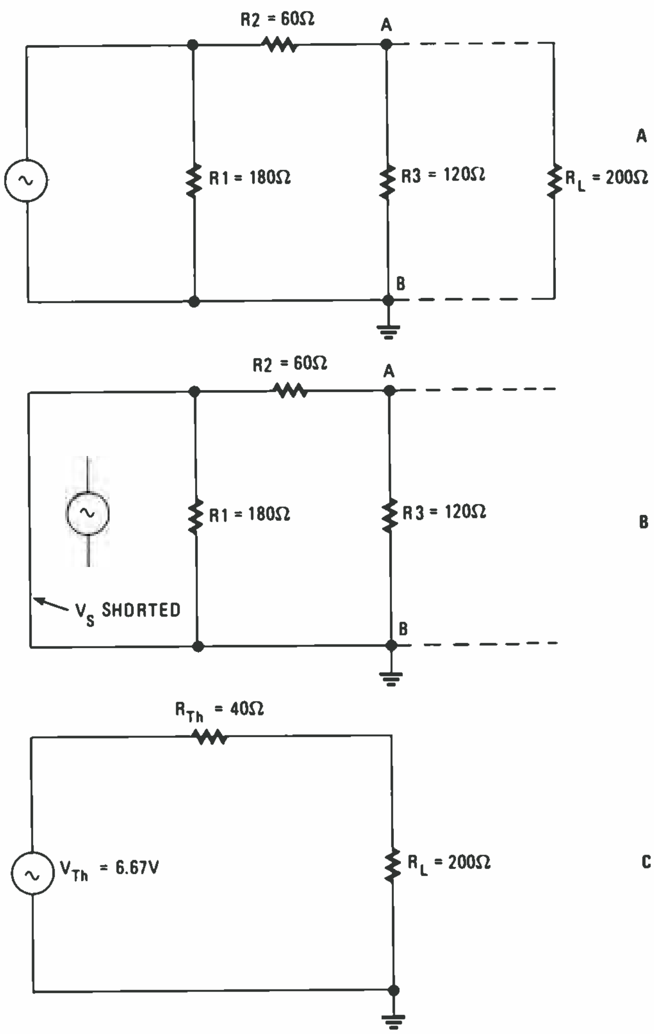

Fig. 17—When you redrew the circuit for various stages of the analysis they should have looked like these.

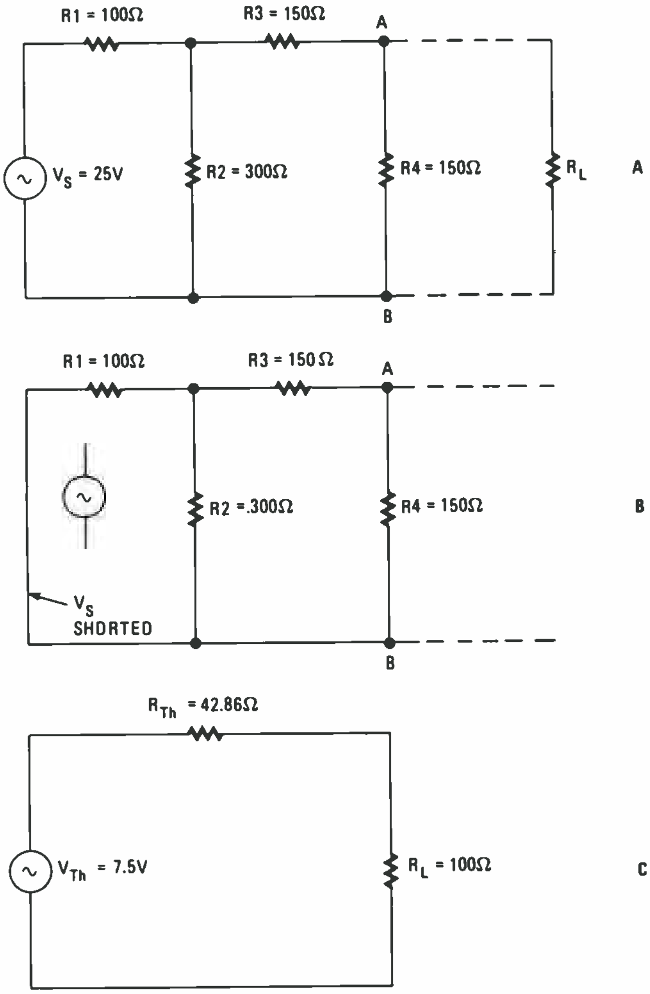

Fig. 18—If you didn't get this problem right, it is strongly recommended that you choose one of the other circuits in the article and try to solve it without referring to the text until you are finished.

Calculate the output voltage across R3:
$V_3 = I_3R3 = .0555(120) = 6.67$ volts.
$V_{th} = V_3 = 6.67$ volts.
c. Calculate the Thevenin's equivalent resistance.
Short V_S. That shorts out R1 and puts R2 and R3 in parallel. See Fig. 17B. $R_{2,3}$ the resistance of R2 and R3 in parallel is:
$R_{2,3} = R2(R3)/(R2 + R3)$
$R_{2,3} = 60(120)/(60 + 120)$
$R_{2,3} = 7200/180 = 40$ ohms
d. The equivalent circuit is shown in Fig. 17C.

2. Refer to Fig. 18.
a. Remove the load resistance R_L.
b. Calculate the Thevenin's equivalent voltage across R4 and terminals A – B. To do that first find the total circuit resistance. R3 and R4 are in series so their total resistance is:
$150 + 150 = 300$ ohms.
That appears in parallel with R2. Two 300-ohm resistances in parallel produce:
$300/2 = 150$ ohms.
That is in series with R1 to produce a total of:
$150 + 100 = 250$ ohms.
Calculate the circuit current with Ohm's law.
$I = V_S/R = 25/250 = .1A$.

Compute the voltage across $R1(V_1)$.
$V_1 = IR1 = .1(100) = 10$ V.
Find the voltage across $R_L(V_2)$. By Kirchhoff's law that is:
$V_2 = V_S - V_1 =$
$25 - 10 = 15$ volts.
Calculate the current in R3 and R4.
$(I_{3,4})$. $I_{3,4} = V_2/R3 + R4 = 15/300 = .05A$.
Calculate the output voltage across R4 and A-B.
(V_4). $V_{th} = V_4 = I_{3,4}(R4) =$
$.05(150) = 7.5$ volts.
c. Calculate the Thevenin's equivalent resistance: Short V_S.
R3 and R4 are in series
$(R_{3,4}) = R3 + R4 = 150 + 150 = 300$ ohms.
Also:
$R_{3,4} = 300$ ohms
in parallel with R2 is
$R_{2,3,4} = 300/2 = 150$ ohms.
With V_S shorted, R1 appears in parallel with R2 and R3/R4 in series. The total resistance is 150 in parallel with 100 or:
$R_{th} = R1(R_{2,3,4}/R1 + R_{234})$
$R_{th} = 100(150)/100 + 250 = 15000/350 =$
42.86 ohms
d. The equivalent circuit is shown in Fig. 18C. ■