Displacement current

A field theory approach

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A study of a capacitor as a transmission line by Catt, Davidson and Walton in the December 1978 issue contains, in the author's opinion, inaccuracies, mainly due to the subject being treated as a circuit theory. This article presents an analysis from a field theory viewpoint and shows the importance of the concept of displacement current.

Displacement current is perhaps one of the most difficult field theory concepts and it has been suggested[1] that Maxwell developed it by direct analogy with his equation

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

It must be borne in mind, however, that this analogy fails when the forces on moving charges are considered. Displacement current is a necessary consequence of Coulomb's law when charges change with time, and the electric field becomes non-conservative.

The fundamental point of Coulomb's law is that this force is transmitted through any medium, i.e., space is just as real a medium as a metal. Consider Coulomb's law:

$$\boldsymbol{F} = \frac{q_1 q_2}{4\pi\epsilon_o r^2}\boldsymbol{a}_r$$

In Fig. 1 we have two conducting spheres. Sphere A has a fixed charge while sphere B is connected to ground. As long as both spheres are stationary there will be a constant force exerted by A on B and vice-versa. Let us now start moving sphere A towards sphere B. For simplicity we will consider changes of force in the y-direction only, using the following formulae:

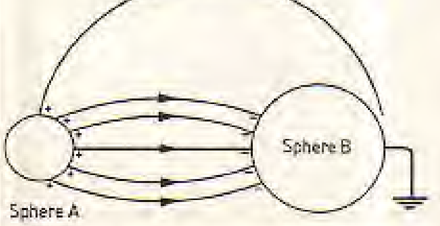


Fig. 1. Two conducting spheres. As long as both spheres are stationary there will be a constant force exerted by A on B and vice-versa.

$$\frac{\partial E_y}{\partial t} = \frac{1}{4\pi\epsilon_o}\frac{\partial}{\partial t}\left(\frac{q_2}{y^2}\right)$$

$$= \frac{1}{4\pi\epsilon_o}\left(\frac{\partial q_2}{\partial t}y^{-2} - q_2\,2y^{-3}\frac{\partial y}{\partial t}\right)$$

therefore:

$$\frac{\partial D_y}{\partial t} = \frac{1}{4\pi}\left(\frac{\partial q_2}{\partial t}y^{-2} - q_2\,2y^{-3}\frac{\partial y}{\partial t}\right)$$

Thus, if the electrostatic energy in the electric field changes, the energy change has to manifest itself in some way. It does so by producing an external flow of current in the conductor connected to sphere B.

It is important to realize that this displacement current does not have the significance of a current in the sense of being the motion of charges. After all, free charge cannot exist in free space, and hence, there cannot be a force proportional to

$$\epsilon_o\frac{\partial \boldsymbol{E}}{\partial t}\times \boldsymbol{B}$$

on the displacement current in empty space. In order to examine the effects of time-changing electric fields three examples will be considered.

For the first example it is required that the charge on a conducting sphere be measured by discharging it on to a large conducting plate connected to an oscilloscope. The resulting voltage pulse is measured and, since the input capacitance of the oscilloscope is known, the charge on the sphere can be calculated. When the resulting pulse is measured and the charge calculated, a serious discrepancy is found to exist between the actual charge on the sphere, which may be found by direct measurement in a Faraday cage, and the charge measured on the oscilloscope; the explanation is interesting.

The energy stored in the electric field is given by

$$W = \frac{1}{2}\iiint_{vol} \boldsymbol{D}.\boldsymbol{E}\,dv$$

As the sphere approaches the plate, the volume of the field is decreasing, so the energy stored in the field has been reduced; but where has the energy gone? As the sphere approaches the plate more negative charge is induced on to the plate and thus more positive charge will flow to ground. At the instant of discharge a pulse is registered on the oscilloscope. This pulse is simply the charge that has not been neutralized by the induced charge on the large conducting plate, i.e., if there was originally +10nC on the sphere and only −8nC induced on the plate then +2nC would flow into the oscilloscope, hence the discrepancy.

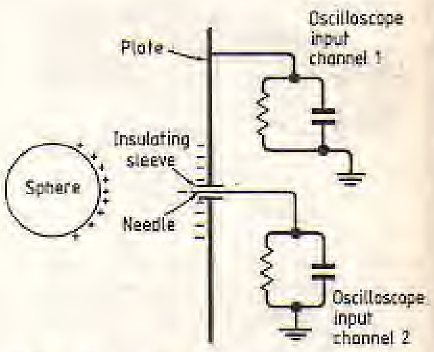


Fig. 2. The set-up used for explaining the discrepancy between calculated and measured electrostatic charges.

The method illustrated in Fig. 2 was used to confirm this theory. In this set-up an extra electrode connected to the oscilloscope's second channel is inserted through a hole in the conducting plate. A protective sleeve insulates this electrode from the plate. Once again the sphere is brought towards the plate but is now allowed to discharge onto the needle. In this case, only −1nC has been induced on the needle so consequently, +9nC will flow into the oscilloscope. The positive pulse measured on the oscilloscope will be almost equal to the charge on the sphere. Similarly, when the discharge occurs, the −8nC induced on the plate will be released since the electric field has collapsed. A pulse of −8nC will be measured on the second channel of the oscilloscope.

The consideration of a capacitor as a transmission line has been discussed[2] in the proposal that displacement current is erroneous. Consider the capacitor in Fig. 3(a): at time $t = 0$ the switch is closed and the capacitor starts to charge. A capacitor cannot charge up instantaneously: it will start to charge with the formation of field line ab, then cd, ef, etc. Hence, the initial current flow, i_1, will be

$$i_1 = \epsilon_o \iint \frac{\partial E_1}{\partial t} ds$$

This current flows until field line ab is formed. At a time t seconds later, a current i_2 will flow shown by

$$i_2 = \epsilon_o \iint \frac{\partial E_2}{\partial t} ds$$

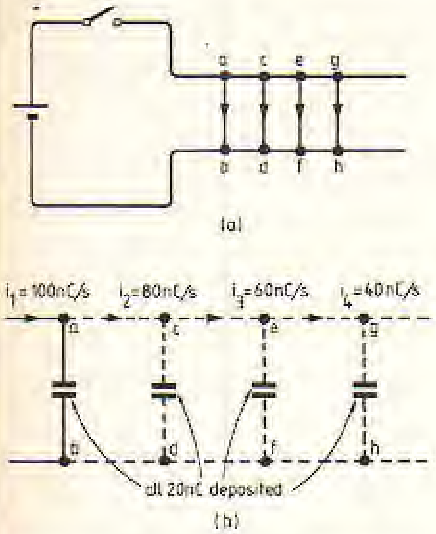


Fig. 3. As a capacitor does not charge up instantaneously, it can be considered to charge up beginning with the formation of field ab, then cd, etc.

Fig. 4. After switch S of 4(a) is closed, 4(b), 4(c) and 4(d) show the charge distribution for charged/uncharged capacitor pairs of various values. Simplified circuits for measuring capacitor discharge are shown in 4(e) and 4(f).

establishing field line cd and so on. Figure 3(b) shows this diagrammatically.

From the above explanation it may be deduced that the transmission line capacitor is in effect an infinite number of small capacitors. I would suggest that this is the reason why it has never been possible to measure inductance in a capacitor, because each capacitor will acquire an infinitely small charge. Obviously this very small amount of moving charge will have an associated magnetic field, but this field will be so weak that it will be undetectable, hence the absence of inductance in a capacitor. It is important to realize that this situation can only arise in a capacitor, because all the applied electrical energy is used in establishing an electric field.

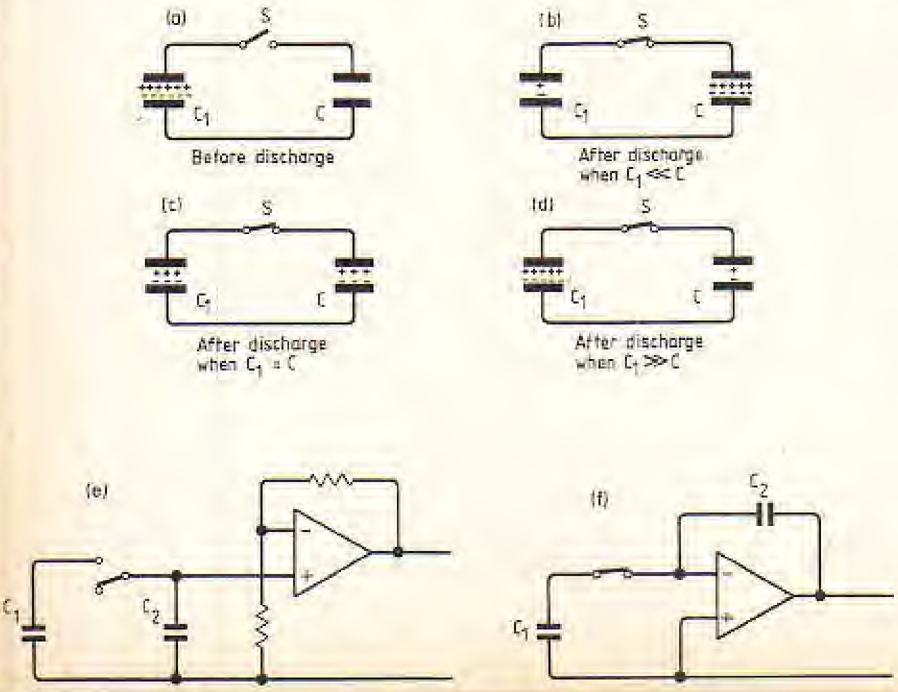
In a standard transmission line with a resistive load the situation is somewhat different. The conductors are spaced well apart from each other so the electric field will be negligible and all the electrical energy will be transferred into the load. In this case electrical energy is transported from one point to another, whereas in the case of the capacitor the energy is distributed over a large area. Inductance now becomes important as a constant time-changing current will produce a changing magnetic field, i.e.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

or in circuit terms,

$$v = \frac{L\mathrm{d}i}{\mathrm{d}t} + ir$$

Finally, in considering the effects of displacement current, it is worth discussing the problem of a charged capacitor being connected to an uncharged capacitor (see Fig. 4) and the mystery of where the 'missing' charge goes[3]. The usual explanation is that the closure of the switch initiates the transfer of energy, producing an



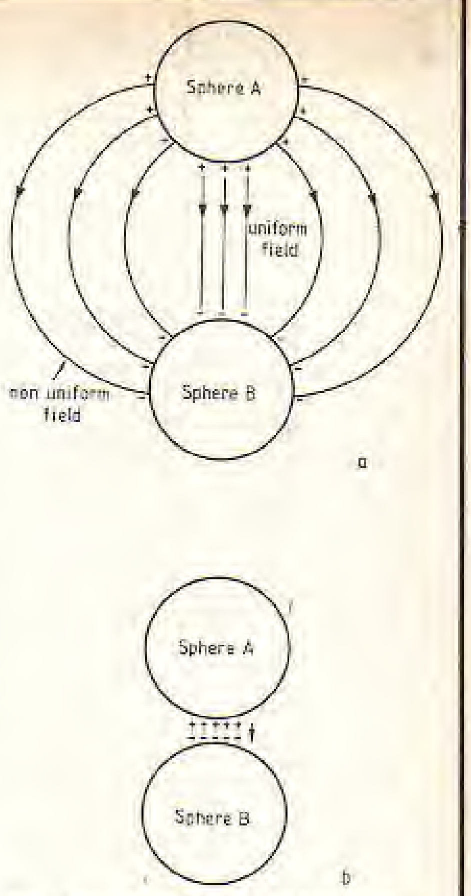


Fig. 5. As spheres A and B of 5(a) move together, ∂E/∂t will change with time on the outer fringes until the total field is uniform as shown in 5(b), resulting in an increase in capacitance between spheres A and B.

oscillation of charge between the two capacitors which finally decays to a steady state.

Consider these two equations for the charge and energy in a capacitor;

$$Q = CV \text{ and } E = \frac{1}{2}\frac{q^2}{C}$$

It is accepted that the charge remains the same before and after the discharge, as can be proved by experiment, but

$$E_1 = \frac{1}{2}\frac{q^2}{C}$$

and

$$E_2 = \frac{1}{2}.\frac{1}{2}\frac{q^2}{C}$$

which would imply an energy loss.

A more thorough study of the equation for the energy stored in a capacitor provides some interesting information. The total energy stored in an electric field is

$$\frac{1}{2}\iiint_{vol} \boldsymbol{D}.\boldsymbol{E}\, dv$$

A parallel plate capacitor is an approximation of a true field, which is represented by two infinite spheres. There are two ways of increasing the capacitance value. One is to move the two spheres closer

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together, causing the charge to move (via the displacement current) as shown in Fig. 5. This method uses much electrostatic energy as the masses of the electrodes are very large compared with the mass of the charge. The weight of 0.02 coulombs is 1.13×10^{-13}kg.

The second method for increasing capacitance is to transport the charges by a conduction current. This method is much more 'energy efficient' as the only losses are those associated with the collision of the charges with ions. Resulting ohmic losses are negligible in short capacitor leads.

The author disagrees with the previously mentioned oscillation explanation, despite the fact that the differential equation for a discharge can be very complex[4], and asks why the same charge is measured before and after the switch is closed? If the circuit did oscillate, the oscillation would obviously decay and the charge would be neutralized by recombination with an equal and opposite charge, with the liberation of heat. Secondly, since the capacitors are in parallel, the charge density will be the same. Consequently, once the charge has redistributed itself, the system will be static.

Finally, it is worth considering the magnitude of current that would have to be present if energy was to be temporarily stored in the inductor. For example, consider a capacitor of 5000μF connected to another of a similar value. Let the voltage be 10V. The energy stored in the capacitor, E, can be found by

$$E = \frac{1}{2}CV^2 = 0.25 \text{ joules}$$

If half this energy were to be stored in an inductor with very short leads of 1μH, then

$$0.125\text{J} = \frac{1}{2} \times 10^{-6} \times I^2$$

so I is 500A.

Conclusion

The energy equation for a capacitor assumes that any change is brought about by letting the field do the work. Charge cannot be created or destroyed, although equal amounts of positive and negative charge may be simultaneously created, obtained by separation and lost by recombination.

References

1. Engineering Electromagnetics, W. H. Hayt, McGraw-Hill 1974, page 340.
2. 'The history of displacement current', I. Catt, M. F. Davidson, D. S. Walton, *Wireless World*, March 1979.
3. 'Did you know?', Epsilon, *Wireless World*, December 1978.
4. High Voltage Engineering, E. Kuffel, M. Abdullah, Pergamon Press Ltd, 1st edition (1970), pages 109-148.