# INTRODUCTORY CIRCUIT ANALYSIS

## Twelfth Edition

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PEARSON

# CONTENTS

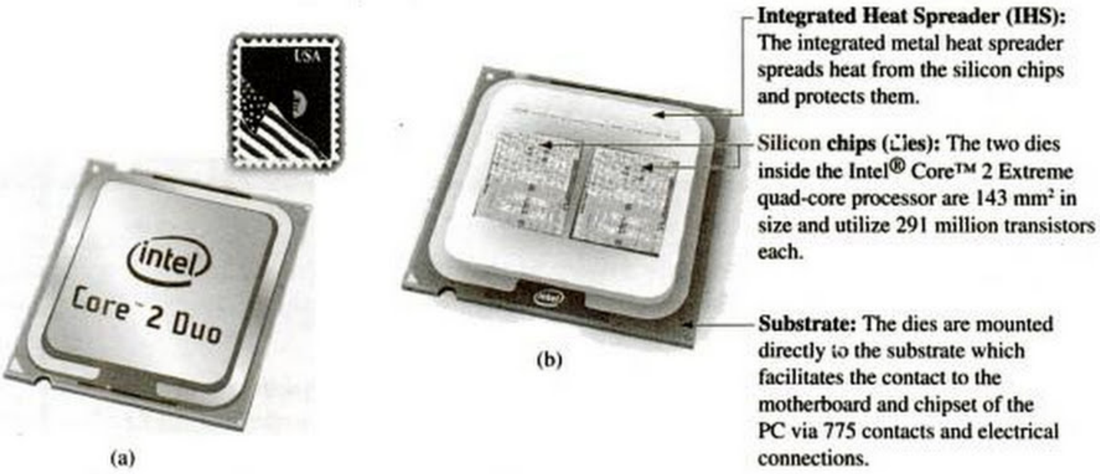



**FIG. 1.1**
*Intel® Core™ 2 Extreme quad-core processer: (a) surface appearance, (b) internal chips.*

transistors in each dual-core chip. The result is that the entire package, which is about the size of three postage stamps, has almost 600 million transistors—a number hard to comprehend.

However, before a decision is made on such dramatic reductions in size, the system must be designed and tested to determine if it is worth constructing as an integrated circuit. That design process requires engineers who know the characteristics of each device used in the system, including undesirable characteristics that are part of any electronic element. In other words, there are *no ideal (perfect) elements* in an electronic design. Considering the limitations of each component is necessary to ensure a reliable response under all conditions of temperature, vibration, and effects of the surrounding environment. To develop this awareness requires time and must begin with understanding the basic characteristics of the device, as covered in this text. One of the objectives of this text is to explain how ideal components work and their function in a network. Another is to explain conditions in which components may not be ideal.

One of the very positive aspects of the learning process associated with electric and electronic circuits is that once a concept or procedure is clearly and correctly understood, it will be useful throughout the career of the individual at any level in the industry. Once a law or equation is understood, it will not be replaced by another equation as the material becomes more advanced and complicated. For instance, one of the first laws to be introduced is Ohm's law. This law provides a relationship between forces and components that will always be true, no matter how complicated the system becomes. In fact, it is an equation that will be applied in various forms throughout the design of the entire system. The use of the basic laws may change, but the laws will not change and will always be applicable.

It is vitally important to understand that the learning process for circuit analysis is sequential. That is, the first few chapters establish the foundation for the remaining chapters. Failure to properly understand the opening chapters will only lead to difficulties understanding the material in the chapters to follow. This first chapter provides a brief history of the field followed by a review of mathematical concepts necessary to understand the rest of the material.

**EXAMPLE 1.11**

a. $(0.00003)^3 = (3 \times 10^{-5})^3 = (3)^3 \times (10^{-5})^3$
$= \mathbf{27 \times 10^{-15}}$

b. $(90{,}800{,}000)^2 = (9.08 \times 10^7)^2 = (9.08)^2 \times (10^7)^2$
$= \mathbf{82.45 \times 10^{14}}$

In particular, remember that the following operations are not the same. One is the product of two numbers in the power-of-ten format, while the other is a number in the power-of-ten format taken to a power. As noted below, the results of each are quite different:

$$(10^3)(10^3) \neq (10^3)^3$$
$$(10^3)(10^3) = 10^6 = 1{,}000{,}000$$
$$(10^3)^3 = (10^3)(10^3)(10^3) = 10^9 = 1{,}000{,}000{,}000$$

## 1.7 FIXED-POINT, FLOATING-POINT, SCIENTIFIC, AND ENGINEERING NOTATION

When you are using a computer or a calculator, numbers generally appear in one of four ways. If powers of ten are not employed, numbers are written in the **fixed-point** or **floating-point notation.**

***The fixed-point format requires that the decimal point appear in the same place each time. In the floating-point format, the decimal point appears in a location defined by the number to be displayed.***

Most computers and calculators permit a choice of fixed- or floating-point notation. In the fixed format, the user can choose the level of accuracy for the output as tenths place, hundredths place, thousandths place, and so on. Every output will then fix the decimal point to one location, such as the following examples using thousandths-place accuracy:

$$\frac{1}{3} = \mathbf{0.333} \qquad \frac{1}{16} = \mathbf{0.063} \qquad \frac{2300}{2} = \mathbf{1150.000}$$

If left in the floating-point format, the results will appear as follows for the above operations:

$$\frac{1}{3} = \mathbf{0.333333333333} \qquad \frac{1}{16} = \mathbf{0.0625} \qquad \frac{2300}{2} = \mathbf{1150}$$

Powers of ten will creep into the fixed- or floating-point notation if the number is too small or too large to be displayed properly.

**Scientific** (also called *standard*) **notation** and **engineering notation** make use of powers of ten, with restrictions on the mantissa (multiplier) or scale factor (power of ten).

***Scientific notation requires that the decimal point appear directly after the first digit greater than or equal to 1 but less than 10.***

A power of ten will then appear with the number (usually following the power notation E), even if it has to be to the zero power. A few examples:

$$\frac{1}{3} = \mathbf{3.33333333333E{-}1} \qquad \frac{1}{16} = \mathbf{6.25E{-}2} \qquad \frac{2300}{2} = \mathbf{1.15E3}$$

Within scientific notation, the fixed- or floating-point format can be chosen. In the above examples, floating was employed. If fixed is chosen

**FIG. 2.16**
*Dell laptop lithium-ion battery: 11.1 V, 4400 mAh.*

the laptops bursting into flames in 2006 and the need for Sony to recall some 6 million computers. The source of the problem was the Li-ion battery, which simply overheated; pressure built up, and an explosion occurred. This was due to impurities in the electrolyte that prevented the lithium ions moving from one side of the battery chamber to the other. Since then this problem has been corrected, and lithium-ion batteries as appearing in Fig. 2.16 are used almost exclusively in laptop computers.

Industry is aware of the numerous positive characteristics of this power source and is pouring research money in at a very high rate. Recent use of nanotechnology and microstructures has alleviated many of the concerns addressed here.

## Solar Cell

The use of solar cells as part of the effort to generate "clean" energy has grown exponentially in the last few years. At one time the cost and the low conversion efficiencies were the main stumbling blocks to widespread use of the solar cell. However, the company Nanosolar has significantly reduced the cost of solar panels by using a printing process that uses a great deal less of the expensive silicon material in the manufacturing process. Whereas the cost of generating solar electricity is about 20 to 30 ¢/kWh, compared to an average of 11 ¢/kWh using a local utililty, this new printing process will have a significant impact on reducing the cost level. Another factor that will reduce costs is the improving level of efficiency being obtained by manufacturers. At one time the accepted efficiency level of conversion was between 10% and 14%. Recently, however, almost 20% has been obtained in the laboratory, and some feel that 30% to 60% efficiency is a possibility in the future. Given that the maximum available wattage on an average bright, sunlit day is 100 $mW/cm^2$, the efficiency is an important element in any future plans for the expansion of solar power. At 10% to 14% efficiency the maximum available power per $cm^2$ would only be 10 to 14 mW. For 1 $m^2$ the return would be 100 to 140 W. However, if the efficiency could be raised to 20%, the output would be significantly higher at 200 W for the 1-$m^2$ panel.

The relatively small three-panel solar unit appearing on the roof of the garage of the home of Fig. 2.17(a) can provide an energy source of 550 watt-hours (the watt-hour unit of measurement for energy will be discussed in detail in Chapter 4). Such a unit can provide sufficient electrical energy to run an energy-efficient refrigerator for 24 hours per day while simultaneously running a color TV for 7 hours, a microwave for 15 minutes, a 60 W bulb for 10 hours, and an electric clock for 10 hours. The basic system operates as shown in Fig. 2.17(b). The solar panels (1) convert sunlight into dc electric power. An inverter (2) converts the dc power into the standard ac power for use in the home (6). The batteries (3) can store energy from the sun for use if there is insufficient sunlight or a power failure. At night or on dark days when the demand exceeds the solar panel and battery supply, the local utility company (4) can provide power to the appliances (6) through a special hookup in the electrical panel (5). Although there is an initial expense to setting up the system, it is vitally important to realize that the source of energy is free—no monthly bill for sunlight to contend with—and will provide a significant amount of energy for a very long period of time.

11. a. What is the resistance of a copper bus-bar for a high-rise building with the dimensions shown ($T = 20°C$) in Fig. 3.45?
    b. Repeat (a) for aluminum and compare the results.

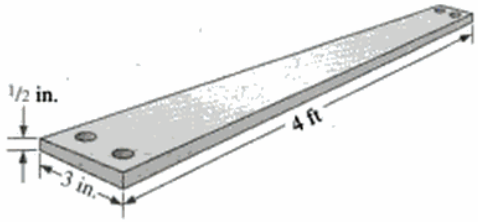



**FIG. 3.45**
*Problem 11.*

12. Determine the increase in resistance of a copper conductor if the area is reduced by a factor of 4 and the length is doubled. The original resistance was 0.2 Ω. The temperature remains fixed.

*13. What is the new resistance level of a copper wire if the length is changed from 200 ft to 100 yd, the area is changed from 40,000 CM to 0.04 in.$^2$, and the original resistance was 800 mΩ?

## SECTION 3.3 Wire Tables

14. a. In construction the two most common wires employed in general house wiring are #12 and #14, although #12 wire is the most common because it is rated at 20 A. How much larger in area (by percent) is the #12 wire compared to the #14 wire?
    b. The maximum rated current for #14 wire is 15 A. How does the ratio of maximum current levels compare to the ratio of the areas of the two wires?

15. a. Compare the area of a #12 wire with the area of a #9 wire. Did the change in area substantiate the general rule that a drop of three gage numbers results in a doubling of the area?
    b. Compare the area of a #12 wire with that of a #0 wire. How many times larger in area is the #0 wire compared to the #12 wire? Is the result significant? Compare it to the change in maximum current rating for each.

16. a. Compare the area of a #20 hookup wire to a #10 house romax wire. Did the change in area substantiate the general rule that a drop of 10 gage numbers results in a tenfold increase in the area of the wire?
    b. Compare the area of a #20 wire with that of a #40 wire. How many times larger in area is the #20 wire than the #40 wire? Did the result support the rule of part (a)?

17. a. For the system in Fig. 3.46, the resistance of each line cannot exceed 6 mΩ, and the maximum current drawn by the load is 110 A. What minimum size gage wire should be used?
    b. Repeat (a) for a maximum resistance of 3 mΩ, $d = 30$ ft, and a maximum current of 110 A.

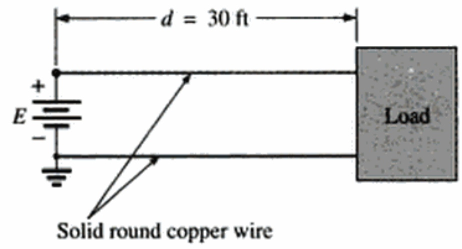



**FIG. 3.46**
*Problem 17.*

*18. a. From Table 3.2, determine the maximum permissible current density (A/CM) for an AWG #0000 wire.
    b. Convert the result of (a) to A/in.$^2$
    c. Using the result of (b), determine the cross-sectional area required to carry a current of 5000 A.

## SECTION 3.4 Temperature Effects

19. The resistance of a copper wire is 2 Ω at 10°C. What is its resistance at 80°C?

20. The resistance of an aluminum bus-bar is 0.02 Ω at 0°C. What is its resistance at 100°C?

21. The resistance of a copper wire is 4 Ω at room temperature (68°F). What is its resistance at a freezing temperature of 32°F?

22. The resistance of a copper wire is 0.025 Ω at a temperature of 70°F.
    a. What is the resistance if the temperature drops 10° to 60°F?
    b. What is the resistance if it drops an additional 10° to 50°F?
    c. Noting the results of parts (a) and (b), what is the drop for each part in milliohms? Is the drop in resistance linear or nonlinear? Can you forecast the new resistance if it drops to 40°F, without using the basic temperature equation?
    d. If the temperature drops to −30°F in northern Maine, find the change in resistance from the room temperature level of part (a). Is the change significant?
    e. If the temperature increases to 120°F in Cairns, Australia, find the change in resistance from the room temperature of part (a). Is the change significant?

23. a. The resistance of a copper wire is 1 Ω at 4°C. At what temperature (°C) will it be 1.1 Ω?
    b. At what temperature will it be 0.1 Ω?

24. a. If the resistance of 1000 ft of wire is about 1 Ω at room temperature (68°F), at what temperature will it double in value?
    b. What gage wire was used?
    c. What is the approximate diameter in inches, using the closest fractional form?

25. a. Verify the value of $\alpha_{20}$ for copper in Table 3.6 by substituting the inferred absolute temperature into Eq. (3.9).
    b. Using Eq. (3.10), find the temperature at which the resistance of a copper conductor will increase to 1 Ω from a level of 0.8 Ω at 20°C.

26. Using Eq. (3.10), find the resistance of a copper wire at 16°C if its resistance at 20°C is 0.4 Ω.

## SECTION 4.5 Energy

**37.** A 10 Ω resistor is connected across a 12 V battery.
- **a.** How many joules of energy will it dissipate in 1 min?
- **b.** If the resistor is left connected for 2 min instead of 1 min, will the energy used increase? Will the power dissipation level increase?

**38.** How much energy in kilowatthours is required to keep a 230 W oil-burner motor running 12 h a week for 5 months? (Use 4 weeks = 1 month.)

**39.** How long can a 1500 W heater be on before using more than 12 kWh of energy?

**40.** A 60 W bulb is on for 10 h.
- **a.** What is the energy used in wattseconds?
- **b.** What is the energy dissipated in joules?
- **c.** What is the energy transferred in watthours?
- **d.** How many kilowatthours of energy were dissipated?
- **e.** At 11¢/kWh, what was the total cost?

**41.** **a.** In 10 h an electrical system converts 1200 kWh of electrical energy into heat. What is the power level of the system?
- **b.** If the applied voltage is 208 V, what is the current drawn from the supply?
- **c.** If the efficiency of the system is 82%, how much energy is lost or stored in 10 h?

**42.** At 11¢/kWh, how long can you play a 250 W color television for $1?

**43.** The electric bill for a family for a month is $74.
- **a.** Assuming 31 days in the month, what is the cost per day?
- **b.** Based on 15-h days, what is the cost per hour?
- **c.** How many kilowatthours are used per hour if the cost is 11¢/kWh?
- **d.** How many 60 W lightbulbs (approximate number) could you have on to use up that much energy per hour?
- **e.** Do you believe the cost of electricity is excessive?

**44.** How long can you use an Xbox 360 for $1 if it uses 187 W and the cost is 11¢/kWh?

**45.** The average plasma screen TV draws 339 W of power, whereas the average LCD TV draws 213 W. If each set was used 5 h/day for 365 days, what would be the cost savings for the LCD unit over the year if the cost is 11¢/kWh?

**46.** The average PC draws 78 W. What is the cost of using the PC for 4 h/day for a month of 31 days if the cost is 11¢/kWh?

***47.** **a.** If a house is supplied with 120 V, 100 A service, find the maximum power capability.
- **b.** Can the homeowner safely operate the following loads at the same time?
  5 hp motor
  3000 W clothes dryer
  2400 W electric range
  1000 W steam iron
- **c.** If all the appliances are used for 2 hours, how much energy is converted in kWh?

***48.** What is the total cost of using the following at 11¢/kWh?
- **a.** 1600 W air conditioner for 6 h
- **b.** 1200 W hair dryer for 15 min
- **c.** 4800 W clothes dryer for 30 min
- **d.** 900 W coffee maker for 10 min
- **e.** 200 W Play Station 3 for 2 h
- **f.** 50 W stereo for 3.5 h

***49.** What is the total cost of using the following at 11¢/kWh?
- **a.** 200 W fan for 4 h
- **b.** Six 60 W bulbs for 6 h
- **c.** 1200 W dryer for 20 min
- **d.** 175 W desktop computer for 3.5 h
- **e.** 250 W color television set for 2 h 10 min
- **f.** 30 W satellite dish for 8 h

## SECTION 4.6 Efficiency

**50.** What is the efficiency of a motor that has an output of 0.5 hp with an input of 410 W?

**51.** The motor of a power saw is rated 68.5% efficient. If 1.8 hp are required to cut a particular piece of lumber, what is the current drawn from a 120 V supply?

**52.** What is the efficiency of a dryer motor that delivers 0.8 hp when the input current and voltage are 4 A and 220 V, respectively?

**53.** A stereo system draws 1.8 A at 120 V. The audio output power is 50 W.
- **a.** How much power is lost in the form of heat in the system?
- **b.** What is the efficiency of the system?

**54.** If an electric motor having an efficiency of 76% and operating off a 220 V line delivers 3.6 hp, what input current does the motor draw?

**55.** A motor is rated to deliver 2 hp.
- **a.** If it runs on 110 V and is 90% efficient, how many watts does it draw from the power line?
- **b.** What is the input current?
- **c.** What is the input current if the motor is only 70% efficient?

**56.** An electric motor used in an elevator system has an efficiency of 90%. If the input voltage is 220 V, what is the input current when the motor is delivering 15 hp?

**57.** The motor used on a conveyor belt is 85% efficient. If the overall efficiency is 75%, what is the efficiency of the conveyor belt assembly?

**58.** A 2 hp motor drives a sanding belt. If the efficiency of the motor is 87% and that of the sanding belt is 75% due to slippage, what is the overall efficiency of the system?

**59.** The overall efficiency of two systems in cascade is 78%. If the efficiency of one is 0.9, what is the efficiency, in percent, of the other?

**60.** **a.** What is the total efficiency of three systems in cascade with respective efficiencies of 93%, 87%, and 21%?
- **b.** If the system with the least efficiency (21%) were removed and replaced by one with an efficiency of 80%, what would be the percentage increase in total efficiency?

***61.** If the total input and output power of two systems in cascade are 400 W and 128 W, respectively, what is the efficiency of each system if one has twice the efficiency of the other?

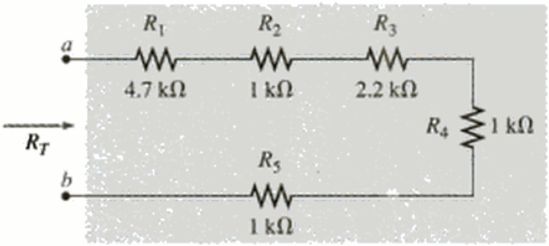



**FIG. 5.9**
*Series combination of resistors for Example 5.3.*

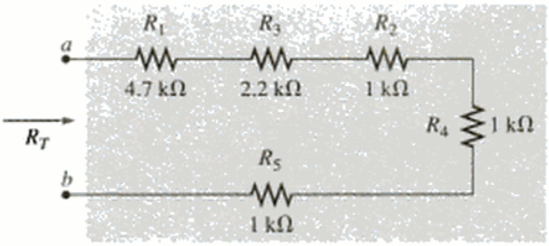



**FIG. 5.10**
*Series circuit of Fig. 5.9 redrawn to permit the use of Eq. (5.2): $R_T = NR$.*

**EXAMPLE 5.3** Determine the total resistance for the series resistors (standard values) in Fig. 5.9.

***Solution:*** First, the order of the resistors is changed as shown in Fig. 5.10 to permit the use of Eq. (5.2). The total resistance is then

$$R_T = R_1 + R_3 + NR_2$$
$$= 4.7\text{ k}\Omega + 2.2\text{ k}\Omega + (3)(1\text{ k}\Omega) = \mathbf{9.9\text{ k}\Omega}$$

## Analogies

Throughout the text, analogies are used to help explain some of the important fundamental relationships in electrical circuits. An analogy is simply a combination of elements of a different type that are helpful in explaining a particular concept, relationship, or equation.

One analogy that works well for the series combination of elements is connecting different lengths of rope together to make the rope longer. Adjoining pieces of rope are connected at only one point, satisfying the definition of series elements. Connecting a third rope to the common point would mean that the sections of rope are no longer in a series.

Another analogy is connecting hoses together to form a longer hose. Again, there is still only one connection point between adjoining sections, resulting in a series connection.

## Instrumentation

The total resistance of any configuration can be measured by simply connecting an ohmmeter across the access terminals as shown in Fig. 5.11 for the circuit in Fig. 5.4. *Since there is no polarity associated with resistance,* either lead can be connected to point *a,* with the other lead connected to point *b.* Choose a scale that will exceed the total resistance of the circuit, and remember when you read the response on the meter, if a kilohm scale was selected, the result will be in kilohms. For Fig. 5.11, the 200 Ω scale of our chosen multimeter was used because the total resistance is 140 Ω. If the 2 kΩ scale of our meter were selected, the digital display would read 0.140, and you must recognize that the result is in kilohms.

In the next section, another method for determining the total resistance of a circuit is introduced using Ohm's law.

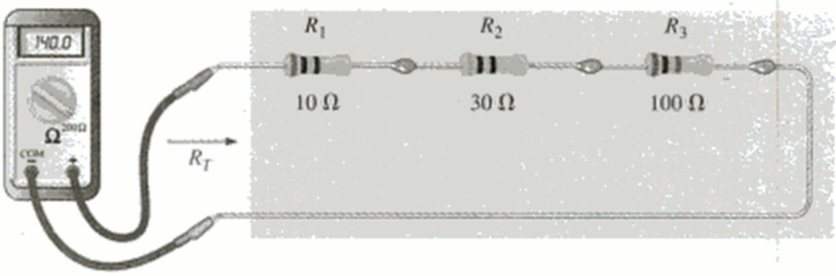



**FIG. 5.11**
*Using an ohmmeter to measure the total resistance of a series circuit.*

as shown in Fig. 5.65(b). The resistance level depends on the type of supply, but it is always present. Every year new supplies come out that are less sensitive to the load applied, but even so, some sensitivity still remains.

The supply in Fig. 5.66 helps explain the action that occurred above as we changed the load resistor. Due to the **internal resistance** of the supply, the ideal internal supply must be set to 20.1 V in Fig. 5.66(a) if 20 V are to appear across the 1 kΩ resistor. The internal resistance will capture 0.1 V of the applied voltage. The current in the circuit is determined by simply looking at the load and using Ohm's law; that is, $I_L = V_L/R_L = 20\text{ V}/1\text{ k}\Omega = 20\text{ mA}$, which is a relatively low current.

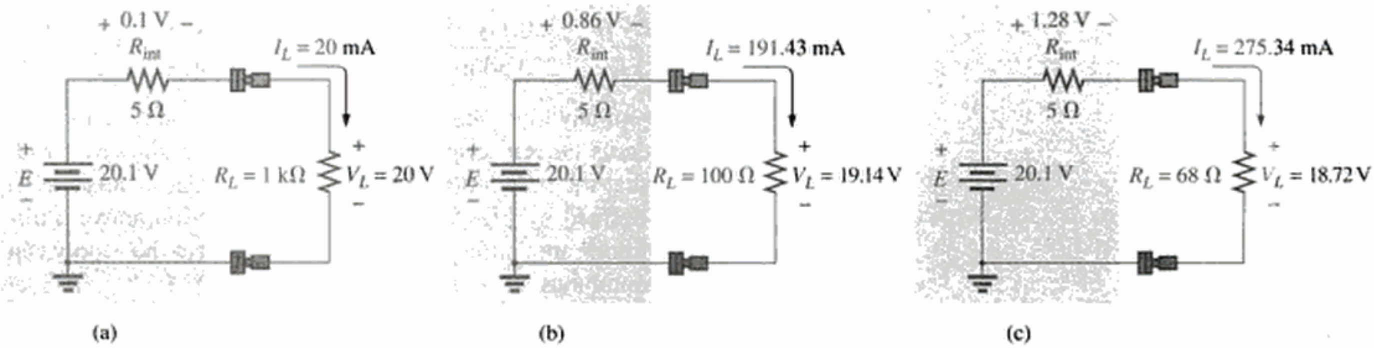



**FIG. 5.66**
*Demonstrating the effect of changing a load on the terminal voltage of a supply.*

In Fig. 5.66(b), all the settings of the supply are left untouched, but the 1 kΩ load is replaced by a 100 Ω resistor. The resulting current is now $I_L = E/R_T = 20.1\text{ V}/105\ \Omega = 191.43\text{ mA}$, and the output voltage is $V_L = I_LR = (191.43\text{ mA})(100\ \Omega) = 19.14\text{ V}$, a drop of 0.86 V. In Fig. 5.66(c), a 68 Ω load is applied, and the current increases substantially to 275.34 mA with a terminal voltage of only 18.72 V. This is a drop of 1.28 V from the expected level. Quite obviously, therefore, as the current drawn from the supply increases, the terminal voltage continues to drop.

If we plot the terminal voltage versus current demand from 0 A to 275.34 mA, we obtain the plot in Fig. 5.67. Interestingly enough, it turns out to be a straight line that continues to drop with an increase in current demand. Note, in particular, that the curve begins at a current level of 0 A.

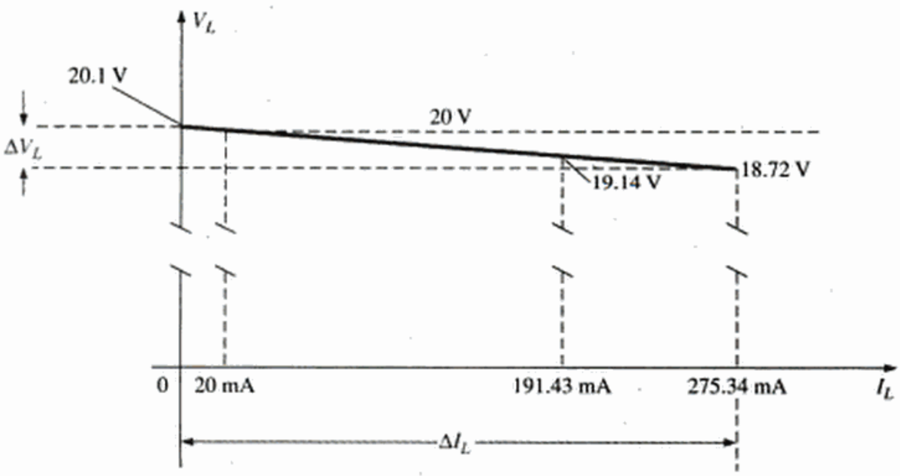



**FIG. 5.67**
*Plotting $V_L$ versus $I_L$ for the supply in Fig. 5.66.*

# Methods of Analysis and Selected Topics (dc)

8

## Objectives

- ***Become familiar with the terminal characteristics of a current source and how to solve for the voltages and currents of a network using current sources and/or current sources and voltage sources.***
- ***Be able to apply branch-current analysis and mesh analysis to find the currents of network with one or more independent paths.***
- ***Be able to apply nodal analysis to find all the terminal voltages of any series-parallel network with one or more independent sources.***
- ***Become familiar with bridge network configurations and how to perform Δ−Y or Y−Δ conversions.***

## 8.1 INTRODUCTION

The circuits described in previous chapters had only one source or two or more sources in series or parallel. The step-by-step procedures outlined in those chapters can be applied only if the sources are in series or parallel. There will be an interaction of sources that will not permit the reduction techniques used to find quantities such as the total resistance and the source current.

For such situations, methods of analysis have been developed that allow us to approach, in a systematic manner, networks with any number of sources in any arrangement. To our benefit, the methods to be introduced can also be applied to networks with only *one source* or to networks in which sources are in *series or parallel.*

The methods to be introduced in this chapter include branch-current analysis, mesh analysis, and nodal analysis. Each can be applied to the same network, although usually one is more appropriate than the other. The "best" method cannot be defined by a strict set of rules but can be determined only after developing an understanding of the relative advantages of each.

Before considering the first of the methods, we will examine current sources in detail because they appear throughout the analyses to follow. The chapter concludes with an investigation of a complex network called the *bridge configuration,* followed by the use of Δ-Y and Y-Δ conversions to analyze such configurations.

## 8.2 CURRENT SOURCES

In previous chapters, the voltage source was the only source appearing in the circuit analysis. This was primarily because voltage sources such as the battery and supply are the most common in our daily lives and in the laboratory environment.

We now turn our attention to a second type of source, called the **current source,** which appears throughout the analyses in this chapter. Although current sources are available as laboratory supplies (introduced in Chapter 2), they appear extensively in the modeling of electronic devices such as the transistor. Their characteristics and their impact on the currents

**EXAMPLE 8.11** Consider the same basic network as in Example 8.9, now appearing as Fig. 8.28.

***Solution:***

*Step 1:* Two loop currents ($I_1$ and $I_2$) are assigned in the clockwise direction in the windows of the network. A third loop ($I_3$) could have been included around the entire network, but the information carried by this loop is already included in the other two.

*Step 2:* Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4 Ω resistor are the opposite for each loop current.

*Step 3:* Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each resistor is determined by $V = IR$. For a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions. If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents are always subtracted from the loop current of the loop being analyzed.

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point $a$)

$$+2\text{ V} - (2\ \Omega)I_1 - \overbrace{(4\ \Omega)\underbrace{(I_1 - I_2)}_{\text{Total current through } 4\ \Omega \text{ resistor}}}^{\text{Voltage drop across } 4\ \Omega \text{ resistor}} = 0$$

Subtracted since $I_2$ is opposite in direction to $I_1$.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point $b$)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

*Step 4:* The equations are then rewritten as follows (without units for clarity):

$$\text{loop 1: } +2 - 2I_1 - 4I_1 + 4I_2 = 0$$
$$\text{loop 2: } -4I_2 + 4I_1 - 1I_2 - 6 = 0$$

and

$$\text{loop 1: } +2 - 6I_1 + 4I_2 = 0$$
$$\text{loop 2: } -5I_2 + 4I_1 - 6 = 0$$

or

$$\text{loop 1: } -6I_1 + 4I_2 = -2$$
$$\text{loop 2: } +4I_1 - 5I_2 = +6$$

Applying determinants results in

$$I_1 = \mathbf{-1\ A} \qquad \text{and} \qquad I_2 = \mathbf{-2\ A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

The actual current through the 2 V source and 2 Ω resistor is therefore 1 A in the other direction, and the current through the 6 V source and 1 Ω resistor is 2 A in the opposite direction indicated on the circuit. The current through the 4 Ω resistor is determined by the following equation from the original network:

$$\text{loop 1: } I_{4\Omega} = I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A}$$
$$= \mathbf{1\ A} \qquad (\text{in the direction of } I_1)$$

Applying Kirchhoff's current law gives

$$0 = I_3 + I_2 + 2\text{ A}$$

and $$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\text{ A} = 0 \longrightarrow \frac{V_2 - V_1}{12\,\Omega} + \frac{V_2}{2\,\Omega} + 2\text{ A} = 0$$

Expanding and rearranging gives

$$V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) = -2\text{ A}$$

resulting in the following two equations and two unknowns:

$$\left.\begin{aligned} V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) &= +4\text{ A} \\ V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) &= -2\text{ A} \end{aligned}\right\} \qquad \textbf{(8.1)}$$

producing

$$\left.\begin{aligned} \frac{7}{12}V_1 - \frac{1}{12}V_2 &= +4 \\ -\frac{1}{12}V_1 + \frac{3}{12}V_2 &= -2 \end{aligned}\right\} \quad \begin{aligned} 7V_1 - \phantom{3}V_2 &= 48 \\ -1V_1 + 3V_2 &= -24 \end{aligned}$$

and $$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = \mathbf{+6\text{ V}}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \mathbf{-6\text{ V}}$$

Since $V_1$ is greater than $V_2$, the current through $R_3$ passes from $V_1$ to $V_2$. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\,\Omega} = \frac{12\text{ V}}{12\,\Omega} = \mathbf{1\text{ A}}$$

The fact that $V_1$ is positive results in a current $I_{R_1}$ from $V_1$ to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\,\Omega} = \mathbf{3\text{ A}}$$

Finally, since $V_2$ is negative, the current $I_{R_2}$ flows from ground to $V_2$ and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\,\Omega} = \mathbf{1\text{ A}}$$

## Supernode

Occasionally, you may encounter voltage sources in a network that do not have a series internal resistance that would permit a conversion to a current source. In such cases, you have two options.

The simplest and most direct approach is to *place a resistor in series with the source of a very small value compared to the other resistive*

The first two areas of application are described in detail in this section. The last are covered in the discussion of the superposition theorem in the ac portion of the text.

The superposition theorem states the following:

***The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.***

In other words, this theorem allows us to find a solution for a current or voltage using *only one source at a time*. Once we have the solution for each source, we can combine the results to obtain the total solution. The term *algebraic* appears in the above theorem statement because the currents resulting from the sources of the network can have different directions, just as the resulting voltages can have opposite polarities.

If we are to consider the effects of each source, the other sources obviously must be removed. Setting a voltage source to zero volts is like placing a short circuit across its terminals. Therefore,

***when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.***

Setting a current source to zero amperes is like replacing it with an open circuit. Therefore,

***when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.***

The above statements are illustrated in Fig. 9.1.

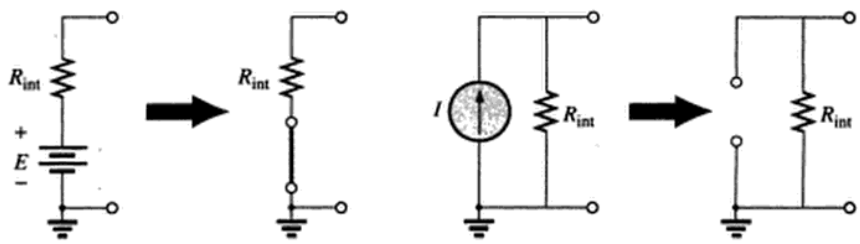



**FIG. 9.1**
*Removing a voltage source and a current source to permit the application of the superposition theorem.*

***Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.***

If a particular current of a network is to be determined, the contribution to that current must be determined for *each source*. When the effect of each source has been determined, those currents in the same direction are added, and those having the opposite direction are subtracted; the algebraic sum is being determined. The total result is the direction of the larger sum and the magnitude of the difference.

Similarly, if a particular voltage of a network is to be determined, the contribution to that voltage must be determined for each source. When the effect of each source has been determined, those voltages with the same polarity are added, and those with the opposite polarity are subtracted; the algebraic sum is being determined. The total result has the polarity of the larger sum and the magnitude of the difference.

To expand on the above conclusion and further demonstrate what is meant by a *nonlinear relationship,* the power to the 6 Ω resistor versus current through the 6 Ω resistor is plotted in Fig. 9.6. Note that the curve is not a straight line but one whose rise gets steeper with increase in current level.

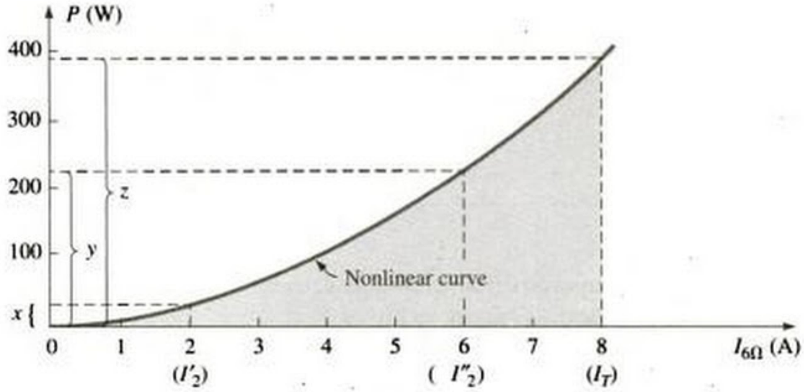



**FIG. 9.6**
*Plotting power delivered to the 6 Ω resistor versus current through the resistor.*

Recall from Fig. 9.4 that the power level was 24 W for a current of 2 A developed by the 36 V voltage source, shown in Fig. 9.6. From Fig. 9.5, we found that the current level was 6 A for a power level of 216 W, shown in Fig. 9.6. Using the total current of 8 A, we find that the power level in 384 W, shown in Fig. 9.6. Quite clearly, the sum of power levels due to the 2 A and 6 A current levels does not equal that due to the 8 A level. That is,

$$x + y \neq z$$

Now, the relationship between the voltage across a resistor and the current through a resistor is a linear (straight line) one as shown in Fig. 9.7, with

$$c = a + b$$

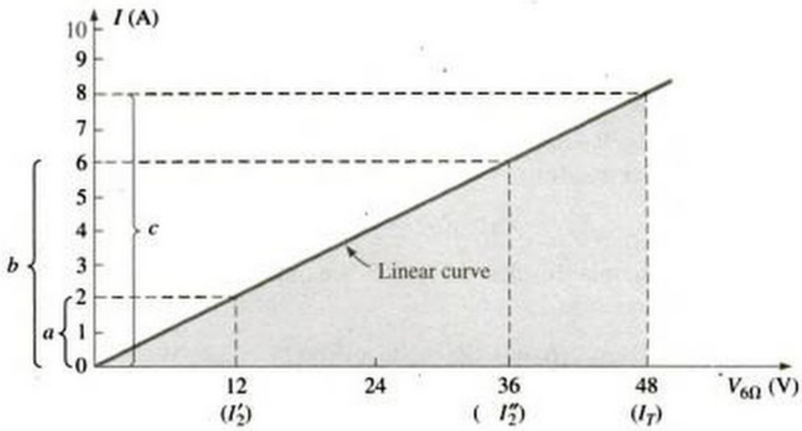



**FIG. 9.7**
*Plotting I versus V for the 6 Ω resistor.*

level of the resulting voltage to establish the measured resistance level. In Fig. 9.28(b), the trickle current of the ohmmeter approaches the network through terminal $a$, and when it reaches the junction of $R_1$ and $R_2$, it splits as shown. The fact that the trickle current splits and then recombines at the lower node reveals that the resistors are in parallel as far as the ohmmeter reading is concerned. In essence, the path of the sensing current of the ohmmeter has revealed how the resistors are connected to the two terminals of interest and how the Thévenin resistance should be determined. Remember this as you work through the various examples in this section.

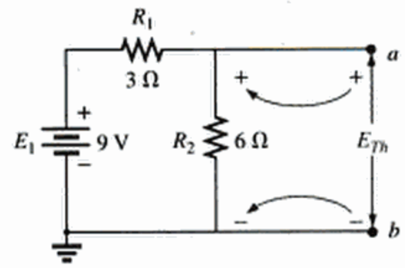



**FIG. 9.29**
*Determining $E_{Th}$ for the network in Fig. 9.27.*

*Step 4:* Replace the voltage source (Fig. 9.29). For this case, the open-circuit voltage $E_{Th}$ is the same as the voltage drop across the 6 Ω resistor. Applying the voltage divider rule gives

$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

It is particularly important to recognize that $E_{Th}$ is the open-circuit potential between points $a$ and $b$. Remember that an open circuit can have any voltage across it, but the current must be zero. In fact, the current through any element in series with the open circuit must be zero also. The use of a voltmeter to measure $E_{Th}$ appears in Fig. 9.30. Note that it is placed directly across the resistor $R_2$ since $E_{Th}$ and $V_{R_2}$ are in parallel.

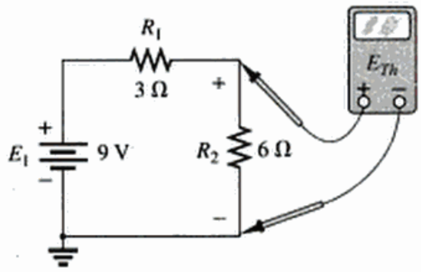



**FIG. 9.30**
*Measuring $E_{Th}$ for the network in Fig. 9.27.*

*Step 5:* (Fig. 9.31):

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$R_L = 2\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$$R_L = 10\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$R_L = 100\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$

If Thévenin's theorem were unavailable, each change in $R_L$ would require that the entire network in Fig. 9.26 be reexamined to find the new value of $R_L$.

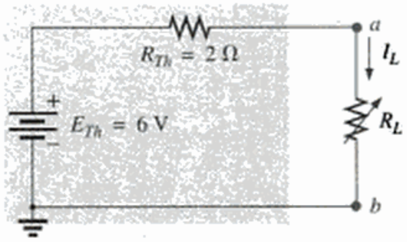



**FIG. 9.31**
*Substituting the Thévenin equivalent circuit for the network external to $R_L$ in Fig. 9.26.*

---

**EXAMPLE 9.7** Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.32.

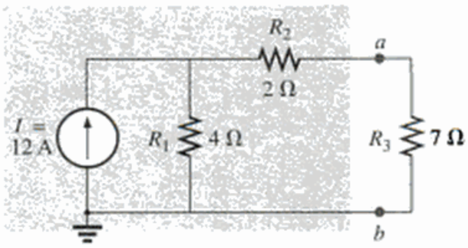



**FIG. 9.32**
*Example 9.7.*

***Solution:***

*Steps 1 and 2:* See Fig. 9.33.

*Step 3:* See Fig. 9.34. The current source has been replaced with an open-circuit equivalent and the resistance determined between terminals $a$ and $b$.

In this case, an ohmmeter connected between terminals $a$ and $b$ sends out a sensing current that flows directly through $R_1$ and $R_2$ (at the same level). The result is that $R_1$ and $R_2$ are in series and the Thévenin resistance is the sum of the two,

$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$

list, choose **Add Library** and add **SPECIAL** to the list. Select the **SPECIAL** library and scroll the **Part List** until **PARAM** appears. Select it; then click **OK** to obtain a rectangular box next to the cursor on the screen. Select a spot near **Rval,** and deposit the rectangle. The result is **PARAMETERS:** as shown in Fig. 9.113.

Next double-click on **PARAMETERS:** to obtain a **Property Editor** dialog box, which should have **SCHEMATIC1:PAGE1** in the second column from the left. Now select the **New Column** option from the top list of choices to obtain the **Add New Column** dialog box. Under **Name,** enter **Rval** and under **Value,** enter **1** followed by an **OK** to leave the dialog box. The result is a return to the **Property Editor** dialog box but with **Rval** and its value (below **Rval**) added to the horizontal list. Now select **Rval/1** by clicking on **Rval** to surround **Rval** by a dashed line and add a black background around the **1.** Choose **Display** to produce the **Display Properties** dialog box, and select **Name and Value** followed by **OK.** Then exit the **Property Editor** dialog box **(X)** to display the screen in Fig. 9.113. Note that now the first value (1 Ω) of **Rval** is displayed.

We are now ready to set up the simulation process. Under **PSpice,** select the **New Simulation Profile** key to open the **New Simulation** dialog box. Enter **DC Sweep** under **Name** followed by **Create.** The **Simulation Settings-DC Sweep** dialog box appears. After selecting **Analysis,** select **DC Sweep** under the **Analysis type** heading. Then leave the **Primary Sweep** under the **Options** heading, and select **Global parameter** under the **Sweep variable.** The **Parameter name** should then be entered as **Rval.** For the **Sweep type,** the **Start value** should be 1 Ω; but if we use ¹ Ω, the curve to be generated will start at 1 Ω, leaving a blank from 0 to 1 Ω. The curve will look incomplete. To solve this problem, select 0.001 Ω as the **Start value** (very close to 0 Ω) with an **Increment** of 1 Ω. Enter the **End value** as 30.001 Ω to ensure a calculation at $R_L$ = 30 Ω. If we used 30 Ω as the end value, the last calculation would be at 29.001 Ω since 29.001 Ω + 1 Ω = 30.001 Ω, which is beyond the range of 30 Ω. The values of **RL** will therefore be 0.001 Ω, 1.001 Ω, 2.001 Ω, . . . 29.001 Ω, 30.001 Ω, and so on, although the plot will look as if the values were 0 Ω, 1 Ω, 2 Ω, 29 Ω, 30 Ω, and so on. Click **OK,** and select **Run** under **PSpice** to obtain the display in Fig. 9.114.

Note that there are no plots on the graph, and that the graph extends to 32 Ω rather than 30 Ω as desired. It did not respond with a plot of power versus **RL** because we have not defined the plot of interest for the computer. To do this, select the **Add Trace** key (the key that has a red curve peaking in the middle of the plot) or **Trace-Add Trace** from the top menu bar. Either choice results in the **Add Traces** dialog box. The most important region of this dialog box is the **Trace Expression** listing at the bottom. The desired trace can be typed in directly, or the quantities of interest can be chosen from the list of **Simulation Output Variables** and deposited in the **Trace Expression** listing. To find the power to **RL** for the chosen range of values for **RL,** select **W(RL)** in the listing; it then appears as the **Trace Expression.** Click **OK,** and the plot in Fig. 9.115 appears. Originally, the plot extended from 0 Ω to 35 Ω. We reduced the range to 0 Ω to 30 Ω by selecting **Plot-Axis Settings-X Axis-User Defined 0 to 30-OK.**

Select the **Toggle cursor** key (which has an arrow set in a blue background), and seven options will open to the right of the key that include **Cursor Peak, Cursor Trough, Cursor Slope, Cursor Min, Cursor Max, Cursor Point,** and **Cursor Search.** Select **Cursor Max,** and the

***20.** For the network of Fig. 9.138, find the Thévenin equivalent circuit for the network external to the 300 Ω resistor.

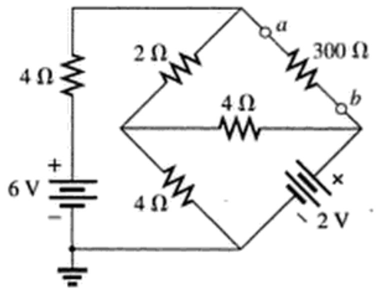



**FIG. 9.138**
*Problem 20.*

## SECTION 9.4 Norton's Theorem

**21. a.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.126.
**b.** Convert the Norton equivalent circuit to the Thévenin form.
**c.** Find the Thévenin equivalent circuit using the Thévenin approach and compare results with part (b).

**22. a.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.127.
**b.** Convert the Norton equivalent circuit to the Thévenin form.
**c.** Find the Thévenin equivalent circuit using the Thévenin approach and compare results with part (b).

**23.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.129.

**24.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.130.

***25.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.131.

***26.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.133.

***27.** Find the Norton equivalent circuit for the network external to the resistor $R$ in Fig. 9.135.

***28.** Find the Norton equivalent circuit for the network external to the 300 Ω resistor in Fig. 9.138.

***29. a.** Find the Norton equivalent circuit external to points $a$ and $b$ in Fig. 9.139.
**b.** Find the magnitude and polarity of the voltage across the 100 Ω resistor using the results of part (a).

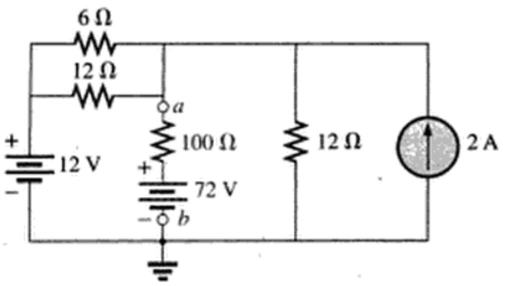



**FIG. 9.139**
*Problem 29.*

## SECTION 9.5 Maximum Power Transfer Theorem

**30. a.** Find the value of $R$ for maximum power transfer to $R$ for the network of Fig. 9.126.
**b.** Determine the maximum power of $R$.

**31. a.** Find the value of $R$ for maximum power transfer to $R$ for the network of Fig. 9.129.
**b.** Determine the maximum power of $R$.

**32. a.** Find the value of $R$ for maximum power transfer to $R$ for the network of Fig. 9.131.
**b.** Determine the maximum power to $R$.

***33. a.** Find the value of $R_L$ in Fig. 9.135 for maximum power transfer to $R_L$.
**b.** Find the maximum power to $R_L$.

**34. a.** For the network of Fig. 9.140, determine the value of $R$ for maximum power to $R$.
**b.** Determine the maximum power to $R$.
**c.** Plot a curve of power to $R$ versus $R$ for $R$ ranging from 1/4 to 2 times the value determined in part (a) using an increment of 1/4 the value of $R$. Does the curve verify the fact that the chosen value of $R$ in part (a) will ensure maximum power transfer?

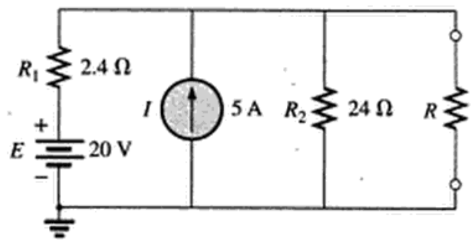



**FIG. 9.140**
*Problem 34.*

***35.** Find the resistance $R_1$ in Fig. 9.141 such that the resistor $R_4$ will receive maximum power. Think!

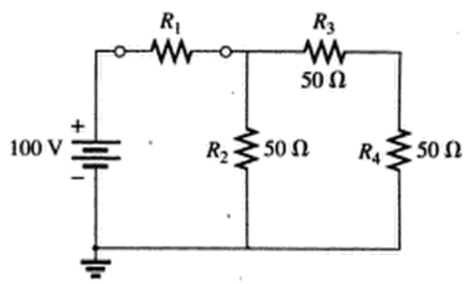



**FIG. 9.141**
*Problem 35.*

***36. a.** For the network in Fig. 9.142, determine the value of $R_2$ for maximum power to $R_4$.
**b.** Is there a general statement that can be made about situations such as those presented here and in Problem 35?

**48.** Find the total capacitance $C_T$ for the circuit in Fig. 10.119.

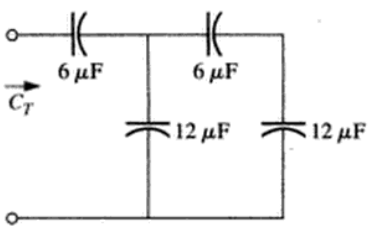



**FIG. 10.119**
*Problem 48.*

**49.** Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.120.

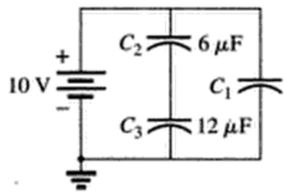



**FIG. 10.120**
*Problem 49.*

**50.** Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.121.

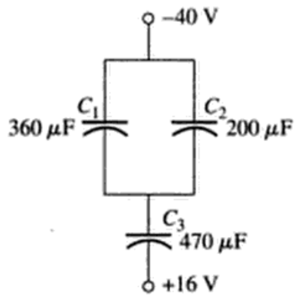



**FIG. 10.121**
*Problem 50.*

**51.** For the configuration in Fig. 10.122, determine the voltage across each capacitor and the charge on each capacitor under steady-state conditions.

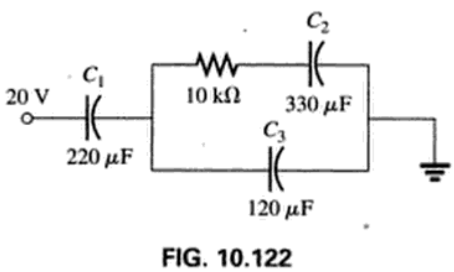



**FIG. 10.122**
*Problem 51.*

**52.** For the configuration in Fig. 10.123, determine the voltage across each capacitor and the charge on each capacitor.

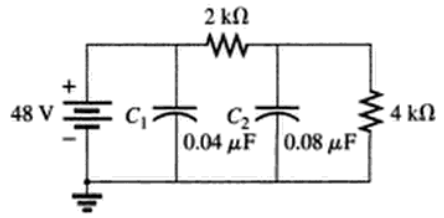



**FIG. 10.123**
*Problem 52.*

## SECTION 10.12 Energy Stored by a Capacitor

**53.** Find the energy stored by a 120 pF capacitor with 12 V across its plates.

**54.** If the energy stored by a 6 μF capacitor is 1200 J, find the charge $Q$ on each plate of the capacitor.

***55.** For the network in Fig. 10.124, determine the energy stored by each capacitor under steady-state conditions.

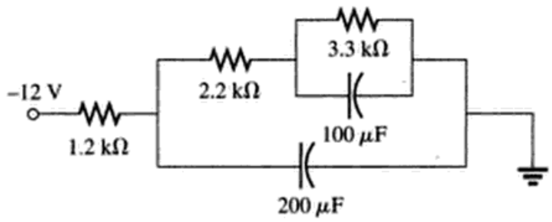



**FIG. 10.124**
*Problem 55.*

***56.** An electronic flashgun has a 1000 μF capacitor that is charged to 100 V.

**a.** How much energy is stored by the capacitor?
**b.** What is the charge on the capacitor?
**c.** When the photographer takes a picture, the flash fires for 1/2000 s. What is the average current through the flashtube?
**d.** Find the power delivered to the flashtube.
**e.** After a picture is taken, the capacitor has to be recharged by a power supply that delivers a maximum current of 10 mA. How long will it take to charge the capacitor?

## SECTION 10.15 Computer Analysis

**57.** Using PSpice or Multisim, verify the results in Example 10.6.

**58.** Using the initial condition operator, verify the results in Example 10.8 for the charging phase using PSpice or Multisim.

**59.** Using PSpice or Multisim, verify the results for $v_C$ during the charging phase in Example 10.11.

**60.** Using PSpice or Multisim, verify the results in Problem 42.


**FIG. 11.1**
*Flux distribution for a permanent magnet.*

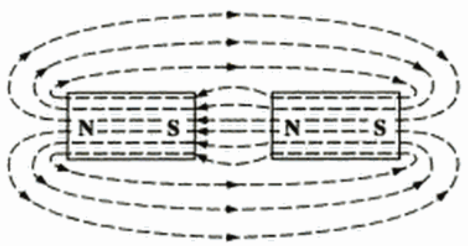



**FIG. 11.2**
*Flux distribution for two adjacent, opposite poles.*

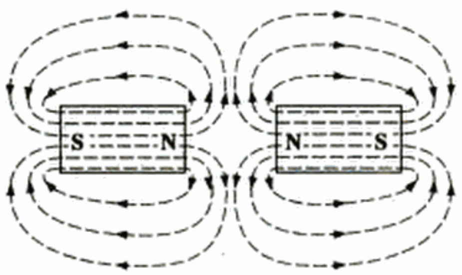



**FIG. 11.3**
*Flux distribution for two adjacent, like poles.*

of the basic concepts of **electromagnetism**—magnetic effects induced by the flow of charge, or current.

A magnetic field exists in the region surrounding a permanent magnet, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in *continuous loops,* as shown in Fig. 11.1.

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar. Note the equal spacing between the flux lines within the core and the symmetric distribution outside the magnetic material. These are additional properties of magnetic flux lines in homogeneous materials (that is, materials having uniform structure or composition throughout). It is also important to realize that the continuous magnetic flux line will strive to occupy as small an area as possible. This results in magnetic flux lines of minimum length between the unlike poles, as shown in Fig. 11.2. The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region. In Fig. 11.1, for example, the magnetic field strength at point *a* is twice that at point *b* since twice as many magnetic flux lines are associated with the perpendicular plane at point *a* than at point *b.* Recall from childhood experiments that the strength of permanent magnets is always stronger near the poles.

If unlike poles of two permanent magnets are brought together, the magnets attract, and the flux distribution is as shown in Fig. 11.2. If like poles are brought together, the magnets repel, and the flux distribution is as shown in Fig. 11.3.

If a nonmagnetic material, such as glass or copper, is placed in the flux paths surrounding a permanent magnet, an almost unnoticeable change occurs in the flux distribution (Fig. 11.4). However, if a magnetic material, such as soft iron, is placed in the flux path, the flux lines pass through the soft iron rather than the surrounding air because flux lines pass with greater ease through magnetic materials than through air. This principle is used in shielding sensitive electrical elements and instruments that can be affected by stray magnetic fields (Fig. 11.5).

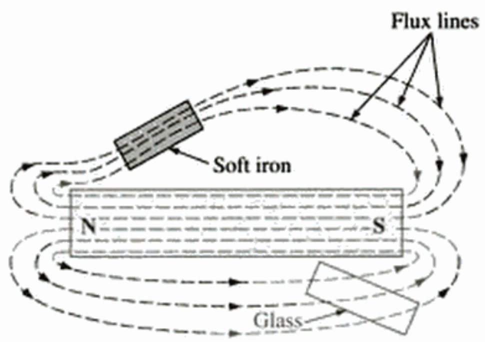



**FIG. 11.4**
*Effect of a ferromagnetic sample on the flux distribution of a permanent magnet.*

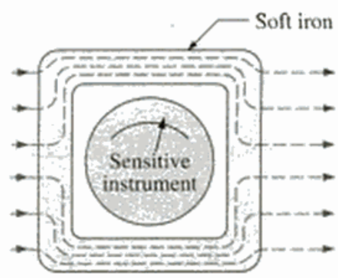
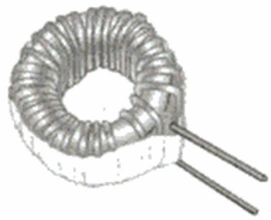



**FIG. 11.5**
*Effect of a magnetic shield on the flux distribution.*

A magnetic field (represented by concentric magnetic flux lines, as in Fig. 11.6) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of *conventional* current flow and noting the direction of the fingers. (This method is commonly called the *right-hand rule.*) If the conductor is wound in a

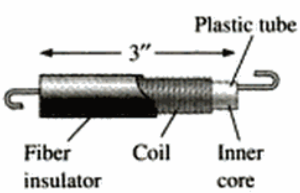
***Type:*** Air-core inductors (1–32 turns)
***Typical values:*** 2.5 nH–1 μH
***Applications:*** High-frequency applications

***Type:*** Toroid coil
***Typical values:*** 10μH–30 mH
***Applications:*** Used as a choke in ac power line circuits to filter transient and reduce EMI interference. This coil is found in many electronic appliances.

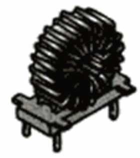


***Type:*** Hash choke coil
***Typical values:*** 3μH–1 mH
***Applications:*** Used in ac supply lines that deliver high currents.

***Type:*** Delay line coil
***Typical values:*** 10μH–50μH
***Applications:*** Used in color televisions to correct for timing differences between the color signal and the black-and-white signal.




***Type:*** Common-mode choke coil
***Typical values:*** 0.6 mH–50 mH
***Applications:*** Used in ac line filters, switching power supplies, battery chargers, and other electronic equipment.



***Type:*** RF chokes
***Typical values:*** 10μH–470 mH
***Applications:*** Used in radio, television, and communication circuits. Found in AM, FM, and UHF circuits.

***Type:*** Molded coils
***Typical values:*** 0.1 μH–100 mH
***Applications:*** Used in a wide variety of circuits such as oscillators, filters, pass-band filters, and others.

***Type:*** Surface-mount inductors
***Typical values:*** 0.01 μH–250 μH
***Applications:*** Found in many electronic circuits that require miniature components on multilayered PCBs (printed circuit boards).

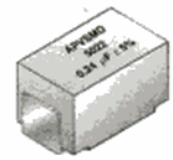


**FIG. 11.22**
*Typical areas of application for inductive elements.*

**FIG. 11.23**
*Variable inductors with a typical range of values from 1 μH to 100 μH; commonly used in oscillators and various RF circuits such as CB transceivers, televisions, and radios.*

## Practical Equivalent Inductors

Inductors, like capacitors, are not ideal. Associated with every inductor is a resistance determined by the resistance of the turns of wire (the thinner the wire, the greater is the resistance for the same material) and by the core losses (radiation and skin effect, eddy current and hysteresis losses—all discussed in more advanced texts). There is also some stray capacitance due to the capacitance between the current-carrying turns of wire of the coil. Recall that capacitance appears whenever there are two conducting surfaces separated by an insulator, such as air, and when those wrappings are fairly tight and are parallel. Both elements are included in the equivalent circuit in Fig. 11.24. For most applications in this text, the capacitance can be ignored, resulting in the equivalent model in Fig. 11.25. The resistance $R_l$ plays an important part in some areas (such as resonance, discussed in Chapter 20)

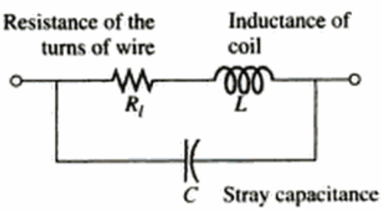



**FIG. 11.24**
*Complete equivalent model for an inductor.*

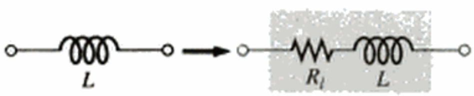



**FIG. 11.25**
*Practical equivalent model for an inductor.*

instant of time, the voltage across the coil can be determined using Kirchhoff's voltage law in the following manner: $v_L = E - v_R$.

Because the waveforms for the inductor have the same shape as obtained for capacitive networks, we are familiar with the mathematical format and can feel comfortable calculating the quantities of interest using a calculator or computer.

The equation for the transient response of the current through an inductor is

$$i_L = \frac{E}{R}(1 - e^{-t/\tau}) \quad \text{(amperes, A)} \tag{11.13}$$

with the time constant now defined by

$$\tau = \frac{L}{R} \quad \text{(seconds, s)} \tag{11.14}$$

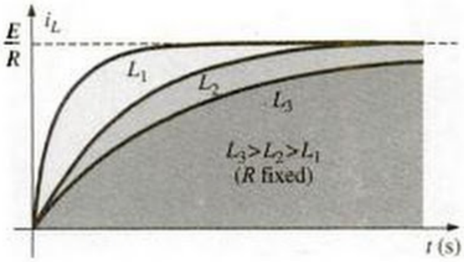


FIG. 11.33
*Effect of L on the shape of the $i_L$ storage waveform.*

Note that Eq. (11.14) is a ratio of parameters rather than a product as used for capacitive networks, yet the units used are still seconds (for time).

Our experience with the factor $(1 - e^{-t/\tau})$ verifies the level of 63.2% for the inductor current after one time constant, 86.5% after two time constants, and so on. If we keep $R$ constant and increase $L$, the ratio $L/R$ increases, and the rise time of $5\tau$ increases as shown in Fig. 11.33 for increasing levels of $L$. The change in transient response is expected because the higher the inductance level, the greater is the choking action on the changing current level, and the longer it will take to reach steady-state conditions.

The equation for the voltage across the coil is

$$v_L = Ee^{-t/\tau} \quad \text{(volts, V)} \tag{11.15}$$

and the equation for the voltage across the resistor is

$$v_R = E(1 - e^{-t/\tau}) \quad \text{(volts, V)} \tag{11.16}$$

As mentioned earlier, the shape of the response curve for the voltage across the resistor must match that of the current $i_L$ since $v_R = i_R R = i_L R$.

Since the waveforms are similar to those obtained for capacitive networks, we will assume that

***the storage phase has passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.***

In addition, since $\tau = L/R$ will always have some numerical value, even though it may be very small at times, the transient period of $5\tau$ will always have some numerical value. Therefore,

***the current cannot change instantaneously in an inductive network.***

If we examine the conditions that exist at the instant the switch is closed, we find that the voltage across the coil is $E$ volts, although the current is zero amperes as shown in Fig. 11.34. In essence, therefore,

***the inductor takes on the characteristics of an open circuit at the instant the switch is closed.***

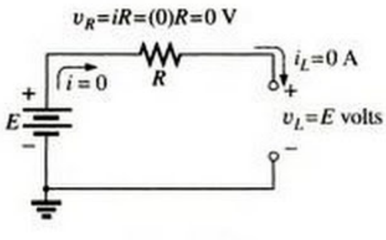


FIG. 11.34
*Circuit in Figure 11.31 the instant the switch is closed.*

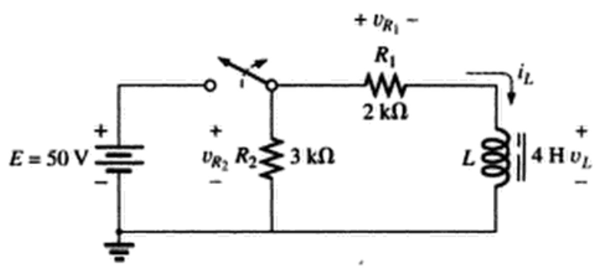



**FIG. 11.44**
*Defined polarities for $v_{R_1}$, $v_{R_2}$, $v_L$, and current direction for $i_L$ for Example 11.5.*

***Solutions:***

a. From Example 11.3:

$$i_L = \mathbf{25\ mA\ (1 - e^{-t/2\ ms})}$$
$$v_L = \mathbf{50\ Ve^{-t/2\ ms}}$$
$$v_{R_1} = i_{R_1}R_1 = i_L R_1$$
$$= \left[\frac{E}{R_1}(1 - e^{-t/\tau})\right]R_1$$
$$= E(1 - e^{-t/\tau})$$

and
$$v_{R_1} = \mathbf{50\ V(1 - e^{-t/2\ ms})}$$
$$v_{R_2} = E = \mathbf{50\ V}$$

b. $\tau' = \dfrac{L}{R_1 + R_2} = \dfrac{4\text{ H}}{2\text{ k}\Omega + 3\text{ k}\Omega} = \dfrac{4\text{ H}}{5 \times 10^3\ \Omega}$
$= 0.8 \times 10^{-3}\text{ s} = 0.8\text{ ms}$

By Eqs. (11.19) and (11.20):

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\text{ k}\Omega}{2\text{ k}\Omega}\right)(50\text{ V}) = 125\text{ V}$$

and
$$v_L = -V_i e^{-t/\tau'} = \mathbf{-125\ Ve^{-t/-0.8\ ms}}$$

By Eq. (11.21):

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

and
$$i_L = I_m e^{-t/\tau'} = \mathbf{25\ mAe^{-t/0.8\ ms}}$$

By Eq. (11.22):

$$v_{R_1} = Ee^{-t/\tau'} = \mathbf{50\ Ve^{-t/0.8\ ms}}$$

By Eq. (11.23):

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(50\text{ V})e^{-t/\tau'} = \mathbf{-75\ Ve^{-t/0.8\ ms}}$$

c. See Fig. 11.45.

where $I_i$ is the starting or initial current. The voltage across the coil is defined by the following:

$$\boxed{\upsilon_L = -V_i e^{-t/\tau'}} \qquad (11.25)$$

with $$V_i = I_i(R_1 + R_2)$$

## 11.8 THÉVENIN EQUIVALENT: $\tau = L/R_{Th}$

In Chapter 10 on capacitors, we found that a circuit does not always have the basic form in Fig. 11.31. The solution is to find the Thévenin equivalent circuit before proceeding in the manner described in this chapter. Consider the following example.

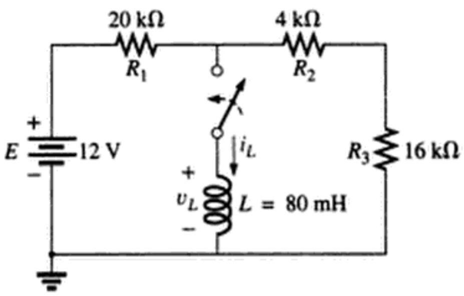



**FIG. 11.46**
*Example 11.6.*

**EXAMPLE 11.6** For the network in Fig. 11.46:

a. Find the mathematical expression for the transient behavior of the current $i_L$ and the voltage $\upsilon_L$ after the closing of the switch ($I_i$ = 0 mA).
b. Draw the resultant waveform for each.

***Solutions:***

a. Applying Thévenin's theorem to the 80 mH inductor (Fig. 11.47) yields

$$R_{Th} = \frac{R}{N} = \frac{20\text{ k}\Omega}{2} = 10\text{ k}\Omega$$

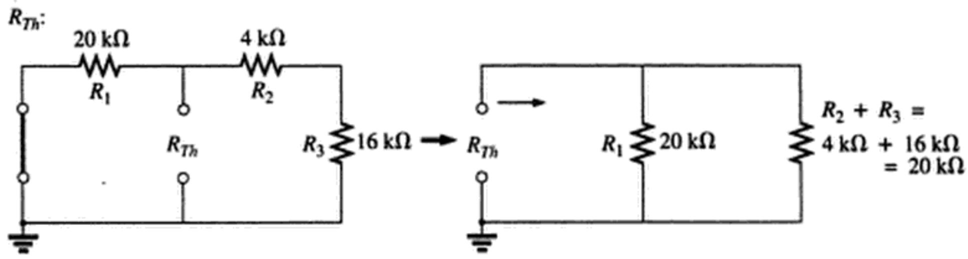



**FIG. 11.47**
*Determining $R_{Th}$ for the network in Fig. 11.46.*

Applying the voltage divider rule (Fig. 11.48), we obtain

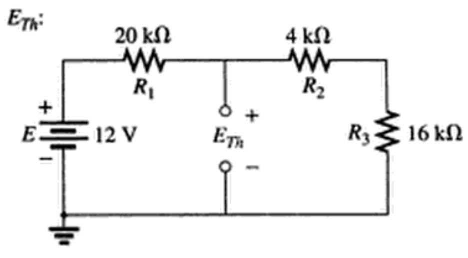



**FIG. 11.48**
*Determining $E_{Th}$ for the network in Fig. 11.46.*

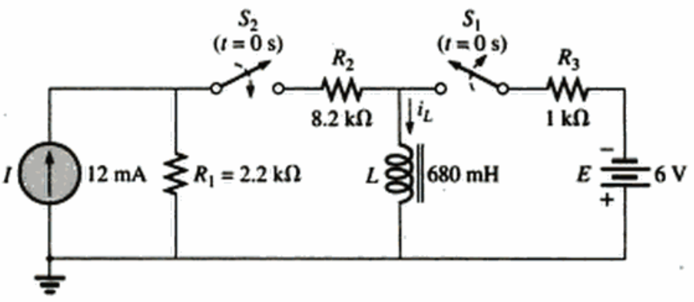



**FIG. 11.51**
*Example 11.7.*

***Solutions:***

a. Using Ohm's law, we find the initial current through the coil:

$$I_i = -\frac{E}{R_3} = -\frac{6\text{ V}}{1\text{ k}\Omega} = -6\text{ mA}$$

b. Applying Thévenin's theorem gives

$$R_{Th} = R_1 + R_2 = 2.2\text{ k}\Omega + 8.2\text{ k}\Omega = 10.4\text{ k}\Omega$$
$$E_{Th} = IR_1 = (12\text{ mA})(2.2\text{ k}\Omega) = 26.4\text{ V}$$

The Thévenin equivalent network appears in Fig. 11.52.

The steady-state current can then be determined by substituting the short-circuit equivalent for the inductor:

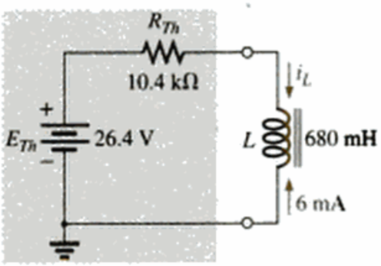



**FIG. 11.52**
*Thévenin equivalent circuit for the network in Fig. 11.51 for $t \geq 0$ s.*

$$I_f = \frac{E}{R_{Th}} = \frac{26.4\text{ V}}{10.4\text{ k}\Omega} = 2.54\text{ mA}$$

The time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{680\text{ mH}}{10.4\text{ k}\Omega} = 65.39\ \mu\text{s}$$

Applying Eq. (11.17) gives

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} \\ &= 2.54\text{ mA} + (-6\text{ mA} - 2.54\text{ mA})e^{-t/65.39\ \mu\text{s}} \\ &= \mathbf{2.54\ mA - 8.54\ mA}e^{-t/65.39\ \mu s} \end{aligned}$$

c. Note Fig. 11.53.

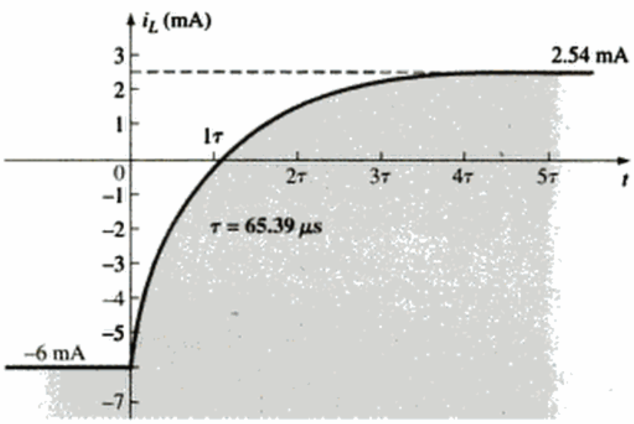



**FIG. 11.53**
*The current $i_L$ for the network in Fig. 11.51.*

## 11.9 INSTANTANEOUS VALUES

The development presented in Section 10.8 for capacitive networks can also be applied to *R-L* networks to determine instantaneous voltages, currents, and time. The instantaneous values of any voltage or current can be determined by simply inserting *t* into the equation and using a calculator or table to determine the magnitude of the exponential term.

The similarity between the equations

$$v_C = V_f + (V_i + V_f)e^{-t/\tau}$$

and

$$i_L = I_f + (I_i - I_f)e^{-t/\tau}$$

results in a derivation of the following for *t* that is identical to that used to obtain Eq. (10.23):

$$\boxed{t = \tau \log_e \frac{(I_i - I_f)}{(i_L - I_f)}} \quad \text{(seconds, s)} \qquad \textbf{(11.26)}$$

For the other form, the equation $v_C = Ee^{-t/\tau}$ is a close match with $v_L = Ee^{-t/\tau} = V_i e^{-t/\tau}$, permitting a derivation similar to that employed for Eq. (10.23):

$$\boxed{t = \tau \log_e \frac{V_i}{v_L}} \quad \text{(seconds, s)} \qquad \textbf{(11.27)}$$

For the voltage $v_R$, $V_i = 0$ V and $V_f = EV$ since $v_R = E(1 - e^{-t/\tau})$. Solving for *t* yields

$$t = \tau \log_e \left(\frac{E}{E - v_R}\right)$$

or

$$\boxed{t = \tau \log_e \left(\frac{V_f}{V_f - v_R}\right)} \quad \text{(seconds, s)} \qquad \textbf{(11.28)}$$

## 11.10 AVERAGE INDUCED VOLTAGE: $v_{L_{av}}$

In an effort to develop some feeling for the impact of the derivative in an equation, the average value was defined for capacitors in Section 10.10, and a number of plots for the current were developed for an applied voltage. For inductors, a similar relationship exists between the induced voltage across a coil and the current through the coil. For inductors, the average induced voltage is defined by

$$\boxed{v_{L_{av}} = L\frac{\Delta i_L}{\Delta t}} \quad \text{(volts, V)} \qquad \textbf{(11.29)}$$

where Δ indicates a finite (measurable) change in current or time. Eq. (11.12) for the instantaneous voltage across a coil can be derived from Eq. (11.29) by letting $V_L$ become vanishingly small. That is,

$$v_{L_{inst}} = \lim_{\Delta t \to 0} L\frac{\Delta i_L}{\Delta t} = L\frac{di_L}{dt}$$

In the following example, the change in current $\Delta i_L$ is considered for each slope of the current waveform. *If the current increases with time, the average current is the change in current divided by the change in*

## SECTION 11.11 Inductors in Series and in Parallel

**35.** Find the total inductance of the circuit of Fig. 11.100.

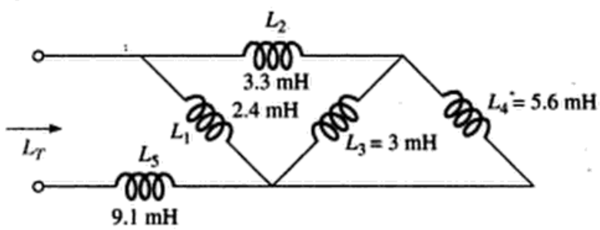



**FIG. 11.100**
*Problem 35.*

**36.** Find the total inductance for the network of Fig. 11.101.

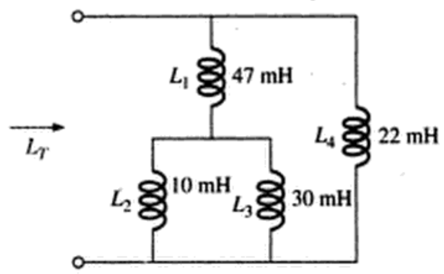



**FIG. 11.101**
*Problem 36.*

**37.** Reduce the network in Fig. 11.102 to the fewest number of components.

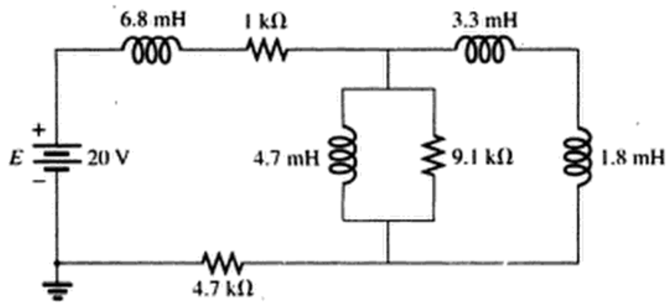



**FIG. 11.102**
*Problem 37.*

**38.** Reduce the network in Fig. 11.103 to the fewest elements.

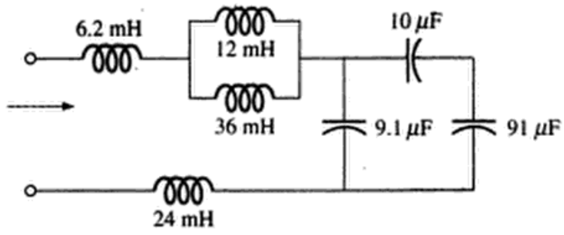



**FIG. 11.103**
*Problem 38.*

**39.** Reduce the network of Fig. 11.104 to the fewest elements.

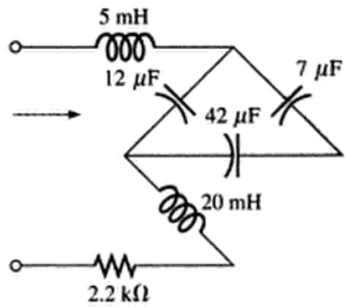



**FIG. 11.104**
*Problem 39.*

***40.** For the network in Fig. 11.105:

**a.** Write the mathematical expressions for the voltages $v_L$ and $v_R$ and the current $i_L$ if the switch is closed at $t = 0$ s.

**b.** Sketch the waveforms of $v_L$, $v_R$, and $i_L$.

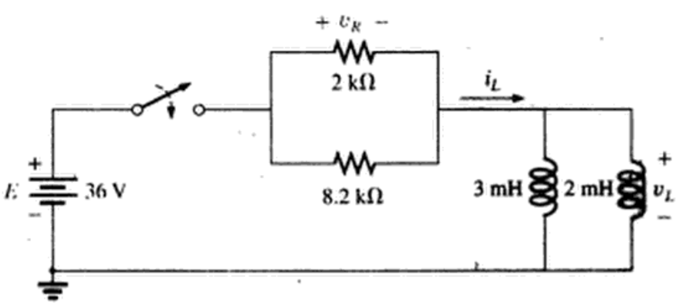



**FIG. 11.105**
*Problem 40.*

***41.** For the network in Fig. 11.106:

**a.** Write the mathematical expressions for the voltage $v_L$ and the current $i_L$ if the switch is closed at $t = 0$ s. Take special note of the required $v_L$.

**b.** Sketch the waveforms of $v_L$ and $i_L$.

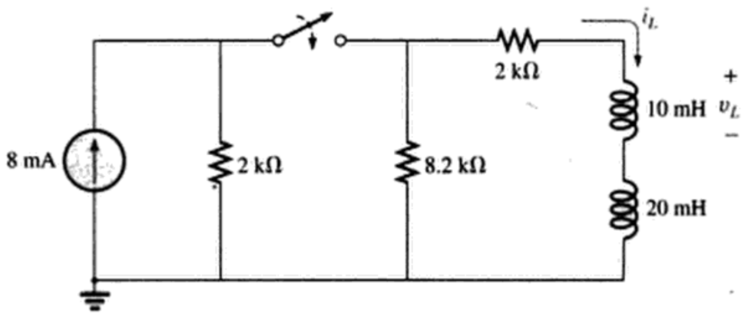



**FIG. 11.106**
*Problem 41.*

***42.** For the network in Fig. 11.107:

**a.** Find the mathematical expressions for the voltage $v_L$ and the current $i_L$ following the closing of the switch.

**b.** Sketch the waveforms of $v_L$ and $i_L$ obtained in part (a).

**c.** Determine the mathematical expression for the voltage $v_{L_3}$ following the closing of the switch, and sketch the waveform.

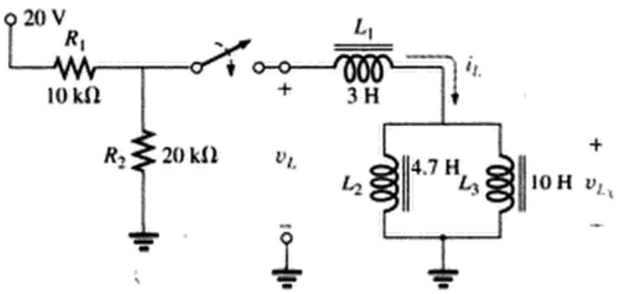



**FIG. 11.107**
*Problem 42.*

## SECTION 11.12 Steady-State Conditions

**43.** Find the steady-state currents $I_1$ and $I_2$ for the network in Fig. 11.108.

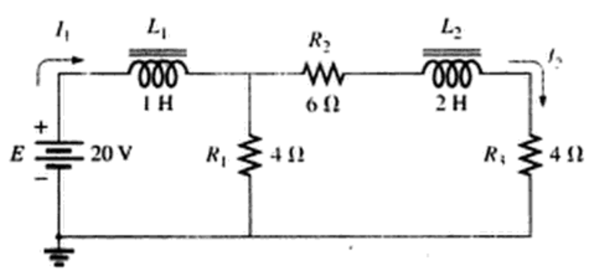



**FIG. 11.108**
*Problem 43.*

**44.** Find the steady-state currents and voltages for the network in Fig. 11.109.

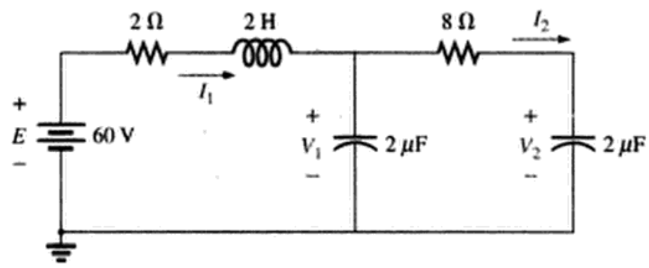



**FIG. 11.109**
*Problem 44.*

**45.** Find the steady-state currents and voltages for the network in Fig. 11.110 after the switch is closed.

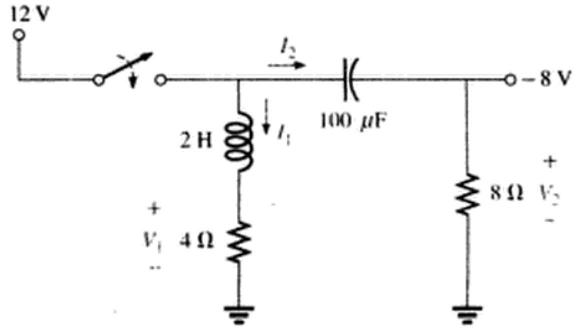



**FIG. 11.110**
*Problem 45.*

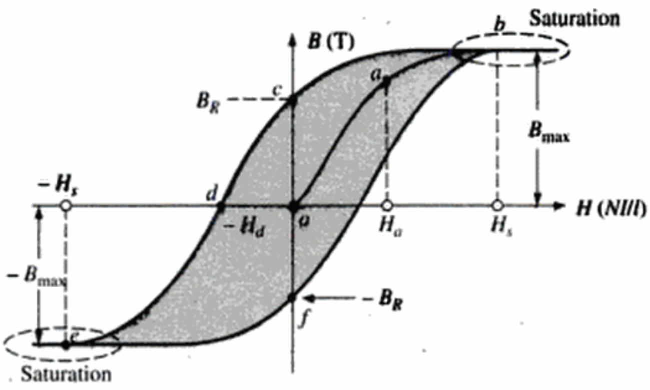



**FIG.12.5**
*Hysteresis curve.*

density $B_R$, which remains when the magnetizing force is zero, is called the *residual flux density*. It is this residual flux density that makes it possible to create permanent magnets. If the coil is now removed from the core in Fig. 12.4, the core will still have the magnetic properties determined by the residual flux density, a measure of its "retentivity." If the current $I$ is reversed, developing a magnetizing force, $-H$, the flux density $B$ decreases with an increase in $I$. Eventually, the flux density will be zero when $-H_d$ (the portion of curve from $c$ to $d$) is reached. The magnetizing force $-H_d$ required to "coerce" the flux density to reduce its level to zero is called the *coercive force*, a measure of the coercivity of the magnetic sample. As the force $-H$ is increased until saturation again occurs and is then reversed and brought back to zero, the path *def* results. If the magnetizing force is increased in the positive direction ($+H$), the curve traces the path shown from $f$ to $b$. The entire curve represented by *bcdefb* is called the **hysteresis** curve for the ferromagnetic material, from the Greek *hysterein*, meaning "to lag behind." The flux density $B$ *lagged* behind the magnetizing force $H$ during the entire plotting of the curve. When $H$ was zero at $c$, $B$ was not zero but had only begun to decline. Long after $H$ had passed through zero and had become equal to $-H_d$ did the flux density $B$ finally become equal to zero.

If the entire cycle is repeated, the curve obtained for the same core will be determined by the maximum $H$ applied. Three hysteresis loops for the same material for maximum values of $H$ less than the saturation value are shown in Fig. 12.6. In addition, the saturation curve is repeated for comparison purposes.

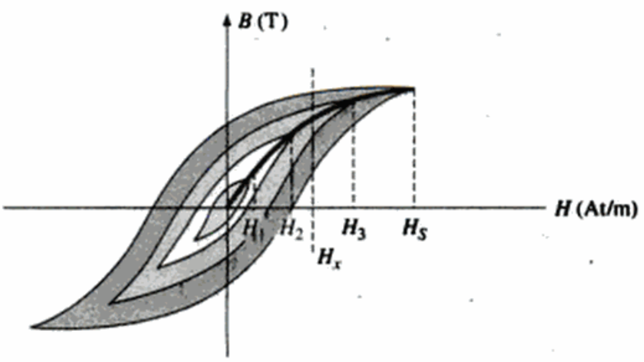



**FIG.12.6**
*Defining the normal magnetization curve.*

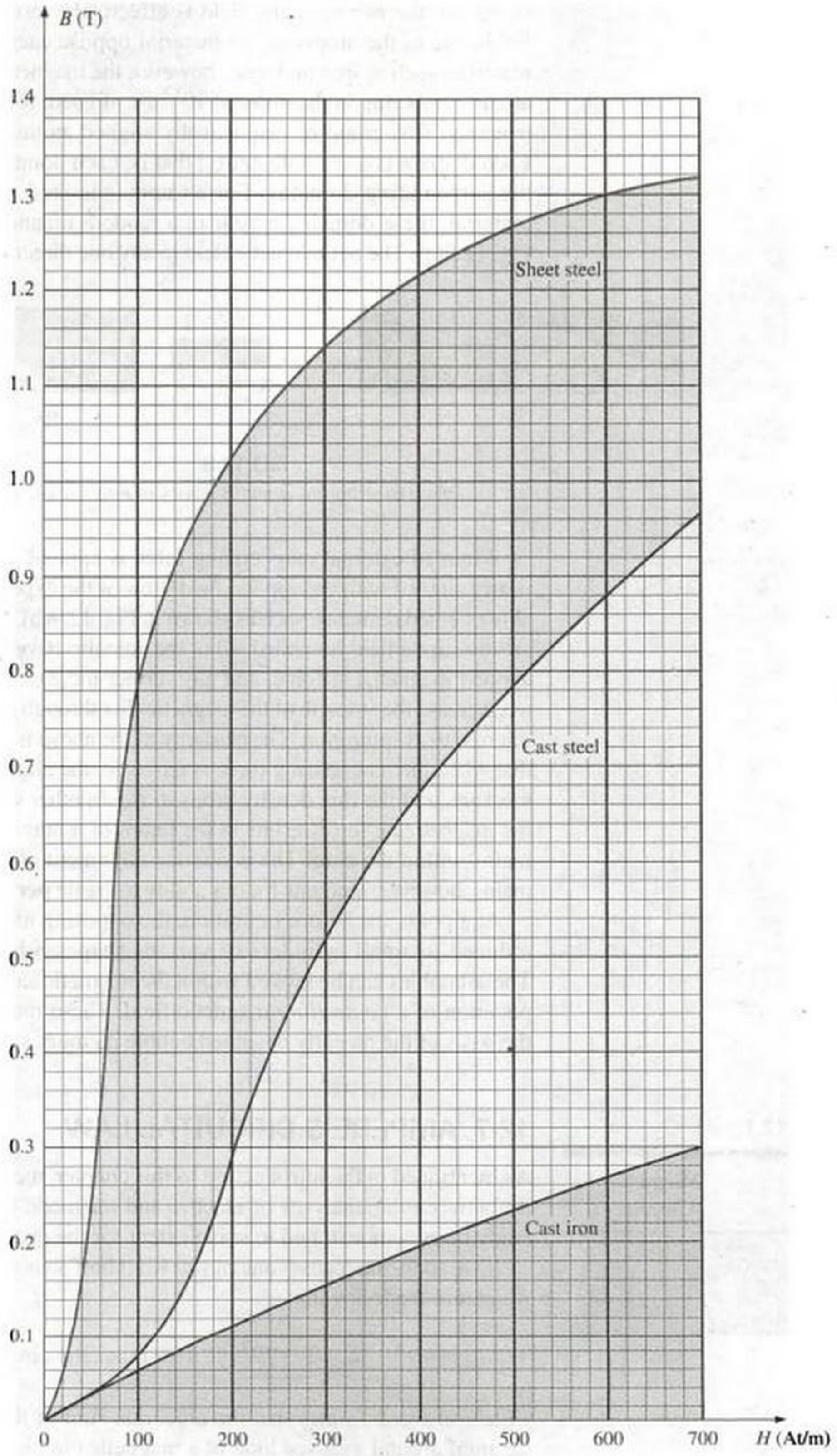



**FIG.12.8**
*Expanded view of Fig. 12.7 for the low magnetizing force region.*

Inserting the above data in Table 12.3 results in Table 12.4.

**TABLE 12.4**

| Section | $\Phi$ (Wb) | $A$ (m$^2$) | $B$ (T) | $H$ (At/m) | $l$ (m) | $Hl$ (At) |
|---|---|---|---|---|---|---|
| *efab* | $3.5 \times 10^{-4}$ | $6.452 \times 10^{-4}$ | 0.542 | 70 | $304.8 \times 10^{-3}$ | 21.34 |
| *bcde* | $3.5 \times 10^{-4}$ | $6.452 \times 10^{-4}$ | 0.542 | 1600 | $127 \times 10^{-3}$ | 203.2 |

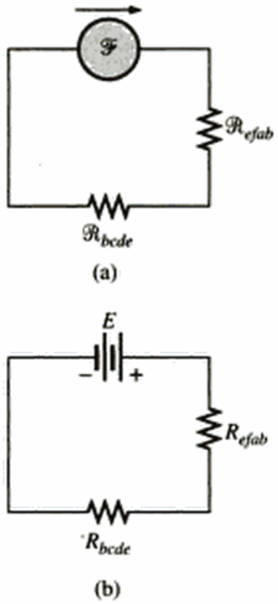



**FIG. 12.15**
*(a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnet in Fig. 12.14.*

The magnetic circuit equivalent and the electric circuit analogy for the system in Fig. 12.14 appear in Fig. 12.15.

Applying Ampère's circuital law, we obtain

$$NI = H_{efab}l_{efab} + H_{bcde}l_{bcde}$$
$$= 21.34 \text{ At } + 203.2 \text{ At} = 224.54 \text{ At}$$

and
$$(50 \text{ t})I = 224.54 \text{ At}$$

so that
$$I = \frac{224.54 \text{ At}}{50 \text{ t}} = \mathbf{4.49\ A}$$

**EXAMPLE 12.3** Determine the secondary current $I_2$ for the transformer in Fig. 12.16 if the resultant clockwise flux in the core is $1.5 \times 10^{-5}$ Wb.

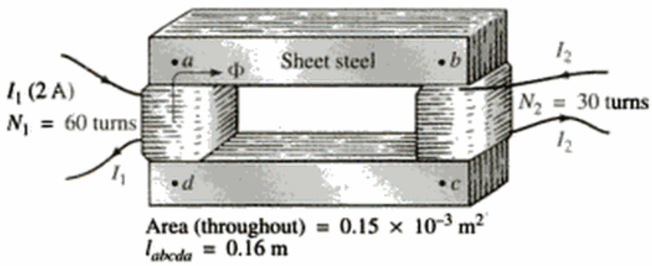



**FIG. 12.16**
*Transformer for Example 12.3.*

***Solution:*** This is the first example with two magnetizing forces to consider. In the analogies in Fig. 12.17, note that the resulting flux of each is opposing, just as the two sources of voltage are opposing in the electric circuit analogy.

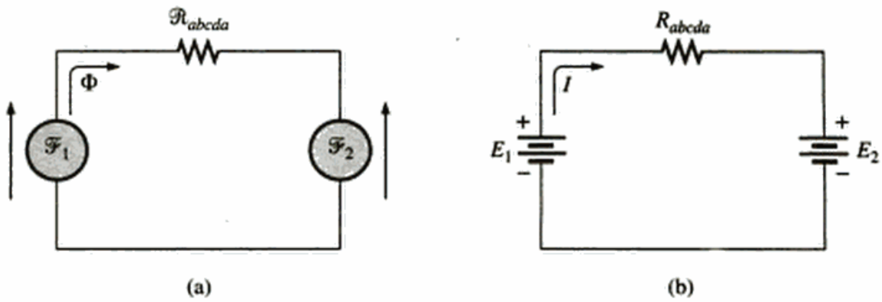



**FIG. 12.17**
*(a) Magnetic circuit equivalent and (b) electric circuit analogy for the transformer in Fig. 12.16.*

Since $$-\sin\omega t = \sin(\omega t \pm 180°)$$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180°)$$

revealing that a negative sign can be replaced by a 180° change in phase angle (+ or −); that is,

$$e = -E_m \sin\omega t = E_m \sin(\omega t + 180°) = E_m \sin(\omega t - 180°)$$

A plot of each will clearly show their equivalence. There are, therefore, two correct mathematical representations for the functions.

The **phase relationship** between two waveforms indicates which one leads or lags the other and by how many degrees or radians.

---

**EXAMPLE 13.12** What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$
b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$
c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$
d. $i = -\sin(\omega t + 30°)$
   $v = 2 \sin(\omega t + 10°)$
e. $i = -2 \cos(\omega t - 60°)$
   $v = 3 \sin(\omega t - 150°)$

***Solutions:***

a. See Fig. 13.31.
   ***i* leads *v* by 40°, or *v* lags *i* by 40°.**

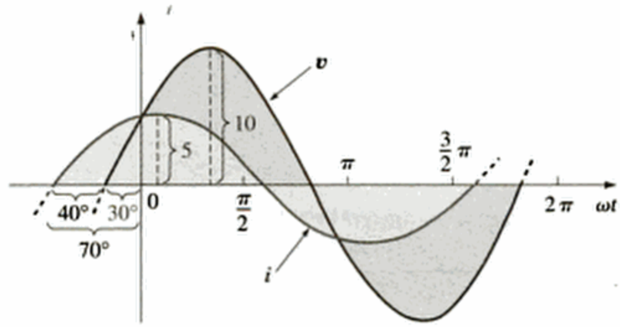



**FIG. 13.31**
*Example 13.12(a): i leads v by 40°.*

b. See Fig. 13.32.
   ***i* leads *v* by 80°, or *v* lags *i* by 80°.**

c. See Fig. 13.33.

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

***i* leads *v* by 110°, or *v* lags *i* by 110°.**