DEALING WITH UN

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As the saying goes, the only two things in life that are certain are death and taxes. The field of electronics supports that adage. Electronics is filled with numerical uncertainties—values that may fall anywhere within a limited range.

Normally a circuit's lack of precision components have little effect on its operation. Indeed, most circuits are designed to tolerate variation. Even the most cautiously designed network, however, can be crippled if deviations from nominal specifications add up.

One of the largest sources of uncertainty is the variation between component values; no two are exactly the same. Of all the 1000-ohm resistors you've ever used, probably none of them were exactly 1000 ohms. They may have been so close that your meter registered 1000, but more sensitive equipment would have shown a deviation.

Variations between component values are not the only culprits. Other causes of uncertainty in electronics include uncalibrated meters, imperfectly regulated power supplies, induced electrical noise, and even environmental effects such as temperature and humidity.

Armed with even a limited knowledge of how to deal with uncertainties in calculations, you can identify potential problems and eliminate them. This awareness can also be a great asset when selecting component tolerances (*e.g.* you'll know when to use a 5% resistor and when to spend the extra few cents for its 1% counterpart).

This article presents a basic overview of how uncertainties should be dealt with for the four primary mathematical operations: addition, subtraction, multiplication, and division. While discussing the four basic operations, the article will demonstrate a generic methodology that can be used to determine how uncertainties should be handled in more complex operations. Before we get to all that, though, a short discussion of the notation used for uncertainties is in order.

Uncertainty Notation. As you're probably aware, numbers with uncertain values can be written as a series of three separate numbers: a target or nominal value, a maximum deviation above nominal, and a maximum deviation below nominal. Because the two deviations are generally the same in magnitude, only two numbers are usually listed.

For example, let's return to the hypothetical 1000-ohm resistor mentioned earlier. If it were a 5%-tolerance component (signified by a gold fourth color band), its value could be listed as 1000-ohm ±5%. That indicates that the component's actual resistance can vary from 1000 ohms by up to 5% in either direction. Because 5% of 1000 is 50, the resistor may actually be anywhere from 950 to 1050 ohms.

Note that the deviation from nominal are usually given as either a percentage (1000 ±5%), but can be specified as an absolute magnitude (1000 ±50).

Addition and Subtraction. Assume two resistors are connected in series, the first a 5% component with a nominal value of 820 ohms and the second a 10%, 360-ohm element. The network's nominal resistance, from the top of the upper component to the bottom of the lower one, would then be 1180 ohms.

The highest possible total resistance would occur if both components were at the upper end of their tolerance bands. True resistance values would then be 861 and 396 ohms, respectively, and the total circuit resistance would therefore be 1257 ohms. Finally, the actual circuit's deviation from its designed value would be 77 ohms. Clearly, the total deviation of 77 ohms is the sum of the absolute deviations for the two individual components, 41 and 36 ohms.

Now let's apply the above procedure to the subtraction operation. Figure 1 shows a segment of a larger circuit, in which the current through a resistor is dependent on the exact values of two independent current sources. By Kirchoff's current law, the current through the resistor (I_r) is the difference between I_1 and I_2; that is:

$$I_r = I_1 - I_2$$

We would expect I_r to have its largest value if I_1 and I_2 were at the upper and lower ends of their respective tolerance bands. Let's say the have a tolerances of ± 50 mA and ±25 mA respectively. So:

$$I_1 = (750\ mA + 50\ mA) = 800\ mA$$

and

$$I_1 = (200\ mA - 25\ mA) = 175\ mA$$

the calculation of I_r yields a result of 625 mA. The actual current flowing through the resistor would therefore be 75 mA greater than the target rate of 600 mA. We conclude that the current through the resistor is 600 mA ±75 mA.

From the results of these two thought experiments we can infer a standard rule: When adding or subtracting two numbers, the total nominal value is the

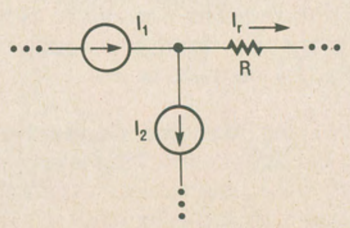

Fig. 1. The uncertainty of component values leads to uncertainties in the values of current through a circuit. The overall uncertainty can really add up.

The tolerance of components can lead to design errors. Learn how to determine when tight component tolerances are needed with this quick tutorial.

CERTAINTIES

sum or difference of the two original values; the uncertainty of the result is the sum of the absolute magnitudes of the two original uncertainties.

Multiplication and Division. With a general rule for addition and subtraction under our belts, we now turn our attention to the more complex operations of multiplication and division. Since multiplication and division have the same relationship to each other as addition and subtraction (they are algebraic inverses of one another), we might expect a single rule to cover both multiplication and division. That does turn out to be the case, so we will therefore only consider multiplication in detail.

Assume P is the power dissipated by a resistor in a direct-current (DC) circuit. The power is the product of the the current (I) through the resistor and the voltage (V) across it. If I = 16 ±2 amps and V = 20 ±5 volts, the nominal value of P is 320 watts. The power will deviate furthest from this value when both the multiplier (V) and multiplicand (I) are at the upper limits of their ranges. In that case, P is 450 watts. The difference between the two 130, or 40.625% of the nominal value.

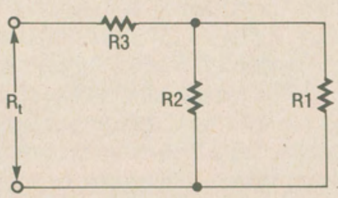

Fig. 2. This example resistor network should help clarify how tolerance values are affected by mathematical operations.

To obtain a minimum value for the component's power both V and I must be at the lower ends of their respective ranges. The calculation becomes:

$$P = 15 \times 14 = 210 \text{ watts}$$

which is 110 (34.375%) below nominal.

Unlike addition, the multiplication operation produces asymmetric (read that unequal) positive and negative uncertainties. A mathematical purist would be forced to indicate separate positive and negative tolerances. Those of us who can afford to lose some accuracy, however, can make an approximation of the uncertainty of the final result.

Averaging the two deviations comes to mind as a good method. When that is done, in both the absolute and percentage forms, the averages are 120 and 37.5%. From that it is easy to see that an approximation for the final uncertainty is just the addition of the percentage uncertainties of the multiplier and multiplicand.

By performing a similar analysis for division it can be shown that the same result applies. Hence we can write a general rule for both operations: When the operation of multiplication or division is to be performed on two numbers, the uncertainty of the result is the sum of the percentage uncertainties of the two original numbers.

Example. Finally, to demonstrate the rules just derived, we'll calculate the total resistance of the series/parallel resistor network shown in Fig. 2. It is suggested that you try this exercise on your own before reading the solution.

Assume the following component values:

$$R1 = 910 \pm 5\% \text{ ohms}$$
$$R2 = 560 \pm 1\% \text{ ohms}$$
$$R3 = 180 \pm 5\% \text{ ohms}$$

Given those values, the parallel combination of R1 and R2 would yield a combined resistance of R_P where:

$$\begin{aligned} R_P &= R1 \times R2/(R1 + R2) \\ &= (910 \pm 5\%) \times (560 \pm 1\%) \div \\ &\quad ((910 \pm 45.5) + (560 \pm 5.6)) \\ &= 509600 \pm 6\%/(1470 \pm 51.1) \\ &= 509600 \pm 6\%/(1470 \pm 3.476\%) \\ &= 346.7 \text{ ohms } \pm 9.476\% \\ &= 346.7 \pm 32.8 \text{ ohms} \end{aligned}$$

Of course, the last step in the calculation of R_t (the total equivalent resistance) is to add the resistance of R1 to the value of R_P just calculated:

$$\begin{aligned} R_t &= R1 + R_P \\ R_t &= 180 \pm 5\% + 346.7 \pm 32.8 \\ R_t &= 180 \pm 9 + 346.7 \pm 32.8 \\ R_t &= 526.7 \pm 41.8 \text{ ohms} = 526.7 \text{ ohms} \\ &\quad \pm 7.94\% \end{aligned}$$

From this example we see how even rather small uncertainties can be amplified through repeated calculations. The tolerance of the total resistance network is nearly 8% even though the individual tolerances are 5% or less.

As mentioned in the introduction, normally this type of error "stack-up" will have a little effect on a circuit's operation. But armed with even the bounded knowledge presented here, you should be able to avoid any problems that may arise from uncertainty. ■