NORTON'S THEOREM and CURRENT SOURCES

By Louis E. Frenzel

Norton's equivalents are not look-alikes from The Honeymooners; *they can help break any circuit down into a current source and resistance*

There are lots of different ways to analyze electronic circuits. In the past two issues, we have shown you how to use Thevenin's theorem and the superposition theorem to convert circuits into equivalent voltage sources. Any complex circuit can be rearranged so that it can be represented by an equivalent voltage source in series with an internal resistance and the load. With the circuit in that simplified form, calculations regarding output voltage and current for different load values can be quickly and easily performed.

But not all circuits respond to such treatment. Some circuits contain current sources rather than voltage sources. For those, other techniques must be used. For example, Norton's theorem can be used to convert a circuit into a simple parallel network containing a current source and a parallel-internal resistance.

In this month's installment we are going to introduce you to Norton's theorem and a method of circuit analysis using it. Later, we will discuss practical current-source circuits and some of their applications.

Current Sources. A current source is a generator that supplies a fixed current to any value of load resistance connected to it. An ideal current source will supply that fixed value of current into an open circuit or a short circuit and any resistance value between the two. Of course, there are no perfect current sources. However, in practice it is possible to construct current sources that will supply a single current to a wide range of loads with little deviation.

Most electronic circuits are designed to be voltage sources. As you may recall from the last two articles of the series, we defined a voltage source as a generator that produces a constant output voltage regardless of the load impedance. In order to achieve that kind of performance, a voltage source must have zero internal resistance. Such perfect voltage sources do not exist, but many do have extremely low internal resistances, and so closely approximate a perfect voltage source. Batteries and electronic power supplies are excellent voltage sources. Any electronic circuit with a low output impedance (low internal resistance) is a good voltage source. With low output impedance, varying load resistance has little or no effect on the output voltage.

There are some applications in electronics that require a constant current rather than a constant voltage. That is where current sources become useful. With a constant output current, varying load resistances have little or no effect on the output current.

A current source is usually represented by the symbol shown in Fig. 1A. The arrow points in the direction of current flow. If we were to observe electron flow (and we will for our discussion), the arrow points in the direction of the electron flow. If conventional current flow (the movement of holes) is assumed, the arrow would point in the direction of that flow.

Just as practical voltage sources have a finite internal resistance, so do current sources. A practical current source is shunted by an internal resistance, designated R in Fig. 1B. In a cur-

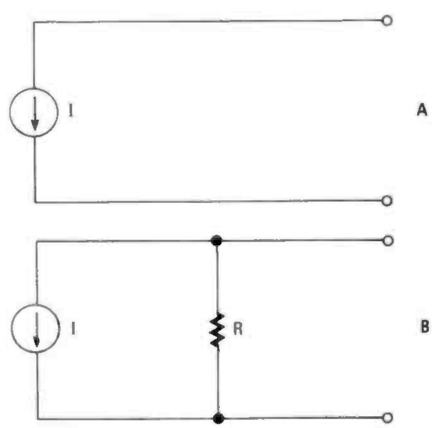

Fig. 1. The standard current source (A) is really like an ideal current source in parallel with its inherent internal resistance (B).

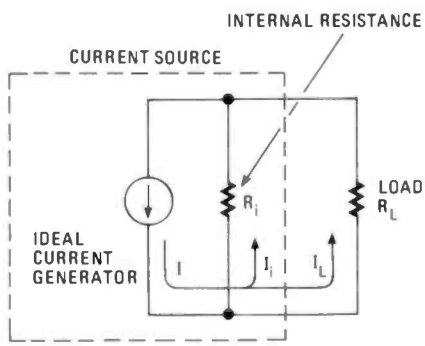

Fig. 2. The internal resistance of a practical voltage source steals some of the current from the supply.

rent source, the internal resistance appears in parallel with the source. A perfect current source will have an infinite value of R. The internal resistance in a practical current source will not be infinite, but will instead be some finite but usually high value. The higher the internal resistance, the better the current source.

You can see why that is by simply looking at the current source in Fig. 2. If it generates a fixed current, I, some of that current will pass through the internal resistance, R_i. The current supplied to the load, I_L, will be less than the current produced by the generator. In Fig. 2:

$$I_L = I - I_i$$

where I_L is the load current, I is the constant current produced by the generator, and I_i is the current through the internal resistance. A high value of internal resistance compared to the load resistance, will only shunt away a small amount of current so that most of the current produced by the generator will pass through the load.

When no load is connected to the current source, then all of the current produced by the generator will flow through the internal resistance. When a load is connected, the current will be divided between the internal resistance and the load.

The primary reason for working with a current source rather than a voltage source is because it is sometimes easier to analyze a network in terms of current. That is particularly true when the circuit you are working with contains many parallel branches. If the circuit is primarily series, then it is usually better to use a voltage source and Thevenin's theorem for analysis.

Norton's Theorem. Norton's theorem states that any linear circuit can be replaced by an equivalent circuit consisting of a current source and its parallel internal resistance connected to the load. Norton's theorem defines the mathematical procedures used to compute the Norton's-equivalent circuit. The process is best illustrated with an example.

Refer to Fig. 3A. That network consists of a battery plus a resistive network connected to a load. Our job is to convert that circuit into its Norton's equivalent. The battery plus the circuit consisting of R1 through R3 will be translated into an equivalent current source with its internal resistance.

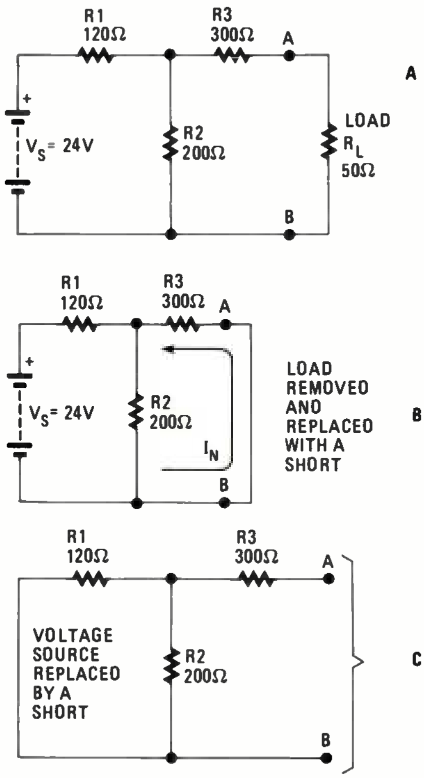

Fig. 3. The steps in calculating the Norton's equivalent of a circuit (A) are reviewed diagramatically here. Drawing B displays the load replaced by a short, and C shows the source replaced by a short.

There are two steps to determining the Norton's equivalent of a circuit. The first step is to calculate the Norton's-equivalent current, I_N. That is usually referred to as the short-circuit current. That is the amount of current that will flow if the load is replaced with a short circuit. It is also the current that a load will see when connected to the Norton's-equivalent generator.

The other part of the calculation is to compute the equivalent internal resistance, R_N. That is the total resistance appearing across the load terminals. Now, let's take a look at the procedures you use to calculate the Norton's-equivalent generator.

The rules for determining the Norton's-equivalent circuit can be summarized as follows:

1. Disconnect the load from the output.
2. Short across the output (load) terminals.
3. Calculate the current in the short. That is the Norton's-equivalent current, I_N.
4. Remove short from output terminals.
5. Replace voltage source with a short.
6. Calculate resistance between the output terminals (with the load still disconnected.) That is the Norton's-equivalent resistance, R_N.
7. Connect a current generator, I_N, in parallel with R_N to form the complete Norton's equivalent.
8. Reconnect the load and make any additional calculations.

Let's apply those steps to the circuit in Fig. 3A. First, we remove the load between terminals A and B. Then, we short terminals A and B. It is through that short that the Norton's-equivalent current will flow. See Fig. 3B. The Norton's-equivalent current, I_N, then is the current through R3 and the short. We can use Ohm's and Kirchhoff's laws in determining that current value.

To do that, we must first calculate the total circuit resistance. That is the parallel combination of R2 and R3 in series with R1:

$$R_T = R1 + R2R3/(R2 + R3)$$
$$R_T = 120 + 200(300)/(200 + 300)$$
$$R_T = 120 + 60{,}000/500$$
$$R_T = 120 + 120 = 240 \text{ ohms}$$

The total current drawn from the voltage source is:

$$I_T = V_S/R_T = 24/240 = .1 \text{ amp}$$

That current produces a voltage across R1 of:

$$V_1 = I_T R1 = .1(120) = 12 \text{ volts}$$

Now, according to Kirchhoff's law, that leaves:

$$24 - 12 = 12 \text{ volts}$$

across R2 and R3. The current in R3 is the Norton's equivalent, which we can calculate by Ohm's law:

$$I_N = 12/300 = .04 \text{ amp}$$

Next we need to compute the Norton's-equivalent resistance, R_N. To do that, we remove the short from between terminals A and B. Then we replace the battery with a short circuit. We can now compute the total resistance of the network between terminals A and B as shown in Fig. 3C. That is the resistance of R1 and R2 in parallel connected in series with R3. The resistance calculations are as follows:

$$R_N = R1(R2)/(R1 + R2) + R3$$
$$R_N = 120(200)/(120 + 200) + 300$$
$$R_N = 24{,}000/320 + 300$$
$$R_N = 75 + 300 = 375 \text{ ohms}$$

Now, the total Norton's equivalent can be drawn. It is a current source with a value of .04 Amp in parallel with the Norton's-equivalent resistance, R_N, of 375 ohms. When the load is recon-

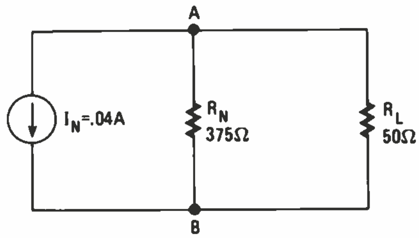

Fig. 4. Finally you will arrive at this Norton's equivalent of the circuit in Fig. 3 after you follow the steps.

nected, the value I_N will flow in the load. See Fig. 4. The load voltage, V_L, is then:

$$V_L = I_N R_L$$
$$V_L = .04(50) = 2 \text{ volts}$$

As you can see, a complex circuit with a voltage source can be reduced to its Norton's equivalent containing a current source. That allows you to make current calculations in a parallel circuit. You can, of course, convert the circuit to its Thevenin's equivalent and work with a voltage source and a series circuit. The choice is yours. In either case, you simplify the original circuit to make analysis and design faster and easier.

Exercise Problem. Now try the procedure yourself.
1. Convert the circuit in Fig. 5 into its Norton's equivalent.

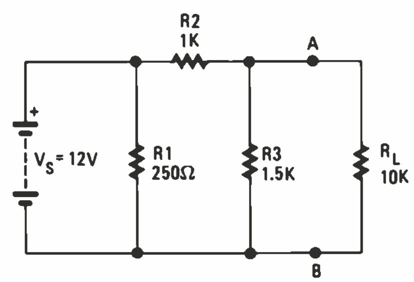

Fig. 5. Remember to use what you've learned to solve for this circuit.

Converting Thevenin to Norton. As you saw in the previous example, a circuit with a voltage source can be converted into a Norton's equivalent with a current source. You can also take a circuit with a current source and convert it into a Thevenin's equivalent with a voltage source. It all depends on the circuit and the kinds of calculations you want to make. You may wish to convert back and forth to make the optimum calculations on a given circuit. The easiest way to do that is to convert between the Thevenin's and Norton's equivalents. Let's consider how to do that.

In the previous example, we converted the circuit in Fig. 3A into its Norton's equivalent shown in Fig. 4. Suppose we want the Thevenin's equivalent instead. We could go back to the original circuit and apply the Thevenin's conversion steps, but that is too time consuming. It's easier to use some simple conversion formulas.

To convert the Norton's equivalent to the Thevenin's equivalent, first remove the load and apply these formulas:

$$V_{Th} = I_N R_N$$
$$R_{Th} = R_N$$

Remember, V_{Th} is the Thevenin's-equivalent voltage while R_{Th} is the Thevenin's-equivalent series resistance.

Using our previous example then:

$$V_{Th} = I_N R_N$$
$$V_{Th} = .04(375) = 15 \text{ volts}$$
$$R_{Th} = R_N = 375 \text{ ohms}$$

So, the Thevenin's equivalent of the circuit in Fig. 4 is given in Fig. 6.

You can also go the other way. Assume the Thevenin's equivalent in Fig. 7A. To get the Norton's equivalent, you use these formulas:

$$I_N = V_{Th}/R_{Th}$$
$$R_{Th} = R_N$$

Applying them with the values in Fig. 7A, we get:

$$I_N = V_{Th}/R_{Th} = 7.5/100 = .075 \text{ amp}$$
$$R_{Th} = R_N = 100 \text{ ohms}$$

The Norton's equivalent is shown in Fig. 7B.

Exercise Problems. Practice the concepts with these problems:
2. Change the Norton's-equivalent circuit you derived from Fig. 5 into its Thevenin's equivalent using the conversion formulas. 3. Convert the circuit in Fig. 8 into its Norton's equivalent using the conversion formulas.

Superposition with Current Sources. In the previous article, I discussed the superposition theorem. It is extremely helpful in simplifying circuits with two or more voltage sources. It can also be used on circuits with two or more current sources.

The superposition theorem states that:
"The current through (or voltage across a component in a linear circuit is the algebraic sum of the currents (or voltages) produced by each current (or voltage) source acting independently."

To use the superposition theorem, you disable all but one source, then calculate the various currents and voltages. Then, you repeat that with the other sources. Finally, you add up

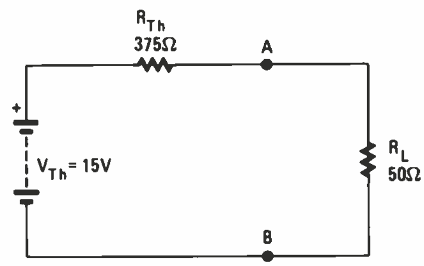

Fig. 6. This Thevenin's equivalent can be generated from the Norton's equivalent in Fig. 5.

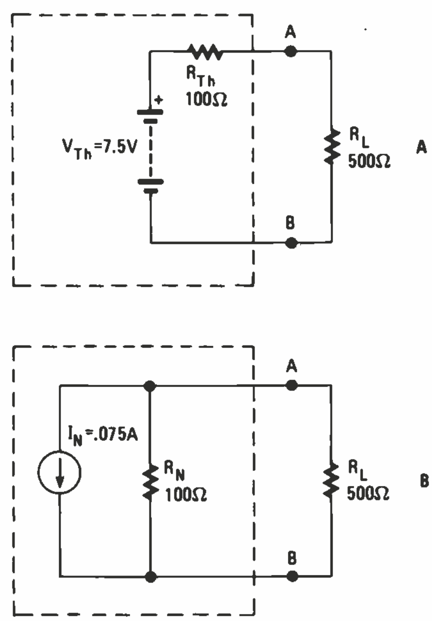

Fig. 7. The Thevenin's equivalent in A can be converted into a Norton's equivalent shown in B.

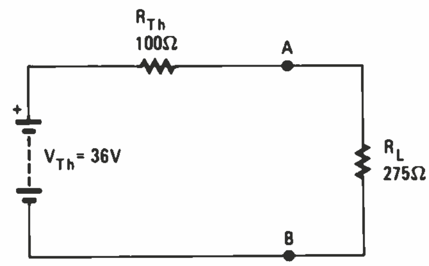

Fig. 8. Use the handy formulas in the text to convert this Thevenin's equivalent into a Norton's equivalent.

the currents and voltages to get their combined effect. Let's see how to do that in a circuit with two current sources.

Refer to Fig. 9A. We want to find the voltage drop across R2. Note that current source I_1 produces current flow in one direction through R2, while source I_2 produces current in the opposite direction.

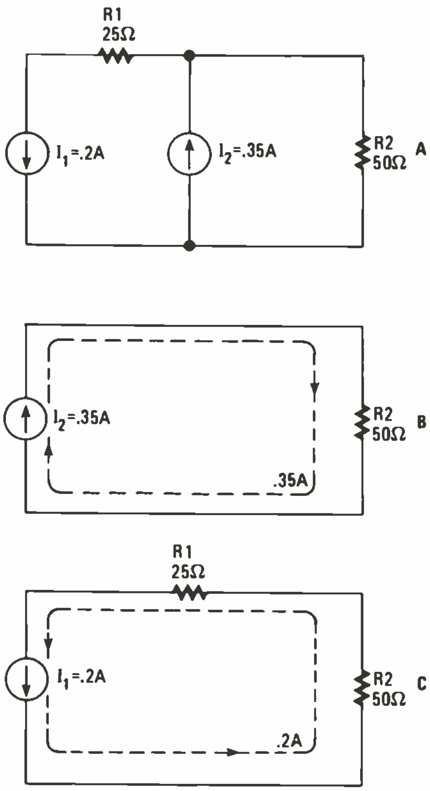

Fig. 9. When analyzing the circuit in A use the superposition theorem and ignore I_1 (as in B) first and I_2 (as in C) second and sum the results.

To solve the problem, we first disable I_1. To disable a current source, you remove it from the circuit and you leave the circuit open at that point. That's like replacing the current source with an infinite resistance. When working with voltage sources, you disable them with a short. You disable a current source with an "open." Anyway, the resulting equivalent circuit is shown in Fig. 9B. Note that R1 is removed since it is effectively not in the circuit. So the current in R2 is .35 Amp.

Now, we put I_1 and R1 back, and disable I_2. The resulting circuit is in Fig. 9C. The current in R2 then is .2 Amp, but in the opposite direction.

The composite current in R2 is then simply the algebraic sum of the two currents. Since the currents flow in opposite directions in R2, they oppose one another. So, the total current is the difference between the two currents, or:

$$.35 - .2 = .15 \text{ amp}$$

The direction of current flow is in the direction of the largest current. So in Fig. 9, the composite current in R2 flows from top to bottom.

Knowing the current in R2, we can find the voltage drop with Ohm's law:

$$V_2 = I_{R2} \times R2 = .15 \times 50 = 7.5 \text{ volts}$$

Exercise Problem. See how easy the process is by doing it yourself.

4. Find the current in R3 in the circuit in Fig. 10.

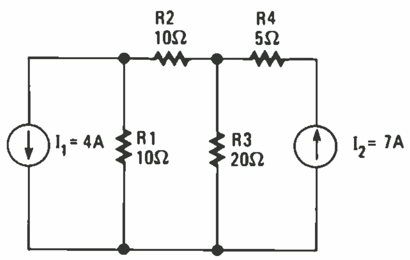

Fig. 10. Make use of superposition when analyzing this circuit.

Practical Current Sources. The basic characteristics of a current source are a constant output current regardless of the load resistance, and very high internal resistance.

Such a current source is difficult to realize with everyday electronic components. Therefore, we settle for a less than ideal current source which, in most cases, will do the job if we restrict the range of loads over which the current source must work. In practical applications, that is usually not a problem. With that in mind, we can now take a look at some of the ways that real-world current sources are implemented.

Voltage Source with Resistance. The easiest way to make a current source is to connect a very large value of resistance in series with a voltage source as shown in Fig. 11. The value of the resistance is made very high compared to the load resistance. In order for the circuit to act like a current source, the internal resistance, R_S, should be at least one hundred times the load resistance. If we assume a load resistance of 100 ohms, then the current-source resistance should be at least 10,000 ohms. In the example, we use an internal resistance of 1 megohms along with a voltage source of 15 volts.

We can use Ohm's law to figure out what the constant current supplied by the current source is. The load current, I_L, is:

$$I_L = V_S/(R_S + R_L)$$

Because R_L is so much lower than R_S, we can virtually ignore its effect. Therefore, the current supplied is very nearly equal to:

$$I_L = V_S/R_S = 15/1{,}000{,}000$$
$$= 15 \text{ microamperes}$$

If we should increase the load resistance from 100 ohms up to 10,000 ohms, suddenly the load resistance now becomes a much larger portion of the total-circuit resistance. Therefore, its effect must be taken into consideration. The total-circuit resistance in that case is:

$$1{,}000{,}000 + 10{,}000 = 1{,}010{,}000 \text{ ohms}$$

We can calculate the load current as before with Ohm's law:

$$I_L = 15/1{,}010{,}000 = 14.85 \text{ microamperes}$$

As you can see, the current in the load is less than the desired 15 microamperes, however, it is very close. For most applications, that value would be adequate. In fact, the current is actually 99% of the current with a zero ohm load. A 1% error is tolerable in most electronic circuits.

To maintain the current so that it is 99–100% of the constant current value, simply be sure that the load resistance is less than one-hundredth of the internal resistance of the source. In Fig. 11, with a 1-megohm source, the maximum value of load resistance is 1,000,000/100 = 10,000 ohms or 10,000

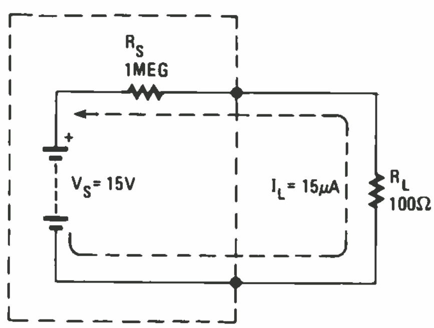

Fig. 11. If the load is restricted to a certain range, a voltage source with a resistor can be used as a simple constant-current source.

ohms. For values of load between zero and 10,000 ohms, the current will be within the 99–100% of the constant current value.

When designing a current source of that type, the job is to select supply voltages and series resistances that will provide the desired amount of current in the load. When very high values of current are required with high impedance, it is often necessary to use very high voltage values. That is usually impractical since most solid-state circuits use low-voltage power supplies. You can get the desired current by using a lower value of resistance, but often that resistance will be too low in value compared to the load to provide the constant-current effect. When that happens, some other form of constant-current source must be used.

Bipolar-Transistor Current Source. The simplest way to make a current source is to use a bipolar transistor as shown in Fig. 12A. A voltage divider made up of R1 and R2 applies a voltage, V_B, to the base of the transistor. That forward biases the emitter-base junction causing the transistor to conduct. Emitter current flows through resistor R_E. The amount of voltage across the emitter resistor is V_E. The value of V_E is equal to the base voltage less the voltage drop across the emitter-base junction, V_{BE}. Since V_{BE} is typically 0.7 volt in most silicon transistors, then V_E can be computed as follows:

$$V_E = V_B - 0.7$$

As an example, suppose voltage divider R1/R2 produces a base voltage of 5.7 volts. The voltage across the emitter resistor then is:

$$V_E = 5.7 - 0.7 = 5 \text{ volts}$$

Now assume that the emitter-resistor value is 2000 ohms. The emitter current then is:

$$I_E = V_E/R_E = 5/2{,}000 = 2.5 \text{ mA}$$

The emitter current flows into the emitter of the transistor, through the base, and into the collector. It then flows through the load resistance, R_L, to the supply. Remember in a high-gain transistor, the collector current is very nearly equal to the emitter current. In most cases, the base current that it uses up is extremely small and can be ignored. So, the collector current, being equal to the emitter current, provides a constant load current of 2.5 mA.

The way to set up a constant-current source is simply to choose the voltage-divider resistors R1 and R2 to provide the correct value of V_B and select a value of emitter resistance that will give the desired constant current with V_E. The nice thing about designing constant-current sources like that, is that you don't have to fool around with all of the transistor parameters such as

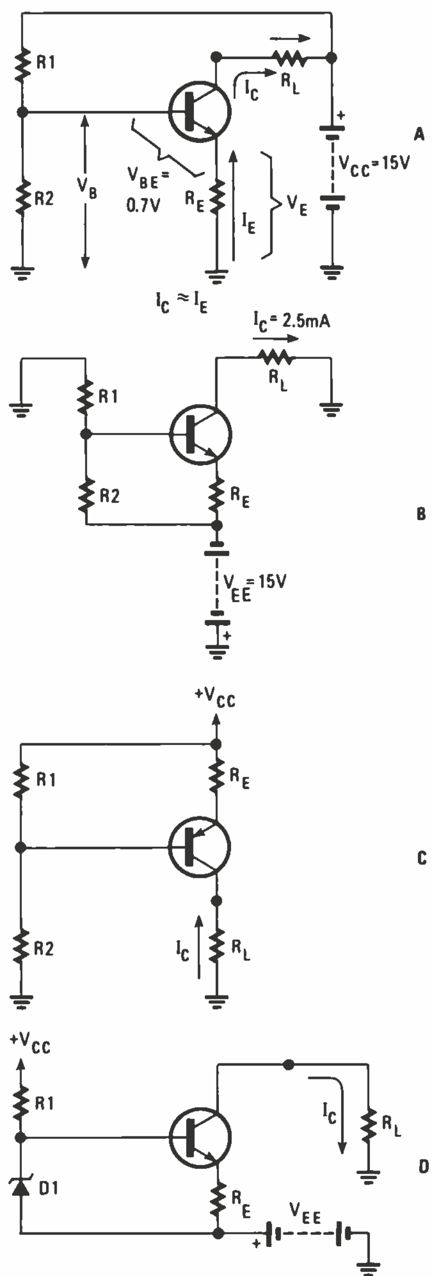

Fig. 12. The current source in A leaves the load floating. If the load must be grounded, the circuit in B can be used. If you also need the current to flow through the load in the opposite direction you can use the circuit in C. An improved current source with a Zener diode is shown in D. The diode sets the base voltage.

current gain. Because the circuit uses heavy negative feedback by way of the emitter resistor, the circuit characteristics are strictly a function of the external applied voltages and resistor values.

The circuit in Fig. 12A acts as a current source over a relatively wide range of load-resistance values. In the circuit, the load can be any value from zero up to approximately 4,000 ohms. By referring to the circuit, you can see why that is so.

When current flows through the circuit, 5 volts is dropped across the emitter resistor, some voltage is dropped across the emitter-collector connection of the transistor, while the remaining voltage is dropped across the load. The total of the three voltages must add up to 15 volts to satisfy Kirchhoff's law. As R_L is made higher and higher, the voltage redistributes itself and soon the relationship no longer holds true. For example, if the load resistance is made 10,000 ohms, the transistor would ordinarily try to force 2.5 milliamperes through it. That, of course, would produce a voltage of 25 volts across R_L. Since the supply voltage is limited to only 15 volts, naturally the circuit won't work. So even though you are limited to a narrow range of load-resistance values, again that is usually not a problem in most electronic circuits.

One of the problems with the circuit in Fig. 12A is that the load is floating. That is, one end is connected to the collector and the other end is connected to the positive terminal of the supply voltage, V_{CC}. In many applications, the load must be grounded. The problem can be corrected by simply rearranging power supplies and grounds in the circuit.

A different version of the circuit is shown in Fig. 12B. Here, nothing has changed except that we have switched from a positive supply voltage to a negative supply voltage connected to the emitter. The resistor values and the constant-current amplitude are the same.

If you need a grounded load, but the current must flow through the load in the opposite direction to that shown in Fig. 12B, you can use the alternate circuit shown in Fig. 12C. That circuit uses a PNP transistor, but otherwise all the resistor values are the same. With that arrangement, current now flows through the load in the opposite direction.

An improved current source is shown in Fig. 12D. That circuit is similar to that shown in Fig. 12B, but a Zener diode, D1, is used to set the base voltage. Resistor R1 sets the bias level through the diode. In critical applications requiring a very constant current, a Zener diode provides a stabilized voltage at the base of the transistor. That ensures that the output current remains constant despite circuit variations. In temperature-critical circuits, one or more standard silicon diodes are usually connected in series with the Zener diode to provide temperature compensation for both the Zener and the emitter-base junction of the transistor.

FET Current Sources. Field-effect transistors make ideal current sources because the drain current remains very constant with wide variations in source-to-drain voltage. That means that you can connect a wide range of load resistances to a basic FET circuit and maintain a constant current through it.

The basic FET constant-current source is shown in Fig. 13A. Here, an N-channel FET is connected so that its source and gate are shorted together. The drain is connected to the load resistance, R_L. The supply voltage, V_{DD}, completes the circuit. With that arrangement, the constant current supplied through the load is equal to the I_{DSS} current value of the FET. Most FET's have an I_{DSS} value in the zero to 10 mA range. By selecting an FET whose I_{DSS} value is the desired constant current, then the simple circuit in Fig. 13A can be used.

Again, the circuit in Fig. 13A contains a floating load. You can rearrange that circuit as shown in Fig. 13B so that the load is grounded. The gate and source are still shorted together, but a negative-source supply, V_{SS}, is used instead of the positive-drain supply as in Fig. 13A. By using P-channel FET's and making other changes in the power supplies, the direction of current flow through the load can generally be anything desired.

Levels of constant current different from the I_{DSS} FET value can be obtained by connecting a resistance in series with the source as shown in Fig. 13C. With the resistor R_S inserted, the constant current supplied to the load will be some value less than I_{DSS} depending upon the value of R_S selected. The higher the value of R_S, the lower the constant current in the load.

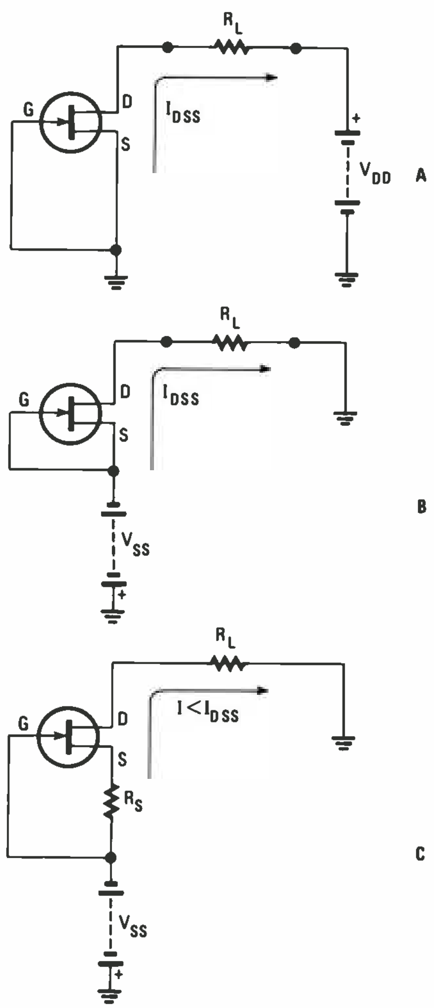

Fig. 13. FET current sources are very common now. Shown are the floating load type (A), grounded load type (B), and a circuit that can be set depending on R_S.

Pre-packaged FET current sources are also available. Those two-terminal devices are often referred to as constant current diodes or current-regulator diodes. They consist of an N-channel or P-channel junction FET with the gate and source connected together and/or with an appropriate source resistor. They are available in a wide range of current values. Such a component provides a convenient means of obtaining a constant-current source in a single two-lead package. The schematic symbols often used to represent constant current diodes are illustrated in Fig. 14. Simply connect one of the diodes in series with the load to the voltage source to provide the desired constant-current value.

Op-amp Current Sources. You can also use an IC op-amp as a current source as shown in Fig. 15A. That configuration is the familiar non-inverting amplifier circuit. Ordinarily an input voltage, V_i, is applied to the non-inverting (+) input and the output is taken from the op-amp output and is designated V_O. That circuit can be used as a current source simply by connecting the load resistance, R_L, between the output and the inverting (−) input as shown in Fig. 15A. Since the op-amp inputs draw little or no current, then the current through the load resistance is equal to the current through the input resistor R1. Because of the feedback provided from the output of the op-amp back to the inverting input by R_L, V_O in the Fig. 15A is a virtual ground. That means the voltage across R1 is essentially equal to V_i. Because of that, you can now calculate the amount of current supplied to the load. That is:

$$I = V_i/R1$$

For example, if V_i is 6 volts and R1 is 4700 ohms, then the constant current through R1 is:

$$I = 6/4700 = 1.28 \text{ mA}$$

That current will also flow through the load.

Fig. 14. Constant-current FET diodes are depicted in either of the two ways shown here. They are treated as diodes.

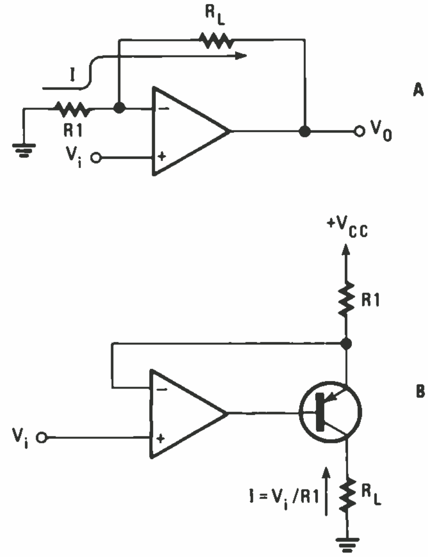

Fig. 15. You can also use an IC op-amp as a current source as shown in A That configuration is the familiar non-inverting amplifier circuit. If you don't want the load to float then the circuit in B is useful.

In that circuit, the load is floating. Since that is often a disadvantage in some circuits, some means is usually required to use a grounded load. One arrangement is shown in Fig. 15B. A PNP transistor is connected to the op-amp output. The load is connected between the collector and ground. The output current is still equal to the value of the input voltage divided by R1. Other arrangements of NPN transistors and different power supply polarities will permit any desired output-current direction to be achieved.

So what do you do with a constant-current source? There are lots of different applications, but there are some widely used ones with which you should be familiar.

LED Driver. Current sources are often used as driver circuits for light-emitting diodes. To cause an LED to turn on, it must be forward biased so that a certain level of current passes through it. The brightness is directly proportional to the current value. When current flows through the LED, a voltage appears across it. For different LED's, that voltage can vary from approximately 1.7 volts to 2.5 volts for a single value of current.

One common way of driving an LED is to use a transistor switch as shown in Fig. 16A. Whenever a positive voltage is applied to the base of the transistor,

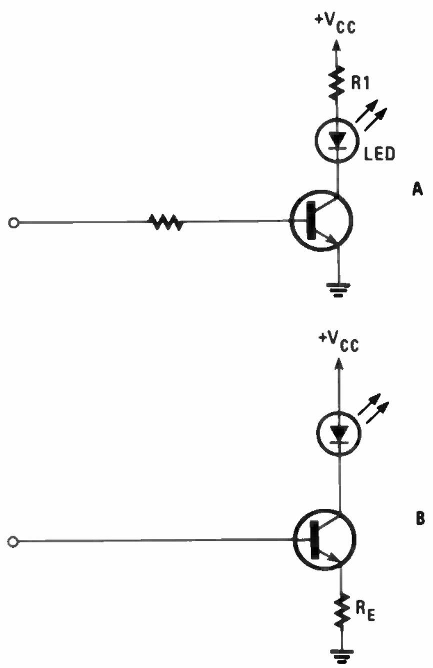

Fig. 16. LED drivers such as voltage drivers (A) and current drivers (B) can be made with one transistor.

the transistor turns on and acts as a very low value of resistance. It effectively grounds the cathode of the LED, turning it on. Resistor R1 is chosen to drop the supply voltage down to a voltage suitable for the LED. However, in using a voltage source to provide a particular LED voltage drop, LED's of the same type may have varying brightness levels because the current will be different in each, depending upon the value of R1 selected. As an example, assume multiple LED's are used to display a binary number. It is esthetically pleasing for all LED's to have the same brightness. The circuit in Fig. 16A doesn't guarantee it.

An improved method of driving an LED is to use a current source as shown in Fig. 16B. Whenever a positive voltage is applied to the base of a transistor, a voltage is developed across the emitter resistor, R_E. That sets the amount of current supplied to the LED. When multiple circuits are used, the current will be the same in each LED. While the voltage drops may vary, the brightness will be consistent.

Differential Amplifiers. A differential amplifier is an amplifier with two inputs and a single output. The output voltage is the difference between the two input voltages multiplied by the gain. A typical differential amplifier is shown in Fig. 17A. Bias is provided to the emitter-base junctions by way of the emitter resistor, R_E, and the emitter supply voltage, $-V_{EE}$.

In order for that circuit to perform correctly, the current supplied by $-V_{EE}$ and R_E must be equally divided between transistors Q1 and Q2. The currents are not always equal because of differences between the emitter-base voltage drops, transistor current gains, and the driving impedances of the inputs V_1 and V_2. Further, the input impedances of V_1 and V_2 should be made as high as possible. They are dependent on the value of the emitter resistor.

The basic approach to achieving those goals is to make the negative-supply voltage and the value of R_E as large as possible. But in practical circuits, there are limits. That is particularly true when differential amplifiers are made in integrated-circuit form as most of them are.

To overcome those problems, most differential amplifiers use a constant-current source in place of R_E. That is illustrated in Fig. 17B. Transistor Q3 is the

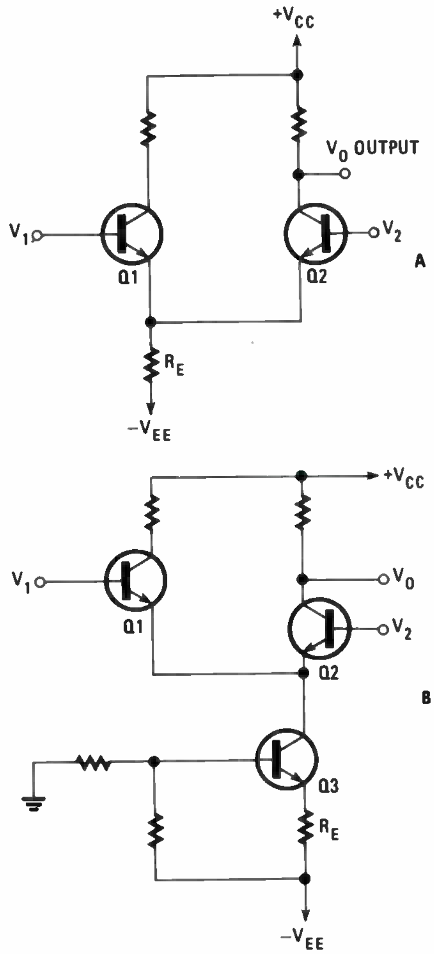

Fig. 17. A differential amplifier such as the one in A, can be improved if it receives constant current from the source. The circuit added to it in B provides it with constant current.

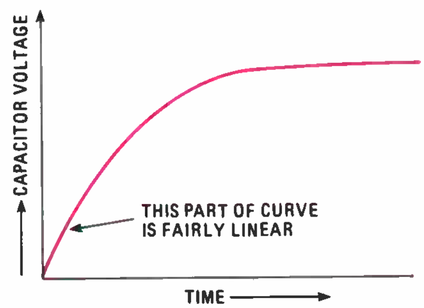

Fig. 18. As you can see, capacitors hardly charge in a linear fashion.

current source as described p' viously. It is biased by a voltage divi in that circuit, but a Zener diod other biasing arrangements may used in more critical application any case, the constant-current sour greatly improves the performance of differential amplifier. Most differenti amplifiers used in integrated-circu

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op-amps and other linear devices use constant-current sources.

Ramp Generation. Another common application for a constant-current source is in the generation of a linear (straight line) voltage ramp. A voltage ramp is required whenever it is necessary to generate a sawtooth or triangular waveform. Such signals are usually produced by charging (or discharging) a capacitor through a large resistance. A typical charge curve is shown in Fig. 18. Because of the exponential shape of the capacitor charge curve, the output is naturally not linear. One way to overcome that problem is simply to use only a narrow portion of the charge curve as shown in Fig. 18. The lower part of the curve is relatively linear but that restricts the output-voltage swing. Improved swings can be obtained by using very high DC voltages and very large values of charging resistance. However, that is often impractical in solid state circuits. The way to overcome that problem is to charge the capacitor with a constant-current source.

The voltage across a capacitor is directly proportional to the charging current (I) and the length of time (t) that the charging takes place, and is inversely proportional to the value of the capacitor (C). Expressed mathematically, we find that the capacitor voltage, V_C, is:

$$V_C = It/C$$

For a fixed value of capacitance and a constant current, I, you can see that the capacitor voltage will rise linearly with respect to time.

For example, if a .2 µF capacitor is allowed to charge for 30 milliseconds with a current of .1 mA, the output voltage will be:

$$V_C = .1 \times 10^{-3}(30 \times 10^{-3})/.2 \times 10^{-6}$$
$$= 15 \text{ volts}$$

With that arrangement, the voltage across the capacitor will rise in a straight line from 0 to 15 volts.

The key to making that relationship work is a constant current to charge the capacitor. A typical circuit for doing that is illustrated in Fig. 19A. Here the PNP transistor Q1 provides a linear charging current for the capacitor. The

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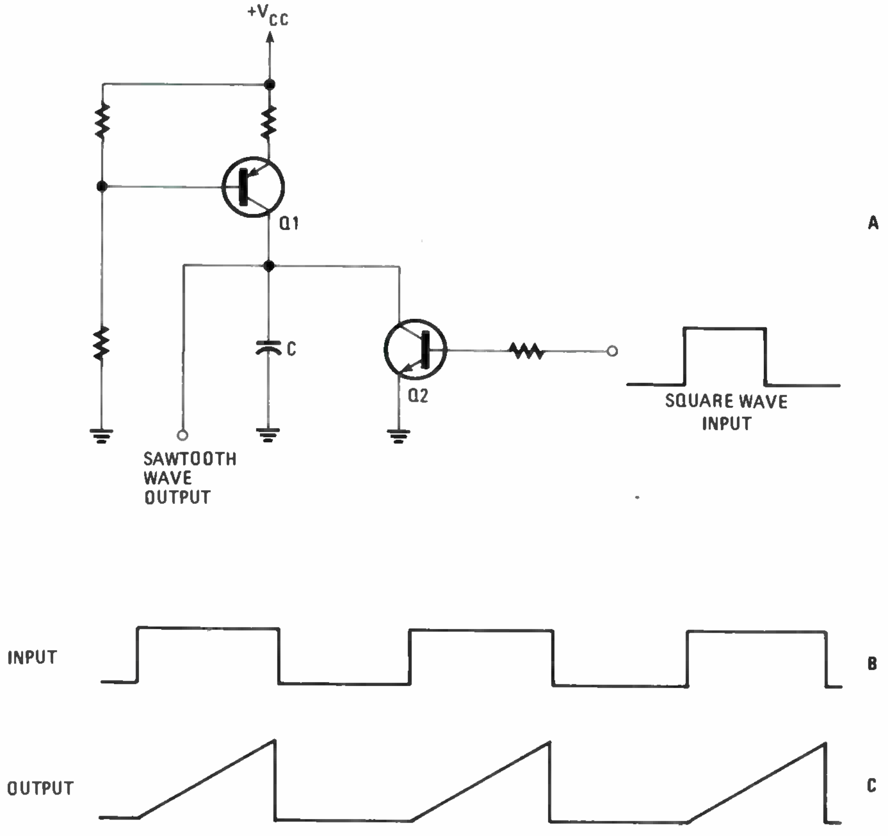

Fig. 19. If the constant-current source (A) charges the capacitor based on the square wave input (B), the output is a sawtooth wave (C).

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voltage across the capacitor will continue to rise until the power supply limitations are exceeded. Usually a transistor switch Q2 is connected across the capacitor to discharge it prior to that point being reached. A square wave used to drive Q (see Fig. 19B) produces a sawtooth output waveform, as shown in Fig. 19C. By using two current sources, one positive and one negative, triangular waves can also be developed. ■

Solutions to Exercises

1. $I_N = 12$ mA, $R_N = 600$ ohms.
See Fig. 20.

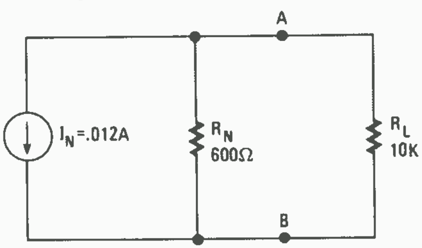

Fig. 20. This is what your solution to exercise 1 should look like.

a. Remove R_L and short A and B.
b. Calculate short current, which is current in R2.
c. The current in R2 is I_2:

$I_2 = V_S/R2 = 12/1000 = .012$ A or 12 mA

Therefore,

$I_N = R2 = 12$ mA

d. Remove short between A and B.
e. Short the voltage source.
f. Calculate resistance between A and B. That is R_N:

$R_N = R2R3/(R2 + R3)$
$R_N = 1000(1500)/(1000 + 1500)$
$R_N = 1{,}500{,}000/2{,}500 = 600$ ohms

The Norton's equivalent is shown in Fig. 20.

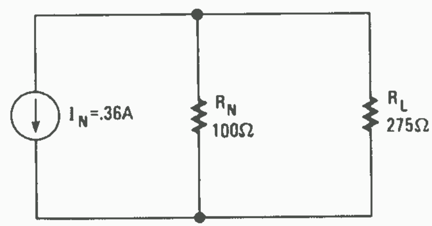

Fig. 21. The Norton's equivalent to problem 3 looks like this.

2. $V_{Th} = 7.2$ volts, $R_{Th} = 600$ ohms
a. $V_{Th} = I_N \times R_N = .012 \times 600 = 7.2$ volts
b. $R_{Th} = R_N = 600$ ohms.

See Fig. 20.

3. $I_N = 360$ mA, $R_N = 100$
a. $I_N = V_{Th}/R_{Th} = 36/100 = .36$ A or 360 mA
b. $R_N = R_{Th} = 100$

See Fig. 21.

4. $I_{R3} = 2.5$ A
a. Disable I_1.
b. Current I_2 flows through resistor R4, then divides, with some of the current flowing in resistors R1 and R2 and the remainder in resistor R3. Since:

$R1 + R2 = 20$ ohms,

and

$R3 = 20$ ohms

I_2 divides equally with 3.5 A in each branch including R3:

$I_{R3} = 3.5$ A

c. Replace I_1.
d. Remove I_2.
e. I_1 flows in R1 and through the combination of R2 + R3. The total resistance of the circuit is R1 in parallel with R2 and R3 in series. Resistor R4 is effectively out of the circuit, so:

$R2 + R3 = 10 + 20 = 30$ ohms
$R_T = 10(30)/(10 + 30)$
$R_T = 300/40 = 7.5$ ohms

f. I_1 produces 4 A in 7.5 ohms. That gives a voltage across R1 of:

$V = I_1 \times R$
$V = 4 \times 7.5 = 30$ volts

g. That 30 volts appears across the R2 + R3 combination of 30 ohms so the current that flows in R3 is:

$I_{R3} = V/R2 + R3 = 30/30 = 1$ A

h. The total current that flows in R3 is the difference of the two currents calculated previously, or:

$I_{R3} = 3.5 - 1 = 2.5$ A ■