Doppler Effect

Fundamentals of the phenomenon as it occurs in sound waves

by 'Cathode Ray'

Just as the foreigner who wants to know the English words for a container, for the activity of a pugilist with his fists or of a mariner with his compass, for the most expensive seats at the opera or the free seats for the jury in the courtroom, for a shrub used for edging, for a Christmas gift for the postman, for a yellow check pattern at busy urban crossroads, and for a cottage used by huntsmen, is astonished to learn that they are all 'box', so some of the readers of *Wireless World* and similar (though of course inferior) literature may be somewhat bewildered by the appearance of the Doppler effect in a variety of quite different contexts. It crops up in radar, in loudspeaker distortion, in radio-wave propagation studies and in astronomy — both radio and optical varieties. Almost invariably, if any explanation at all for the term is vouchsafed, one is told that it is what causes the drop in pitch of the sound emitted by a rapidly moving vehicle as it goes past. Even if the writer goes so far as to offer some explanation of why this happens it is usually so brief (in order not to divert too much attention from his main theme) as to leave plenty of room for misunderstanding.

For instance, one can easily get the idea that the pitch-drop is due to the change in the speed with which the sound waves reach the listener; faster when the source of sound is approaching, and slower when it is going away. After all, that is what happens with bullets fired from a moving vehicle. And it is usually taken for granted that the Doppler effect in radio and light waves is the same in principle as with sound, and that exactly the same formulae apply, when allowance is made for the difference in wave speeds.

Even Doppler himself seems to have lacked a completely correct grasp of the effect named after him. He, by the way, was an Austrian physicist who lived from 1805 to 1853 and first pronounced upon his effect in 1842. Basically he had the right idea, but he went much too far in crediting it with the differing colours of the stars. (Temperature is mainly responsible for that, though of course Doppler effect is of very great importance to astronomers, but it appears as a shift in the positions of the spectral lines on the frequency scale.)

Confirmation of the Doppler theory by experiment was carried out (not by Doppler) in 1845. We can pleasurably visualize the modus operandi as described by Alexander Wood in his book *'Acoustics'*: 'The first experiments were carried out by Buijs Ballot on a single-track railway between Utrecht and Maarsen. A trumpet was carried on the locomotive and three others were used by groups posted at the side of the track. The trumpets were sounded alternately on the locomotive and at the side of the track, and the apparent change of pitch was observed both for a moving source and for a moving observer by musicians whose estimate of small intervals of pitch was considered to be reliable.'

To keep things simple — or at least not so complicated as they could be — let us assume two things. One is that all the speeds to be considered are uniform, or constant. The other is that all movements are directly towards or away from the observer — or listener, if you prefer. Short of actual suicide, anyway. That is to ensure that the observed rise or fall in pitch will itself be constant. If the track of the source of sound passes wide of us the change in pitch is gradual. During 1944-5, when we were cast unwillingly in the role of observer in experiments carried out by the Nazis, we became quite clever at judging whether there was any need to dive for shelter when a V1 (flying bomb or doodlebug) was heard approaching. An early and gradual drop in pitch of the guttural note, and one could carry on unconcernedly — 'I'm all right, Jack'. A constant and rapidly swelling note, and one got underground at the double, or even treble.

First, let us suppose that we, the observers, are stationary, and so is the air — there is no wind. A source of sound of constant frequency f hertz (Hz) is also stationary, some distance away. It might be easiest to suppose it is a motor bike with the engine running in neutral, so that the sound is in the form of pulses and we can draw the peaks of the air-wave pulses as circles surrounding the source (Fig. 1). (This is a variant of the stone dropped in the pond, beloved by writers of elementary books on the principles of radio.) The sound pulses are numbered in the order in which they are emitted; No. 1 is just assaulting our ears at the instant depicted, when No. 9 is on the point of emerging from that part of the machine they call the silencer. The radial distance between successive peaks is the wavelength of the sound, denoted by λ as in radio. If the speed or velocity of sound in metres per second (m/s) is called V, then in one second f waves or pulses have spread out over V metres. So the length of each of them (λ) is V divided by f:

$$\lambda = \frac{V}{f}$$

But you can forget about that for a little while, because the pitch of a sound depends on its frequency, not its wavelength. If you have any doubt about that, don your skin-diving kit and listen to a constant-frequency sound under water,

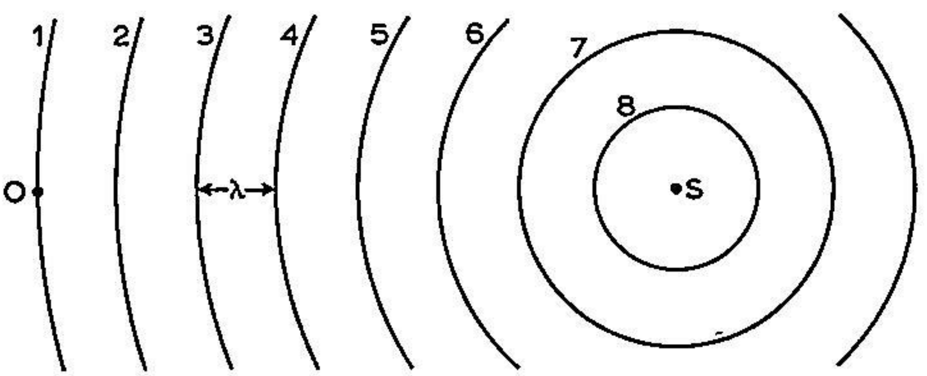

Fig. 1 The first few wave peaks emitted from a stationary source of sound, S. The first of them has just reached a stationary observer O.

in which sound travels more than four times as fast as in air so the waves are more than four times longer. Yet the pitch is the same as in air.

In the conditions assumed, the sound waves reach our ears with the same frequency as they were emitted from the source. Obviously. Mere distance has no effect on the pitch of the sound we hear, though it mercifully reduces its intensity. The statement just made, though plausible, is however not quite accurate. And I don't mean the 'mercifully' bit, which betrays that my motor-cycling days are over, and might be contested by those for whom they are not. The fact is that pitch is a subjective thing (that is to say, is produced by our own senses, nerves and brain) and depends slightly on loudness as well as on frequency. But, like the trumpeters of Utrecht, let us ignore that complication and assume that pitch is a measure of the frequency of the sound reaching the ear.

Next, keeping our fixed distance from the revving engine, let us suppose that a steady wind springs up. If it blows direct from the source to us, then its speed (call it v_m, for velocity of the medium, the air) is added to V, so that the sound waves come to us at a speed of $V + v_m$ m/s. The time they take, though shorter than with no wind, is the same for all the waves, so the frequency with which they reach us is the same as that with which they were generated, whatever the direction and speed of the wind, so long as it is steady.

Now let us suppose that the motor bike is coming straight for us at a speed of v_s m/s, and that its rider is cleverly managing to keep its engine revs the same as when it was standing still. The important thing to keep in mind is that once a sound has been committed to the air its progress is in no way affected by what its source then does, whether it stays still or moves. So nothing about wave No. 1, emitted when the machine was instantaneously in the same position as in Fig. 1, has changed. But by the time No. 2 starts, $1/f$ seconds later, the source has moved nearer. The distance nearer is equal to speed × time, v_s/f metres, represented in Fig. 2 by the distance between S positions 1 and 2. So wave-front No. 2 is drawn from that new centre, 2. Similarly for the other six shown. Because the waves are closer together along the line of motion towards us at O, they reach us at shorter intervals of time. In other words, to us their frequency is higher than f.

With the stationary source, the wavelength, as we have already noted, was V/f metres. Now we have found that in our direction it is shorter by v_s/f metres. So as far as we are concerned the wavelength is not V/f but $\frac{V}{f} - \frac{v_s}{f}$

The corresponding frequency, which we will call f', is V divided by the wavelength:

$$f' = V \div \frac{V - v_s}{f} = \frac{fV}{V - v_s}$$

If the motor bike happens to be rather unusually fast, so that $v_s = V$ (a condition known as Mach 1 among aviators, who do this sort of thing and think little of it)

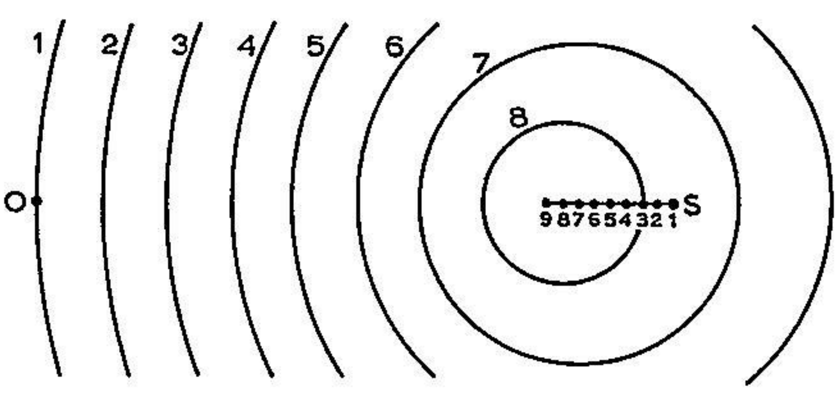

Fig. 2. In this situation the observer is still stationary, but the source is moving towards O at constant speed v_s.

the wave starting-point scale stretches all the way from S to O, and what we observe is nothing at all until the arrival of wave No. 1. Its arrival is likely to be somewhat blurred by the arrival of all the other waves simultaneously *and* the source itself. That could well bring our observations to an abrupt end; but if the rider manages, in spite of his hurry, to achieve a near miss, we get nothing worse than a sonic boom. Looking back at the formula we see that making $v_s = V$ drives f' to infinity, which is another way of putting it.

If a source of sound *exceeds* Mach 1 ($v_s > V$), then it arrives before any of its sound, which then follows in reversed order in time, corresponding to negative values of f'. After a few months to enable us to study the better-known aspects of the Doppler effect by means of the V1, the Nazis extended our course of study into its more bizarre manifestations by organizing the supersonic V2. The fact that it arrived before any of its associated sound precluded one from taking evasive action, but enabled those not on target, if they were interested, to note the unusual acoustical accompaniments.

When the sound source has gone past, the sign of v_s is of course reversed, so the apparent frequency (if we are still alive to hear it) is

$$f' = \frac{fV}{V + v_s}$$

and this is clearly lower than f.

To take some reasonable figures, suppose f is 50 Hz, corresponding to 3000 r.p.m. with a single-cylinder two-stroke. At 15°C and normal atmospheric pressure, V is 342 m/s. If the speed is 45 m.p.h., v_s is 20.1 m/s. So when v_s is towards us, $f' = (50 \times 342)/(342 - 20.1) = 53.15$ Hz. This drops to 47.25 Hz when v_s is away from us. The sound we hear is thus about 6% sharp or flat, amounting to about a semitone, according to whether its source is coming or going. So the total drop in pitch is about a whole tone.

If now we reverse the procedure, moving ourselves at 45 m.p.h. past a standing noise maker, we might expect exactly the same thing to happen. The relative speed between the source of sound and ourselves is the same. In the diagram, Fig. 3, the wave fronts are of course the same as in Fig. 1. The corresponding positions of the observer — us — are shown on the left. After a period of one wave, during which time wave No. 2 moves to the shown position of No. 1, we are no longer there but have already been passed by it on our way to observer position 2. So the *observed* time interval between waves 1 and 2 — and between every succeeding pair of waves — is less than the interval between them at the source. So the observed frequency is higher, as expected. And if we tried it we would again notice a drop of about a whole tone as we passed the source of sound. But to make sure let us again go through the calculation in detail.

Because we are moving towards the source of the sound, relative to us the speed of the sound is $V + v_o$, where v_o means the speed of the observer. Frequency being velocity divided by wavelength, the frequency to us is not just V/λ but $(V + v_o)/\lambda$. And λ, as we have noted, is V/f.

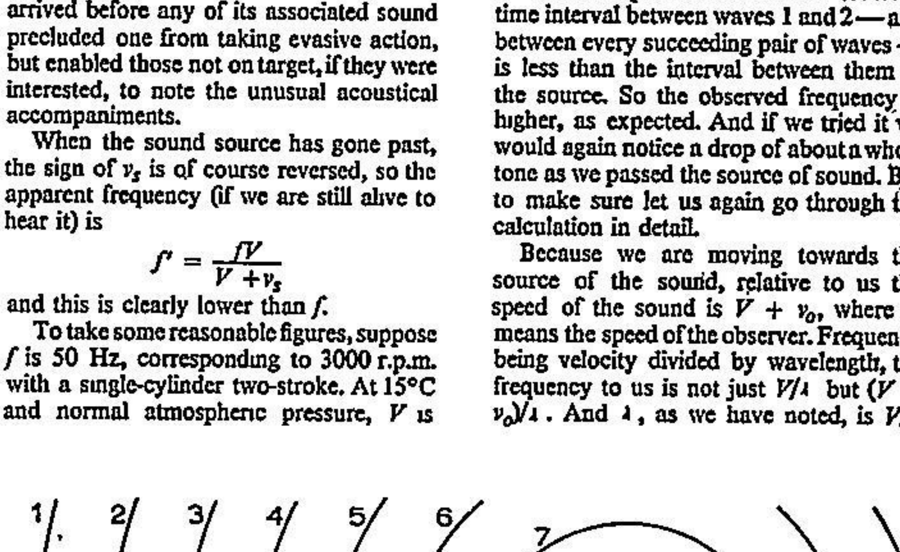

Fig. 3. Here it is the source that is stationary and the observer moving towards it at constant speed v_o.

So the observed frequency, f', is now

$$\frac{f(V + v_o)}{V} \text{ or } f\left(1 + \frac{v_o}{V}\right)$$

If we compare this with the result for the moving source we can see it is not the same. Using the same values as in our example, with v_o also 45 m.p.h. we find the observed frequencies are 52.95 and 47.05. These are equally above and below 50, whereas with the moving source the high frequency was 3.15 Hz above 50 and the low one 2.75 below. The total drop in Hz seems to be 5.90 in both cases however, and so they are, to sliderule accuracy. But if we took the trouble to be more precise we would find a difference of about 1 in 3000 — hardly enough to be noticeable even to the reliable musicians of Utrecht (or anywhere else). And if we reckon the total drop in frequency as the ratio of the low to the high, it is *exactly* the same, being $(V - v_s)/(V + v_s)$ in one case and $(V - v_o)/(V + v_o)$ in the other. Musical intervals (of pitch) are such ratios, so even an ideal musician would always hear exactly the same interval for a given relative speed between him and the source of sound. But he would have to know which was moving in order to determine the absolute pitch of frequency of the sound at the source.

The inequality in the sharpening and flattening of the pitch of sound when it is the source that moves, quite small in our example, is much greater if the ratio of v_s or v_o to V is larger. When equal (Mach 1) f' goes up to infinity and down to zero.

It may still be puzzling some (especially those who are thinking of the magic word 'relativity' and are remembering that not long ago in connection with Fig. 1 we satisfied ourselves — I hope — that the speed, if any, of the air made no difference) why the Doppler effect for a given relative speed between source and observer should depend in the slightest degree on which was said to be stationary and which moving. After all, that is only a convention, depending on the 'frame of reference' (as Einstein called it) one chooses. In such matters as these one usually chooses the surface of the earth. But someone else would be perfectly entitled to choose the motor bike as the origin in his frame of reference, and say it was always stationary and the observer always moving. Who has never been confused when in a train in a station, looking at another train alongside?

The explanation is that the rule about movement of the air making no difference is true only when there is no relative movement between source and observer. Although we worked out the formula for Fig. 2 on the basis of a moving source, we could equally well have treated it as a variety of the Fig. 3 situation, the only difference being the existence of a 45 m.p.h. wind blowing from O to S so that we, the observers, would feel no breeze in our faces as we were swept along, and might almost imagine we were standing still.

If it hadn't been that I find it clearer, and believed you might too, to deal with one thing at a time, I could have saved a lot of space by following the usual textbook line and deriving one formula to cover all cases — moving source, observer and medium. It is

$$f' = f\left\{\frac{V \pm v_m \pm v_o}{V \pm v_m \mp v_s}\right\}$$

You use the upper of the + and − signs for $\left\{\begin{matrix} v_m \\ v_o \\ v_s \end{matrix}\right\}$ when the $\left\{\begin{matrix} \text{medium} \\ \text{observer} \\ \text{source} \end{matrix}\right\}$ is moving towards the $\left\{\begin{matrix} \text{observer} \\ \text{source} \\ \text{observer} \end{matrix}\right\}$. For Fig. 2, v_m and v_o are zero, and when we omit them from the above formula we get the one we found for that case. Similarly for Fig. 3, where v_m and v_s are zero. But if in Fig. 3 we choose to regard the observer as stationary, then v_o is zero and $v_m = v_s$, giving the same result.

The textbooks also go into the modifications that have to be made when motion is not directly towards and away from the observer, and into what happens when observation is via a moving reflector. This last condition has practical importance with radio waves, as you may have noticed if you failed to spot the police radar in time to slow down. But I think we'd better leave radar along with radio waves for another article. Meanwhile, lest it be thought that sound Doppler is completely out of place in these pages, I will briefly describe what it is liable to do to the reproduction of sound by moving diaphragms, as in almost all loudspeakers.

Suppose that in order to punish, in a manner befitting the offence, a motorist who was addicted to excessive use of his horn, he was condemned to sit in the driving seat, with his horn permanently on, while his car was mounted on a mechanism that kept on making it alternately surge forward with highly promising acceleration and then frustratingly retreat to the starting point. If we, in order to study his reactions, took up our position along the line of his oscillation, but well clear of its peak displacement, enough has already been said to enable us to forecast that reception of the horn's steady note would be marred by what in the context of gramophones and tape recorders would be called wow. It will perhaps help us to imagine what we would hear if, instead of his horn, the motorist was playing a flute solo or some other musical composition in the middle and upper ranges of frequency. It would be unpleasantly distorted.

This is roughly what takes place when programmes are reproduced by a single moving-coil loudspeaker. Although the amplitudes through which the diaphragm moves at the very lowest frequencies hardly compare with those in the unconventional punishment just described, they are vastly greater than at high musical frequencies — which nevertheless are much more audible. The *velocity* of the low-frequency movement could well be comparable. Frequency modulation of, say, a 1000 Hz note at 50 Hz is unpleasant.

That is one reason why in any fi system professing to be hi the reproduction of the full frequency range is usually divided up among two or more diaphragms, or the amplitude of movement is reduced by an impedance-matching enclosure, or both.

H.F. Predictions — May

The probability of a skywave path existing can be judged from the value of working frequency relative to HPF (10%) and FOT (90%). These curves are constructed by applying an ionospheric index to standard charts of ionospheric characteristics and are for practical purposes independent of equipment parameters. Actual communication however is only possible if the resulting signal-to-noise ratio exceeds a required threshold value. The number and variations of factors determining s/n ratio are such that it is not possible to draw generalized curves for the 10% and 90% probability of achieving all service thresholds. Individual cases would have the same form as the chart LUFs (which are the 90% curves for commercial telegraphy using several kilowatts, directional aerials and good sites assuming 100% skywave probability at all frequencies) but would be displaced vertically up or down. Circuit reliability is the product of skywave and threshold probabilities, for example at the intersection of FOT and LUF on the charts reliability of the stated service is predicted to be 81%