E-Z MATH

NETWORK ANALYSIS: SUPERPOSITION THEOREM

By Louis E. Frenzel

If you can understand addition, then understanding superposition will be a snap!

THIS MONTH WE'LL CONTINUE OUR DISCUSSION OF techniques for network analysis. Last month, we introduced Thevenin's theorem which is widely used to reduce a circuit to an equivalent voltage source. In this installment, we present the superposition theorem, which allows you to analyze circuits with two or more voltage sources.

Today many electronic circuits derive their power from two or more voltage sources. Analyzing such circuits is tricky using conventional techniques, but when you use the superposition theorem, the process is fast and nearly painless. Combined with Thevenin's theorem, the superposition techniques will give you an extra-powerful tool for simplifying and analyzing more complex circuits. Everything that you need is contained within this lesson including a brief review of Thevenin's theorem in case you missed it last month. So, let's begin.

The Superposition Theorem

Basically, what the superposition theorem says is that the current through a component or a voltage across it is a combination of the effects produced by the multiple voltage sources. More specifically, that current or voltage is the algebraic sum of the individual currents or voltages produced by the voltage sources acting independently on the component. That leads to a conclusion about how we might be able to implement the theorem on practical circuits. If we simply disable all but one voltage source, then compute the various currents and voltage drops, then repeat the process with the other voltage sources, the total voltage or current associated with a component is obtained by just adding the individual currents or voltages.

The process of the superposition theorem is as follows:

1. Disable all but one voltage source in the circuit. You do that by replacing it with a short circuit. Any internal impedance associated with the voltage source should be retained in the circuit.

2. Calculate the total-circuit resistance and the various currents through, and voltages across, each component using Ohm's and Kirchhoff's laws.

3. Repeat Steps 1 and 2 for each voltage source in the circuit.

4. Combine the currents or voltages for the desired component algebraically by adding them together. The result will be the desired current or voltage.

To see how that works, let's start with a simple example. Take a look at the circuit in Fig. 1A. It consists of two voltage sources, one of 5 volts and one of 12 volts. Those sources are connected to a pair of resistors that form a voltage divider. We wish to determine the output voltage at point A, the center of the resistors, with respect to ground. Using the superposition theorem, we first calculate the output voltage at point A first with the 5-volt source, V_1, connected. Then we make the same calculation with the 12-volt source, V_2, connected. The true output voltage is then simply the algebraic sum of the two voltages we calculated independently.

To begin the analysis of the circuit in Fig. 1A, first replace the 12-volt source with a short. The resulting circuit is shown in Fig. 1B. The result is simply a voltage divider across the 5-volt source. The output voltage at A with respect to ground is simply the voltage across the resistor R2. We can use the familiar voltage-divider formula to calculate that voltage. It is:

$$V_A = V_1R2/(R1+R2) = 5(3.3K)/(1K+3.3K)$$
$$= 5(3.3K/4.3K) = 5(0.767) = 3.84 \text{ volts}$$

The output voltage at A with respect to B then with V_2 shorted is simply +3.84 volts. See Fig. 1B.

The next step is to replace V_2 and remove V_1. The equivalent circuit for that is illustrated in Fig. 1C. The 5-volt source is replaced with a short. That leaves the 12-volt source V_2 driving the voltage divider made up of R1 and R2. In that case, the output voltage at A is the voltage across R1 rather than R2. We can still use the voltage-divider formula to find the output voltage, but the formula has to be rearranged. The output voltage then is:

$$V_A = V_2R1/(R1+R2)$$
$$= -12(1K)/(1K+3.3K) = -12(1K)/(4.3K)$$
$$= -12(.233) = -2.79 \text{ volts}$$

To complete the problem, we simply combine the two voltages we calculated independently. We do that by adding them algebraically:

$$3.84 + (-2.79) = 1.05 \text{ volts.}$$

As you can see, the output is a positive 1.05 volts.

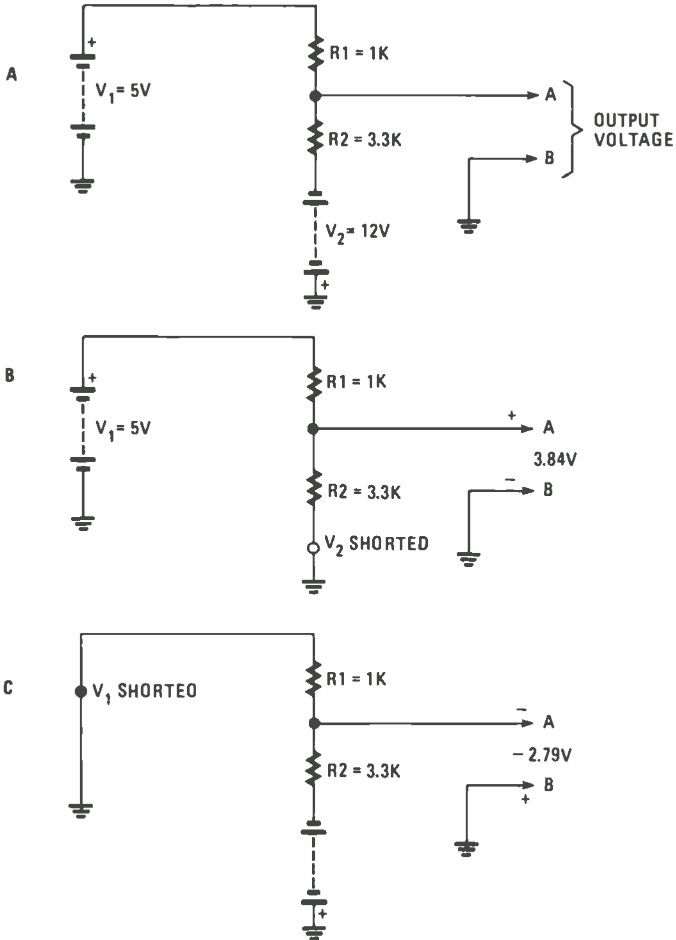

Fig. 1—The analysis of a circuit with more than one voltage source must be performed by examining the effects of each source independently with the others shorted.

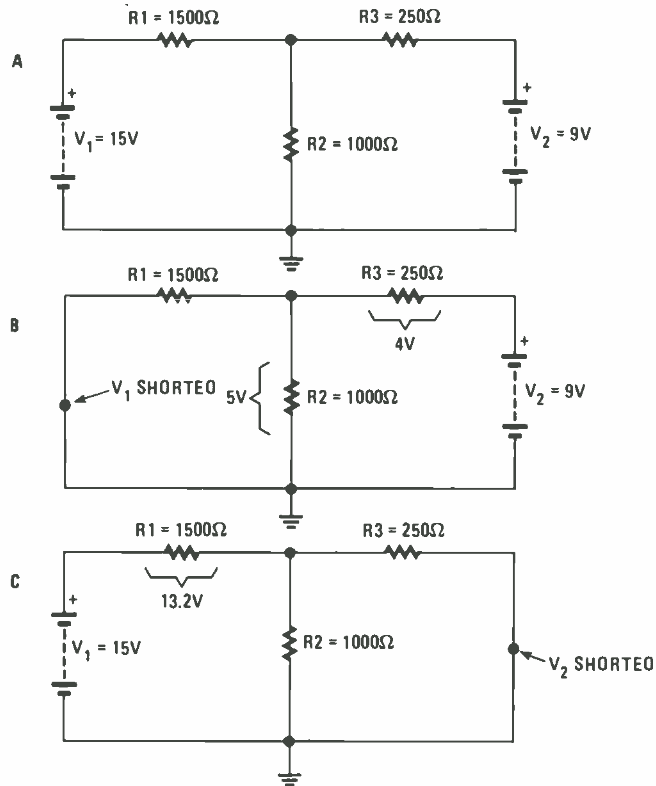

Fig. 2—Using the voltage divider relationship is common practice when analyzing circuits containing more than one voltage source since each source is shorted at some point.

Let's take another more complex example. Refer to the circuit in Fig. 2A. Again we have two voltage sources involved, therefore, the superposition theorem is needed. Our objective is to determine what the current is in resistor R2. One way to look at the circuit is as a voltage divider made up of R1 and R3 connected to two voltage sources. Resistor R2 is the load.

We can start by replacing V_1 with a short. That produces the equivalent circuit shown in Fig. 2B. Effectively R1 and R2 are in parallel and that combination is in series with R3 across the 9-volt source V_2.

For starters, we need to compute the total circuit resistance. That is done by first finding the parallel equivalent of R1 and R2. We use the familiar parallel resistor formula. Where the total resistance of R1 and R2 in parallel is designated $R_{1,2}$:

$$\begin{aligned} R_{1,2} &= R1R2/(R1+R2) \\ &= 1500(1000)/(1500+1000) \\ &= 1500000/2500 = 600 \text{ ohms} \end{aligned}$$

The total resistance of the circuit R_T then is simply $R_{1,2}$ in series with R3 or:

$$R_T = R_{1,2} + R3 = 600 + 250 = 850 \text{ ohms}$$

We can now calculate the total-circuit current I_T using Ohm's law:

$$I_T = V_2/R_T = 9/850 = .016 \text{ amperes}$$

Our objective is still to find the current through R2. We first find the current through R2 with V_2 applied and V_1 disabled. We can do that with standard Ohm's and Kirchhoff's law calculations. For example, knowing the total-circuit current, we can compute the voltage drop across R3:

$$V_{R3} = I_T R3 = .016(250) = 4 \text{ volts}$$

With 4 volts across R3, then by Kirchhoff's law we know that there must be 5 volts across R1 and R2. Remember Kirchhoff's law says that the sum of the voltage drops around the circuit is equal to the source voltage. In this case, the 5 volts across R1 and R2 adds to the 4 volts across R3 to give us 9 volts, the value of V_1.

Finally, we can calculate the current in R2 because we know the voltage across it and its resistance value:

$$I_{R2} = V_{R2}/R2 = 5/1000 = .005 \text{ ampere}$$

We now need to repeat the procedure, but with V_2 disabled. We replace V_2 with a short to produce the equivalent circuit shown in Fig. 2C. Now R3 is in parallel with R2. We again compute the total-circuit resistance. We do this by finding the parallel combination of R2 and R3 in parallel which we designate $R_{2,3}$:

$$\begin{aligned} R_{2,3} &= R2R3/(R2+R3) \\ &= 1000(250)/(1000+250) \\ &= 250000/1250 = 200 \text{ ohms} \end{aligned}$$

The 200-ohm equivalent resistance is in series with R1, producing a total-circuit resistance of:

$$R_T = R_{2,3} + R1 = 200 + 1500 = 1700 \text{ ohms}$$

The total-circuit current can be found by using Ohm's law:

$$I_T = V_1/R_T = 15/1700 = .0088 \text{ ampere}$$

That current is flowing through R1, therefore, we can find the voltage drop across it:

$$V_{R1} = I_T R1 = .0088(1500) = 13.2 \text{ volts}$$

The voltage across R2 can be found by simply subtracting the voltage across R1 from V_1 giving us 1.8 volts. Kirchhoff's voltage law says that the sum of the voltages around a circuit is equal to the source voltage. In this example we know the source voltage $V_2 = 15$ volts and one voltage drop that makes it up. To find the unknown drop, we subtract the known drop from the source voltage:

$$V_{R2,3} = V_2 - V_{R1}$$
$$= 15 - 13.2 = 1.8 \text{ volts}$$

With 1.8 volts across R2, we can now find its current, again by Ohm's law:

$$I_{R2} = V_{R2}/R2 = 1.8/1000 = .0018$$

Now we know the current in R2 produced by both sources independently. To find the total current in R2 then, we simply add the two currents algebraically. In the example circuit, both currents are flowing in the same direction, therefore, they will add rather than oppose one another. The total current in R2 then is:

$$I_{R2} = .005 + .0018 = .0068 \text{ amperes}$$

Example Problem

Now check your own understanding of this process. The problem below will give you a handle on the procedure. Don't look at the answer at the end of the article until you work the complete problem.

1. Refer to Figure 3. Calculate the current through R2.

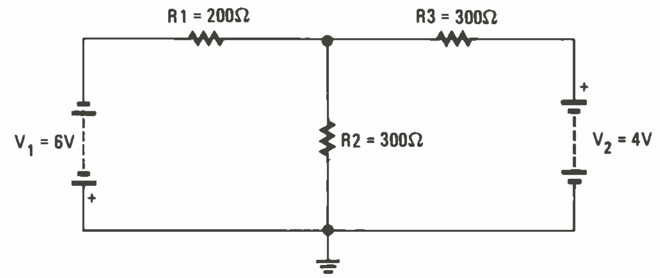

Fig. 3—Remember to follow the steps outlined when analyzing this practice example. Refer to the text as necessary but don't refer to the answer until you are done.

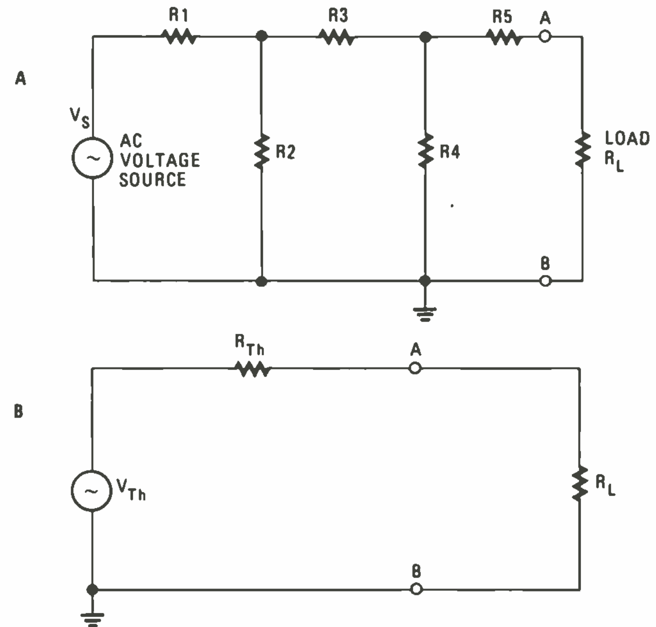

Fig. 4—No matter how complex a linear circuit is, the Thevenin's equivalent will simplify analysis.

Thevenin's Theorem Review

Thevenin's theorem is widely used to simplify electronic circuits for the purpose of analyzing their operation or designing them. Thevenin's theorem says that an entire network containing a voltage source plus various circuit elements (resistors, capacitors, and inductors) can be replaced by a single voltage source in series with an impedance called the Thevenin's equivalent voltage V_{Th} and resistance R_{Th}.

Figure 3A shows a complex network containing an AC voltage source, various resistive elements, and the load. That circuit can be replaced by the Thevenin's equivalent voltage and resistance as illustrated in Fig. 3B. With the Thevenin's equivalent in place, the same voltage appears across load terminals A and B as with the original circuit. Because the Thevenin's equivalent is much simpler, analysis of the load voltage and current is faster and easier to determine.

To translate a circuit into its Thevenin's equivalent is known as "Theveninizing" the circuit. The process of determining the Thevenin's equivalent is as follows:

1. Remove the load from the output terminals A and B in the original circuit of Figure 3A.
2. Calculate the voltage between terminals A and B without the load using standard Ohm's law and Kirchhoff's law techniques. That is the Thevenin's equivalent voltage V_{Th}.
3. Replace the voltage source V_S with a short. Then compute the total resistance between terminals A and B. That is the Thevenin's equivalent resistance R_{Th}.
4. Redraw the circuit consisting of the Thevenin's equivalent voltage source V_{Th} in series with the Thevenin's equivalent resistance R_{Th}. See Fig. 3B.

See last month's installment for further details.

Superposition with Thevenin

You have seen how the superposition theorem helps you to find the voltage or current in a circuit with two or more voltage sources. While the superposition method permits you to analyze those complex circuits, the calculations are still messy and time consuming. One way to simplify them further is to employ Thevenin's theorem along with the superposition method. By doing that, you can reduce your two voltage-source circuit down into an equivalent single voltage source with a series resistance. When analyzing the operation of the circuit with different values of load resistance, the process is helpful. An example will show what we mean.

Take a look at the circuit in Fig. 5A. It has voltage sources of 12 and 5 volts. The load resistor is R2, 500 ohms. Let's see how we can use the superposition method to produce the Thevenin's equivalent of this circuit.

First, we remove the load resistance from between terminals A and B. The remaining circuit shown in Fig. 5B is the one we will Theveninize. We begin by applying the superposition method of replacing one of the voltage sources with a short and calculating the load voltage. Replacing V_2 with a short produces the circuit in Fig. 5C. That is simply a voltage divider made up of R1 and R3 connected across the 12-volt source, V_1. The voltage between A and B (V_{AB}) is the voltage across R3. Using the voltage-divider formula:

$$V_{AB} = V_1(R3)/(R1+R3)$$
$$= 12(180)/(120+180)$$
$$= 12(180)/300 = +7.2 \text{ volts}$$

Note the voltage is positive because V_1 is positive with respect to ground.

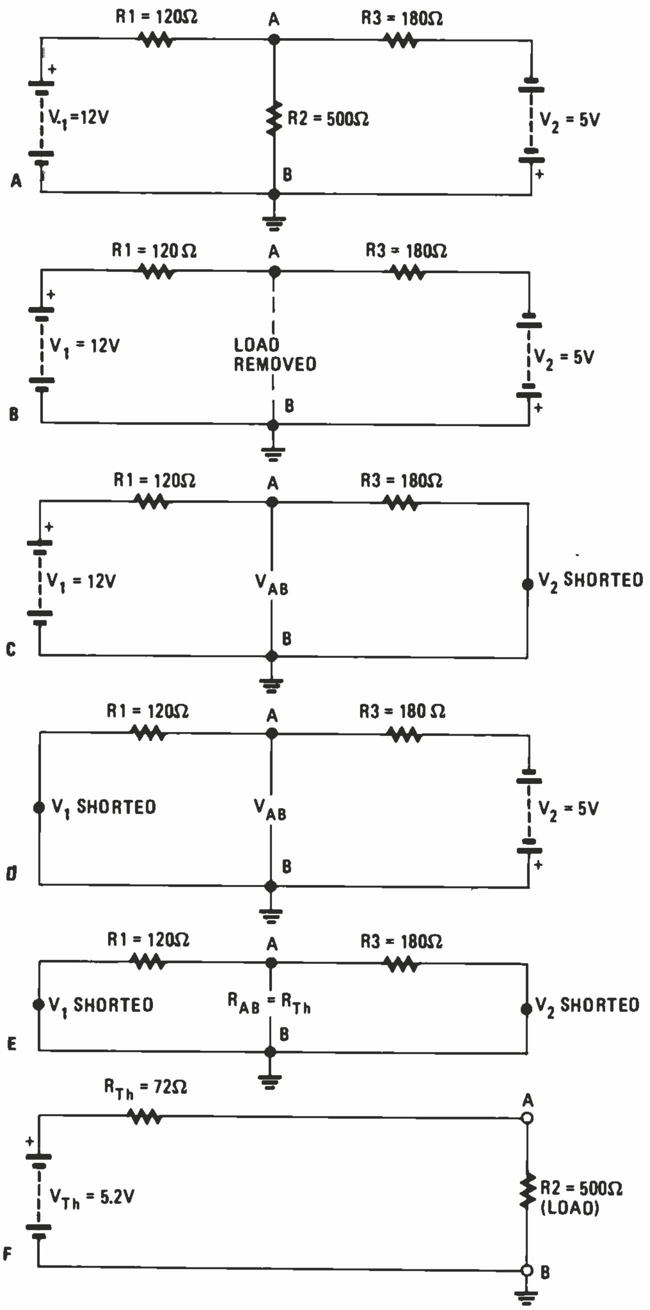

Fig. 5—Superposition can be used to help Theveninize a circuit like A by allowing you to determine the Thevenin voltage by shorting the power supplies and summing their effects before determining the Thevenin resistance.

Next we want to find the equivalent voltage across the output terminals produced by V_2 with V_1 shorted. Shorting V_1 produces the equivalent circuit shown in Fig. 5D. That is just a voltage divider made up of R1 and R3 connected across V_2. The output across terminals A and B is the voltage across R1. Again, the conventional voltage-divider formula can be used. In that case, the polarity of V_2 with respect to ground is negative, therefore, we label V_2 as being negative:

$$V_{AB} = V_2(R1)/(R1+R3)$$
$$= -5(120)/(120+180)$$
$$= -5(120)/300 = -2 \text{ volts}$$

Here the voltage is negative because V_2 is negative with respect to ground.

The composite output voltage across terminals A and B is simply the sum of the two voltages we just calculated. In that case, V_{AB} equals:

$$V_{AB} = +7.2 + (-2) = +5.2 \text{ volts}$$

That is the Thevenin's equivalent voltage V_{Th}.

Now we can find the Thevenin's equivalent resistance. To do that, both voltage sources are replaced with shorts producing the circuit shown in Fig. 5E. Resistors R1 and R3 are connected in parallel across terminals A and B. The resistance between A and B is R_{AB} and is the Thevenin's equivalent resistance:

$$R_{AB} = R_{Th} = R1R3/(R1+R3)$$
$$= 120(180)/(120+180)$$
$$= 21600/300 = 72 \text{ ohms}$$

Now we can draw the complete Thevenin's equivalent for the original circuit in Fig. 5A. It is shown in Fig. 5F. A 5.2-volt source is in series with a 72-ohm resistance with the load connected to terminals A and B. The equivalent circuit will, of course, produce exactly the same output voltage with varying loads as the original circuit.

Exercise Problem

Check your knowledge of the superposition process by working the following problem.

2. Theveninize the circuit given in Fig. 1. Determine V_{Th} and R_{Th}.

Practical Voltage Sources

In order to use the superposition method and Thevenin's theorem in circuit analysis, the power sources in the circuit must be voltage sources. As we indicated in the previous article on Thevenin's theorem, a perfect voltage source is one that has a zero internal resistance. In other words, the output impedance of the source is zero. That means that when it supplies current to a load, there will be no internal voltage drop. And, therefore, the output voltage that is available will be equal to the full value capable of being supplied by the voltage source.

In reality, there are no perfect voltage sources. Very few voltage sources even approach perfection. Probably the closest is a battery. While its internal resistance is not zero, it is very small, so little voltage is dropped across it. However, as a battery is used, it deteriorates chemically. As it does, its internal impedance rises. When current is drawn from the battery in its weakened condition, some voltage will be lost across the higher internal resistance. That's why a battery's output voltage declines with use.

Not all voltage sources, of course, are batteries. Therefore, we must take a look at some practical electronic circuits and see how they are used as voltage sources. To do that, we must develop a working definition of voltage sources so that we can see if a particular circuit qualifies as a voltage source or not.

For our discussion here, we will define a voltage source as one whose internal resistance, or output impedance, is much less than the load resistance. To be more specific, we will assume that a good voltage source is one whose internal resistance is less than one-tenth of the load resistance. For example, if our load is 100 ohms, then a good voltage source for driving the load would have an internal resistance of less than one-tenth that, or less than 10 ohms. A better voltage source is one whose internal resistance is less than 1% of the load resistance, but such superior voltage sources are not usually necessary in electronic circuits.

Power Supplies

Power supplies are usually good voltage sources. Their internal resistance is low so that their output voltage remains essentially constant with load variations. Just keep in mind that all power supplies do have a finite value of internal resistance. That internal resistance in turn determines the regulation of the power supply. Regulation, of course, is a figure that indicates the percentage of output voltage change between no load and full load conditions of a power supply. It is calculated with the expression:

$$\%\ \text{reg} = (V_{NL} - V_{FL}/V_{FL}) \times 100$$

Here V_{NL} is the output voltage of the power supply with no load and V_{FL} is the output voltage with a full (maximum) load. For example, if $V_{NL} = 6$ volts and $V_{FL} = 5$ volts, the regulation is:

$$\%\ \text{reg} = ((6 - 5)/5) \times 100 = (1/5) \times 100 = 20\%$$

The lower the percentage of regulation, the lower the change in output voltage for no load to full load conditions. What that means essentially is that the lower the percent regulation, the lower the internal resistance of the power supply and the better the voltage source it is. A perfect voltage source has a percent regulation of zero. In a practical power supply, its internal resistance is responsible for the output-voltage variation from no load to full load.

What makes up the internal resistance of a power supply? Actually, a lot of factors contribute to it. Take a look at the simple power supply shown in Fig. 6. It uses a transformer T1 to step down the AC-line voltage to a lower AC voltage. A bridge rectifier converts the lower AC into pulsating DC. A large capacitor C1 smoothes out the pulsations to create a nearly-pure DC.

Just looking at the circuit, it is not obvious where the internal resistance lies. There are several sources of internal resistance in the power supply. Those are the winding resistances of the primary and secondary windings of the transformer, the voltage drop across the rectifier diodes, and the effectiveness of the filter capacitor. When a load is connected to a power supply, current is drawn from it and voltage drops appear across the transformer windings and the diodes. The load also causes the filter capacitor to discharge more between half cycles and, therefore, the output voltage will drop producing an effect similar to an internal resistance. By proper design, all of those factors can be minimized. Nevertheless, the power supply ends up with a rather large equivalent internal resistance which may be detrimental in those applications where the load voltage must remain constant with load-resistance variations.

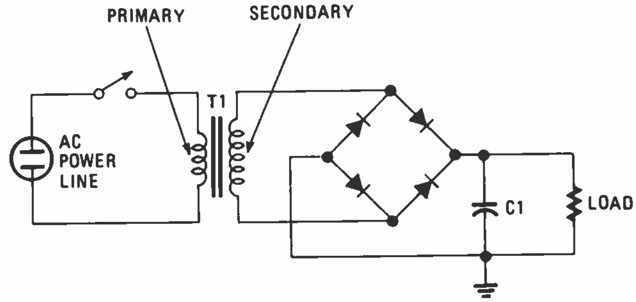

Fig. 6—A common power supply has an internal impedance that is a composite of the transformer-winding resistance, diode voltage drops, and capacitor impedance.

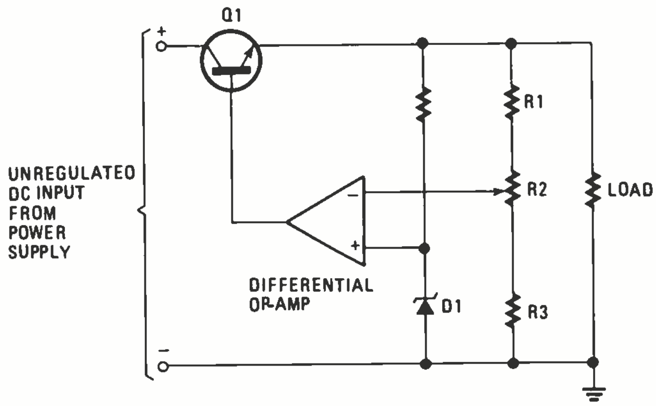

Fig. 7—A feedback voltage regulator greatly reduces a power-supply's output impedance by responding to the load.

The way to compensate for or offset the effect of high internal impedance in a power supply is simply to add a regulator circuit to it. A regulator circuit is a sophisticated electronic circuit with feedback that senses changes in the output voltage and thereby adjusts the output voltage automatically to maintain it at a fixed level. A typical regulator is shown in Fig. 7. Its input comes from the output of the power supply in Fig. 6. Most regulators use a variable series impedance between the power-supply output and the load. That is usually a bipolar transistor such as Q1 whose conduction is varied to change its resistance, and thereby vary the output voltage. Changes in the power-supply output voltage caused by changing load current or varying power-line voltage are sensed by the regulator.

In Fig. 7, the voltage divider made up of R1, R2, and R3 taps off a part of the output and applies it to one input of an op-amp. The other input to the op-amp comes from Zener diode D1 which is used as a voltage standard or reference. The voltage across D1 remains constant because of Zener action. The op-amp compares the output voltage sample to the reference voltage and amplifies the difference to create base drive for Q1. The regulator circuit then adjusts the base drive to the transistor causing it to conduct more or less as required to maintain a constant load voltage.

If the output voltage goes down due to a load increase, the circuit causes Q1 to conduct more. Its resistance decreases, therefore, less voltage is dropped across it and more appears at the output. Thus the original decrease is compensated for.

Such electronic regulators work very rapidly and maintain a very constant output voltage. Such electronic regulators work very rapidly and maintain a very constant output voltage. The effect is as if the internal resistance has been reduced to an extremely low value. When power-supply output voltage variations are a problem, adding a regulator effectively decreases the internal resistance. Regulated power supplies, like batteries, are extremely good, but not perfect voltage sources.

Transistor Circuits

Many transistor circuits also serve as voltage sources of varying degrees of quality. Transistor amplifiers, for example, all have an internal resistance which to a load appears as an output impedance. Take the simple common-emitter amplifier circuit shown in Fig. 8A. Its output impedance is just slightly less than the value of the collector resistor R_c. In the circuit shown, the output impedance or internal resistance of the circuit as seen by the load is 1000 ohms. Its equivalent circuit is shown in Fig. 8B.

In general, an amplifier would not be considered a good voltage source, but remember that that can only be determined by considering the load resistance in comparison to the output impedance. If the load resistance is 10K ohms or greater, then the circuit is a good voltage source by our previous definition. For values less than 10K ohms, the circuit is not a good voltage source.

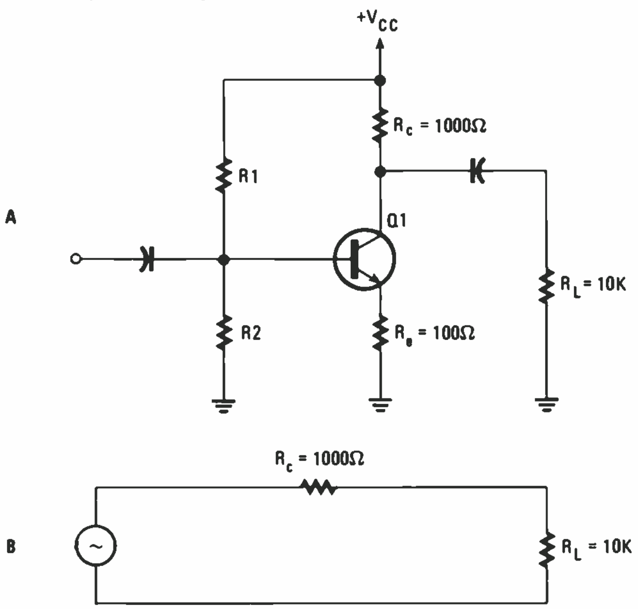

Fig. 8—The output impedance of a common-emitter amplifier is approximately equal to the value of the collector resistor's.

When the output of one amplifier is connected to the input of another as in Fig. 9A, the output resistance R_c forms a voltage divider with the input resistance of the next stage R_i as the equivalent circuit in Fig. 9B shows. The resulting voltage-divider action causes the input voltage to the second stage (Q2) to be lower than the actual voltage delivered by the previous amplifier (Q1). Such voltage-divider action can offset a considerable amount of the gain produced by the amplifier if the effect is not taken into consideration.

One way to overcome the loss due to voltage-divider action when cascading stages is simply to make the output impedance of the driving amplifier lower compared to the input resistance of the driven amplifier. In the amplifier of Figure

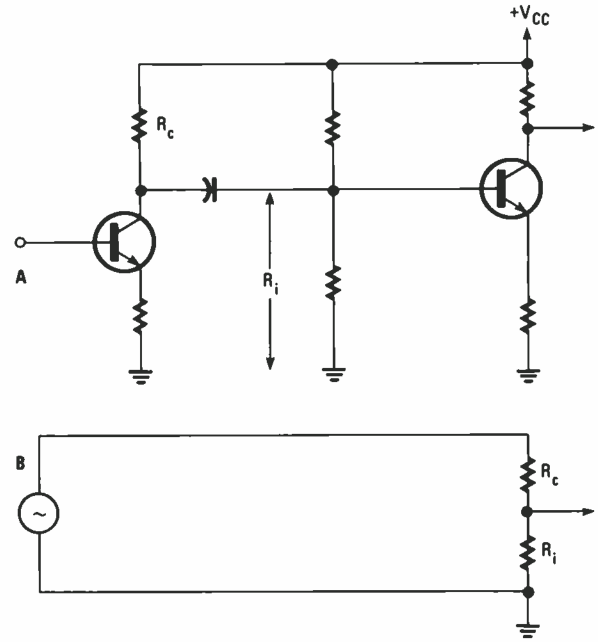

Fig. 9—Cascading amplifier stages causes each stage to load the previous stage. The loads must be taken into account when designing the overall circuit.

9A, that means making the value of collector resistor R_c smaller.

While that can be done, it has several detrimental effects. For example, lowering the value of R_c decreases the gain of Q2 and increases the power consumption of the circuit. Both of those are undesirable characteristics although the output impedance is reduced. Such trade-offs are common in electronic-circuit design. Usually the choice of a collector resistor is some optimum value that is a balance between low output impedance, high gain and minimum current consumption. An alternative or additional technique is to work on making the input resistance R_i equal to Q2's or higher.

Lowering Amplifier Impedance

One way to lower the output impedance of the amplifier is simply to use an emitter follower circuit between the amplifier and the load. A typical emitter-follower circuit (Q2) is shown in Fig. 10A. That is a common-collector amplifier circuit whose primary characteristics are a high input impedance, low output impedance, and unity gain. The high input impedance minimizes the voltage-divider loss between Q1 and Q2. While the emitter follower does not provide voltage amplification, it does produce the same voltage at its output that appears at its input. A lower output impedance for the same voltage level allows much lower load resistances to be driven.

In most amplifier designs, the biasing resistors R1 and R2 in Fig. 10A are usually eliminated and the emitter follower is connected directly to the collector of the driving amplifier, which provides not only the signal input, but also the correct DC bias level. That is illustrated in Fig. 10B. The output is taken from across the emitter resistor. Such a circuit usually results in an output impedance of several-hundred ohms, whereas the output impedance of the driving amplifier, Q1 may be several thousand ohms. While the emitter follower itself does not provide any voltage gain, its low output impedance minimizes the overall gain lost to voltage-divider action in the cascaded stages.

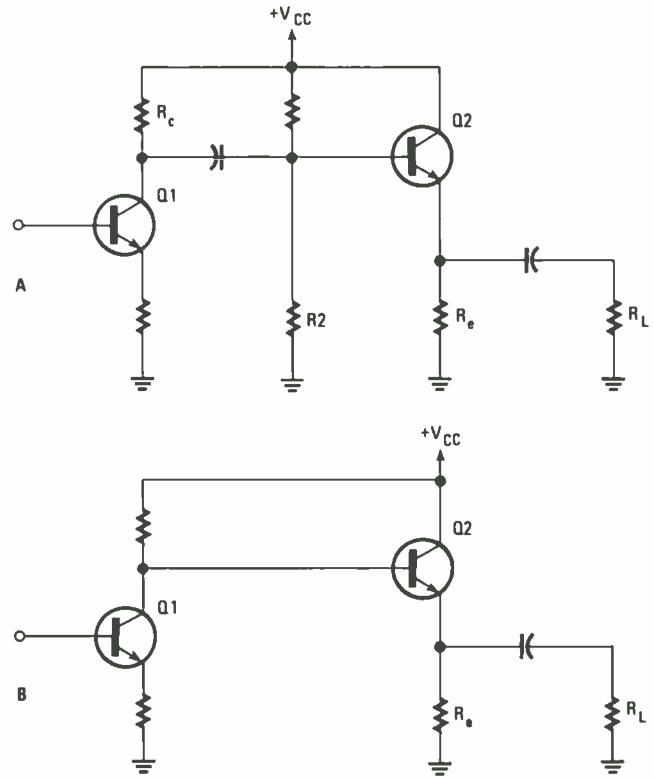

Fig. 10—Using an emitter-follower stage is an excellent way to lower the output impedance, and allow the amplifier to amplify a signal without having to drive the output.

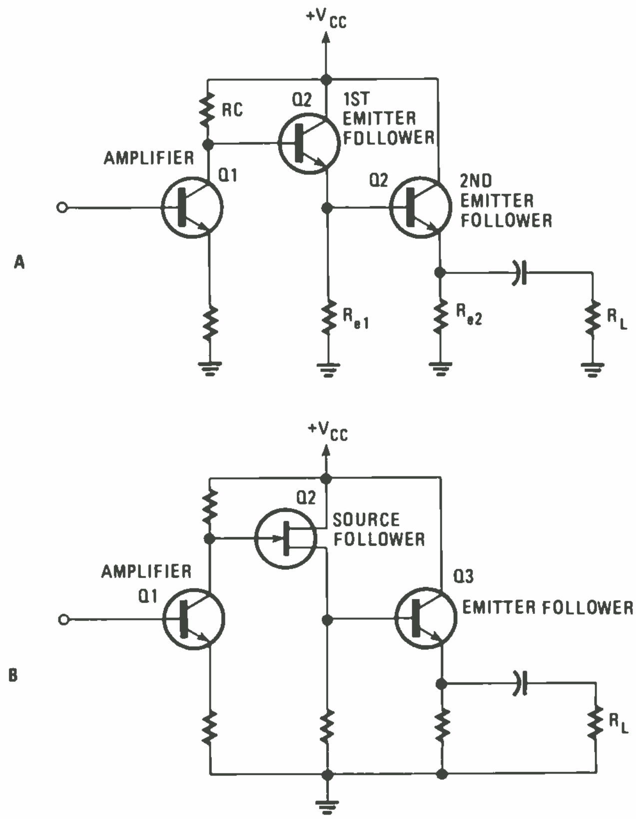

Fig. 11—Emitter followers can themselves be cascaded to produce a circuit with lowered output impedance.

Additional Stages

If even lower output impedance is needed, several emitter-follower stages may be cascaded as shown in Fig. 11A. One stage, Q1, will get the output impedance down to several hundred ohms. The next stage, Q2, will reduce that even farther to some value less than 100 ohms. A popular combination is to use a field-effect transistor as a source follower, and follow it with an emitter follower for further output-impedance reduction. See Figure 11B. The FET stage Q2 minimizes loading on the amplifier circuit while providing some decrease in output impedance. The bipolar stage Q3 reduces the output impedance even more.

One technique that is similar in effect to cascading emitter followers is to use the popular Darlington connection shown in Fig. 12. There two bipolar transistors, Q2 and Q3, are connected in such a way that they appear as a single very high gain transistor called a Darlington pair. With very high gain, the Darlington connected device, when used in an emitter-follower circuit, produces very high input impedance and extremely low output impedance.

Another way to reduce the output impedance of a circuit is to add a power amplifier to it. A widely used power amplifier is the popular complementary-symmetry circuit shown in Fig. 13. It is a class-B amplifier where each transistor supplies one-half of the signal to the load. Transistor Q2 supplies the positive half-cycles, and Q3 supplies the negative half-cycles. Such power amplifiers have extremely low output impedance.

Complementary-symmetry amplifiers like the one in Fig. 13 are commonly used in audio power amplifiers that must drive speakers. Speakers have inherently very low impedances of 4, 8, or 16 ohms. In order to drive a speaker properly, the driving amplifier must have a very low output impedance.

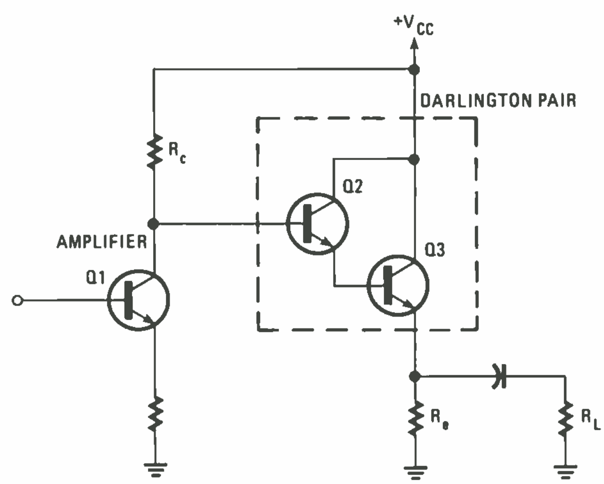

Fig. 12—Here we show the use of a Darlington pair to lower the output impedance of an emitter-follower amplifier.

Such low impedances are easily accomplished with power-amplifier circuits such as that shown. Such amplifiers are excellent voltage sources.

Transformers

Another technique for lowering the output impedance of an amplifier is simply to use a transformer. Recall that a transformer, because of its turns ratio, can be used to match impedances. The windings can be chosen so that the output impedance is much less than the input impedance. The relationship between the impedance ratio and turns ratio is expressed in the formula:

$$Z_p/Z_s = (N_p)^2/(N_s)^2$$

Where Z_p is the primary impedance, Z_s is the secondary or load impedance, N_p is the number of turns on the primary, and N_s is the number of turns on the secondary.

Rearranging the formula to solve for the turns ratio in terms of the impedance ratio, we get:

$$N_p/N_s = Z_p/Z_s$$

For example, assume we have an 8-ohm speaker load but the output impedance of our amplifier is 800 ohms. We can match the two with a transformer. The turns ratio needed is:

$$N_p/N_s = 800/8 = 100 = 10 \text{ or } 10\text{-to-}1$$

The transformer needs a turns ratio of 10-to-1 or 10 times as many primary turns as secondary turns. The transformer makes the amplifier appear to have lower output impedance than it really does.

At one time, transformers were widely used in power amplifiers to achieve low output impedance. A typical class-A audio amplifier is shown in Fig. 14. While such amplifiers are still used in some small radios and audio amplifier circuits, today most of those circuits have been replaced by transformerless power amplifiers such as the complementary-symmetry circuit described previously.

Op-Amps

Op-amps are very popular for implementing a variety of amplifier, signal-processing, and signal-generating circuits. The op-amp, as you recall, is a very-high gain differential amplifier that is normally used with input and feedback circuits of various types to set the characteristics of the circuit.

Most op-amps are designed with power-amplifier output stages for low output impedance. Typical open-loop output impedance is usually less than 100 ohms. However, when negative feedback is used (as in most applications), the output impedance is decreased considerably. The actual amount of output impedance depends on the feedback circuit and the amount of overall circuit gain. For example, in the typical inverting-amplifier stage of Figure 15, the output impedance may only be 10 ohms.

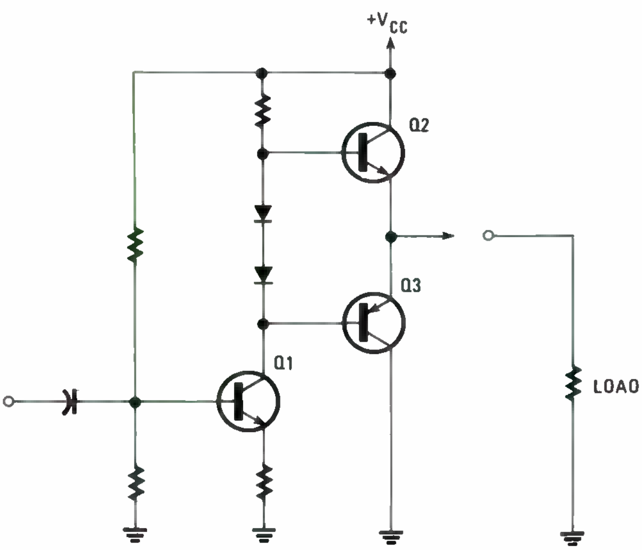

Fig. 13—A Basic complimentary-symmetry power amplifier has a very low output impedance. It makes a good last stage.

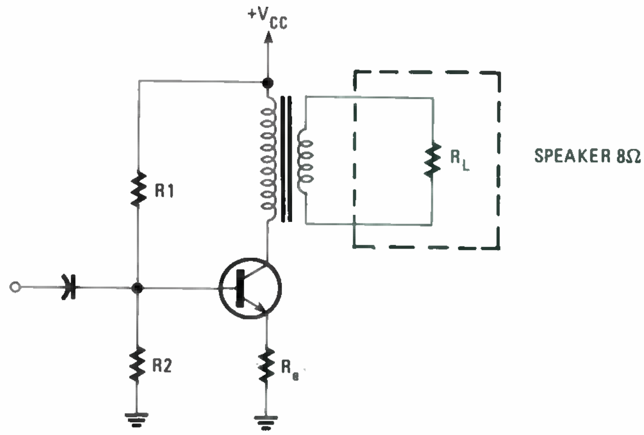

Fig. 14—Using impedance-matching transformers is another way of reducing the output impedance of a driving stage.

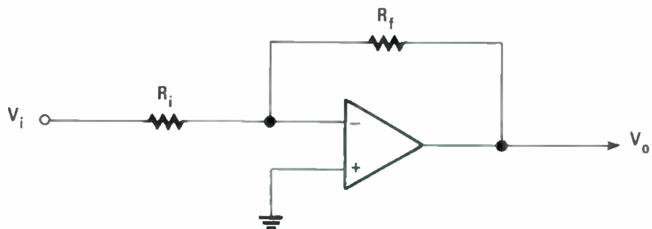

Fig. 15—Op-amp inverters not only have an inherently low output impedance, but they also have high input impedance.

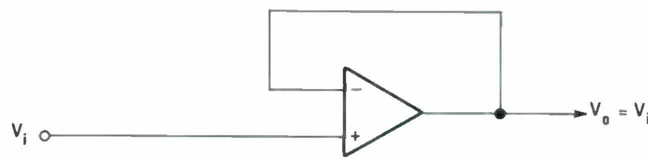

Fig. 16—Op-amp followers typically have an output impedance of less than 1 ohm. But their input impedance is extremely high so as not to load the preceding stage.

For even lower values of output impedance, an op-amp follower can be used. The op-amp follower, as shown in Fig. 16, has 100% feedback from the output to the inverting (−) input. That produces a circuit similar in performance to the simple emitter or source follower. The input impedance is extremely high while the output impedance is extremely low. The amplifier gain is unity. With such a configuration, the output impedance is usually decreased to much less than 1 ohm. For applications requiring a very-high quality voltage source, use op-amp circuits, particularly the follower.

Voltage-Source Calculations

The most important thing to remember is that practical electronic circuits have output impedance. Most circuits have a finite value of internal resistance and, therefore, are not perfect voltage sources. However, if they meet the criterion stated earlier for a good voltage source, regardless of their output impedance, then the circuits will work well. Just keep in mind that the internal resistance must be taken into consideration when forming superposition and Thevenin's calculations.

Earlier in the discussion we indicated that to perform the analysis, the voltage source is usually replaced by a short circuit. When practical voltage sources are involved, that is

(Continued on page 106)

(Continued from page 81)

not the case. Instead of replacing the voltage source with a short, it must be replaced with a resistance whose value is equal to the internal resistance of that voltage source. For example, in analyzing a circuit with an emitter follower whose output impedance is 50 ohms, the voltage source would be replaced with a 50-ohm resistor in performing superposition or Thevenin's calculations. ■

Answers

1. Short V_1. Calculate the current in R2:
a. Find the parallel resistance of R1 and R2. $R_{1,2} = R1R2/(R1 + R2) = 200(300)/(200 + 300) = 60000/500 = 120$ ohms b. Find the total-circuit resistance R_T. $R_T = R_{1,2} + R3 = 120 + 300 = 420$ ohms c. Find the total-circuit current I_T. $I_T = V_2/R_T = 4.5/420 = .0107$A d. Find the voltage across R3. $V_{R3} = I_T R3 = .0107(300) = 3.21$ V. e. Find the voltage across R1 and R2. $V_{R1,2} = V_2 - V_{R3} = 4.5 - 3.21 = 1.92$ V. f. Find current in R2. $I_{R2} = V_{R1,2}/R2 = 1.29/300 = .0043$ A. In this case, the current through R2 flows from bottom to top because of the polarity of V_2. Let's call this a positive current or $+ .0043$ A.

2. Short V_2. Calculate current in R2. a. Find the parallel resistance of R2 and R3. Since R2 = R3:
$R_{2,3}R2/2 = 300/2 = 150$ ohms b. Find the total-circuit resistance. $R_T = R_{2,3} + R1 = 150 + 200 = 350$ ohms c. Find the total-circuit current. $I_T = V_1/R_T = 6/350 = .0171$ A. d. Find the voltage across R1. $V_{R1} = I_T R1 = .0171(200) = 3.42$ volts e. Find the voltage across R2 and R3. $V_{R2,3} = V_1 - V_{R1} = 6 - 3.42 = 2.58$ volts f. Find the current in R2. $I_{R2} = V_{R2,3}/R2 = 2.58/300 = .0086$ A.

This current flows in R2 from top to bottom because of the polarity of V_1. Let's call this a negative current to distinguish it from the other current, or $- .0086$. The total current in R2 is the algebraic sum of the two previously calculated currents:

$$I_{R2} = + .0043 + (- .0086)$$
$$= - .0043$$

The total current is the difference between the two currents and it is negative because the negative current is greater. The total net current in R2 flows from top to bottom.

2. Since the circuit in Fig. 1 has no load, the output voltage calculated earlier, 1.05 volts, is the Thevenin's equivalent voltage:

$$V_{Th} = + 1.05$$

The Thevenin's equivalent resistance is the parallel combination of R1 and R2 or:

$$R_{Th} = R1R2/(R1 + R2)$$
$$= 1000(3300)/(1000 + 3300)$$
$$= 3300000/4300 = 767.4 \text{ ohms}$$

■