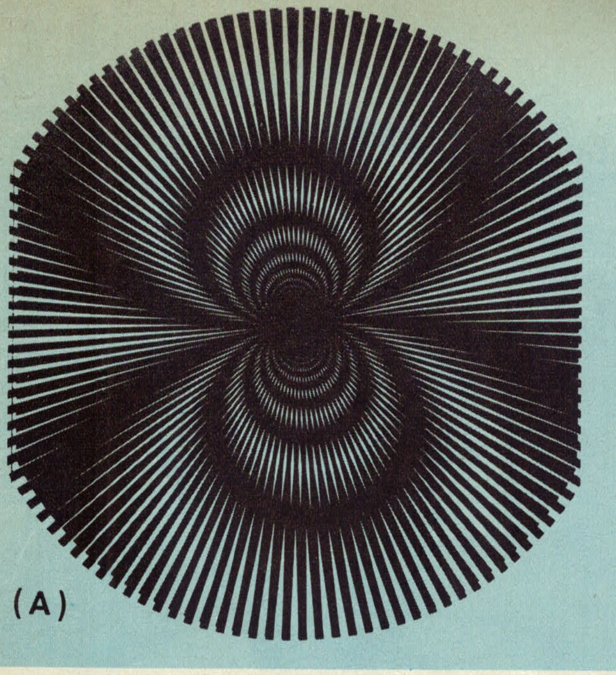


(A)

MOIRE

ancient craft in the service of modern technology

by Elmar Laisk

Most people see moire patterns during the course of everyday life, perhaps without realising what they are or even giving them any thought. The wind blows a nylon curtain into folds, and a series of irregular bright and dark bands appear; across the road you see a line of railings, and behind it another similar line of railings and as you walk along, peculiar optical effects are apparent; a comb sits on a mirror, and patterns of light and dark bars appear. These are but three common examples of moire fringes from among many that may be encountered almost daily. These moire fringes form fascinating patterns that seem to possess a life of their own, changing in shape, revolving, swelling and contracting continually with the slightest movement of the observer or the material observed.

Such a fascinating phenomenon is hardly likely to be neglected by those concerned with the visual arts. Moire patterns are now used widely in "op art": on television (particularly in the preparation of commercials) and in situations where weird physchedelic effects are desired.

However, moire techniques have more serious applications than this. Technology has also adaped the phenonemon to be useful in various technical applications: For example, in the digital control of program-operated machines; in non-destructive testing for stresses and deformations; for accurate determinations of small linear and angular displacements. in the science of metrology; and in optical measurements^{4,9,12}. Apparatus producing moire effects has also proved to be useful in psychological examinations.

In many ways, moire patterns are helpful also in teaching; for instance demonstrating the principles of interference, phase velocity, holography, microwave propagation in waveguides, radiation pattern of dipoles, electric field configurations and flow lines in moving liquids.

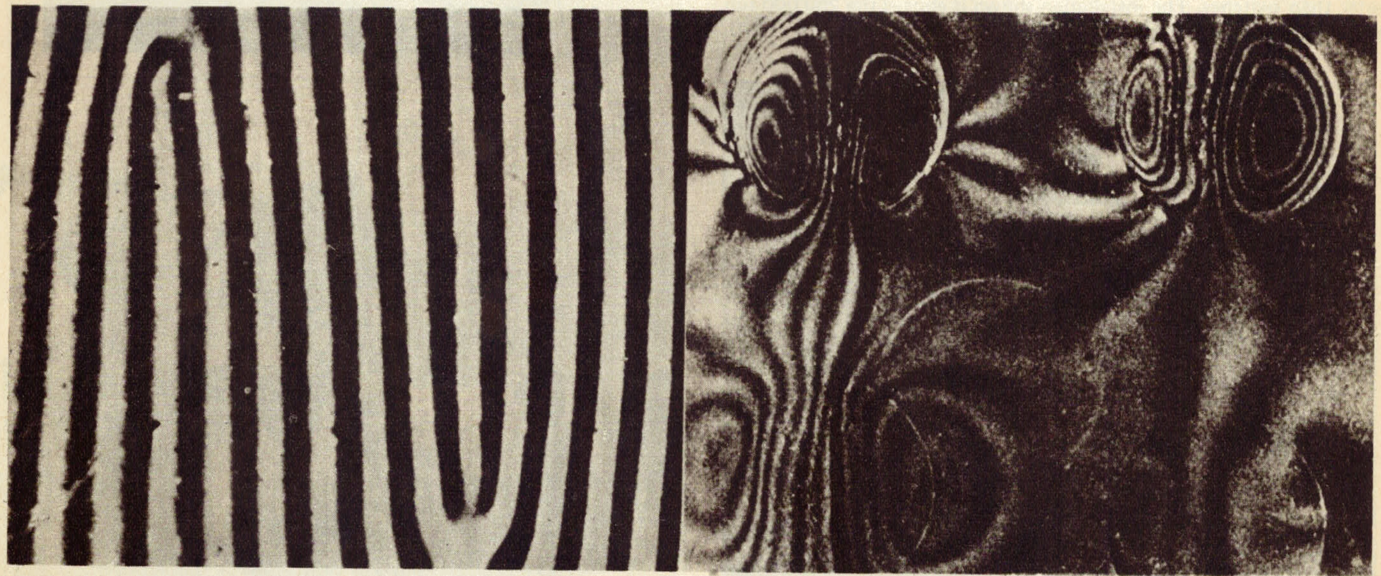
In electronics, moire pattern visual aids can simulate beating, mixing and modulation processes, the superhet principle, the radar moving target indicator, the backward wave oscillator and radiation patterns of phased arrays.

With moire techniques, one can study graphically certain differential and integral equations and solve problems in geometry and Fourier analysis.

On the other hand, moire fringes can present problems in circumstances where they are not wanted. Common instances of these problems are the appearance of moire patterns on the TV screen; the "beating" between two half-tone screens in the printing industry; and the occasions when people find irritating the moire patterns made by finely woven materials, such as nylon curtains.

The term "moire" is derived from the French name for the Chinese and Thai fabrics that shimmer mysteriously with subtle colouring effects, known to us as watered silk.

BELOW: Two rotational patterns produced by two crystal layers ("dislocations") irradiated with x-rays in a silicon interferometer. Additional interest in these pictures stems from the "beat" produced in the screening process used to prepare the picture for printing. The picture is copied directly from "Physics Today" (August, 1970) and the 130 screen of the original is interfering with the 150 screen used in the printing of this magazine.



PATTERNS

This "watered" effect of silk, mohair, and cotton fabrics is brought about by several well-kept secret trade processes originated several thousand years ago by Chinese craftsmen. The process usually involved the folding of a lattice-woven glossy fabric, combined with the application of heat, vapours, and heavy pressures. Besides the references to moire in ancient Chinese writings, there is mention of a particular dress worn by the Queen of Sheba that enhanced her vital statistics and serpentised the movements of her body and limbs.

The first really scientific description of moire fringes was given by the English physicist Lord Rayleigh in 1874, when he proposed a sensitive test of accuracy for diffraction gratings. Nearly 50 years later, the Italian V. Ronchi designed practical tests for abberations in lenses and mirrors (1922). Then in 1950 Sir Thomas Merton (UK) proposed a simple but very sensitive moire fringe counting system suitable for industrial measurement. This system was realised in the National Physical Laboratory in England by T. Guild, who developed gratings and photoelectric fringe counting methods (1956) which were immediately adopted by industry[3,4].

Various publications by Gerald Oster[2,5,6,7] and especially a moire kit devised by him[6], introduced moire techniques into art and science teaching, where they were immediately successful.

Figures 1A and C display moire fringes produced by two rulings intersecting at a small angle. The dark fringes are due to the obstruction of the light passing successively though both rulings. Since the fringes represent intensity modulation of light, the slit-and-bar rulings are also called "amplitude gratings". The appearance of the moire fringes is unrelated to the wavelength of light and depends only on the line spacing (pitch), closeness and the alignment of the two rulings.

With very fine rulings exceeding 2000 lines per inch, the diffraction and interference effects become significant. The fringe appearance then depends on whether diffused, collimated, monochromatic, polarised or coherent laser light is used for illumination. However, very fine rulings are seldom employed for moire purposes in industrial applications.

For purely geometric moire fringes, the two gratings must be in actual contact, or, if it is required that the two should be able to slide, lubricated with a thin layer of oil.

The importance of closeness is strikingly exemplified by means of gratings made from Letratone (artists shading screen), made by Letraset Ltd of the UK, and available in Australia from Letraset Australia Pty Ltd, Sydney. Gratings made from Letratone 4 and 1 negative format (white dots on a black ground) are inserted into the beam of a slide projector; if in contact, they produce moire effects whereas, if they are aligned at a distance, colour effects are produced. Almost pure colours can be produced from white light by moire filters of this type.

Figure 3 shows how fringes are formed from two rulings intersecting at an angle θ. The fringe width can be determined by the formula

$$m = P/\theta$$

where

m is the moire fringe width in inches

p is the pitch of the rulings, in inches

θ is the angle between the two rulings, in radians.

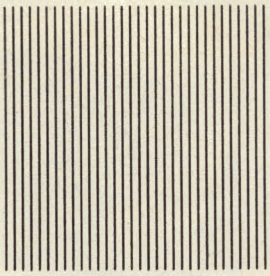
For example, if the pitch of the rulings is .01 inch (almost unresolvable by the eye) and the inclination is .0175 radian (1 degree) then the moire fringe width will work out to 0.57 inch. At an angle of only .0029 radian (10 minutes of arc) the fringe width equals 3.45 inches, a magnification of 345 times relative to the pitch.

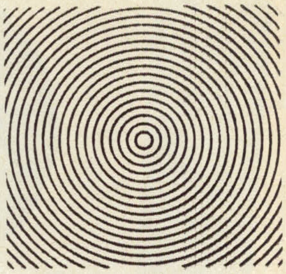
In consequence, if one ruling is moving 0.010 inch per second (3 feet per hour) horizontally, the fringes will move 3.45 inches per second (about 0.2 mile per hour) vertically. A small photocell behind the rulings will then record a sinusoidal variation of light intensity across each fringe. By counting the fringes which pass the photocell, the traversed length of the ruling can be determined numerically within a fraction of the pitch of the reference grating.

Figure 1B shows a second type of moire fringe produced by parallel alignment of two rulings with slightly different pitches, p and q respectively. Again, an inverse law holds: – the smaller the difference d between pitches, the wider the fringe width m, namely $m = pq/d \approx p^2/d$ or $p \times (p/d)$, which means that the fringe width represents a magnification of (p/d) times the pitch p.

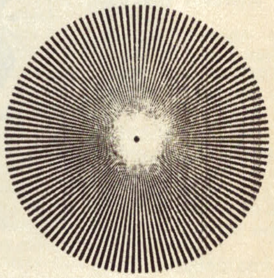
These moire fringes run parallel to the lines, and are called vernier fringes. They are analogous to the beats produced between

Typical patterns for moire experiments

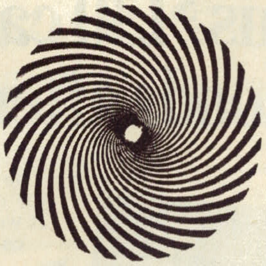




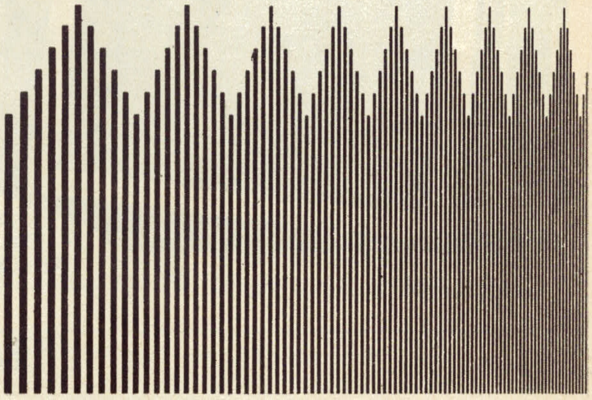
LEFT: Superposition of two line gratings, representing periodic waves, demonstrates interference and standing waves. Gratings with slightly different spacing demonstrate beats, ie, intermediate frequency in superhet receiver.
RIGHT: Equispaced concentric rings represent point source of waves. Superposition of two such ring patterns produces fringes, similar to those in Young's double-slit experiment.
Superposition of these two patterns (lines and rings) represents Fresnel diffraction at a straight edge, producing spherical waves.



Radial lines represent field lines of central force, or point charge. Superposition of two such patterns causes fringes which represent field lines (polar diagram) of a dipole, as shown on the facing page.



This logarithmic spiral combines the features of a radial grating and logarithmically spaced concentric grating.

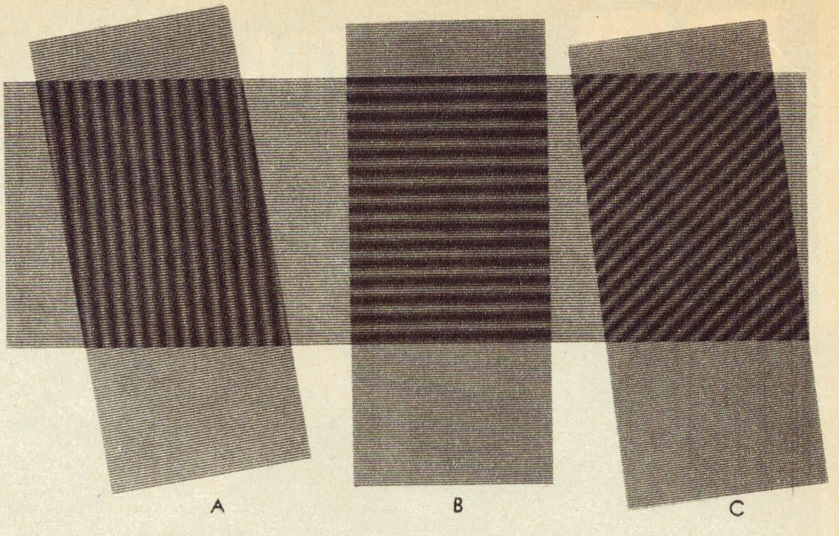


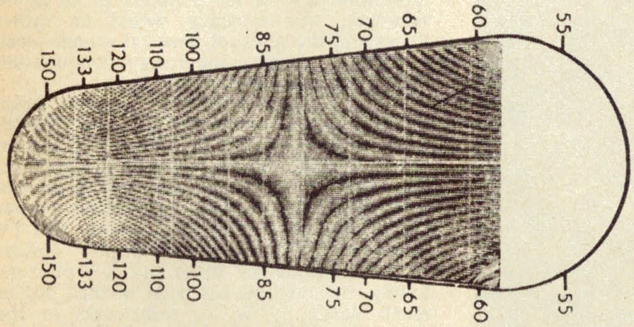
Logarithmic grating. Overlapping this pattern with its copy produces fringes which represent the performance of an exponential microwave horn.

Moire fringes from gratings

RIGHT: Figure 1. Moire fringes produced by two grating which have (A) same pitch and an angle of rotation of 10 degrees; (B) small difference in pitch and no rotation; (C) small difference in pitch and a rotation of 6 degrees combined.

BELOW: Figure 2. A "universal" type screen determiner. This is constructed by connecting equispaced points along a line to a common origin which lies on the axis to the perpendicular, to form a fan-shaped configuration. When overlaid on a half-tone picture, a four-pointed star moire pattern is formed. The screen present on the picture is indicated by the arms of the star which point to the appropriate numerals.





two tones of nearly equal pitch, or generally, between any two oscillations of slightly different frequency.

For instance, two rulings having 1000 and 1001 lines per inch, with a difference d in pitch of 10^{-6} inch, will produce beat fringes 1 inch apart. This fringe width represents a magnification of 1000 times the pitch (= 0.01in); or a million times, $(p/d)^2$, ot the error (10^{-6}in) in the pitch. At such magnification any microscopic error in the rulings will be visible to the unaided eye.

If one of the gratings is fastened to a shaft which slides, say, .01in/sec (=3ft per hour), then the fringes will travel 10in/sec (= 0.6mph), very much faster than the physical velocity involved.

It is noteworthy that the beat fringes will travel either forward or backward depending on whether the pitch ratio p/q is less than or larger than 1, where p is the pitch of the moving grating. The direction depends also on the transmission (=bright/dark spacings) ratio of the two gratings. The effect can be observed with gratings cut from Letratone LT73 ("frequency" 64 lines per inch) and LT72 ("frequency" 56lpi), which produce 8 moire fringes per inch (= "intermediate frequency").

Moire fringes will often form compound patterns if the two gratings have disparity in both pitch and angular alignment at the same time. This situation often occurs in non-destructive stress analysis. The method involves transferring an exact copy of a reference grating to the surface of the test piece, using photographic, photo-resist or transfer techniques. As the test piece is loaded, the transferred grating will deform with the surface, while the reference grating above it will not be deformed. Moire fringes will be apparent when the gratings are viewed in collimated light. These fringes are related directly to the displacement field from which can be computed cartesian strain components. The components of stress can then be determined from the strain field for the material, for which the modulus of elasticity, Poisson ration and modulus of rigidity are known.

For measurement of contour irregularities on a finished surface, a technique has been evolved which requires only one grating. Using collimated light a shadow of the test grating is projected on to the workpiece and viewed, through the grating, usually by means of a camera, from a carefully selected and controlled viewing angle. Any departure of the surface from true flatness or true curvature according to what was required, becomes immediately evident. A similar technique employs a highly reflective coating on the workpiece to provide a reflection of the grating, rather than a shadow.

In a more refined reflection moire setup ("multiple source method") it is possible to record contours of a curved surface directly. The method has been extended for solving dynamic problems – for instance, recording vibrations of plates and membranes in flexural modes (Chladni figures) so that they can be evaluated numerically.

Commercial moire gratings consist of highly accurate linear arrays with about 1000 lines per inch. The master gratings are made on high precision ruling machines. The production models are produced from these expensive master rulings by several replication methods.

Although one or two ruling devices in the world are capable of ruling a fantastic 250,000 lines per inch for diffraction purposes, such fineness would be quite useless in industry. Only in a few non-destructive testing processes is a frequency of 10,000 to 20,000 lines per inch required in order to measure strains of about 100 microinches per inch.

Moire gratings up to 15ft long with standard pitches from 10 to 1000 lines per inch are available commercially. A Ferranti counter with digital output employs 2500 lines per inch gratings. It gives four counts per fringe and an accuracy of 0.0001 inch.

The half tone screens used in the printing industry by photo-engravers have, at most, about 200 lines per inch. Beyond that, photographic or photo-resist techniques can be used to obtain inexpensive but satisfactory moire gratings up to 1000 lines per inch, or more.

For experimental purposes, commercially available Ronchi rulings, and moire sets are available from Edmund Scientific Co, Barrington, NJ 08007, USA; and from Proops Bros, 52 Tottenham Court Road, London W1. Also, as already mentioned, workable grating can be made from Letratone, from which patterns LT 71, 72, 73, 57, 920 and 171 are suitable. With some practice, it is possible to make a moire vernier from Letratone 73 (64 lines per inch) as a reference grating, and a piece of the same material carefully stretched to 63 lines per inch as the moving grating. The material is carefully fixed to pieces of clear plastic material, such as Perspex.

Moire generating patterns shown on page 13 are more useful in art and science teaching. For an op artist, they offer an almost infinite variety of fascinating patterns and forms, especially if the sets are in basic colours. In teaching maths and science, they offer facilities for a large number of analogues and demonstrations, some of which are indicated in the table.

In industry, the use of very high precision moire techniques and interference methods is impractical. The great advantage of the moire fringe method is that it can be adapted, with ease, using coarser ruling and lesser angles, to any accuracy. Moire fringes with rulings up to 1000 lines per inch (40 lines per mm) can be seen and counted without special optical equipment. Nevertheless, an accuracy of about 0.0001in can be achieved with such cheap and robust moire fringe attachment an any machine tool.

For the production of sharp moire fringes it is essential that the spacings in the two overlayed grids are nearly the same, because the contrast and the separation of the moire fringes decreases as the difference in spacing between the two generating grids increases. This effect provides, for instance, an easy method for the determination of the spacing of an unknown printing screen by means of a variable space comparison screen (figure 2).

On the other hand, the rate of widening of moire fringes increases enormously as the difference either in pitch or in angular alignment of the two gratings becomes very small. The graphical expression of such an inverse law – the smaller the difference, the wider the fringes – is a hyperbola. In consequence, a linear change is pitch of inclination between the two gratings results in hyperbolic moire fringes, whereas a linear moire fringe is the focus of points of constant pitch.

Such extraordinary resolving power of moire fringes relative to small changes enables one to measure linear displacements of the order of 0.0001in and angular differences about one second of arc quite simply. With a more refined moire technique it should be possible, in theory, to measure angles of the order of 10^{-10} radians. In comparison, an optical telescope can resolve two stars when their separation exceeds 10^{-4} radians. Several high-resolution moire "optical levers" have been devised.

The theoretical limit of linear resolutions with moire techniques is predicted to be about 10^{-12} centimetres, which is about one ten-thousandth of the diameter of an atom. However, this accuracy would be rather illusory in the physical sense; at atomic level, other methods involving Bragg reflections and light interference are preferred.

As emphasised before, moire fringe methods provide extremely sensitive tests for errors and distortions in gratings and arrays. Diffraction gratings are tested by this method. The same method is now extended to provide a convenient method of testing raster distortions and scanning errors in TV monitors, TV cameras and display devices such as cathode ray tubes and X-Y plotters. Lately, the method has been worked out in much detail by W. C. Sobolevski in the Electrical Engineering Department at the University of Adelaide.

It appears that, in principle, moire methods could be applied to the detection of counterfeit money. Australian currency notes

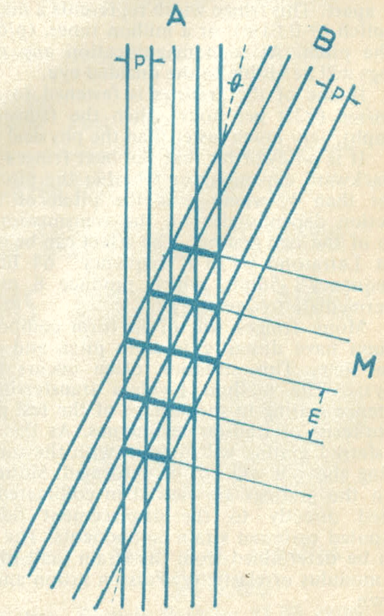


Figure 3. Showing how moire fringes are formed from two gratings. Grating A, with line pitch P, is laid over an identical grating B at a small angle θ. The moire fringes M, with pitch m, occur at the loci of the intersections.

have a linear grating running through the pattern which readily forms fringes even with rough test screens devised from Letratone 49 and 56. Any variations in these rulings which might be present in counterfeit money would be apparent using more discriminating tests screens. For practical application, some kind of test jig which allowed speedy comparison could be devised.

It is quite simple to build a moire fringe attachment for a vernier caliper or micrometer so that measured lengths can be displayed numerically on a relay-counter. The two gratings can be made from Letratone 71 or 73, stuck to two pieces of clear plastic. The fringe counting device consists, essentially, of a small cadmium sulphide photocel, a prefocused torch cell and a PMG counter.

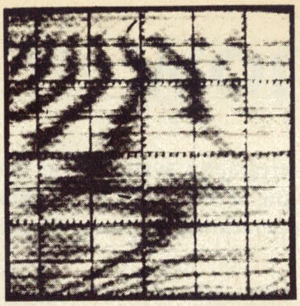
The above attachment makes also an accurate indicator for linear expansion and stress. In a similar manner, an angular moire indicator made from radially ruled lines of Letraset 195, 196 or 197 can provide a numerical indicator for a sensitive torsion spring balance or electrometer.

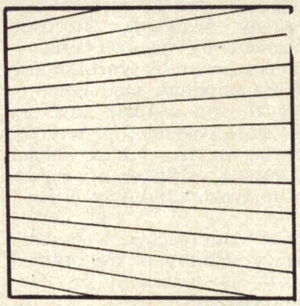
It is possible to build many special effects devices with moire gratings: for example, a light beam modulator, a visible sound standing wave indicator (Kundt tube), a psychedelic illuminator or even a moire microscope. The latter employs two widely seperated dot gratings – one negative, the other positive, illuminated from a point source. The gratings are aligned so that, normally, they do not pass light. If a small transparent object of varying refractive index is placed near the light source, the light rays will be deflected and a contour of the object becomes visible on the screen. The same principle applied in collimated light enables one to visualise temperature gradients, variations in concentration and flow lines in liquids and gases.

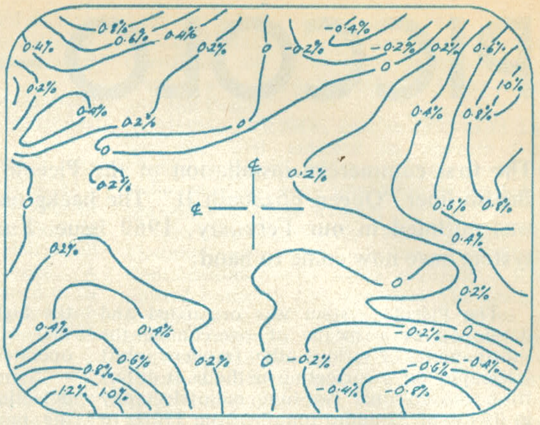
Moire fringes will appear when two seemingly irregular meshes having some hidden common pattern are viewed against each other in experimentally found alignment. In principle, one of the meshes could be a regular pattern or half-tone screen with either constant or variable pitch. From the appearance of the moire fringes the features of the hidden pattern (= signal in noise) can be deduced.

However, the method can also be applied in reverse. For instance, if a complex mesh, such as a fingerprint or a contour map, is viewed through a transparent overlay carrying a regular pattern, then the resulting simple moire pattern will exhibit specific features. These characteristic features can be used for classifying and referencing fingerprints, folk-weave patterns, weather maps or landscapes.

A moire fringe Fourier analyser has been proposed by A. Lohmann. The purpose of the analysis is, of course, to determine the amplitude and frequency of various sinusoidal waves which constitute an inharmonic waveform such as a square pulse. The analyser consists of two illuminated identical grids producing moire fringes whose period varies with the angle between the grids, and the light transmission varies sinusoidally with their linear shift. These moire fringes are imaged on a mask into which the square waveform is cut, and the Fourier coefficient can be determined from the transmitted light while shifting and rotating the grids.







The moire fringe method of testing for TV picture distortions. Moire fringes (top left) resulting from raster distortions are obtained by placing a screen grating on the face of the TV screen (lower left). The diagram at right shows typical equal-distortion contours derived from moire fringes on a TV screen.

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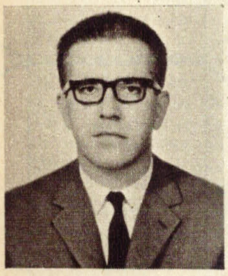
(Continued on page 150)

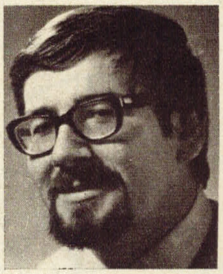
Pioneers in modern moire techniques

PROFESSOR P. S. THEOCARIS has written over 100 pages on various applications of moire fringes. In 1970 he published a comprehensive book on moire techniques. He has been awarded highest degrees from universities in Athens, Brussels and Paris, and has help fellowships and professorships at the Massachusetts Institute of Technology, Brown University and Pennsylvania State University. At present he is Professor of Mechanics at the National Technical University of Athens.



Professor Theocaris says that his present research involves a different moire method for the study of body and gravity forms around tunnels, real time holography to investigate crack propagation modes in metals, and a reflected shadow method for the evaluation of stress in plastics.

VICTOR C. SOBOLEWSKI, born in Cracow, Poland, received a first class honours BE degree in 1965, and is now working for a PhD degree at the University of Adelaide. His work on moire fringes was prompted by the need to determine very precisely the distortions of a TV-type display combined with an imaging lens and scanning system of a vidicon camera.



Of his work in this direction he writes: "Basically, moire fringe metrology depends on the comparison of a simple two-dimensional pattern representing the quantity to be measured with a similar accurate standard pattern, either a "hard copy" transparency or one generated electronically. The resultant moire fringes are indicative of differences between the two, greatly magnified.

"The simplicity of the method and its use results in very low cost, while the ease with which the results can be interpreted makes it possible even for amateurs to measure precisely CRT and TV display linearity.

"Scanning and display linearity of a computer-driven displays for precision graphics applications and, in military applications, of radar PPI with standard and non-standard scan patterns, have been measured with previously unobtainable accuracy. Moire methods can equally be used to measure non-linearity of images in imaging devices such as simple photographic cameras or TV cameras.

"By mixing signals requiring analysis with standard electronically generated signals and observing the displayed results, information about the signal or the signal processing hardware, otherwise not readily obtainable, can be readily seen and evaluated".

PROFESSOR GERALD OSTER is currently Professor of Biophysics at the Mount Sinai School of Medicine at the City University of New York. He graduated from Brown University and received a PhD from Cornell University in 1943. He had conducted research at the Rockefeller Institute for Medical Research, Massachussetts Institute of Technology, Princetown University, and in London, Strasbourg and Paris, prior to being appointed Professor at the Polytechnic Institute of Brooklyn.

His many published articles on moire patterns, and his book "The Science of Moire Patterns" in particular, have made moire popular as an art form all over the world. Several of his moire constructions have been exhibited in art museums, and he is currently concerned with the visual psychological effects of moire.

MOIRE FRINGES . . . from p 17

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