BY CECIL DEISCH • TELLABS OPERATIONS INC

Modeling skin effect in Spice

SKIN EFFECT CAUSES INCREASED LOSSES AS FREQUENCY INCREASES. IT ALSO CAUSES CHANGES IN SIGNAL VELOCITY, DEGRADING SIGNAL FIDELITY, ESPECIALLY THE EYE OPENING OF HIGH-SPEED DATA SIGNALS ON LONG SIGNAL PATHS ON PC BOARDS AND BACKPLANES.

Skin effect involves the interplay between flux linkages and currents in and on conductors. At dc and low frequencies, current density is essentially uniform throughout the cross section of a conductor. The fact that current flows on the interior of the conductor implies that a magnetic flux also exists within the conductor. Thus, at low frequencies, the inductance of a conductor is higher because magnetic flux is both inside and outside the conductor.

However, as frequency increases, current density within the conductor varies in such a way that it tends to exclude magnetic flux inside the conductor. This situation results in an apparent increase in resistance of the conductor because more of the current is concentrated near the surface and edges of the conductor, and it also causes the effective inductance of the conductor to decrease as frequency increases. These two effects become especially important when modeling the performance of high-speed data signals. A Spice model can easily accommodate the frequency variation of the skin resistance, which usually increases as the square root of frequency, such that simulation losses accurately agree with measured losses. However, it's also important to model the change in conductor inductance with frequency because this change affects the signal velocity and, hence, time of flight. This effect is especially important when modeling gigahertz data signals as they travel across the backplane.

SIGNAL DEGRADATION

Digital data signals of several gigahertz greatly degrade after they have traveled, say, 30 in. from one end of a backplane to the other end. The travel causes the "eye" opening of the signal to close severely and become corrupt, making decoding error-prone. The fact that the higher frequency signal components have greater attenuation than the low-frequency components causes much of this eye closure. Another factor is the time of arrival of the various frequency components. Because the conductor's inductance is slightly less at higher frequencies, the velocity of propagation is slightly greater, which implies that the higher frequency signal components arrive at the far end slightly before the bulk of the lower frequency components. This varying time-of-arrival, or dispersion, factor affects the zero-crossing times and further degrades the eye opening. Both the skin effect's attenuation and the time of arrival of the high-frequency components can dramatically corrupt the signal and lead to a nearly closed eye pattern.

Because skin effect and the change in inductance are inextricably linked, a Spice model also links these two effects. The simplest model for an incremental length of transmission line is the basic series L_1 and a lossless shunt, C, with some resistor R_1 in series with the inductance (**Figure 1**). R_1 can be either fixed or frequency-dependent to account for skin-effect losses. However, this simple model has no provision to change signal velocity. In another model, part of the series inductance has a shunt resistance across it (**Figure 2**). It's easy to see that, at low frequencies, the total inductance is simply the sum of L_1 and L_2, and the loss due to shunt resistor R_2 is negligible because its impedance is so much higher than that of L_2. As frequency increases, R_2 comes more into play and causes more losses as the impedance of L_2 increases. Thus, L_2 and R_2 perform the same function that occurs in a conductor—that is, as currents inside the conductor exclude some of the internal magnetic-flux lines, the apparent inductance decreases, and the losses increase. By juggling the values of L_1, L_2, R_1, and R_2, this simple circuit mod-

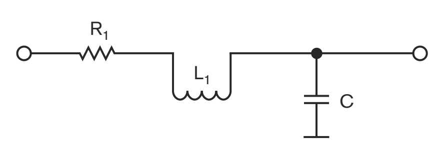

Figure 1 This basic model has no provision to change signal velocity with frequency.

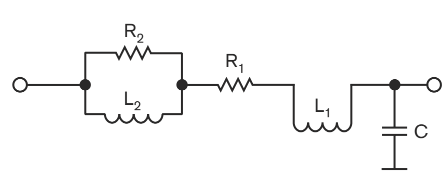

Figure 2 L_2 and R_2 increase losses and velocity as frequency increases.

els the resistance and inductance of an actual conductor over a frequency range greater than 25-to-1.

If accuracy over a wide frequency range is insufficient, you can add L_3 and R_3 (**Figure 3**). The sum of L_1, L_2, and L_3 represents the low-frequency inductance of the conductor, and you can adjust the values of all six of the series components to give an even better model. For most applications, one or two extra series sections give accurate models, but you can add series sections to achieve the required accuracy. Modeling a long signal path that carries high-frequency signals requires many of these incremental sections. A good rule is to account for at least the third harmonic and sometimes as high as the seventh harmonic of the highest data rate. This rule implies that the delay through each incremental section should be no greater than one-fifth the period of the highest harmonic. It's also a good idea not to make all incremental sections of equal length because this step can lead to certain high-frequency anomalies in the simulation waveforms. Vary the length of each incremental section, but make none greater than one-fifth the period.

TABLE 1 APPROXIMATE RESISTANCE OF THE CENTER CONDUCTOR

Frequency (MHz)	Handbook resistance values (ohm/ft)	Model resistance values (ohm/ft)	Model inductance values (nH)
1	0.13	0.15	83+7.7
5	0.24	0.22	83+6.8
10	0.32	0.34	83+5.1
50	0.7	0.67	83+1.7
100	0.96	0.89	83+1.3
500	2.2	2.4	83+0.31
1000	3.1	2.8	83+0.09

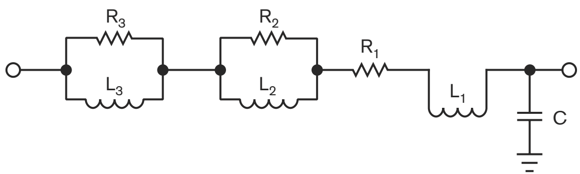

Figure 3 If accuracy over a wide frequency range is insufficient, you can add L_3 and R_3.

A SIMPLE EXAMPLE

Although designers have much interest in how data signals degrade as they travel along a backplane, the mathematics are intractable, so another model uses a simpler coaxial cable (**Figure 4**). The center conductor is #30 AWG, the relative dielectric constant of the interior insulation is 2.46, and the inside diameter of the outer sheath is 37 mils (0.037 in.). This arrangement gives a characteristic impedance of 50Ω. At dc and low frequencies, #30 AWG wire has a resistance of approximately 0.1Ω/ft. **Table 1** shows the approximate resistance of the center conductor (**Reference 1**). A quick calculation shows that the resistance increase is approximately the square root of frequency above 10 MHz.

According to standard tables, the center conductor of this coaxial cable has an inductance of approximately 83 nH/ft. But this inductance occurs at high frequencies at which virtually all current flows on the surface of the inner conductor and all magnetic flux lines are outside the conductor. However, at low frequencies, the inductance of the center conductor is significantly higher.

For a given total conductor current, flux lines outside the conductor are constant, independent of frequency or current distribution within the conductor. However, at low frequencies, current flows throughout the conductor, and a substantial number of flux lines are inside the conductor. For this model, more than 25% of the flux lines are inside the conductor. Because total inductance relates to the number of flux lines encircling the current, inductance is higher at low frequencies. However, the low-frequency inductance is not 25% higher because these internal flux lines don't enclose the entire conductor current. Low-frequency inductance is typically 10 to 15% higher. These circumstances imply that the inductance of the center conductor cannot be greater than its dc value of about 92 nH and cannot be less than 83 nH/ft, which you can model as 83 nH in series with approximately 9 nH. You can subdivide this 9 nH into several sections with appropriate shunt resistors to give skin-effect resistance that varies close to the square root of frequency.

THE SPICE MODEL

The model in **Figure 5** applies to the earlier-described coaxial cable and provides excellent agreement with the calculated skin resistance over the 1-MHz to 1-GHz frequency range. This model requires just two sections of resistance-shunted inductance and has an "error" of only ±12% from **Reference 1**'s values over this range (**Figure 6**). Although the skin-effect resistance of actual conductors, including backplane conductors, closely approximates the square root of frequency, the relationship isn't exact. This model uses fixed-value resistors and has no need for frequency-dependent resistance. Although this model gives per-foot values, the incremental sections for high-speed signals need to be 0.3 in. or less, which implies that you must divide the values by 40. A 30-in. length would require 100 of these incremental sections.

This small change in effective inductance as frequency increases is not insignificant. Consider a data rate of 2 Gbps over a 30-in. length. Using inductance numbers from **Table 1** indicates that the 1-GHz-frequency components arrive at the far end approximately 125 psec sooner than the 10-MHz components. Because the bit interval is 500 psec, this difference in arrival time amounts to an approximately 45° phase

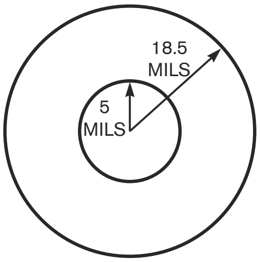

Figure 4 This model uses a simple coaxial cable.

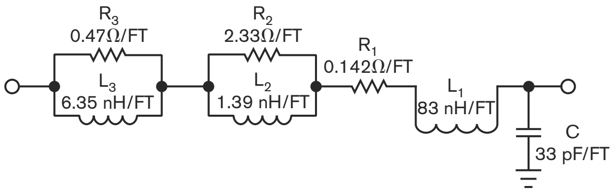

Figure 5 This model applies to the earlier-described coaxial cable and provides excellent agreement with the calculated skin resistance over the 1-MHz to 1-GHz frequency range.

shift, which greatly degrades the eye. **Figure 7** shows an alternative model with a slightly different topology and the same accuracy and error-versus-frequency curve as the previous model.

This article covers only the change in inductance of a conductor that causes high-frequency components of the data signal to arrive sooner than the low-frequency components. This model assumes a constant shunt capacitance, but some published data indicates that the relative dielectric constant of backplane material decreases slightly as frequency increases. This effect would also cause high-frequency components to travel faster, further degrading the eye opening. EDN

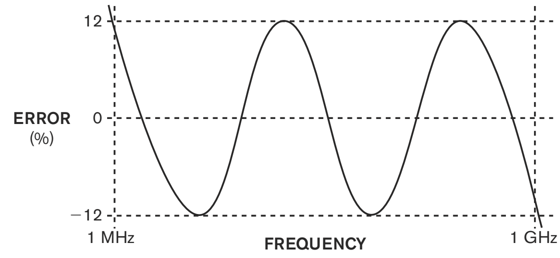

Figure 6 The Spice model in Figure 5 agrees with Table 1 within 12% over a 1000-to-1 frequency range.

REFERENCE

1 *Reference Data for Radio Engineers, Fifth Edition*, Howard W Sams and Co, 1968.

AUTHOR'S BIOGRAPHY

Cecil Deisch is a staff engineer with Tellabs; he was previously at Bell Labs. He holds a bachelor's degree in electrical engineering from the South Dakota School of Mines and Technology (Rapid City) and a master's degree in electrical engineering from New York University (New York). His interests include thermodynamics, economics, and mathematics.

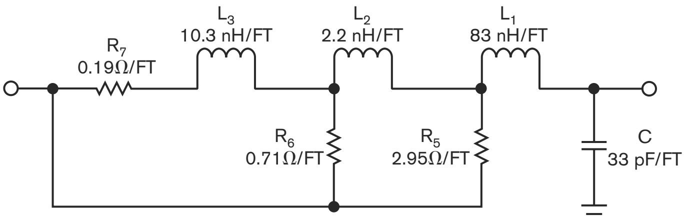

Figure 7 This model has a slightly different topology but the same error-versus-frequency (Figure 6) curve as the model in Figure 5.