

SCIENCE NOTEBOOKTM

Programmed Lesson on Inductive and Capacitive Circuits

This self-teaching lesson presents the basic theory necessary for understanding ac circuits using inductors and capacitors. Impedance, reactance, and resonance will become familiar terms

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Inductance and Capacitance are important concepts in circuit theory associated with energy storage. Inductors and Capacitors are also practical circuit components just about as readily available as resistors. Most electronic circuits require the use of capacitors or both inductors and capacitors along with other components to perform their intended function. Therefore, an understanding of their characteristics is essential for those involved in electronics. In this lesson, voltage and current relations for inductors and capacitors will be discussed. From these relationships, practical circuit applications will be noted.

As with previous programmed lessons, start by reading frame 1 and then examine the answers to the question at the end of the frame. Choose an answer you think is correct and go to the frame indicated. Then proceed through the lesson as directed.

1 Inductance can be described as the ability of a wire or conductor to produce induced voltage when the current varies. This voltage, resulting from electromagnetic induction, appears across the conductor similar to an IR drop across a resistor. A coil of wire has more induced voltage than an equivalent length of straight wire. Therefore, practical inductors are just coils of wire around a suitable core, manufactured to have a definite value of inductance. The schematic symbol for an inductor resembles a coil as shown in Figure 1.

The voltage induced across an inductor is calculated from the expression, $e = L \frac{\Delta i}{\Delta t}$, where L is the symbol for inductance in henries. The $\frac{\Delta i}{\Delta t}$ factor represents the time rate of change of current. Induced voltage is also called the counter e.m.f. or back e.m.f. If the applied voltage increases or decreases, the back e.m.f. opposes this change. Thus, an inductor in a circuit opposes any change in the current.

The magnetic field associated with an inductor stores energy. This energy, usually expressed in the unit of joules, is equal to $\frac{1}{2}LI^2$.

QUESTION: What is the value of the back e.m.f. induced across a coil of 0.3 henry, when the current increases from 80 ma to 100 ma in 10 μ sec?

- 1) 0.6 volt . . . Go to frame 5
- 2) 30 volts . . . Go to frame 15
- 3) 600 volts . . . Go to frame 10

2 Your answer is incorrect. You forgot to carry along the units when finding the stored energy, $\frac{1}{2} CE^2$. For this problem, C is equal to 1×10^{-6} farad. Return to frame 10 and choose another answer.

3 Your answer is incorrect. The $\frac{\Delta E}{\Delta t}$ factor equals $\frac{25}{1 \times 10^{-6}}$ volts/sec. Multiply this factor by the capacitance in farads to obtain the charging current. Note that 200 pF equals $200 \times 10^{-12}F$ or $2 \times 10^{-10}F$. Return to frame 6 and choose another answer.

4 Your answer is correct. How do we account for voltage and current relationships in circuits that are not purely inductive or purely capacitive? After all, most circuits contain combinations of resistors with capacitors or inductors, or contain all three components. In ac circuits, the term "impedance" is used to describe RLC combinations that impede current. The capital letter Z is used to represent impedance, which is specified by both magnitude and phase angle. Admittance, designated Y, is the reciprocal of impedance. For RC or RL series circuits, the magnitude of the impedance is found using the expression, $Z = \sqrt{R^2 + X^2}$. In this expression X represents either the capacitive or inductive reactance. Using impedance, Ohm's law is stated as $E = IZ$ or $e = iZ$ since small letters are usually used to represent ac quantities. Phase angles between voltage and current lie somewhere between 0° and plus or minus 90° , depending on the relative magnitudes of reactance and resistance. The exact angle may be found from the trig function for the tangent, where $\tan \theta = \frac{X}{R}$. In an RL series circuit, current lags the voltage and the phase angle lies between 0° and minus 90° . In an RC series circuit, current leads the voltage so that the phase angle lies somewhere between 0° and plus 90° . In both circuits the exact angle is determined by using the above formula.

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QUESTION: For the circuit shown in Figure 6, does the amplitude of the output increase, decrease, or remain the same as the frequency increases?

- 1) output amplitude increases . . .
Go to frame 9
- 2) output amplitude decreases . . .
Go to frame 17
- 3) output amplitude remains the same . . . Go to frame 13

5 Your answer is incorrect. The units must be changed to amps and seconds before using the formula, $e = L \frac{\Delta i}{\Delta t}$. For this problem, current changes 20×10^{-3} amp in 10×10^{-6} sec. Recalculate the back e.m.f. using powers of ten notation and choose another answer to frame 1.

6 Your answer is correct. Note the similarity for calculating energy storage in capacitors and inductors. In a capacitor stored energy is found from $\frac{1}{2} CE^2$ whereas, stored energy in an inductor equals $\frac{1}{2} LI^2$.

Capacitors and inductors produce opposite effects when used in circuits. Remember that an inductor opposes changes in current due to its counter e.m.f. The capacitor on the other hand opposes changes in voltage by maintaining circuit current. Capacitive current is found using the expression, $i = C \frac{\Delta E}{\Delta t}$. Actually, current does not flow through the capacitor but allows current to flow in a circuit when it charges or discharges.

QUESTION: What amount of capacitive current flows when the voltage across a 200 pF capacitor charges 25 volts in 1 μ sec?

- 1) 500 ma . . . Go to frame 3
- 2) 50 ma . . . Go to frame 19
- 3) 5 ma . . . Go to frame 12



FIGURE 1

THE SCHEMATIC SYMBOL FOR AN INDUCTOR RESEMBLES THAT OF A COIL, ILLUSTRATING THE BASIC CONSTRUCTION.

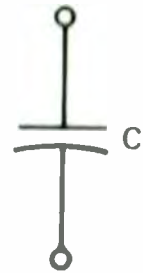


FIGURE 2

SCHEMATIC SYMBOL FOR A CAPACITOR, CAPACITORS ARE SOMETIMES REFERRED TO BY THEIR OLD NAME "CONDENSERS".

7 Go directly to frame 21.

8 Your answer is incorrect. Reactances in series can be added but capacitors in series are calculated in the same manner as resistors in parallel. Thus for capacitors in series, use the formula $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$. For two capacitors in series, this formula re-

duces to $C_s = \frac{C_1 \times C_2}{C_1 + C_2}$. Conversely, capacitors in

parallel are added similarly to series resistors, so that $C_p = C_1 + C_2 + C_3 + \dots$. Apply the above formulas and choose another answer to frame 12.

9 Your answer is incorrect. Consider the expression for capacitive reactance where $X_c = \frac{1}{2\pi fC}$.

As the frequency increases the reactance decreases. At any particular frequency we can treat the magnitude of X_c as a resistor of so many ohms. As the frequency increases this resistor gets smaller and smaller. From voltage divider principles the output then decreases.

The RC circuit in Figure 6 is known as a low-pass circuit, or low-pass filter, since low-frequency signals are relatively unattenuated. Higher frequency signals become attenuated and may be blocked from appearing in the output. Go to frame 17 and begin reading with the second sentence.

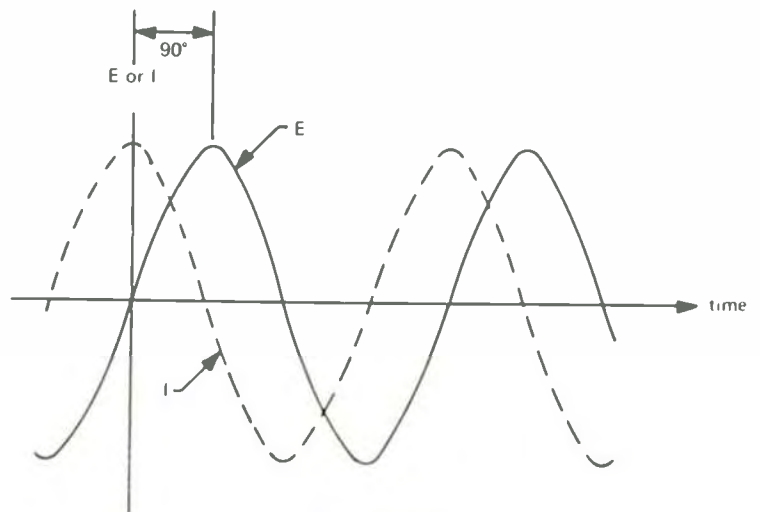


FIGURE 3

IN AN AC CIRCUIT CONTAINING ONLY CAPACITORS, THE CURRENT LEADS THE VOLTAGE BY 90°.

10 Your answer is correct. A capacitor is formed when two metallic plates are separated from each other by a thin layer of insulating material called a dielectric. The schematic symbol for a capacitor is shown in Figure 2. If a battery is placed across the capacitor plates, current cannot flow due to the dielectric material. However, the potential across the capacitor will equal the battery voltage. An electrostatic field exists across the capacitor plates and the capacitor is said to be charged. If the battery is removed, the parallel plates remain charged until an external circuit allows the plates to discharge. The charge on the plates is expressed using the relationship $Q = CE$, where Q represents the charge in coulombs. In this expression, C denotes the value of capacitance in farads and E is the voltage across the capacitor plates. The farad is such a large value of capacitance that microfarads ($\mu F = 10^{-6}F$) and picofarads ($pF = 10^{-12}F$) are more common units. A capacitor stores energy in its electrostatic field and is calculated using the expression $\frac{1}{2}CE^2$.

QUESTION: How much energy is stored in a $1 \mu F$ capacitor charged by a dry cell to 6 volts?

- 1) 1.8×10^{-5} joule . . . Go to frame 6
- 2) 18 joules . . . Go to frame 2
- 3) 3×10^{-6} joule . . . Go to frame 16

11 Your answer is incorrect. The power factor is always less than 1. Review the formula for power factor in frame 14 and choose another answer.

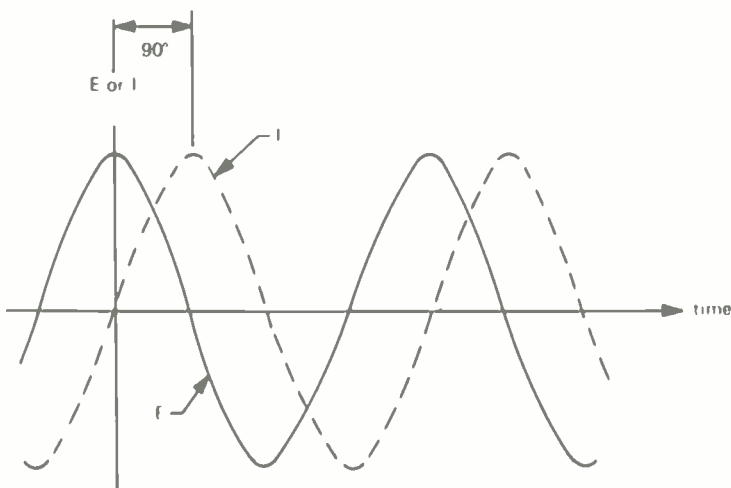


FIGURE 4
CURRENT LAGS THE VOLTAGE BY 90°
IN PURELY INDUCTIVE CIRCUITS

12 Your answer is correct. Opposition to the flow of alternating current in capacitive and inductive circuits is called reactance. For sinusoids, capacitive reactance is expressed in ohms using the formula, $X_c = \frac{1}{2\pi fC}$. Capacitive reactance has a different value for each frequency and decreases as the frequency increases. For purely capacitive circuits, reactance is treated similar to resistance. Thus Ohm's law can be applied as $E = IX_c$ and the reactances added in series or parallel like resistors. Note that reactance is treated like a resistance, not the capacitor. An important difference exists, however, between resistive and reactive circuits. For reactive circuits, current and voltage changes are not in step and the two are said to be out of phase. In a circuit containing only capacitors the current leads the voltage by 90° . Waveforms illustrating the leading phase angle are shown in Figure 3.

For purely inductive circuits, the opposition to changes in current is called inductive reactance. For a given sinusoid frequency, the reactance in ohms is found from $X_L = 2\pi fL$. Again we can apply Ohm's law as $E = IX_L$ or treat the inductive reactance as resistance, adding in series or parallel combinations. In purely inductive circuits the current lags the voltage by 90° as shown in Figure 4.

The reciprocal of reactance is called susceptance designated by B . For capacitors $B_c = 2\pi fC$, and for inductors $B_L = \frac{1}{2\pi fL}$.

QUESTION: For the circuit in Figure 5, what is the total series capacitance and equivalent parallel inductance?

- 1) $C_s = \frac{1}{2} \mu f$, $L_p = 1 \text{ mh}$. . .
Go to frame 4
- 2) $C_s = 2 \mu f$, $L_p = 1 \text{ mh}$. . .
Go to frame 8
- 3) $C_s = \frac{1}{2} \mu f$, $L_p = 4 \text{ mh}$. . .
Go to frame 20

Your answer is incorrect. From Ohm's law, the magnitude of the output voltage is $e_o = \frac{X_c e_{in}}{\sqrt{R^2 + X_c^2}}$

Consider how X_c varies as the frequency increases and choose another answer to frame 4.

14 Your answer is correct. At resonance, current and voltage are in phase since only resistance is left in the series circuit.

In ac circuits, power is dissipated only by resistors since, as was previously mentioned, capacitors and inductors both store energy. Power dissipated by

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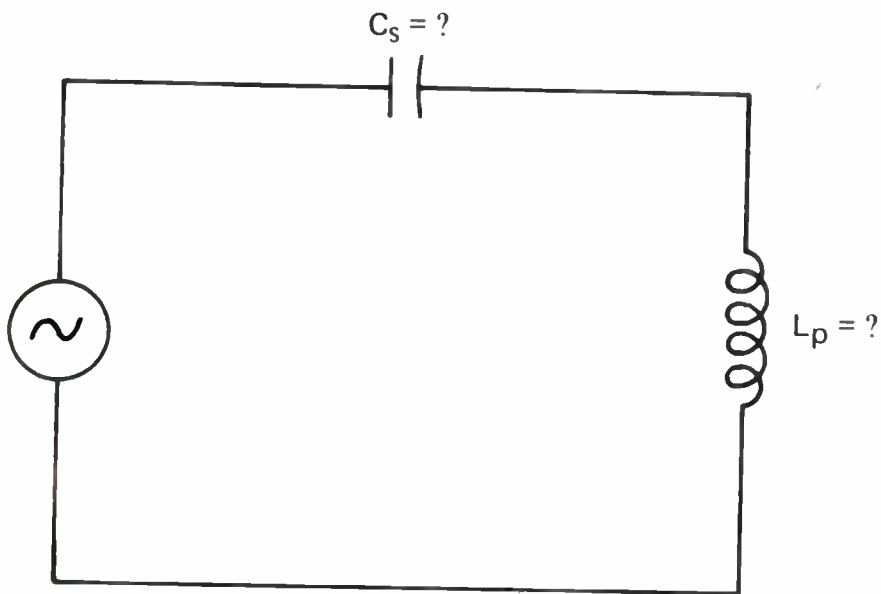
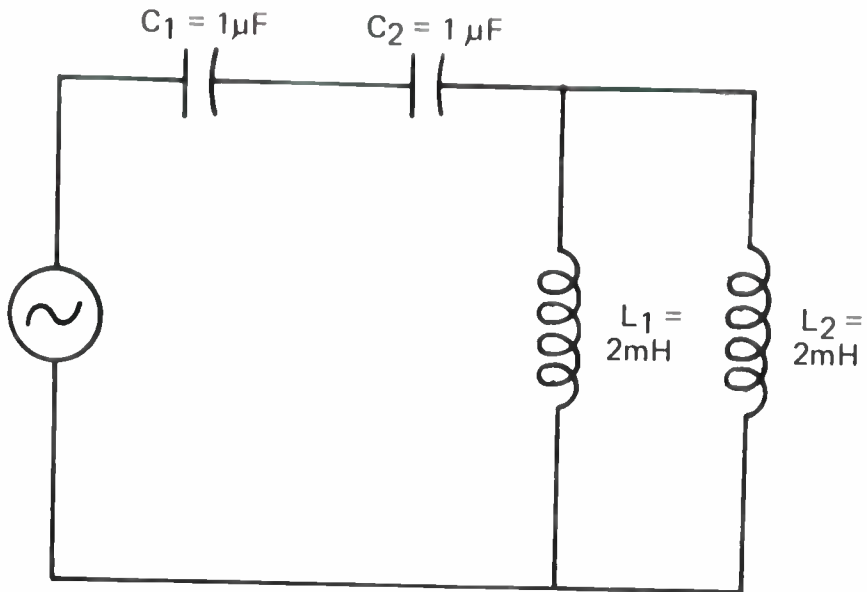


FIGURE 5
FIND C_S AND L_P FOR THE EQUIVALENT
CIRCUIT — FRAME 12

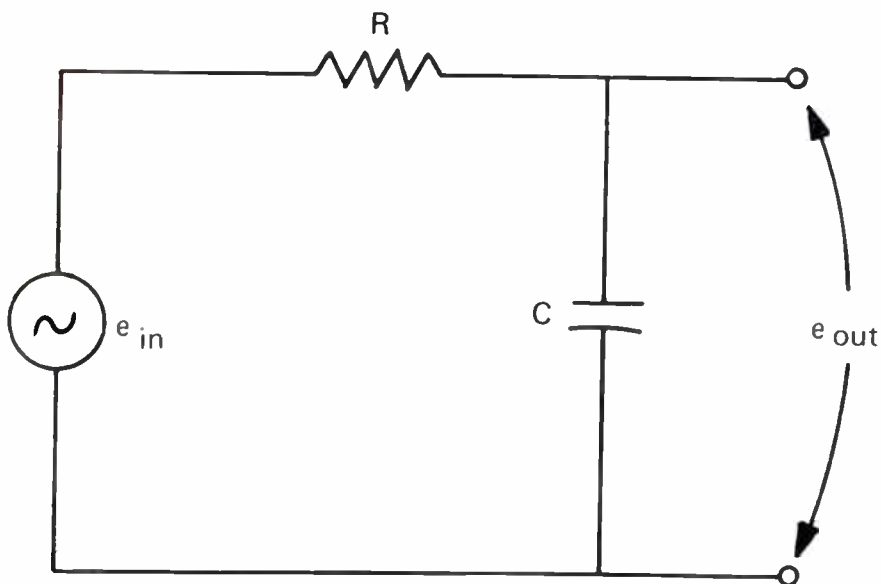


FIGURE 6
RC CIRCUIT FOR FRAME 4

the circuit resistance is called "real power" and is calculated as $I_{rms}^2 R$ or E_{rms}^2 / R . Another formula for real power is $E_{rms} I_{rms} \cos \theta$, where the RMS values for current and voltage are multiplied by the cosine of the phase angle. The $\cos \theta$ factor is called the power factor and has a numerical value between 0 and 1. For series circuits:

$$\text{Power factor} = \cos \theta = \frac{R}{Z}$$

If a circuit is mostly resistive, the power factor is close to 1. At the opposite extreme, a purely reactive circuit has a power factor of 0.

QUESTION: What is the power factor of a circuit containing a 300 resistor in series with an inductive reactance of 400?

- 1) 0.75 . . . Go to frame 18
- 2) 1.33 . . . Go to frame 11
- 3) 0.60 . . . Go to frame 22

15 Your answer is incorrect. Current changes 20 ma in 10 μ sec. The $\frac{\Delta i}{\Delta t}$ factor equals $\frac{20 \times 10^{-3}}{10 \times 10^{-6}}$ or 2×10^3 amp/sec. Multiply this value by 0.3 henry and choose another answer to frame 1.

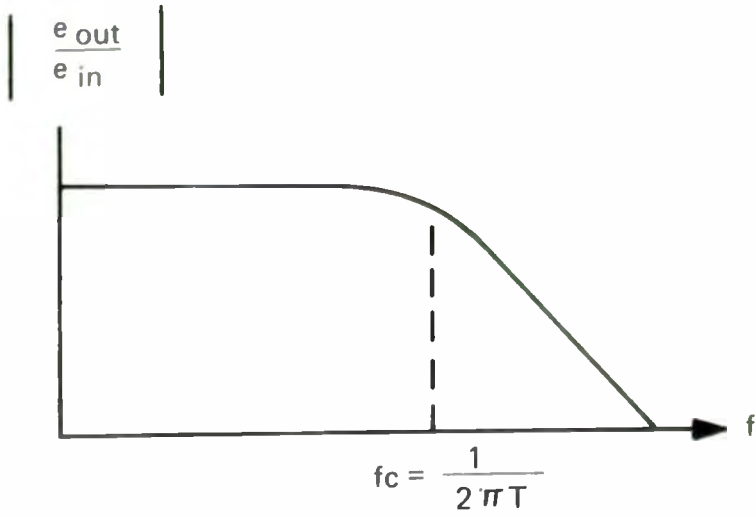
16 Your answer is incorrect, you forgot to square the voltage term. Recalculate the stored energy and choose another answer in frame 10.

17 Your answer is correct. RC and RL circuits can be used as either low-pass or high-pass filters. The four possible configurations are shown in Figure 7 along with graphs of magnitude versus frequency. A high-pass filter passes high frequencies but attenuates lower frequencies. Alternately as we saw in the previous frame, a low-pass filter passes lower frequencies but reduces the amplitude of higher frequencies. The frequency where the voltage output is reduced 0.707 or 3db (decibels) from its midrange value, is called the cutoff frequency. The "time constant" of the particular circuit determines the cutoff frequency as shown in the figure.

For series RLC circuits, the impedance is found from the expression $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\tan \theta = \frac{X_L - X_C}{R}$. If X_L is greater than X_C

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LOW PASS



HIGH PASS

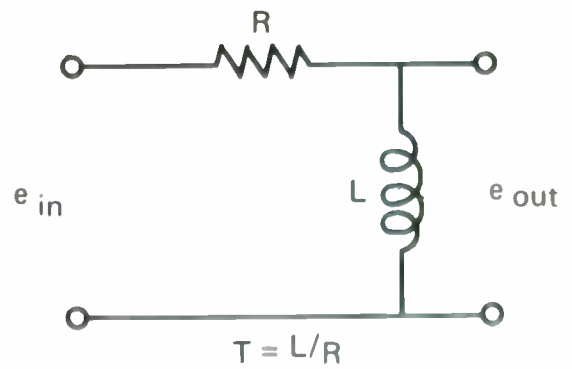
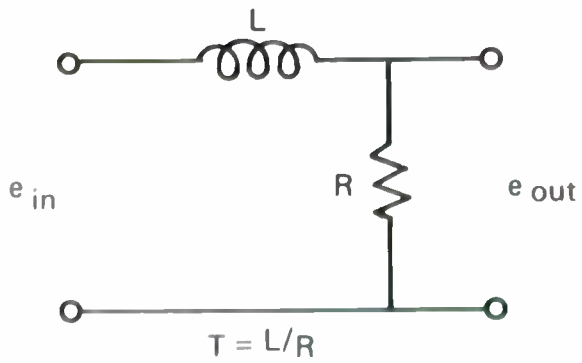
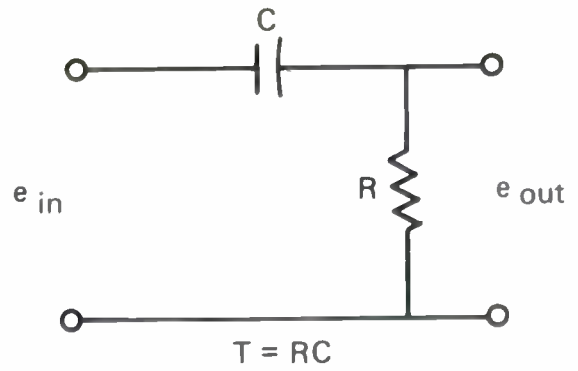
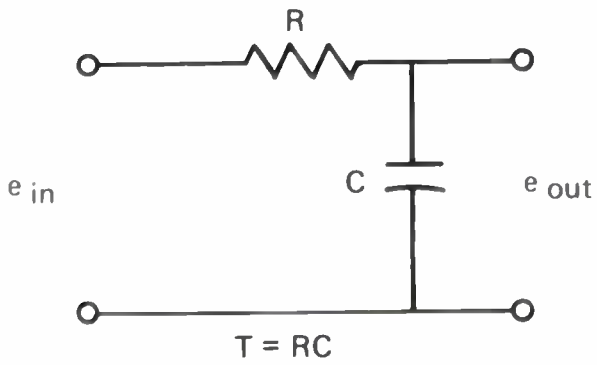
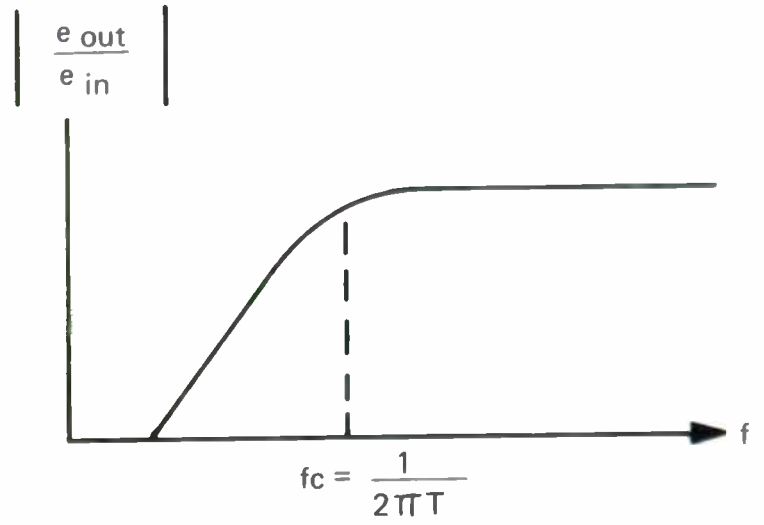


FIGURE 7

SIMPLE RC AND RL FILTERS

A Programmed Lesson on Inductive and Capacitive Circuits (continued)

the RLC circuit has an impedance which is inductively reactive. When X_c is greater than X_L the impedance "looks" capacitive. At one particular frequency X_L will be equal to X_c causing the reactive terms in the impedance formula to cancel one another. This frequency is known as the resonant frequency and in a series circuit, current will be a maximum. The series components act like a bandpass filter as shown by the frequency versus amplitude graph in Figure 8.

QUESTION: What is the resonant frequency for the circuit in Figure 8?

- 1) 5 Hz . . . Go to frame 21
- 2) 1.6KHz . . . Go to frame 14
- 3) 100KHz . . . Go to frame 7

18 Your answer is incorrect. Note that the power factor is the ratio of resistance to impedance not reactance. The impedance for the series circuit is determined from the expression $Z = \sqrt{R^2 + X^2}$. Find this impedance and recalculate the power factor. Then choose another answer to frame 14.

19 Your answer is incorrect. The numbers used in this problem are typical values encountered in electronics, illustrating the need for the use of powers of ten in calculations. For multiplication, powers of ten are added, for division they are subtracted. The 200 pF capacitor can be expressed as $200 \times 10^{-12}F$ or $2 \times 10^{-10}F$. Likewise, 1 μ sec equals 1×10^{-6} sec. The $\frac{\Delta E}{\Delta t}$ factor is equal to 25×10^6 volts/sec. Multiply this value by the capacitance to obtain the charging current. Then select another answer to frame 6.

20 Your answer is incorrect. Inductors in series and parallel combinations can be reduced in exactly the same manner as resistors. For series inductors $L_s = L_1 + L_2 + \dots$, and for parallel inductors $\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$. For two inductors in parallel, the above expression reduces to $L_p = \frac{L_1 \times L_2}{L_1 + L_2}$. Find the equivalent parallel inductor for Figure 5 using the above formula and choose another answer to frame 12.

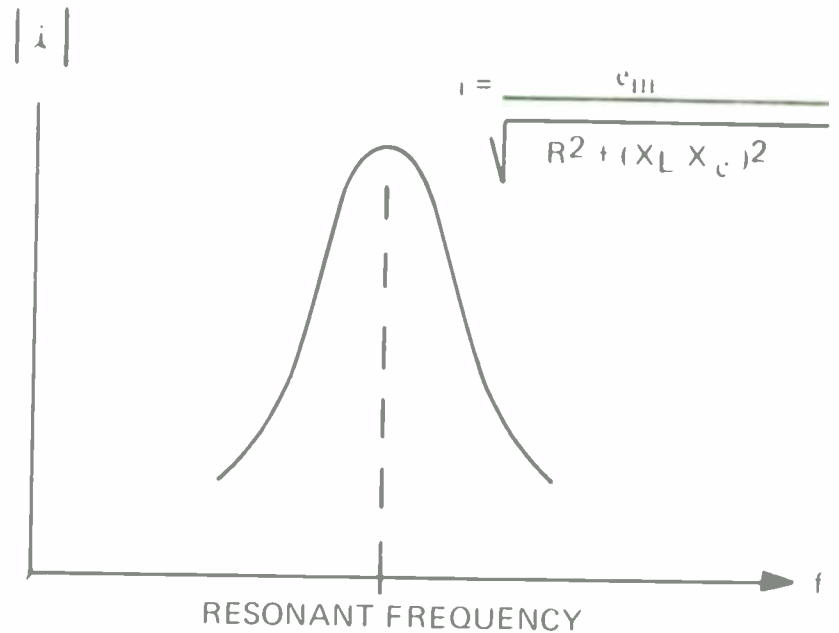
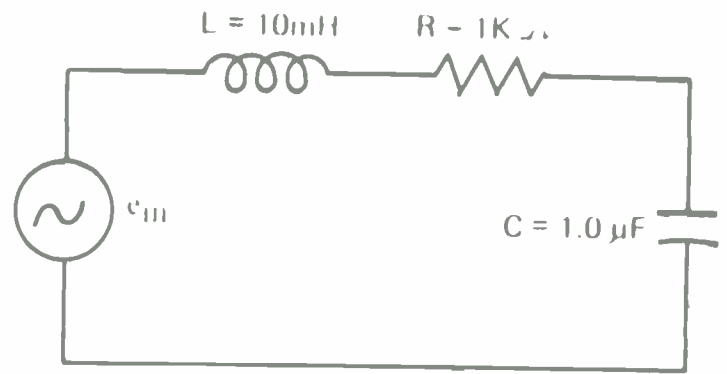


FIGURE 8
SERIES RLC CIRCUIT FORMS A BANDPASS FILTER — FRAME 17

21 Your answer is incorrect. The resonant frequency occurs when $X_L = X_c$. Substituting $2\pi fL$ and $\frac{1}{2\pi fC}$ for X_L and X_c respectively, and then solving for frequency results in an expression for the resonant frequency as $f = \frac{1}{2\pi\sqrt{LC}}$. Use this formula and the values for L and C from Figure 8 to find the resonant frequency. Then choose another answer to frame 17.

22 Your answer is correct. In this lesson we have concentrated primarily on series circuits. Parallel and series-parallel RLC combinations are of course possible and often used in circuit applications. In general, impedances may be combined following the same rules as for resistors. However, calculations become cumbersome and exact expressions for impedance and phase angle may be difficult to obtain. Handbooks are available which show various RLC combinations with expressions already determined for impedance and phase angle. One particularly good basic reference is Allied Radio Shack's, "Electronics Data Handbook." This handy and inexpensive handbook has condensed formulas, trig tables, and other commonly used electronic data.