Magnetism and Magnetic Units

Understanding the basic relationships, with special reference to SI units

by "Cathode Ray"

The other day I saw—on 'Nationwide', I believe—something about a shopkeeper who persisted in doing business in £sd. (Even he admitted that he wouldn't actually refuse decimal coins. What he thought of paint by the litre and timber by the metre, assuming he was a DIY man, wasn't revealed, probably because his opinion of them wouldn't have been unusual enough to rank as news.)

SI* units, or at least those included in the mksA system, have been with us far longer than decimal coinage. The mks (metre-kilogram-second) system was proposed by Prof. G. Giorgi as long ago as 1901, and although more than 30 years passed before much notice was taken of it, when the break came (as it did in electrical engineering—after the addition of the ampere—more than 20 years ago) the change-over was much faster than the most optimistic had expected. Yet there is still a pocket of resistance that goes on using cgs units though all others have stopped. I mean the people concerned with magnets and magnetism.

Practically everybody *uses* magnets, in such things as loudspeakers, magnetic pick-ups and microphones, tape heads and television receivers for example, but not many are so much involved with them as to have to use magnetic units, or, more correctly perhaps, units of magnetism. May be it is because these are a relatively small group, confined largely to Sheffield†, completely single-minded in their devotion to the task of producing ever better magnets, that they are out of touch with the rest of the technological world in this (to them) unimportant matter. Like the Japanese sergeant found in some remote spot in Indonesia, they don't know that the (units) war has been over for 20 years. To be fair, one must admit that there are other possible reasons for this backwardness. It is all very well for the rest of the technological world to be self-righteous about their own acceptance of SI units; their volts and amps and watts and even henries were completely unaffected by the change. In so far as magnetic magnitudes have to be considered by some, this was usually a small part of their whole world and the new units could be accepted without too much upheaval. But for specialists in magnetism, cgs units were part of their tradition, and much greater mental adjustment was required. And even now, when challenged they can claim more than mere mental inertia as an excuse: with some justification they can retort that reckoning flux density per square metre is not strikingly appropriate in this day and age of microelectronics. Square centimetres are much nearer the mark, especially in the loudspeaker magnet trade. Their reasonableness in pleading against the inconvenience of having to specify a typical magnet flux as, say, 0.0015 webers may at this point be adulterated by a certain amount of low commercial cunning, since 150,000 maxwells is much better calculated to impress potential customers. Another argument that will undoubtedly be raised is the convenience of the cgs permeability of air being equal to 1, instead of $4\pi/10^7$ as in SI.

So the magnet trade at least may be hard to convince. Perhaps a better line to take with them than extolling the virtues of SI (which they will have difficulty in seeing, even if they want to see them, which is unlikely) is the negative approach—to point out that there is no more future for cgs units than for £sd coinage. Their sons—and daughters—are being brought up on SI, and most fathers don't like to be seen as squares in their own business. And even their hi-fi customers, looking up the current loudspeaker lists as I am just now, may soon be wondering what these gauss and maxwells—and even 'lines'—are. When the magnet men realize they are talking an archaic language to the new generation of big money spenders they will change.

The readers I have in mind are not the members of the magnet trade, nor the young who know only SI, but those who were brought up on cgs and are not yet too handy with SI, together with all who are hazy about magnetic quantities of any kind and their relationships to the familiar amps and volts and ohms.

So first of all I will show how magnetic circuits correspond to electric circuits. I know that this is an extremely unoriginal procedure, found in nearly all the elementary books. I used it myself in the September 1947 issue, but even if you had been born by then you would hardly remember it. And I know that superior persons, looking for a chance to demonstrate their superiority, will point out that this is a false analogy, since magnetic flux corresponds to electric flux, not current. But practically nobody outside the classroom, and few of those inside it, are really familiar with electric flux and elastance. It is a basic principle of teaching that the obscure should not be explained in terms of the more obscure. So I'm going to liken magnetic flux to electric current, with the warning that there is a more perfect analogy to come later.

I hopefully assume that everyone who is still with us understands Ohm's Law. No; I'm not thinking of the pedantic aspects of it that were my subject in the August 1953 issue and can be seen to this day in "Second Thoughts on Radio Theory". All I mean is the relationship between volts, ohms and amps ($I = E/R$), and how resistance depends on the dimensions and resistivity of the circuit or part of a circuit concerned. So, in Fig. 1, the resistance of the bit of wire is

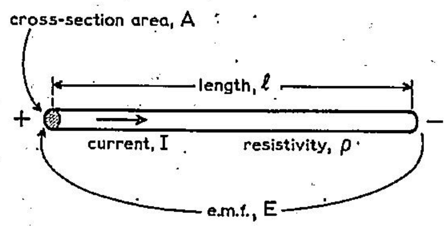

Fig. 1. Ohm's law applied to a piece of wire to find the current flowing through it, given the dimensions and resistivity of the wire and the e.m.f. applied to it.

directly proportional to its length l and to the resistivity ρ of the metal, and inversely proportional to the cross-sectional area A:

$$R = \frac{\rho l}{A} \qquad (1)$$

This is true whatever the units of R, l and A. But the value of ρ depends on those units. In SI the basic unit of length is the metre, so ρ is the resistance between two opposite faces of a metre cube of the material, and in the equation l must be in metres and A in square metres, or metres² as we are encouraged to write it. There is nothing to stop

*Système Internationale d'Unités.

†To forestall indignant retorts, or even physical assault, from citizens of Sheffield, I would assure them that I have no wish to bring their city into contempt. By all accounts it is an admirably progressive one, not least in the reduction of atmospheric pollution.

us reckoning A in square millimetres (mm^2) if we prefer, so long as we allow for this deviation by dividing by 10^6. For ordinary circuit materials ρ is a constant at any one temperature, which is more or less what Ohm was on about. (He didn't know anything about volts, amps, or even ohms.) For metals ρ increases slightly as the temperature rises. For a lot of other things it falls. And for electronic devices it depends mainly on V or I, but of course Ohm knew nothing about them.

One must admit that this resistance formula (1) is not very often used in practice. The resistance of wire is given in tables, and the resistance of resistors is shown by the colour code they bear. If in doubt one can easily measure the resistance with the usual multirange meter. The resistances of electronic devices cannot be calculated by the formula, because ρ is unknown; anyway, one is not usually interested in their resistances as such so much as in the varying relationship between E and I, given by characteristic curves. The main purpose of eqn. 1 is to provide a clear picture of how units of resistance depend on circuit dimensions.

So much for the recapitulation. Now for the analogy. To change over to a magnetic circuit, for electromotive force E volts put magnetomotive force F amps (yes!), for current I amps put magnetic flux Φ webers (Wb), for resistivity ρ put reluctivity ν, and for resistance R ohms put reluctance S amps per weber (A/Wb). (Note: ohms could be called volts/amp, which would make the resemblance of form still closer. Incidentally, in specifying the full-scale current drain of voltmeters, their manufacturers call amps ohms per volt, but in this case the reason is unknown.)

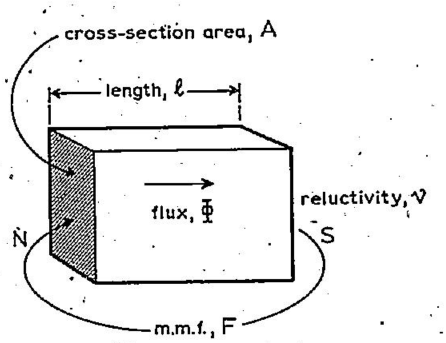

Fig. 2. This is a magnetic analogue of Fig. 1, showing how the magnetic flux in a block of (say) iron can be calculated.

In Fig. 2 we have, say, a piece of iron such as a pole-piece forming part of a magnetic circuit. Following the same reasoning as for Fig. 1 we get

$$S = \frac{\nu l}{A} \qquad (2)$$

In both diagrams A has deliberately been made constant throughout the length l to avoid bringing in mathematical complications that would distract attention from the main principle. Although for our theoretical purposes A and l could have been made the same sizes in Fig. 2 as in Fig. 1, in practice magnetic circuits are generally made short and fat because (1) the object is usually to make Φ as large as possible, and (2) whereas the resistivity of the space surrounding an electric circuit is usually high enough for practically no current to leak into it, reluctivity is never very low so leakage of magnetic flux could be considerable in a long narrow circuit. There is no such thing as a magnetic insulator.

In case anyone is puzzled by reluctivity it might be helpful to reveal that it is the reciprocal of the better known permeability, μ; i.e., $\nu = 1/\mu$. If you prefer you can put permeability in Fig. 2 and substitute the corresponding quantity conductivity, γ, in Fig. 1. But I thought we might make a bad start if we encountered this rather unfamiliar quantity so soon.

Permeabilities or reluctivities, take your choice, are almost the same for all materials —including empty space—other than those called ferromagnetic, for which μ can be many thousands of times greater and varies enormously according to the degree of magnetization. In fact, such materials correspond very much to electronic devices in electric circuits; characteristic curves are needed, and electronic current and magnetic flux are both limited by saturation.

Before we can tackle magnetic units we have to consider how Φ and F, and other magnetic quantities not shown in Fig. 2, are related to current and voltage. We must make perfectly sure we don't confuse these *relationships* with the *analogy* we have just been considering. It would have been better if we could have illuminated magnetic quantities in Fig. 2 by some analogy with totally unrelated quantities, say the flow of tomato chutney along a pipeline on its way to the bottling department; but chutney-motive force is not a sufficiently familiar concept to come within our basic principle of education, and there are other flaws in the analogy. It happens that Ohm's Law is clearer and simpler and better known than any other valid analogy I could call to mind. But now, having I hope got a clear picture of Fig. 2, let us forget about Fig. 1.

We all know that when an electric current flows it sets up a magnetic field around itself (Fig. 3). And that the strength of this field is directionally proportional to the current. Does it depend on anything else? As a one-time famous broadcaster would so rightly have said, it all depends on what you mean by a magnetic field. I've used the term as vaguely as I suspect many people, even some readers of *Wireless World*, think about it. That is exactly why I'm trying to clarify the matter. There are various approaches, but as we have already established a magnetic 'Ohm's Law' let us begin there, without stopping yet to explain exactly what is meant by a magnetic field.

Whatever it is it can be supposed to be caused by what we already know as a magnetomotive force, hereafter to be abbreviated to m.m.f. in line with e.m.f. It in turn is caused by electric current, and depends on nothing else. That is, if you follow the modern practice and count the total current around which the m.m.f. is considered. So if there are 50 wires close together, each carrying 0.1A (usually because the wire is wound into a 50-turn coil) the effective current is 5A. Formerly one would have said 5 ampere-turns. The main object of SI being to exclude all illogical constants in the relationships between the basic units, the SI unit of m.m.f. has been so chosen that it is numerically equal to the current that creates it. That is why the name of the unit of m.m.f. is the same as that for the basic unit of current—the ampere.

M.m.f. is not directly useful, but only as a cause of magnetic flux; just as e.m.f. is not directly useful for creating magnetism, but only as a means of making the current flow. And just as the amount of current a given e.m.f. will cause to flow in a circuit is decided by the resistance of the circuit, so

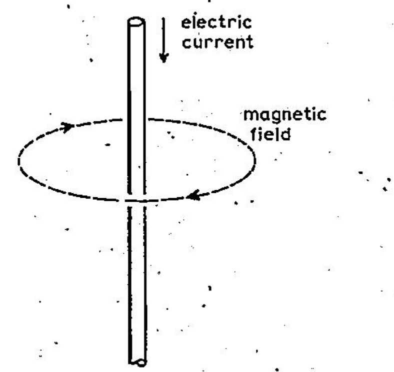

Fig. 3. The basic relationship between an electric current and a resulting magnetic field

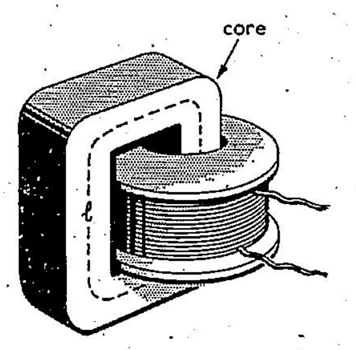

Fig. 4. Here the magnetic circuit linked with a current-carrying coil is assumed (for simplicity) to be confined to a high-permeability core of uniform cross-sectional area A and mean length l.

the amount of flux a given m.m.f. will cause in a magnetic circuit is decided by the reluctance of that circuit. In practice one usually looks at it from the other end: knowing that a certain amount of flux has to be provided, how much m.m.f.—in terms of current and number of turns—is needed?

This can be quite difficult. The shape of a magnetic circuit is usually decided by what it is for. In any case the whole circuit around the current cannot be of the ideal rectangular shape shown in Fig. 2. Assuming that one wants to produce the maximum flux for the minimum m.m.f.—in other words to have as little reluctance as possible—eqn. 2 shows that we would choose one of the special alloys with a very low ν, or high μ. Makers of these alloys supply data showing the values of μ under various conditions. One of the many forms of core made of such materials is shown in Fig. 4. It is quite possible to make A constant throughout, or nearly so; and although l varies according to distance from the centre an average figure can be used, and so the reluctance of the whole circuit can be calculated reasonably well.

It is seldom as simple as this. Very often, as in electric motors and generators, loudspeakers and moving-coil meters, the flux has to pass through an air gap to be of any use. When the gap is of such a shape that A and l are constant, its reluctance can easily be calculated, μ for air being known very accurately, though one has to allow for edge effects. Because μ for the core is usually so enormous in comparison, the core reluctance can sometimes be neglected, so letting one off the problem of ascertaining it. Another help is to remember that just as resistances in series add up, so do reluctances, and one can split up the magnetic circuit into separate parts, each needing a certain m.m.f. to carry a given flux. (This is analogous to Kirchhoff's voltage law.)

You may be bursting to tell me that most of the magnets in which *Wireless World* readers are likely to be interested are permanent magnets, for which no current is needed. Actually they too require current to cause the required m.m.f., but the molecules of the magnet material itself are so aligned that the electrons circulating in them constitute the necessary current. (In all other materials the alignment is random or in direct opposition, so the magnetic effects of these tiny currents cancel out.) One would have to be rather unusually bright at physics to predict the effective m.m.f., but fortunately the suppliers of permanent magnets also provide all the necessary data. The units used are (or should be) the same as for electromagnets; the theory is too much to push in here and now, and in any case can be understood more easily when we have covered magnetism generally. I may get around to it later, but meanwhile if you can see the March 1961 issue you will find it all there.

If you look up magnet or magnet core data you are likely to find most of it in terms of B and H, with Φ and F and S hardly mentioned, if at all. Even μ may not be specified directly, although it seems to be the most important factor in reluctance. To understand these omissions, let us take a look at a curve of Φ against F for some magnetic material such as iron (Fig. 5). The slope of this curve will be Φ/F. Our magnetic 'Ohm's Law' is

$$\Phi = \frac{F}{S} = \frac{F\mu A}{l}$$

So $$\frac{\Phi}{F} = \frac{\mu A}{l} \qquad (3)$$

The dimensions of the piece of iron, A and l, being fixed, we see that the slope is proportional to μ. To find the actual value of μ we would have to multiply the slope by l and divide by A. This way of presenting the data is silly, because we are not interested in the figures for the piece of iron that the manufacturer's lab people happened to use for their tests, but in the properties of that particular material, which we can then use to tell us about a piece of the size and shape we might want to use. One way would be to measure a unit cube of the material, so that l and A were both $=1$. But this would restrict the method of measurement very inconveniently, especially with SI units, for a metre cube of iron weighs about 8 tons.

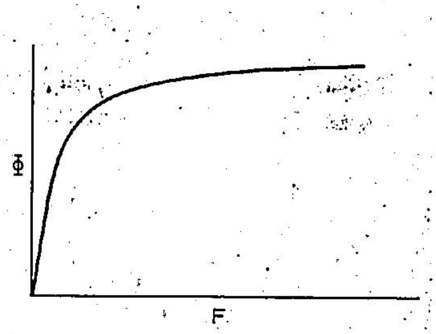

Fig. 5. A graph of flux against m.m.f. for a ferromagnetic material would apply to only one particular size and shape. But by suitable choice of scales of flux density against magnetic field strength the same graph is made to apply to that material in any size and shape.

A better idea is to have units that will refer to unit dimensions of the material. So instead of Φ, the total flux, we use the flux passing through unit cross-sectional area: the flux density, denoted by B, in Wb/m^2, called teslas (T); and what is called magnetizing force or magnetic field strength, H, in A/m. Rearranging eqn. 3 we get

$$\frac{\Phi}{A}\cdot\frac{l}{F} = \mu$$

So $$\mu = \frac{B}{H} \quad \text{or} \quad B = \mu H$$

For the reason just explained I didn't bother to provide Fig. 5 with scales, but if B is written in place of Φ and H in place of F then numerical scales would apply to that material in general, regardless of size or shape. (There are exceptions, called anisotropic materials, 'anisotropic' meaning that their properties are not the same in all directions, like wood having different properties along and across the grain.)

Sometimes one comes across data curves showing μ directly in terms of H or B. From the typical B/H curve shape in Fig. 5 we can see that the permeability (=slope) begins high and continues so over a range, beyond which it falls off rapidly towards a certain flux density, called saturation, which is not much more than for air. Under these conditions there would be a lot of leakage flux outside the iron.

Since most magnetic data and calculations are in terms of B and H, referring back to Fig. 1 we may wonder why the same policy is not adopted there, replacing current by current density and e.m.f. by electric field strength. Well, if I had started from the more strictly appropriate analogy, comparing magnetic fields with electric fields, that is just what one would do. Because one is interested in electric fields mainly in non-conducting spaces (inside a cathode-ray tube, for example) current is replaced by electric flux, which is treated like magnetic flux and reduced to flux density or displacement. For an overall grasp of electric and magnetic theory it is very helpful to consider this analogy in detail, but I assumed that from a more practical standpoint most people are familiar with electric circuits and would like to be clearer about magnetic circuits *and* fields.

While we are on about fields we might look again at Fig. 3. If the current flowing through the wire (or group of wires) is called I, we now know that the m.m.f. F encircling the wire—at *any* distance from it —is equal to I, both I and F being reckoned in amps. But because the path length around —call it l again—is proportional to the distance r from the axis of the wire, being in fact equal to $2\pi r$, the m.m.f. is spread over a greater circular length as the distance from the current is increased. So the magnetic field strength

$$H = \frac{F}{l} = \frac{I}{2\pi r}$$

In words, it is inversely proportional to the distance from the current that causes it. We are assuming—in case you didn't know— that the whole of the space around the wire has the same permeability and contains no currents or magnets to upset the cylindrical distribution of field around the wire.

If your information on magnetism was obtained some time ago you may have been wondering why I've about come to the end of this exposition without having ever mentioned 'unit magnetic pole'. Most of the books used to base their treatment of magnetism on it. The more honest of them admitted that no such things exist, which is why I've ignored them. It is rather different with the analogous electrical concept, unit electric charge at a point, because electrons and protons are as near as you like mobile point charges. Another item that has been perhaps conspicuous here by its absence is the 'line of magnetic force', so much used in 'explaining' magnetic fields. They don't exist either, and can be actually misleading if they are allowed to convey the impression that the spaces between are any less magnetic than the lines themselves. But, like the lines cartoonists draw radiating from persons experiencing intense emotion, they at least help one to visualize something that does exist. In particular, they show on a diagram the directions along which a magnetic field acts; for example, in Fig. 3, in circular paths around the current. If there were such things as mobile magnetic poles of negligible size, these are the paths along which they would be moved.

No; I haven't forgotten that I set out to enlighten any who are still groping in cgs twilight. The fact that cgs units don't fit in with the familiar electrical units such as volts and amps has already been mentioned as one of their disadvantages. Another is the fact that there are two cgs systems of units, one based on unit electric charge and the other on unit magnetic pole, and their units differ from one another and from the practical units by factors usually of many millions. Another snag is that unit charge and unit pole were each said to give rise to a flux of 4π units. The reason for this apparently odd choice was that unit flux density was defined to exist at unit distance from the unit point source of flux. The surface area of the sphere of unit radius is 4π units, so if the flux emerging through unit area of the surface is 1 the total flux must be 4π. By starting on this basis, the originators of the cgs systems eliminated the factor 4π precisely where one ought to find it—in a situation of spherical geometry. The result was that the factor 4π, expelled from where it rightly belonged, broke out in places where its presence could not be justified by the geometry; for example, in the formula for a parallel-plate capacitor.

And in the relationship between current and m.m.f. My electrical engineering tutor, whenever a student was stuck at a problem, sat down opposite him, scribbled on a sheet of paper with a circular motion to represent a current-carrying coil; then repeatedly smiting its interior with the point of the pencil to represent end views of lines of force, hissed 'Magnetomotive force is point four pi times the current enclosed!' This relationship took into account the irrational 4π and the fact that the electromagnetic cgs unit of current was 10A. Nowadays even the densest student should be able to retain the SI relationship 'Magnetomotive force is equal to the current enclosed' without having to be constantly reminded of it.

Fig. 4 shows that interrelated current and magnetic flux are like adjacent links of a chain. We have considered how current in the coil causes an m.m.f. linking the current path. Faraday's greatest discovery was that a *change* in magnetic flux causes an e.m.f. linking the flux path. The electromagnetic unit of e.m.f. was quite logically defined as that induced when interlinked flux was changing at unit rate (1 maxwell) per second. But unfortunately this turned out to be $1/10^8$V, or 0.01μV, which is small even by circuit noise standards. The electrostatic cgs unit of e.m.f., by contrast, is about 300V, because the ratio between the units of e.m.f. in the two systems is equal to the speed of light in centimetres per second. To the uninitiated this might seem as irrelevant as the diameter of the earth or the price of beer. The connection lies in the fact that in both cgs systems the permeability and permittivity of empty space (μ_0 and ε_0) are both fixed as 1. Now one just can't have it both ways like this. The reason is that the speed of light (**c**) is equal to $1/\sqrt{\mu\varepsilon}$ for the medium in which it is travelling, so in space is $1/\sqrt{\mu_0\varepsilon_0}$. The only way to make μ_0 and ε_0 both 1 is to choose units of length and time such that $\mathbf{c} = 1$. If the second is retained as the unit of time, then the unit of length must be 299,792,800 metres. Anyone who proposed this as the standard would have no political future.

The inevitable result of making unit length 1cm at the same time as $\mu_0 = \varepsilon_0 = 1$ was the emergence of two cgs systems, depending on whether μ_0 or ε_0 was chosen as basic, in which units of the same quantities differed by factors of **c** or $\mathbf{c}^2$. And the real values of μ_0 and ε_0, which actually are related to **c**, had to be hidden away in the sizes of the various units. So most of them are wildly impractical. The emcgs unit of resistance, for example, is 0.001 microhm, while the escgs unit is about a million megohms. SI works on a different principle. By changing over to the metre and kilogram for length and mass, and using the ampere as the unit of current, all the 'practical' electrical units became parts of it, and new magnetic units emerged from them on the same principles. And so the SI unit of m.m.f. is equal to the current enclosed instead of 0.4π times it. And when the magnetic flux is changing at unit rate per second the e.m.f. induced along a linked path is 1 volt.

Does this mean that π no longer appears in electromagnetic equations? Not at all; it means it appears where it logically ought to—as 2π in cylindrical geometry and 4π in spherical geometry, but not in rectangular geometry. The cgs systems were as confusing as a system of measures would be in which the unit of length was such as to make the surface area of a sphere one unit of length-squared.

Of course there is always a snag. Instead of the convenient values of 1 for space permeability and permittivity we have $4\pi/10^7$ and approximately $1/(36\pi \times 10^9)$ respectively. So π and large powers of 10 get back in by the rear entrance! However, it is easier to remember these two values than to have to remember the correct constants for innumerable formulae. If dirt has to be swept under carpets, it is better to have it swept under two already dirty ones if we can rely on there being none anywhere else. There is even something to be said for μ_0 and ε_0 not being 1. When they were, students were often led to suppose that H and B were more or less the same thing and μ just a multiplier to take account of the properties of magnetic materials. Then they got into difficulties with the dimensions of equations.

What, then, are the dimensions of μ and ε? The best clue to ε is the way the capacitance between two parallel plates is calculated. It is proportional to A, the area of the space between the plates, and to ε, the permittivity of whatever occupies that space. And it is inversely proportional to l, the (uniform) distance between the plates. (Edge effects are neglected, or counteracted in some way.) So in any regular system of units

$$C = \frac{A\varepsilon}{l}$$

Therefore

$$\varepsilon = \frac{Cl}{A}$$

In SI units, C is in farads, l in metres and A in metres^2. So ε is farads $\times$ metres $\div$ metres^2, or farads per metre. Going back to the electrical circuit analogy, we would find in the same way that conductivity (γ) was in siemens (formerly mhos) per metre, and $1/\gamma$ (=resistivity, ρ) was ohm-metres. An alternative that used to be used was ohms per metre cube, and similarly for the other things; but this looks as if it restricted the measurement to a piece of a particular shape and size of the material tested.

As the analogue for capacitance is inductance we start to get at μ from there. The inductance (L) of a coil—say the one in Fig. 4—is equal to the flux linked with it when unit current flows through it. If we neglect flux in the surrounding air, and use eqn. 3 we have, when F is one unit and Φ is therefore equal to L,

$$\mu = \frac{Ll}{A}$$

So μ is in henries per metre.

To sum up, here is a table of the SI magnetic units:

Quantity	Symbol for quantity	Unit	Abbrevn. for unit		emcgs equivt.
Magnetomotive force	F	Ampere	A	In practice, the ampere-turn	0.4π gilberts
Magnetic field strength	H	Amp. per metre	A/m	$=F/l$	$4\pi 10^{-3}$ oersteds
Magnetic flux	Φ	Weber	Wb	$=AB$	10^8 maxwells
Flux density	B	Tesla	T	$=\mu H$	10^4 gauss
Permeability	μ	Henry per metre	H/m	$=B/H$	$10^7/4\pi$ greater
Permeability of space	μ_0	Henry per metre	H/m	$-4\pi 10^{-7}$	ditto (=1)

PUBLICATION DATE

We regret it has not yet been possible for us to get back to publishing on the third Monday of the preceding month. The February issue will not, therefore, appear until February 2nd.