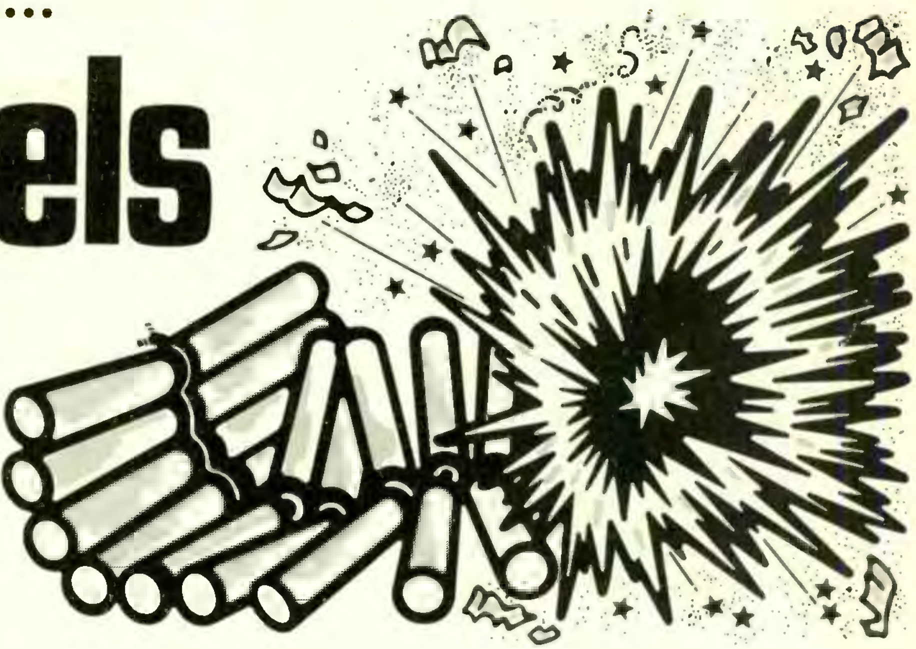
Understanding and Using....

Decibels



By Marvin D. Lamb

We are sure that you have seen many equipment specifications—from dynamic range of amplifiers to the output level of signal generators—given in dB's. While you may know that a dB is a decibel, do you really understand how to use the unit properly?

☐IN ELECTRONICS, POWER, VOLTAGE, OR OTHER LEVELS ARE often specified in decibels (dB). But because those levels aren't *always* given in decibels, conversions to and from that unit are often necessary. While you may have a calculator handy for such occasions, there are some easily learned "rules of thumb" that will enable you to conduct the conversions in your head. We'll talk not only about those conversion rules in this article—we'll also discuss some basic facts about the decibel. So after reading this, you should be more familiar with dB's, and should even be able to mentally compute dB levels.

What Is a dB?

The decibel is the standard unit for expressing relative power levels. For example, the gain of a system—the ratio of the system's output power (P2) to its input power (P1)—can be expressed in dB's:

$$n\ dB = 10 \log_{10}\left(\frac{P2}{P1}\right)$$

Some of you are, perhaps, unfamiliar with the term "$\log_{10}$" that appeared in the preceding equation. It stands for the common logarithm, or the logarithm to the base 10. (When using common logarithms, the subscript "10' is often omitted, as we will do in the remainder of the article.)

The logarithm is the inverse operation of raising a 10 to a power. For example, the expression "n = log x" means: n equals the power that 10 has to be raised to so that it equals x. In other words, $10^n = x$ For example: log 10 = 1 since $10^1 = 10$. In a like manner, log 15 = 1.176 since $10^1 \times 10^{.176} = 15$.

Power ratios are not the only thing that decibels can be used to express—voltage and current ratios can also be expressed in decibels. In that case, n dB = 20 log V_2/V_1 or n dB = 20 log I_2/I_1. In the strictest sense, the decibel can be used to express voltage and current ratios only when the voltages or currents are measured at places having matching impedance. We'll talk more about that later.

You may wonder why the logarithm of ratios would be a useful way to measure different quantities. Its usefulness stems from the fact that power and audio levels are related on a logarithmic basis. That is, if the power increases by a factor of 4, that doesn't mean that the audio level (voltage) will increase by a factor of 4. Instead it increases as the square root of the power increase.

That can be shown as follows: Presume you have a 4-ohm resistor dissipating 4 watts. Using the formula $P = IE = E^2/R$, you can determine that the voltage across the resistor = $\sqrt{PR} = \sqrt{16}$ = 4 volts. If the same resistor dissipates 16 watts, then $E = \sqrt{PR} = \sqrt{64}$ = 8 volts. So while the power increases by a factor of 4 (from 4 watts to 16 watts) the voltage across the resistor increased by $\sqrt{4} = 2$—the square root of the power increase (from 4 volts to 8 volts).

The unit originally used to express power ratios was the "bel," which was defined: n bels = log (P2/P1). (The term "bel" was derived from the last name of Alexander Graham Bell, an early investigator of sound levels.) Even before the days of amplifiers and low RF levels, it was felt that the bel was too large a unit for practical measurements, so the decibel (1/10 bel) was adopted as the international standard unit where n dB = 10 log (P2/P1).

Because the decibel measures the ratio of two power levels, a reference power level (P1) must be specified. For example,

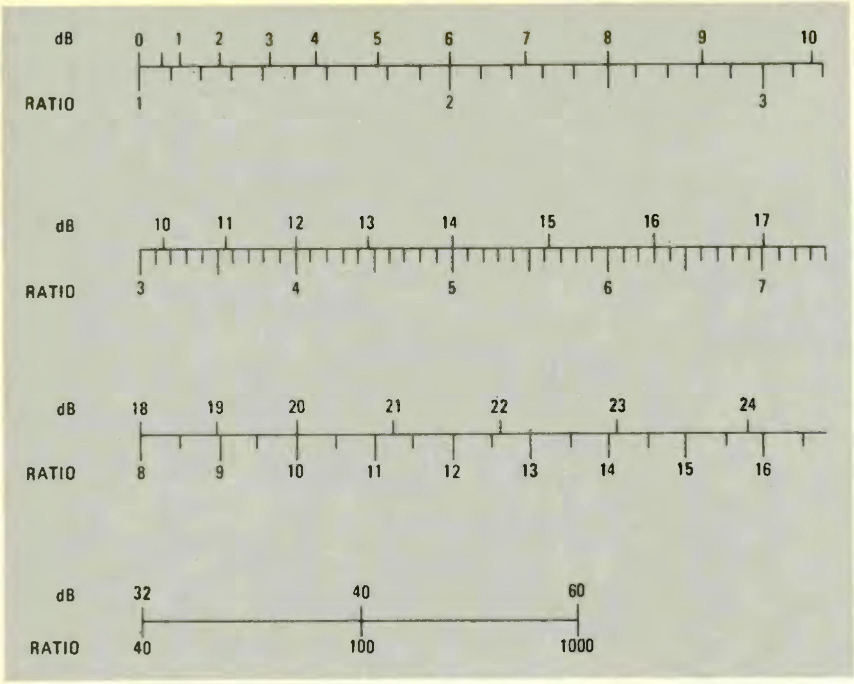


FIG. 1—THIS CHART CAN BE USED as a memory aid or simply to obtain ball-park figures. Note that the ratios shown are for voltage measurements. If you need to determining dB levels corresponding to power measurements, square the ratios shown. (For example, a power ratio of 4 corresponds to a voltage ratio of $\sqrt{2}$ = 6 dB.)

while you could say that the *gain* of an amplifier is 6 dB, you could *not* say that the amplifier's maximum *output* is 22 dB.

There are, however, several *derived* decibel units that can be used. When making power measurements, for example, the unit dBm is often used, where dBm is a power level referenced to 1 milliwatt. Several other decibel abbreviations are often used. Their abbreviations will consist of the letters dB plus additional letters to specify the reference level. Table 1 lists some of those units.

The decibel is used effectively in measurements covering a wide range of levels. The frequency response of amplifiers and filters, for example, is usually expressed as a graph of voltage in decibels as a function of the frequency in Hertz. Most analog AC voltmeters with dB scales are calibrated in dBmV (decibels referred to 1 millivolt across some particular impedance).

TABLE 1—DERIVED DECIBEL UNITS

dBF	Decibels above 1 femtowatt (10^{-15}), or 0.223 μV
dBj	Relative RF signal levels. 0 dBj = 1000 μV
dBk	Decibels referred to 1 kilowatt
dBm	Decibels referred to 1 milliwatt
dBmV	Decibels above 1 millivolt
dBμV	Decibels above 1 microvolt
dBRAP	Decibels above reference acoustical power (10^{-16} watt)
dBv	Decibels relative to 1 volt
dBW	Decibels referred to 1 watt

dB's and Voltage Measurements

If we want to use decibels to relate to the voltage levels of the system, we have to look at another factor: The circuit impedance becomes important when determining dB reference levels. (That's because $P = E^2/R$, or $P = I^2R$ and, therefore, for a given power, the voltage depends on the impedance.)

For example, how would we determine what 0 dBm means for audio measurements? We know that the characteristic impedance for audio transmission lines is 600 ohms. We can also determine what voltage level corresponds to 600 ohms dissipating 1 milliwatt. Recalling that $E = \sqrt{PR}$, we have $E = \sqrt{(.001\ W)\ (600\ \Omega)} = \sqrt{.6} = 0.7746$ volts.

The characteristic impedance of RF generators, attenuators, etc. in the U.S. is 50 ohms (75 ohms in Europe). Thus, 0 dBm for RF measurements is 1 milliwatt into 50 ohms, and the reference voltage is 0.2236 volts.

Decibels and Voltage Ratios

In the following examples, we'll look at how to work with dB ratios, and how the dB ratios correspond to voltage ratios. Using voltage ratios yield answers twice as big as those we'd get using power ratios. Remember, n dB = 10 log (P2/P1) for power levels, while n dB = 20 log (V2/V1).

Ratios are most often used when comparing input and output voltages. For example, if we wanted to determine the gain in decibels of an amplifier having an input of 10 millivolts and an output of 1.0 volt, we would write:

$$\begin{aligned} n\ dB &= 20 \log \left(\frac{1.0\ V}{10\ mV}\right) \\ &= 20 \log (100) \\ &= 20 \times 2 = 40\ dBV \end{aligned}$$

When measuring attenuation ratios, we have a more interesting computation. For example, presume we want to determine the attenuation in decibels of a device with 1.0-volt input and 20-millivolt output. Using our method developed so far, we would proceed as follows:

$$\begin{aligned} n\ dB &= 20 \log \left(\frac{.02}{1}\right) \\ &= 20 \log (.02) \\ &= 20\,(-2.3010) = -40.602\ dB \end{aligned}$$

(Incorrect)

Unfortunately, that is *not* the correct answer. The problem arises because of the *negative characteristic* of the log of a number less than 1. So in the case above, we must write the log with a negative characteristic as a positive number. That's done by adding 10 to the negative log and then subtracting 10. The net effect is zero change to the number but puts it into a form that we can handle. We'll do that for the previous problem as follows:

TABLE 2—EASILY REMEMBERED FACTORS

dB	Voltage ratio	Power ratio
3	$\sqrt{2}$	2
6	2	4
10	$\sqrt{10}$	10
12	4	16
14	5	25
20	10	100
40	100	10,000
60	1000	1,000,000

$$\begin{aligned} n\text{ dB} &= 20\,(8.3010 - 10) \\ &= 166.02 - 200 = -34\text{ dB} \end{aligned}$$

which is the correct answer.

A less cumbersome method of achieving the same result is to arrange the division so that a number greater than 1 always results. Just don't forget to add a minus sign if the circuit is an attenuator. For the previous example:

$$\begin{aligned} n\text{ dB} &= 20 \log\left(\frac{1}{20 \times 10^{-3}}\right) \\ &= 20 \log (50) \\ &= 20 \times 1.669 = 34\text{ dB} \end{aligned}$$

The Factor Method

Now we'll discuss a method that will let you find the answer to the above problems in your head—although you first have to memorize what we'll call combination factors. The factors or ratios are listed in Table 2. With those ratios or factors memorized, you should be able to derive almost all the rest—without referring to any tables! If you don't want to memorize anything, you can always keep Table 2 handy. But we're sure that after you've used the table for a while, you'll find that you've memorized it! Note that the ratios given are for voltage measurements—use the square of the ratio for power measurements.

Some of the ratios are exact, and a few are very close approximations. But all will put your quick answer well within 0.1 dB—which should be sufficient for answers "off the top of your head."

TABLE 3—dB VERSES VOLTAGE RATIO

dB	Voltage ratio (factor)	dB	Voltage ratio (factor)
½	$\frac{3\sqrt{2}}{4} = 1.06$	11	$\frac{5\sqrt{2}}{2} = 3.53$
1	$\frac{\sqrt{5}}{2} = 1.12$	12	4
2	$\frac{2\sqrt{5}}{5} = 1.26$	13	$2\sqrt{5} = 4.47$
3	$\sqrt{2} = 1.414$	14	5
4	$\frac{\sqrt{10}}{2} = 1.58$	15	$4\sqrt{2} = 5.66$
5	$\frac{5\sqrt{2}}{4} = 1.77$	16	$2\sqrt{10} = 6.32$
6	2	17	$5\sqrt{2} = 7.07$
7	$\sqrt{5} = 2.236$	18	8
8	2.5	19	9
9	$2\sqrt{2} = 2.83$	20	10
10	$\sqrt{10} = 3.16$	21	$5\sqrt{5} = 11.18$

Let's look at a problem and use the factor method to solve it. What is the attenuation in decibels of a system with an attenuation factor of 50?

We can find the answer by taking a few simple steps. We know that $50 = 10 \times 5$. We remember (or look at Table 2, to find) that a ratio of 10 corresponds to 20 dB, and a ratio of 5 corresponds to 14 dB. Adding the two figures gives us 34 dB—the same answer we came up with previously. The rule here is that when voltage ratios (the factors) are multiplied, their corresponding decibel values are added together.

Let's look at another problem and another rule for using the factor method: Using the factor method, determine the voltage ratio for a gain of 7 dB.

We have two approaches: First, we know that 7 dB = 10 dB − 3 dB (both of those factors are listed in the table). The rule to remember is that when we subtract units of decibels, we divide the factors, and vice-versa. Thus, our solution is:

$$\frac{\text{Factor for 10 dB}}{\text{Factor for 3 dB}} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$$

Our second approach: We know that 7 dB is one-half of 14 dB. Thus, the factor for 7 dB is the square-root of the factor corresponding to 14 dB. The rule here is that when a decibel value is multiplied by a number, the corresponding factor is raised to that number (in this case, ½). That's best shown by another example: as 3 dB increases to 12 dB, the corresponding factor should increase by a power of 4. Indeed, if you look at the table, you can see that the factors increase from $\sqrt{2}$ to 4, which is $(\sqrt{2})^4$. Another example: For a decrease from 60 dB to 20 dB, the factor decreases from 1000 to $(1000)^{1/3}$; which is equal to 10.

Let's go through a couple more examples. What is the difference between an amplifier with a 7 dB gain and one with a 6 dB gain? With the factors of 6 dB and 7 dB known (from the table and the previous example), 1 dB can be computed as their difference. So we have: 7 dB − 6 dB = $\sqrt{5}/2$.

What factor corresponds to 4 dB? Well, 4 dB can be thought of as 14 dB − 10 dB = $5/\sqrt{10} = \sqrt{10}/2$. It can also be thought of as 10 dB − 6 dB = $\sqrt{10}/2$.

In a similar fashion, all the dB ratios (or factors) from 1 dB to 20 dB can be figured by memorizing just a few. After 20 dB, the ratios repeat, but are multiplied by a factor of 10. For example, the factor for 24 dB is ten times the factor for 4 dB, or $10 \times \sqrt{10}/2$. The factor for 40 dB is therefore ten times that for 20 dB, or $10 \times 10 = 100$.

A good exercise is to list all the factors for decibel values from 1 to 20 using Table 2 and the rules we have discussed. Compare your results to the factors listed in Table 3.

Whole-number Factors

Certain dB ratios provide whole-number factors. The factors are easy to remember (especially if you use them frequently) and can greatly help you to determine dB/voltage levels. The most common ones are given in Table 4.

The Sixth Root of Two

Another convenient memory device to determine the dB ratios uses the sixth root of 2. (That's for voltage measurements—use the third root of 2 for power measurements.) We merely establish $2^{1/6}$ as being equal to 1 dB. That ⅙ power of

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2 is multiplied by the dB level desired.

For example, to determine the factor for 3 dB, you have $2^{3/6} = 2^{1/2}$, which is the square root of two, or 1.414. In a similar manner, we can determine that 6 dB $= 2^{6/6} = 2$. The factor for 9 dB is $2^{9/6} = 2\sqrt{2}$.

It's not necessary to determine the sixth root of two. The usefulness of the method is that it often provides a more common factor of a whole number times the square-root of 2. Then the problem can often be done simply. To find the voltage ratio corresponding to 15 dB, we take $15 \times 1/6 = 2\frac{1}{2}$. Two raised to $2\frac{1}{2}$ can be written as $2^2 \times 2^{1/2}$ or $4\sqrt{2}$.

The error using this method is only 0.4% when taken up to 100 dB! The sixth-root of 2 is 1.122461 while the true factor for 1 dB is 1.122018. The method doesn't work easily for those dB levels that would result in some power of 2 other than $3 \times n/6$, such as $2^{1/3}$. (Then you won't get a multiple of the square root of two.

By using the methods we have discussed in this article, you should be able to work with decibels as the need arises. If you remember our few simple rules about adding dB's, multiplying factors, etc., you can do voltage-ratio-to-dB conversions quite rapidly. Table 2 lists the only voltage ratios, or factors, you need to remember to construct the entire Table 3, which lists all of the dB (voltage) ratios from ½ to 21 dB. (Note that 21 dB, as we discussed before, is 10 times the value of 1 dB.) All you'll probably need to remember from Table 2 are the ratios for 3, 6, and 10 dB. If you still have trouble determining ratios that correspond to dB levels, then Fig. 1 can be cut out and used as a handy reference.

TABLE 4—WHOLE-NUMBER FACTORS

Ratio	dB		
1.5	3½	7	17
2	6	8	18
3	9½	9	19
4	12	10	20
5	14	40	32
6	15½		

If you want to use Fig. 1 to find the dB value corresponding to a ratio larger than 16, then divide the ratio by 10, 100, or 1000 so that it is smaller than 16. Look up the resultant, and add 20, 40, or 60, respectively, to the result. For example, to find the dB level corresponding to a voltage ratio of 130: $130/10 = 13$. From Fig. 1, we see that a ratio of 13 corresponds to about 22.3 dB. Adding 20 yields 42.3 dB.

You can also find the ratio corresponding to a dB level over 24 by subtracting 20, 40, or 60 so that the dB level is less than 24. Then look up that value, and multiply the resulting ratio by 10, 100, or 1000 respectively. As usual, an example should clear things up: How would you find the voltage ratio corresponding to 93 dB? We know that $93 = 80 + 13$. (We chose 80 because it is equal to $60 + 20$, both of which we know how to work with—but we could also have used $40 + 40$, etc.) That yields $1000 \times 10 = 10{,}000$. Using Fig. 1 we find that 13 dB is a ratio of about 4.5. Therefore, 93 dB corresponds to a voltage ratio of $4.5 \times 10{,}000 = 45{,}000$. ■