Accurate sine-waves

Analogue implementation of the Darwood accurate sine-wave oscillator algorithm

by D. H. Follett

In response to Mr Darwood's 'Accurate sine-wave oscillator' article in the June'81 issue of *Wireless World*, the author shows here that the digital implementation used to generate accurate sine and cosine functions can be replaced by simple analogue circuits. A prototype circuit operated over three decades with ±1dB amplitude variation, less than 1° error between the quadrature outputs and around 1% or less distortion. The circuit requires only four quad-i.cs.

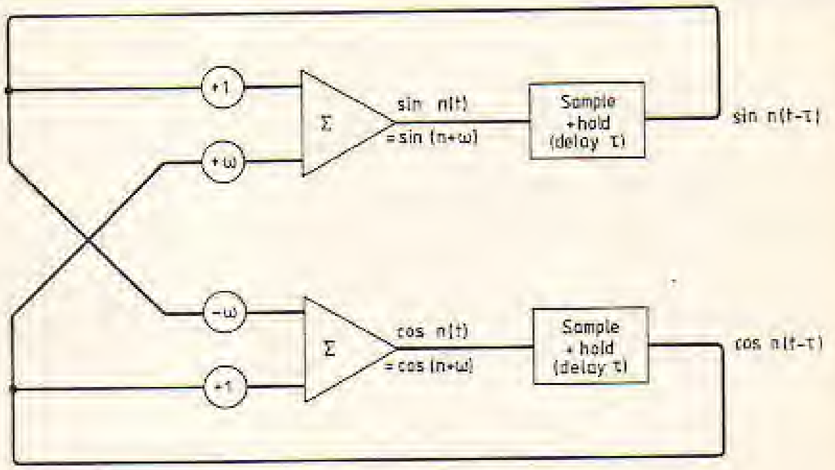


Fig. 1. Block diagram illustrating application of the equations for values of sin n(t) and cos n(t).

The algorithm recently described by N. Darwood[1] for generating sine and cosine functions with digital implementation may also be produced by analogue means, as will be shown. The circuit is really a form of recursive digital filter but I am unrepentant in calling it an analogue implementation since no digitization in the proper sense occurs.

The prototype operated over three decades of frequency with amplitude variations of ±1dB, distortion about 1% or less, and phase error between outputs less than 1°. Only four cheap quad i.cs are required, although Fig. 2 shows six i.cs, since the four dual op-amps can be replaced by two quad op-amps such as the LF347.

Principle

Referring to the original article for fuller explanation, each new value of sin$n$ is computed from the previous value by adding a fraction ω of cos$n$;

$$\sin(n+\omega)=\sin n+\omega\cos n$$

Note that ω does not have its usual significance. If we write $\sin(n+\omega)$ as $\sin n(t)$, the value of sin$n$ at time t, and sin$n$ as sin$n$ $(t-\tau)$, the value at time τ earlier, then

$$\sin n(t)=\sin n(t-\tau)+\omega\cos n(t-\tau)$$

Similarly,

$$\cos n(t)=\cos n(t-\tau)-\omega\sin n(t-\tau)$$

These equations are applied as shown in the block diagram, Fig. 1. The value of sin$n$ is sampled and held so that the output of the sample and hold circuit is effectively delayed by the sampling interval τ and represents the value of $\sin n(t-\tau)$. To this is added the fraction ω of the cos function to generate the new value of the sine function, $\sin(n)t$. A similar process is used for cos$n$.

Referring again to Mr Darwood's article, the sine and cosine outputs are synthesized with $2\pi/\omega$ steps per cycle so that the output frequency is $2\pi/\omega$ times less than the sample clock frequency.

Implementation

The circuit was designed to operate in the audio-frequency range with values as shown in Fig.2. During the first half of each clock cycle (Q high) the values of sin and cos are transferred to the first pair of hold capacitors while the previous values are held on the second pair. In the second half cycle the values are transferred to the second pair of capacitors. This avoids having to use very short sample times and ensures that loops are never closed while settling. The 100kΩ resistors determine the fraction ω; the optimum fraction seems to be about one-tenth, giving about 60 steps per cycle. Although the circuit will run with 400 steps per cycle, distortion occurs because circuit errors become comparable with the step size. More accurate sample-and-hold i.cs and higher accuracy resistors will decrease circuit errors.

The diode-resistor networks around A_2 and A_6 provide a small degree of non-linearity sufficient to stabilize the oscillation amplitude. If only one output is required and exact phase quadrature thus unimportant, one network and the associated preset can be omitted, leaving only the 10kΩ feedback resistor.

Setting up

Initially, the 1kΩ presets should be set to zero to prevent oscillation and the null offsets adjusted. The presets should then be advanced until oscillation occurs and then adjusted for phase quadrature. The output amplitudes should be about 4 to 5V peak-to-peak. Finally, the null offsets may need readjusting. The output frequency is not critical during the adjustments, but should be between 50 and 5,000Hz.

Results

Figure 3 shows oscillograms for frequencies between 5Hz and 16kHz obtained by varying only the clock frequency. Single time-base sweeps were used for all but the 16kHz frequency as the steps are not, in general, synchronized with the output. In Fig. 4(a), the 500Hz signal is expanded to show the steps more clearly, while Figs 4(b) and (c) show Lissajous figures resulting from x-y display of the two outputs to illustrate the quadrature accuracy obtainable between 5Hz (b) and 5kHz (c).

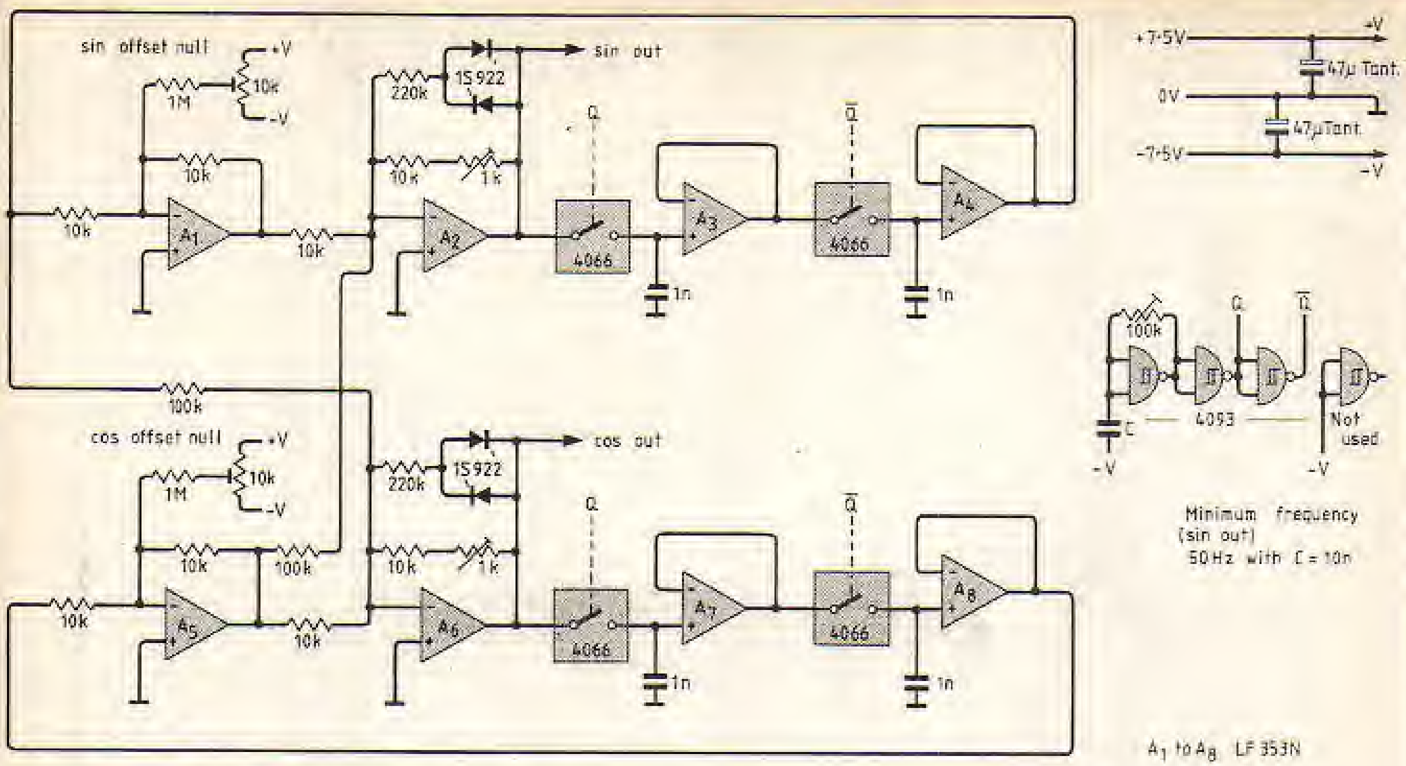
The upper frequency for constant amplitude is about 16kHz but frequencies lower than 5Hz can be obtained by increasing the 1nF hold capacitors proportionally. These capacitors must of course be polystyrene types, or similar, to minimize dielectric soaking effects.

If the clock is replaced by a voltage controlled oscillator the circuit can also be used as a sweep oscillator.

Comparisons

Out of interest, two other oscillators based on recursive filters were compared to the circuit described here using the same building blocks.

The first used direct implementation of the second-order differential for a series-tuned circuit (equivalent to the state-variable filter in band-pass mode). This circuit



used two op-amps fewer but was limited to about 30 steps per cycle, as second order terms make circuit errors more critical.

Phase-shift oscillators using three or four cascaded single-pole filters were also tried. These oscillators were more docile than those previously described, but four sections were required to obtain quadrature outputs and almost twice the component count of the circuit described here gave an inferior performance. D.c. stability was, however, better as feedback at d.c. is negative and thus reduces offsets.

The circuit described here is sensitive to offsets because of positive feedback at d.c., so offset null adjustment is included.

Reference

1. N. Darwood, 'Accurate sine-wave oscillator', *Wireless World*, June 1981, pp 69-78.

***Fig.2.** Circuit diagram of the oscillator and clock. Op-amps A_1 to A_8 were four LF353N i.cs in the prototype but they can be replaced by, say, two LF347 quad i.cs if required. These op-amps have j.f.e.t. inputs.*

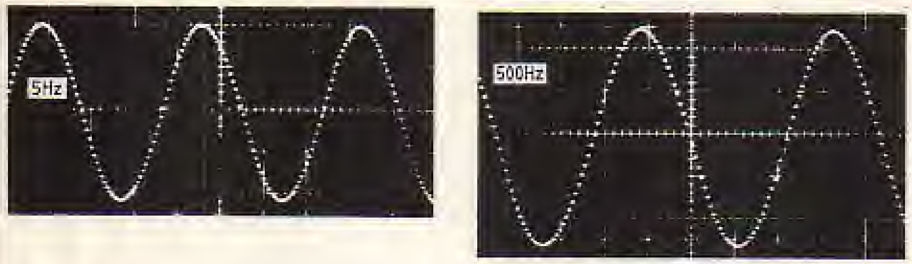


***Fig.3.** Oscillograms for oscillator frequencies between 5Hz and 16kHz. A single sweep time-base was used for all but the 16kHz wave-form in order to show the steps (the steps are not necessarily synchronous with the output frequency).*

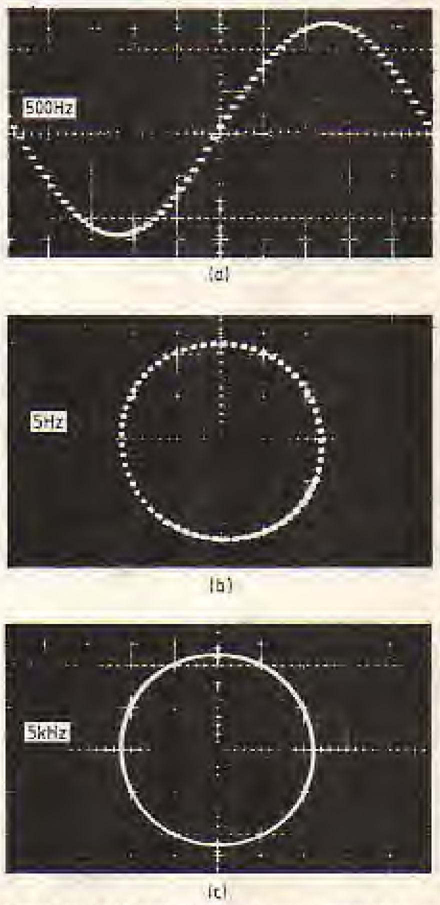


***Fig.4.** (a) is the 500Hz sweep expanded to show the steps more clearly. (b) shows a Lissajous figure resulting from the two oscillator outputs at 5Hz, and (c) is as (b), but at a frequency of 5kHz.*