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Number 23 on your Feedback card

Secrets of Transmission Lines

Part 7: Impedance matching.

In the previous chapter, we saw that a mismatched line can be corrected by placing a stub at the appropriate point on a transmission line. In this, the last chapter of the series, we will be looking at some techniques for impedance matching.

The use of stubs for matching is generally confined to UHF and microwave frequencies. At 2 MHz, a quarter-wave stub is 123 feet long in air dielectric line, and 80 feet long in polyethylene insulated line. A lumped parameter circuit, coils and capacitors, would be more convenient and probably cheaper at these frequencies.

We also saw that it is frequently convenient to use admittance parameters, as well as the use of the Smith Chart, in transforming impedance to admittance and vice versa. The program in **Table 1**, written in BASIC, is a quick way to perform the inversion. Note that the two circuits may be equivalent, but they can have very different values. Let us assume a frequency of 4 MHz. Suppose that we measure an impedance of 40 + j60 ohms. This is a 40 ohm resistor in series with a 23.9 µH inductor. Transforming this to admittance, we obtain 7.69 - j11.5 mmho, which is a 130 ohm resistor in parallel with a 34.6 µH inductor. Note that both the resistor and inductor values have changed significantly. While the component values have changed significantly, both circuits have the same power factor and phase angle. If concealed inside a box with only the two terminals brought out, it would be impossible to distinguish the circuits if measurements were made only at 4 MHz. Of course, measurements at other frequencies would permit distinction.

```
10 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
20  '                      Y/Z INVERSION
30  '                     BY JACK KEUCKEN
40  '                        03/16/95
50 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
60  CLS
70  INPUT "IS INPUT DATA Z OR Y?";A$
80  IF A$ = "Y" GOTO 300
81  IF A$ = "y" GOTO 300
90  INPUT "ENTER R";R
100 INPUT "ENTER X";X
110 DEN = ((R*R)+(X*X))
120 G = R/DEN
130 B = -X/DEN
140 PRINT "ADMITTANCE IS";G;"+J";B;" "
150 INPUT "PRESS ANY KEY TO CONTINUE";B$
160 GOTO 60
290 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
291 '                  Y TO Z TRANSFORMATION
300 INPUT "ENTER CONDUCTANCE IN MHOS";G
310 INPUT "ENTER SUSCEPTANCE IN MHOS";B
320 DENY = ((G*G)+(B*B))
330 R = G/DENY
340 X = -B/DENY
360 PRINT "IMPEDANCE IS";R;"+J";X;" "
370 INPUT "PRESS ANY KEY TO CONTINUE";B$
```

Table 1. Y/Z inversion program in GWBASIC.

The transmission line equations

With the wide availability of personal computers, the most common means of solving the transmission line equation is by computer rather than graphically on the Smith Chart . The program in **Table 2**, also written in BASIC, is a means of solving the transmission line equations. Lines 200 and 210 are included to prevent division-by-zero errors. The program asks for the line Zo and defaults to 50 ohms if none is entered. The line electrical length is called for in wavelengths. This permits one to work in either feet or meters. Conventionally, wavelengths are given in meters; however, you may work wavelengths in feet if you choose. The length must be corrected for the velocity of propagation on the line.

$\lambda = 300/F$ meters
eqn (7-1)

where
λ = wavelength
F = frequency in MHz

For example, let us assume that we have a Teflon-insulated cable that is 2.5 meters (8.2 feet) long. (To convert meters to feet, multiply by 3.28.) At 10 MHz, $\lambda = 300/10 = 30$ meters. The cable electrical length is 2.5/(30*.65) = 0.128 wavelengths. Teflon cable has a Vp of .65. You can look up the cable Vp in the manufacturer's data or a handbook.

For another example, let us consider in **Table 3** the results for a transmission line with an electrical length of 2.5 meters at a variety of frequencies terminated in a 10 ohm resistor.

The Smith Chart illustration in **Fig. 1** shows this data and tells us why the Smith Chart has hung on in popularity. I doubt that there are more than three people in the world who could look at these data columns and realize by inspection that they represent a 5:1 VSWR circle. On the other hand, it is pretty obvious from the Smith Chart figure that the data path is concentric about the center of the chart.

Notice that the electrical lengths listed are all positive and go toward the generator. If you enter any of the R and X values along with the negative of the electrical length, you will get back to the 10 + j0 with some small truncation errors.

The fact that the chart spirals clockwise with increasing frequency is not to be neglected either. If your data ever shows a counterclockwise spiral with increasing frequency over any significant span, there is something dreadfully wrong with your measurements. For this to happen, we would have to have inductors whose reactance decreases with increasing frequency, and capacitors whose reactance increases with increasing frequency .

Note that a terminating load that is purely resistive will have a constant VSWR with changing frequency; however, a termination with a reactive part will have a VSWR that varies with frequency. For example, a termination with a series inductor that measured 10 + j10 ohms at 10 MHz would look like 10 + j20 at 20 MHz and 10 + j30 at 30 MHz. Obviously, the VSWR is increasing with increasing frequency. By a similar token, a termination with a series capacitor would have a VSWR that decreased with decreasing frequency

You will frequently use negative line lengths with this program, since the more common case is that you have the impedance bridge and signal generator on the ground and the antenna

```
100 REM*****************************************************************************
110 REM                    TRANSMISSION LINE EQUATIONS
120 REM                              BY JACK KEUCKEN
130 REM                                    09/12/97
140 REM*****************************************************************************
150 CLS
160 PI = 3.14159265#
170 INPUT "ENTER LINE Z0";Z0
171 IF Z0 = 0 THEN Z0 = 50
180 INPUT "ENTER  REAL PART OF LOAD";ZR
190 INPUT "ENTER IMAGINARY PART OF LOAD";ZI
200 IF ZR = 0 THEN ZR = .000001
210 IF ZI = 0 THEN ZI = .000001
220 INPUT "ENTER LINE LENGTH IN WAVELENGTHS";BL
230 PRINT "A LINE OF";Z0;"OHMS WITH AN ELECTRICAL LENGTH OF";BL;
"WAVELENGTHS"
240 IF ZI<0 THEN J$ = "–J" ELSE J$ = "+J"
250 PRINT "TERMINATED IN A LOAD OF";ZR;J$;ABS(ZI);"OHMS"
260 BL = 2*PI*BL: REM CONVERT TO RADIANS
270 REM*****************************************************************************
280 REM                              THE CALCULATION
290 AN = TAN(BL)
300 ZI2 = Z0*AN
310 IMN = ZI+ZI2
320 NUM = SQR((ZR*ZR)+(IMN*IMN))
330 RED = Z0–(ZI*AN)
340 IMD = ZR*AN
350 DEN = SQR((RED*RED)+(IMD*IMD))
360 PHNUM = ATN(ZR/IMN)
370 PHDEN = ATN(RED/IMD)
380 MAG = NUM/DEN
390 ZS1 = Z0*MAG
400 PH = –(PHNUM–PHDEN)
410 RZS = ZS1*COS(PH)
420 IZS = ZS1*SIN(PH)
490 IF IZS<1 THEN J$ = "–J" ELSE J$ = "+J"
500 PRINT "THE INPUT IMPEDANCE IS";RZS;J$;ABS(IZS);"OHMS"
510 V1 = Z0–ZR
520 V2 = SQR((V1*V1)+(ZI*ZI))
530 V3 = Z0+ZR
540 V4 = SQR((V3*V3)+(ZI*ZI))
550 V5 = V2/V4
560 PRINT "THIS CORRESPONDS TO A VSWR OF";((1+V5)/(1–V5))
570 STOP
2200 RUN
```

Table 2. *Transmission line equations program in GWBASIC.*

f (MHz)	Len. el.	R	x
10	0.0833	13.2	+j 27.3
20	0.167	35.9	+j 74.5
30	0.25	250	+j 0
40	0.33	35.9	–j 74.5
50	0.417	13.1	–j 27.2
60	0.5	9.98	–j 0

Table 3. *Data for a transmission line with an electrical length of 2.5 meters.*

will be in the air at the other end of a transmission line. You insert a negative line length to rotate the impedance back to the antenna so that you can see what it takes to match the antenna.

Sometimes you have the situation where you have an impedance measurement and you would like to know the VSWR. This can be done graphically on the Smith Chart, or you can use the computer program with a very small line length — like 0.000001 wavelength.

The right place for matching

The very best place for impedance matching is always right at the discontinuity. There are three reasons for this:

1. The longer the transmission line, the more the data are smeared out by the transmission line effect. If the impedance is matched right at the discontinuity, it will nearly always have the maximum matched bandwidth.
2. High VSWRs de-rate the cable.
3. Line loss effects are multiplied on the mismatched line.

Let us address ourselves to the latter two effects. The actual solutions to the transmission line equations with real lossy transmission line is another order of magnitude more sophisticated and will not be tackled here. However, from some simpler considerations we can begin to make an estimate.

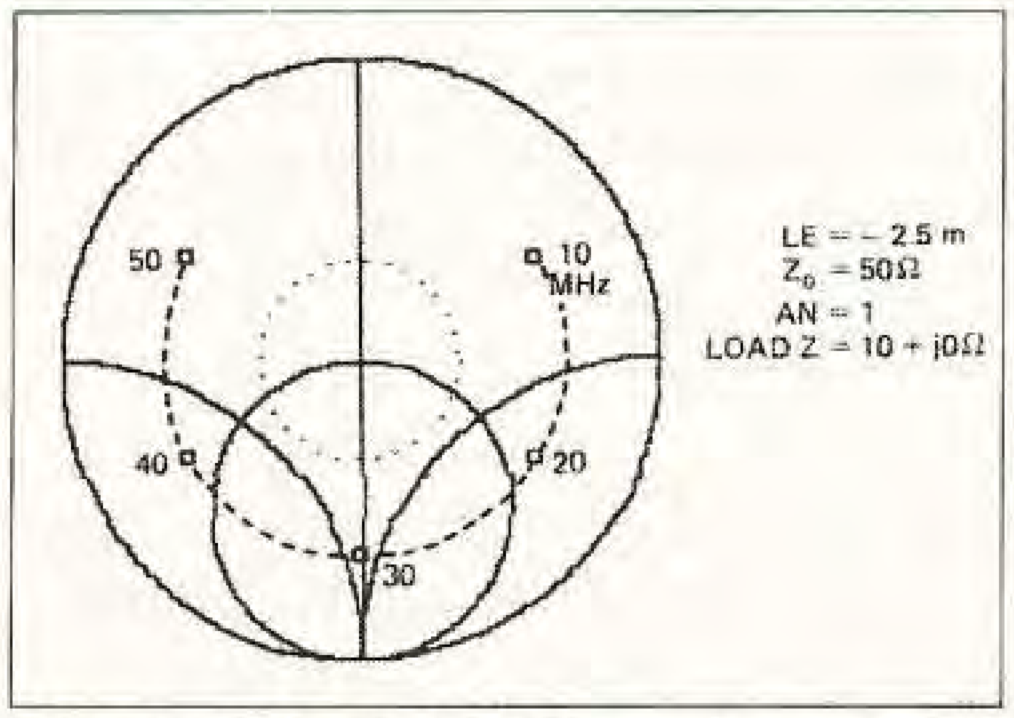

Fig. 1. *This Smith Chart shows a 5:1 VSWR circle.*

We can begin by considering point 2 above. Let us presume that we have 1,000 watts and that we have a lossless antenna coupler or transmitter tank circuit. On a matched 50 ohm termination, our hundred watts would require:

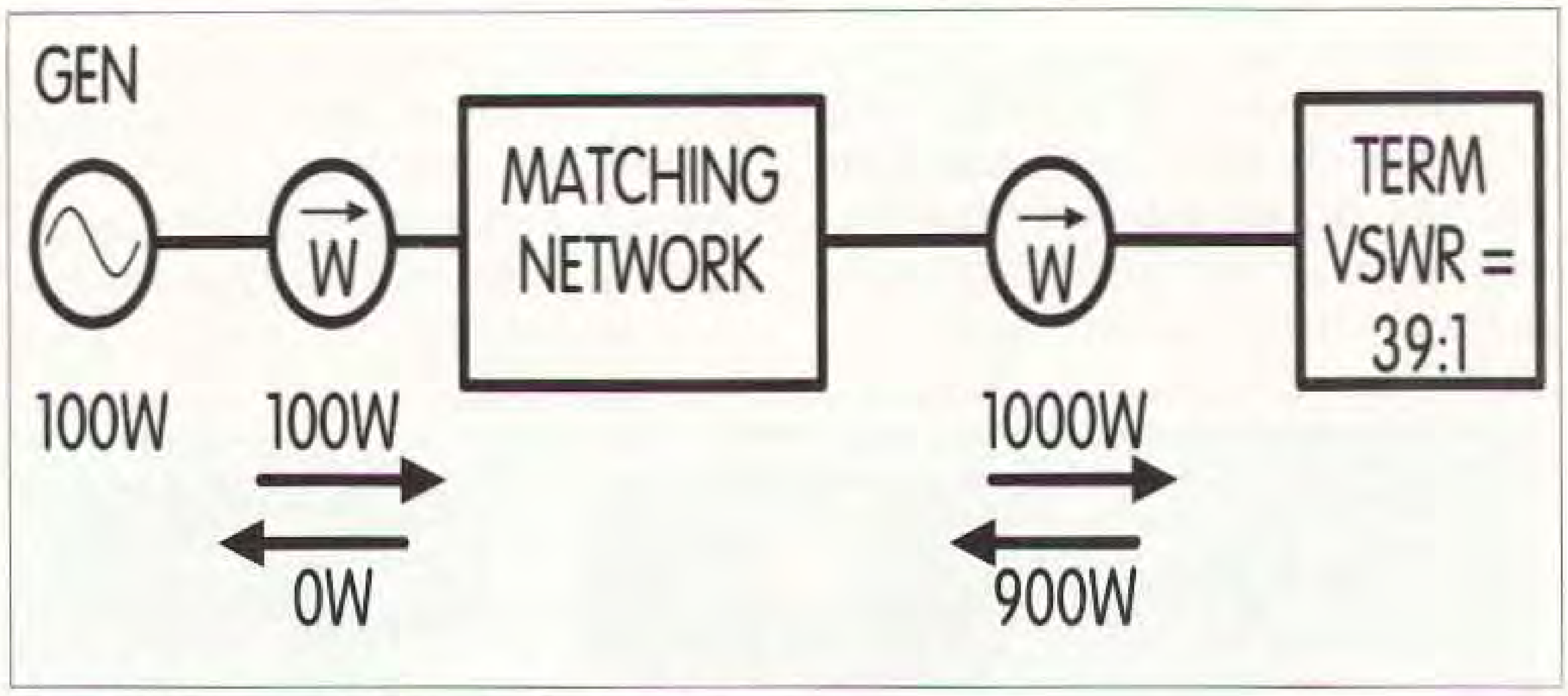

Fig. 2. Voltage multiplication by standing waves.

1,000 watts = (V*V)/50 = 223.61 VRMS = 316.23 V peak

We saw that a 5:1 VSWR will give a peak resistance of 250 ohms. Then:

1,000 watts = (V*V)/250 = 500 VRMS = 707 V peak

The voltage and current are both increased by the square root of the VSWR. At 1,000 watts input, the current is increased from 4.47 amperes to 10 amperes by the 5:1 VSWR.

The peak voltage is of interest since this is what causes arcing. Conversely, the current-induced breakdown is caused by thermal effects, so the RMS values are of interest.

In a case that can easily arise, a centerfed half-wave dipole cut for 80 meters becomes a centerfed full-wave dipole on 40 meters. This is actually a very effective antenna because it acts as two half-waves in phase, and has significantly higher gain than a half-wave. The impedance at the center of this antenna on 40 meters is typically about 2,000 ohms. This is a VSWR of 40:1; the peak voltage for a kilowatt is multiplied up to 2,000 V peak, and the current to 28.27 amperes RMS. The VSWR has multiplied the voltage and current from levels that are easily handled on RG-58U cable and BNC connectors to values that will almost certainly destroy them. An open wire or ladderline of 300 or 400 ohms impedance could be used here. The VSWR and the losses are both lower.

To drive this point home a bit more, let us examine a case similar to the 80/40 meter dipole. In **Fig. 2** we see a generator, a directional wattmeter, a lossless matching network and another directional wattmeter, and a termination with a 39:1 VSWR. For a 39:1 VSWR, the reflection coefficient is 95%, which says that the forward power to backward power is in a 10:9 ratio. In order for the load to dissipate 100 watts, the forward power must be 1,000 watts and the backward power 900 watts.

The lossless matching network must turn the backward power around and add it in phase to the incoming 100 watts from the generator. This is a technique that is frequently used to test components for power handling when one does not have a source of RF equal to the power rating for which the component must be tested.

Losses

Considering a similar setup, let us assume that the transmission line has a loss of 1 dB for the length and frequency in use. A 1 dB loss means that only 79% of the power leaving the matching network reaches the load. With a 39:1 VSWR, 90% of that is reflected, and only 79% of the reflected power gets back to the matching network. That would amount to 0.79*0.9*0.79 = 0.56. Instead of a VSWR of 39:1, the matching net would view a VSWR of 7:1. Without belaboring the point too much, it is easy to see that the VSWR also multiplies the losses in the line. For a realistic evaluation of the power loss, we would have to consider the losses in the matching network that will not be zero. If the line loss is 3 dB and the line is terminated in an open or a short circuit, voltage reflection coefficient at the line input is 0.5 and the VSWR is only 3:1. Fig. 19-5 in the *Radio Amateur's Handbook,* 1999 edition, gives additional line losses as a function of VSWR.

Lumped element matching

At HF frequencies, it is quite common to use lumped elements — that is, resistors, capacitors, and transformers — to match impedances.

Referring to **Fig. 3(a)**, we see a rudimentary Smith Chart in impedance coordinates. We can always add a Yo circle to a Smith Chart with a compass. The circle passes through the Zo point and the Z = 0 point. In the figure, the antenna is designated by a large surveyor's mark. It is situated in a point of lower than 50 ohms resistance and a rather large capacitive reactance. The situation is similar to the impedance

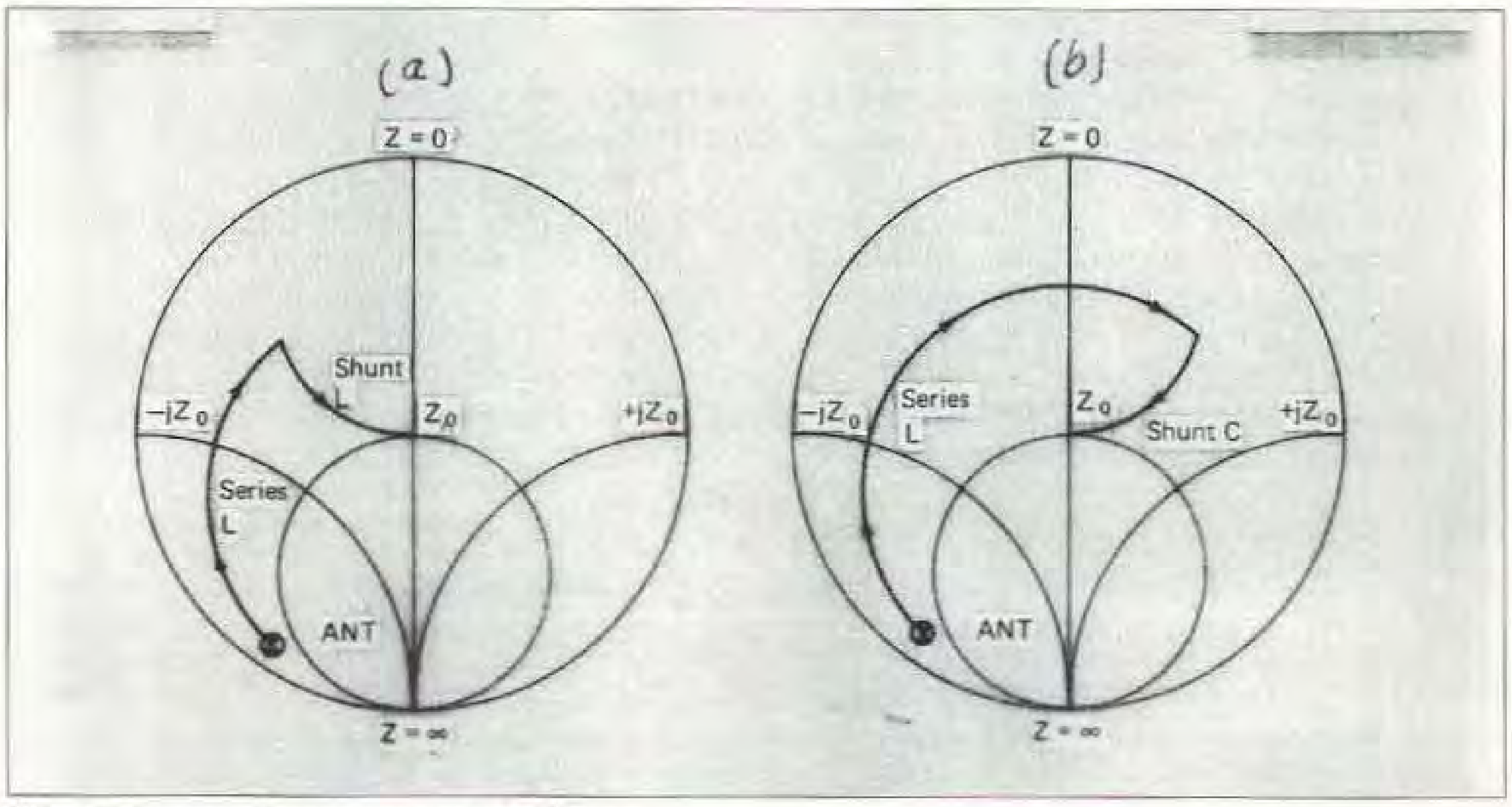

Fig. 3. Lumped parameter matching.

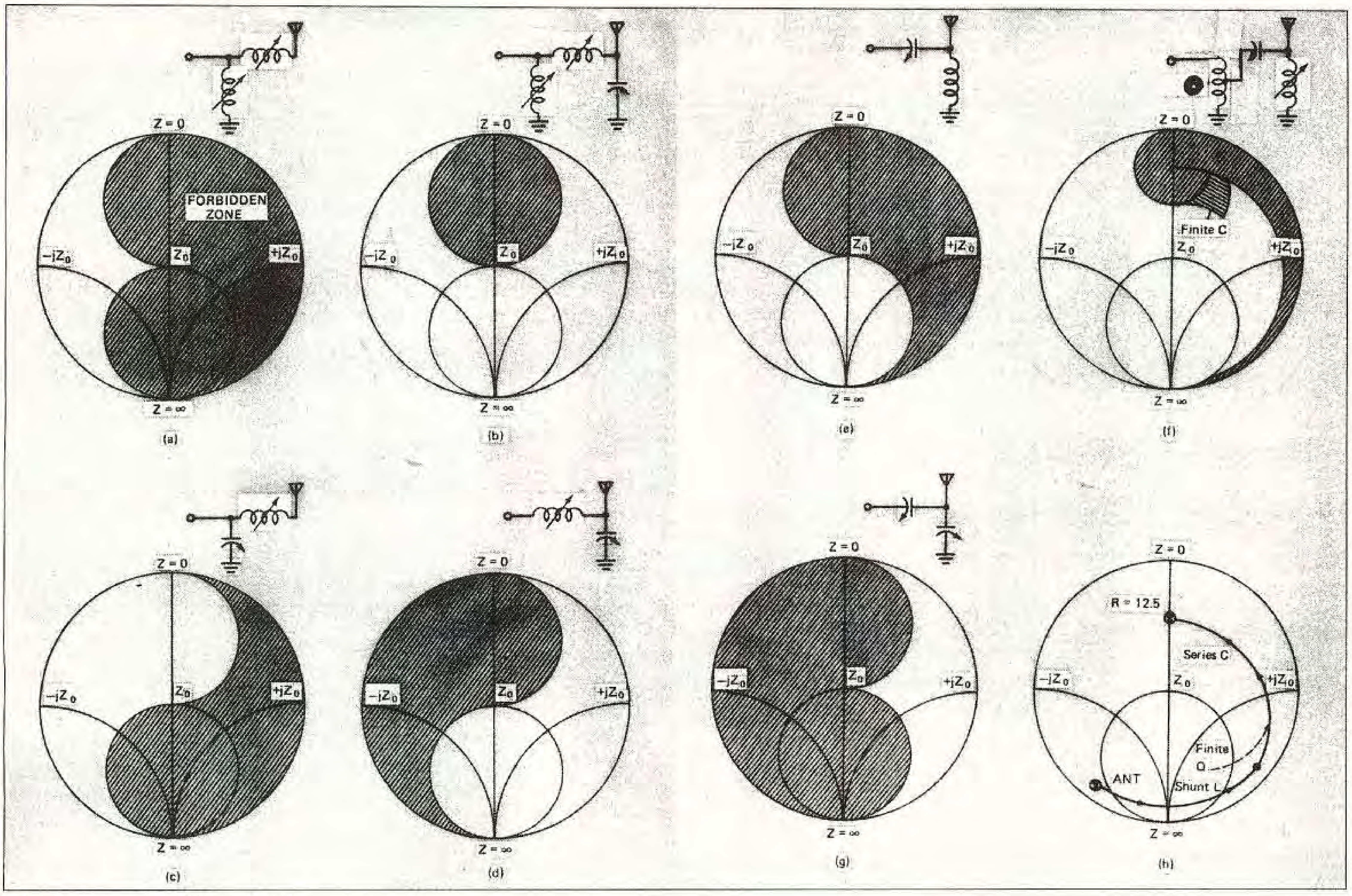

Fig. 4. Some matching circuits.

of an electrically small mobile whip — for example, a 12-foot whip at 7 MHz.

If we add a series inductance to the whip, the impedance point moves along a constant resistance line in the direction of less and less capacitive reactance. We keep adding inductive reactance in series until we reach the Yo line (probably 20 mmho, if the chart is Zo = 50 ohms). At this point, we add shunt inductance until the point impedance reaches the Yo (and Zo) point and the antenna is matched.

In **Fig. 3(b)**, we add more series inductance until the antenna crosses the zero reactance line and meets the Yo curve on the inductive side d. In this case, shunt capacitance will bring the point in to a match.

Fig. 4 shows some matching circuits, including the areas on the Smith Chart where they cannot be used. For example, the circuit at **Fig. 4(a)** cannot be used in any of the shaded areas because the antenna is already inductive or the resistance cannot be moved to a matching line. In **Fig. 4(b)**, the addition of a shunt capacitor permits the swing of inductive data at the right (inductive) side over to the left (capacitive) side, where the two-inductor circuit of **Fig. 3** could do the job.

Fig. 4(c) shows a forbidden region on the inductive side because the antenna is already too inductive. The inverse of this arrangement is in **Fig. 4(d)**. It is noteworthy that the forbidden

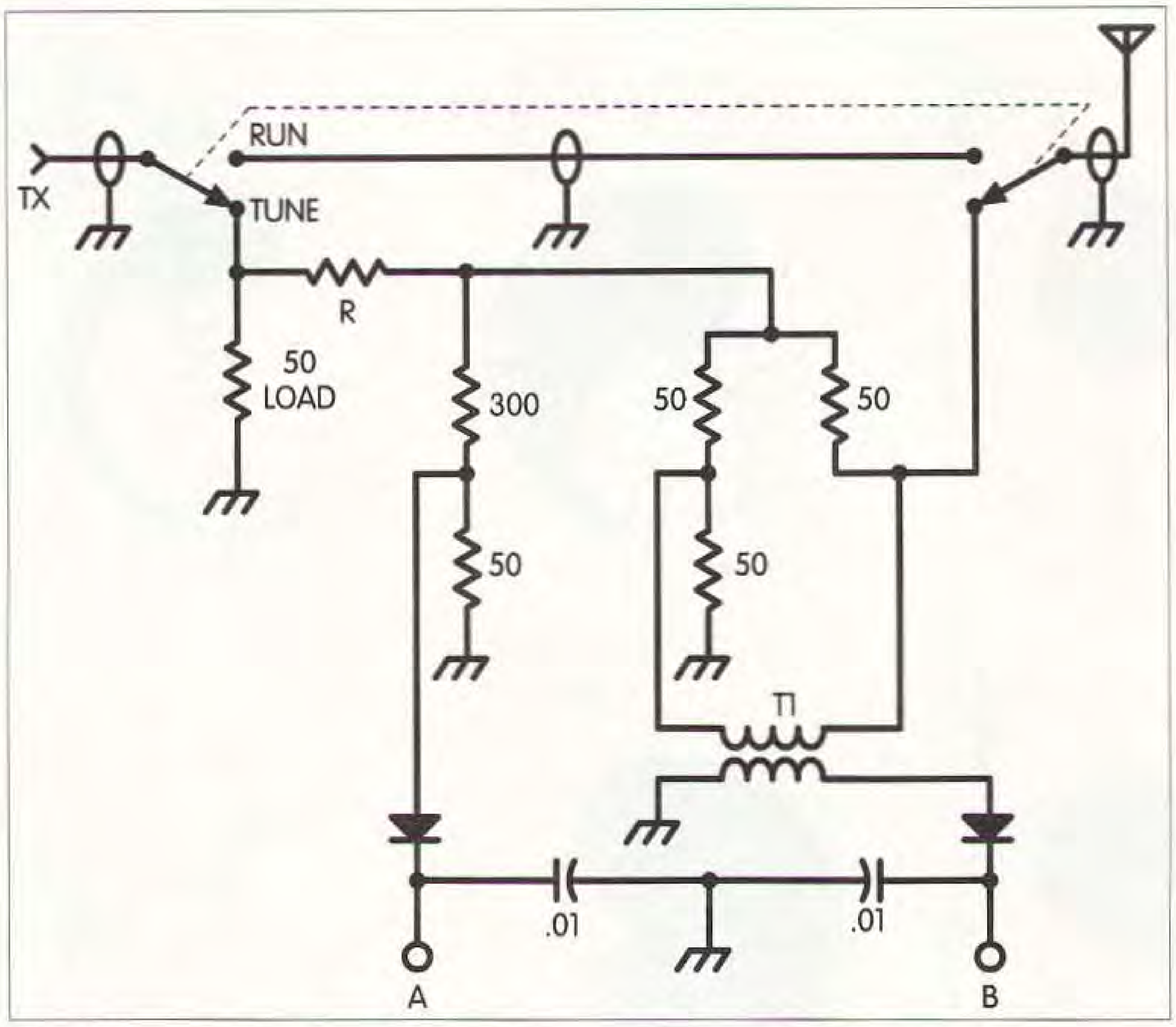

Fig. 5. Micro tuner.

regions form the "yin" and "yang" figures found in Chinese philosophy and literature.

Fig. 4(e) is useful for matching electrically small antennas; however, it may not handle an antenna that goes through the quarter-wave resonance at 37 + j0 ohms, which lies within the forbidden zone. This problem is corrected in **Fig. 4(f)** by the addition of a 4:1 impedance transformer. In this case, we match the antenna to 12.5 ohms and then step up to 50 ohms with an autotransformer.

The circuit of **Fig. 4(g)** is useful mainly for matching inductive antennas, such as loops.

Harking back to the circuit in **Fig. 4(f)**, this is a very useful circuit that I have employed many times for matching mobile, marine, and aeronautical antennas. With the proper range of components, it will handle these antennas from 1.8 to 30 MHz. **Fig. 4(h)** shows the path. The shunt inductor takes the antenna over to the 12.5 ohm line on the inductive side, whereupon the series capacitor carries it up to the 12.5 + j0 point to be transformed to 50 ohms by the 2:1 ratio transformer. The limitation in matching range due to finite capacitance is shown in (f), and the effect of a finite Q in the inductor is shown in (h). With electrically small antennas, the effect of the losses in the elements must be included in the design.

There are, of course, many other possible tuning networks, including "Tee" and "Pi" types, that can be solved using the Smith Chart techniques. In general, the lowest loss and broadest bandwidth networks will be those that move the load toward the match point most directly — that is, in the direction of decreasing VSWR.

Line transformers

One of the techniques used for impedance matching is the line transformer. We earlier observed that the resistance values on a mismatched line went from Zo/VSWR to Zo*VSWR. On the 50 ohm line, the 10 ohm resistor produced 10 ohms, and a quarter-wave away, 250 ohms. This can be used to transform impedances. If we have a 10 ohm load and wish to transform it to 50 ohms, a line 1/4 wave long with a Zo equal to the geometric mean between the two will do the trick:

$$Zo = \sqrt{(50*10)} = 22.36 \text{ ohms}$$

This technique is very frequently used for microwave stripline and waveguide circuits, but it is used at HF only for the rare occasion when a line of the correct Zo happens to be available.

The micro tuner

With the previous discussion, we have shown a number of networks using variable elements to match various loads. In general, all of the elements must be of the proper value in order for an impedance match to be obtained. It can be time consuming and irritating to others to emit a carrier while searching for a tuned condition. My answer to this is what I refer to as a micro tuner, which reduces the emitted carrier to a few milliwatts, thereby minimizing the interference to others. It also serves as a protective device that prevents the transmitter from seeing any extreme mismatches which might damage it.

Fig. 5 shows the general arrangement. The transmitter feeds into a double pole double throw switch. For the 160 through 10 meter range I have used a small relay for this function, something with a rating of 120 V and a few amperes. The relay functions as a tune/operate switch.

In the tune position, the relay goes to the 50 ohm dummy load constructed in the first chapter. This load absorbs most of the power during tune-up and presents a stable load to the transmitter, preventing any damage.

The resistor labeled R taps off a few milliwatts for the bridge and the power sample. It should be selected based upon the power your transmitter produces during tuning.

The bridge resistors are quarter-watt carbon film types such as Tech America 900-0187; the 300 ohm resistor is a half-watt T.E.900-0366 or equivalent. The diodes are preferably germanium RF or switching types such as 1N3666; however, silicon 1N914 or 1N4146 types will also work. The germanium types have a lower forward drop and are more sensitive.

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The transformer T1 is a bifilar-wound toroid. It is not critical. It only needs to present an open circuit reactance of about +j500 ohms at the lowest frequency you intend to use. I have used a Ferronics 11-220-K core (μ = 125) about 3/8-inch in diameter and 3/32-inch thick with 40 bifilar turns of #30 enameled wire. It measured about 74 μH per side at 1 to 4 MHz.

The bridge is a fixed Wheatstone type that has the antenna as one leg. When the antenna looks like 50 + j0 ohms, the bridge output voltage as measured at B falls to zero. For all VSWRs below about 2:1, the voltage at B is lower than the forward voltage sample at A. The meter for reading voltages A and B should have an impedance no higher than 10k. A voltage comparator and an LED can compare the voltages A and B and light whenever the VSWR is less than 2:1.

In operation, the relay is flipped into the "tune" position and the transmitter keyed. With only a few milliwatts radiating, the tuner is tuned until the voltage at B is nulled. The transmitter is unkeyed and the relay switched to "run." Your fellow hams will thank you for the micro power tune-up.

Please feel free to use the two GWBASIC programs provided in this article. Due to their short length they are easy to key in if you are interested. 73