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Meeting Your Match

The fine points of understanding matching networks.

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When we connect a load to a source, be it an antenna or an amplifier stage, we expect power to be delivered to the load. Whether the power delivered is the maximum available from the generator (source) or not depends on the resistance of the load relative to the internal resistance of the generator.

The maximum power is delivered to a conjugate matched load. A conjugate matched load has the same magnitude as the generator's internal impedance, but with the opposite phase angle. The reactive component of the load is then in resonance with the reactance of the generator. When the load impedance has the same magnitude and phase angle as the generator, the load is said to be matched on an image impedance basis. The term "image" arises from the fact that impedances on the two sides of the output terminals are images of each other. Only when the load and generator are purely resistive are the image match and the conjugate match the same.

In a voltage amplifier, say in a receiver IF, voltage is the important concern, not power. Therefore, the amplifier load impedance is maximized at the expense of power output. Under these conditions, matching is not important. However, in a transmitter amplifier chain, power usually is critical and the amplifiers matched for maximum power output.

It's all well and good to say the maximum available power from a generator is obtained when the load is a conjugate match to the generator. If you designed the amplifier, you know what the impedance should be, but what is the output impedance of an unknown generator? The internal impedance of a generator can be found with a couple of measurements and some calculation. **Fig. 1** shows a generator with an unknown impedance. R_g is the generator's open circuit voltage divided by the short circuit current, that is, $R_g = E_g/I_g$. When the generator cannot be safely operated unloaded or shorted, the internal resistance can be calculated by noting the voltage across two different values of load. The reactance, either inductive or capacitive, is tuned out when the voltage across an arbitrary load is maximum. The voltage across the load is measured, the load is changed, and the voltage across the second load resistance is measured. The two different loads and load voltages produce two equations with two unknowns, E_g and R_g, which can be solved simultaneously.

For example, if the voltage across a 400 Ω load is 30 V and the voltage across a 200 Ω load is 20 V, the two equations for E_g are:

$$E_g = \frac{(E_{L1} + E_{L1}R_g)}{R_{L1}} = \frac{(E_{L2} + E_{L2}R_g)}{R_{L2}}$$

Solving for R_g produces:

$$R_g = \frac{R_{L1}R_{L2}(E_{L1} - E_{L2})}{(E_{L1}R_{L2} - E_{L2}R_{L1})}$$

For the values of the loads and load voltages in the example, R_g = 400 Ω and E_g = 60 V.

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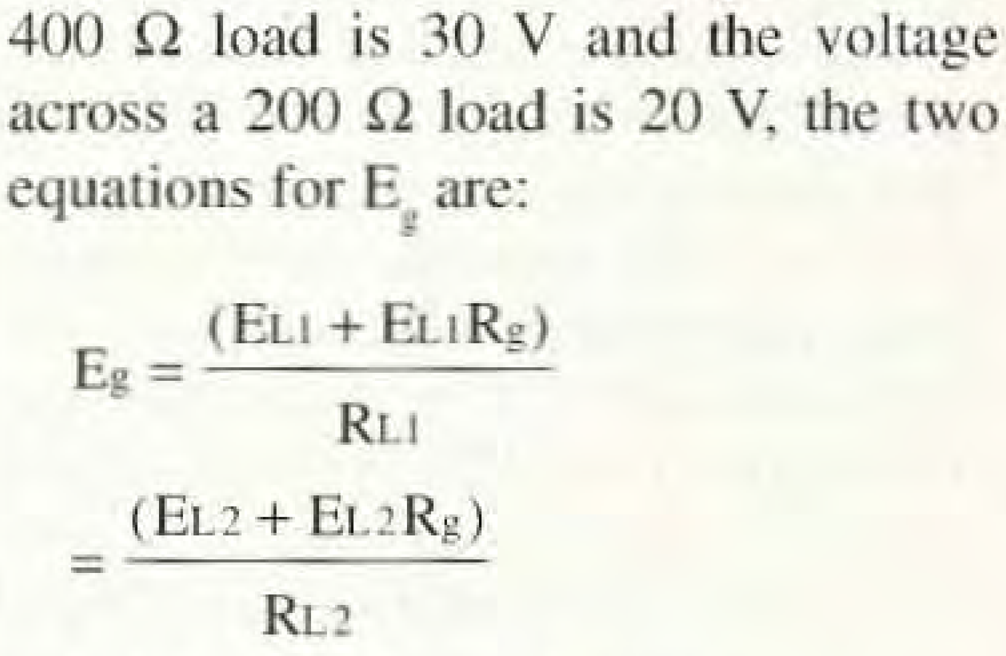

Fig. 1. *The generator impedance can be calculated when the generator's reactance is "tuned out."*

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The relationship between load impedance, generator impedance, output voltage, and output power is shown in "Impedance and Power." As seen in **Table 1**, the maximum power is delivered to the load when $R_L = R_g$ and maximum voltage across the load occurs when the load impedance is maximum. To realize the desired load resistance requires an impedance transformer.

Transforming the load impedance can be accomplished with wound transformers or with suitable reactive networks. At low frequencies, the reactances required are not practical, and the shortcomings of wound transformers must be accepted. At RF, reactive matching networks are the norm. The impedance transformation of a wound transformer is fixed and determined by its construction—primarily the turns ratio. Therefore, wound transformers just transform the magnitude of the load impedance and seldom provide an image match or conjugate match.

The power output of a transformer or reactive network ideally is equal to the input power. Actually, the output power is a little less than the input power because of losses in the resistance of the windings and core losses. The equations for matching networks which follow assume lossless inductors and capacitors.

Matching networks made with discrete reactive components in the form of an "L", "π", or "T", or their combinations, are commonly used at RF. An "L" network is a "π" or "T" with one element reduced to zero or infinity. **Fig. 2a** shows a generic "π" network that matches the load R_o to the generator R_I. Usually the series arms and shunt arms have opposite signs. When the reactance Z_b in the "π" of **Fig. 2a** is infinite, the resulting network is an "L" as shown in **Fig. 2b**. **Fig. 2c** shows a "T" network. When the reactance Z_a in the "T" is zero, the resulting network is also an "L" as shown in **Fig. 2d**. Both the transformation ratio and phase shift through the network can both be selected in either a "π" or "T". In the "L", the phase shift is determined by the load resistance R_o and the series arm. The phase shift ß through the "L" is $\tan^{-1}(Z_c/R_o)$, the angle whose tangent is Z_c/R_o.

The equations for the lossless reactances in an image matching "L" network are:

$$Z_1 = R_1\sqrt{[R_o / (R_1 - R_o)]}$$

(input shunt arm)
[Equation 1]

and

$$Z_2 = \sqrt{[R_o (R_1 - R_o)]}$$

(series arm)
[Equation 2]

The generator can feed either R_I or R_o, but R_o must be less than R_I.

In most impedance matching applications phase shift through the network is not important and the "L" network is satisfactory. But where phase shift is important, a full "π" or "T" may be required. The design equations for the "π" are:

$$Z_a = \frac{jR_1R_2\mathrm{Sin}\beta}{[R_2\mathrm{Cos}\beta - \sqrt{(R_1R_2)}]}$$

$$Z_b = \frac{jR_1R_2\mathrm{Sin}\beta}{[R_1\mathrm{Cos}\beta - \sqrt{(R_1R_2)}]}$$

$$Z_c = j\mathrm{Sin}\beta\sqrt{(R_1R_2)}$$

where ß is the phase shift through the network. When ß is 90°, Sinß = 1 and Cosß = 0, and the equations for the low-pass configuration reduce to

$$Z_a = -j\sqrt{(R_1R_2)}$$

$$Z_b = -j\sqrt{(R_1R_2)}$$

and

$$Z_c = j\sqrt{(R_1R_2)}.$$

The network behaves like a quarter-wave transmission line transformer.

The design equations for the "T" are:

$$Z_a = \frac{-j[R_1\mathrm{Cos}\beta - \sqrt{(R_1R_2)}]}{\mathrm{Sin}\beta}$$

$$Z_b = \frac{-j[R_2\mathrm{Cos}\beta - \sqrt{(R_1R_2)}]}{\mathrm{Sin}\beta}$$

$$Z_c = \frac{j\sqrt{(R_1R_2)}}{\mathrm{Sin}\beta}$$

where ß is the phase shift through the network.

When ß is 90°, Cosß = 0, and Sinß = 1, the equations reduce to:

$$Z_a = -j\sqrt{(R_1R_2)}$$

$$Z_b = -j\sqrt{(R_1R_2)}$$

and

$$Z_c = +j\sqrt{(R_1R_2)}.$$

Again, the network behaves as a quarter-wave transmission line.

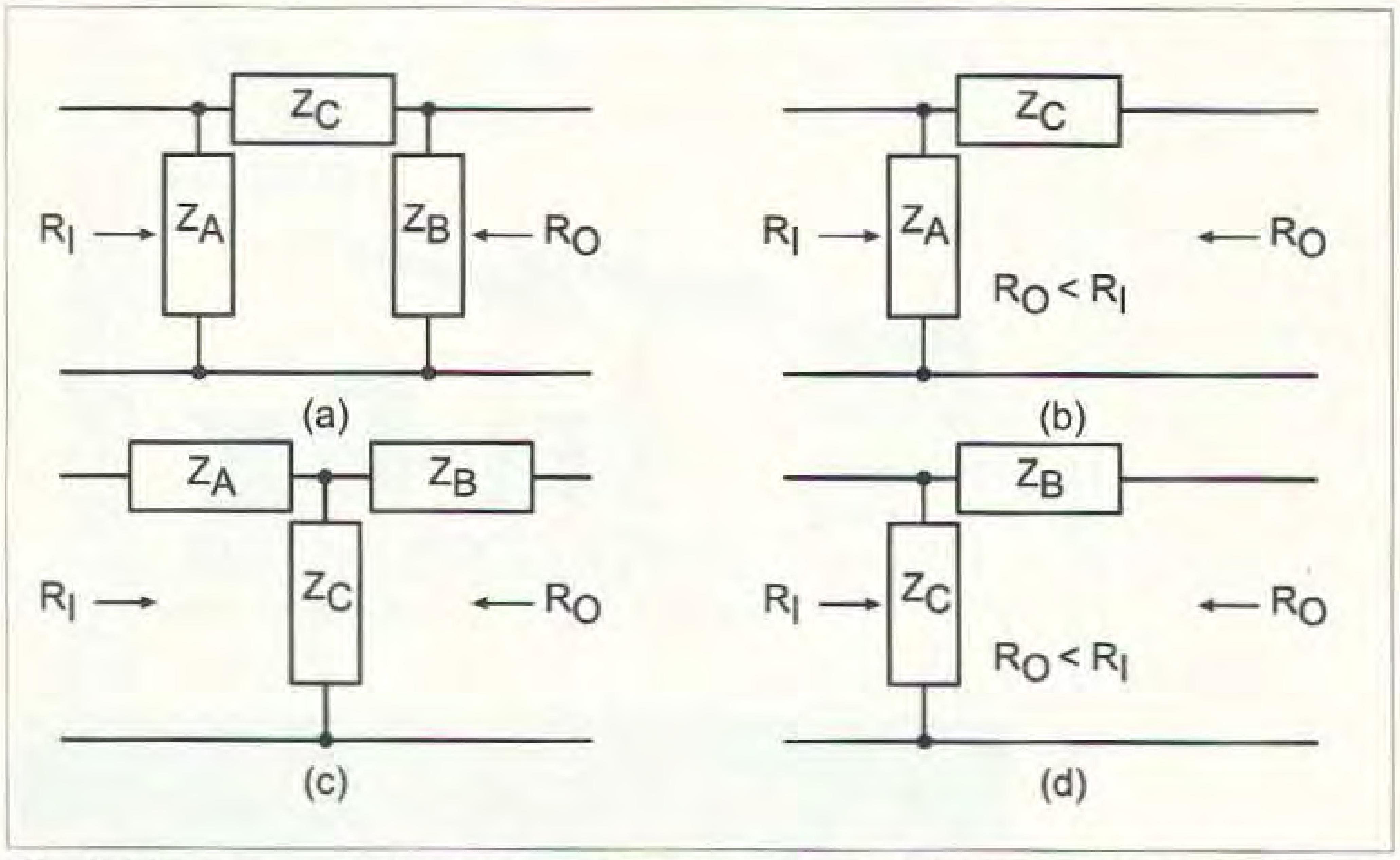

Fig. 2. *There are many forms of matching networks. (a) A "π" network. (b) An "L" network derived from the "π." (c) A "T" network. (d) An "L" derived from the "T."*

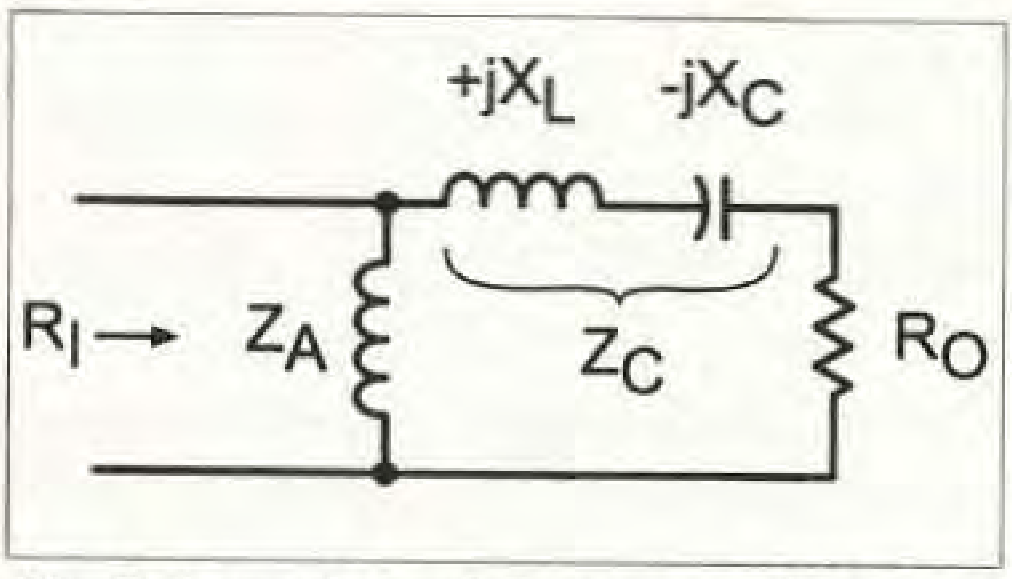

Fig. 3. A tuned circuit increases selectivity.

For an "L" to produce a resistance when looking into Z_a requires Z_a and Z_c to have opposite signs. Most often Z_c is a capacitor and Z_a an inductor, so that DC is blocked from the load and Z_a provides a series DC feedpath for the generator (plate or collector of an amplifier). Unfortunately, when Z_c is a capacitor, the "L" is a high-pass filter and there is no attenuation of harmonics. If harmonics must be suppressed, Z_c can be a series-tuned circuit operated off-resonance to provide the appropriate total reactance for Z_c.

The 3 dB bandwidth of a tuned circuit can be expressed as:

$$BW_{3dB} = \frac{F_o}{Q}$$

where

F_o = center frequency

and

$$Q = \frac{X_L}{R} = \frac{1}{2\pi F_o C R_o}.$$

Making Z_c a series-tuned LC circuit as shown in **Fig. 3** can produce a narrow bandpass response and suppress harmonics. The net value for Z_c is $jX_L - jX_c$. The voltage across capacitor is equal to the current in the load times $-jX_c$. The current in the load I_L is

$$\sqrt{\frac{P_o}{R_o}}.$$

Even with moderate power, the voltage across X_c can be surprisingly high.

For example, when you desire to transform 50 Ω to 800 Ω with a high pass "L", equations 1 and 2 show a series arm Z_c of –j194 and a shunt input arm of +j207. When you want to reduce the second harmonic, a series-tuned circuit operating below its resonant frequency has a capacitive reactance which can replace Z_c.

The response of a single tuned circuit falls 6 dB for every doubling of bandwidth. To achieve about 20 dB of harmonic suppression requires a Q slightly greater than 10. The reactance X_L then must be about j500 and the reactance X_c must be -j694 to produce the net reactance of -j194 required for the impedance transformation. When the power output is 200 W into 50 Ω, the current in the load and the series arm of the "L" is 2 A_{rms} or 2.828 A_{pk} while the voltage across the capacitor is 1.96 kV.

The network equations given in equations 1 and 2 produce a resistive input impedance, but a conjugate match will probably require an additional reactive component to tune out the generator's reactance. When the generator output has a capacitor to ground, a conjugate match requires an inductance in parallel to resonate the generator's capacitance. That additional inductance can be incorporated into Z_a.

For example, when the value of Z_a needed is j207 and the generator has a shunt capacity whose reactance is –j1000, the generator must be paralleled with +j1000 to tune out the generator's reactance. For +j1000 in parallel with $+jZ_a$ to be +j207 requires jZ_a to be +j261.

The "L" network is the simplest circuit for matching a load to a generator, and when the generator is a tube or transistor, the "L" can also provide DC

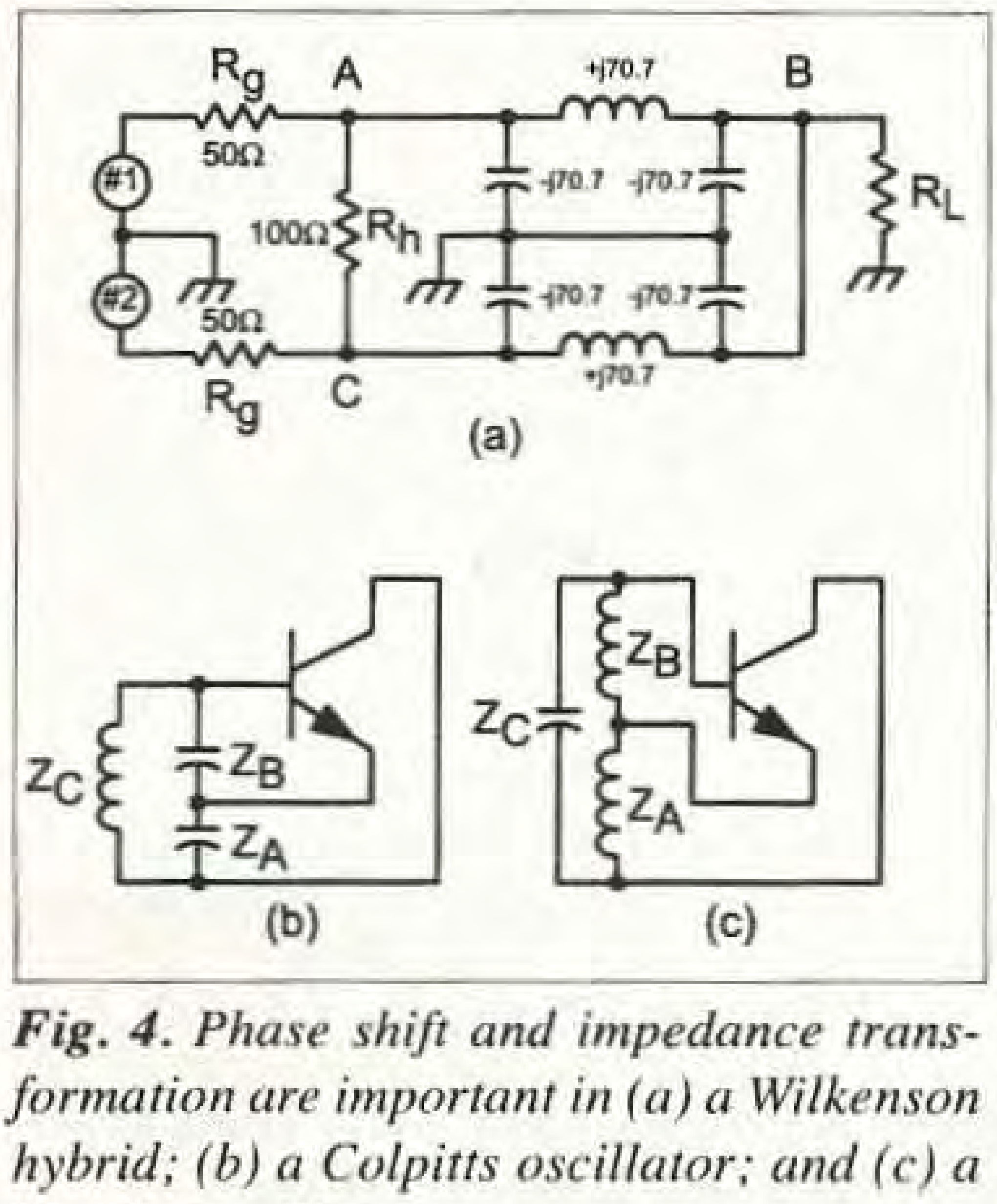

Fig. 4. Phase shift and impedance transformation are important in (a) a Wilkenson hybrid; (b) a Colpitts oscillator; and (c) a Hartley oscillator.

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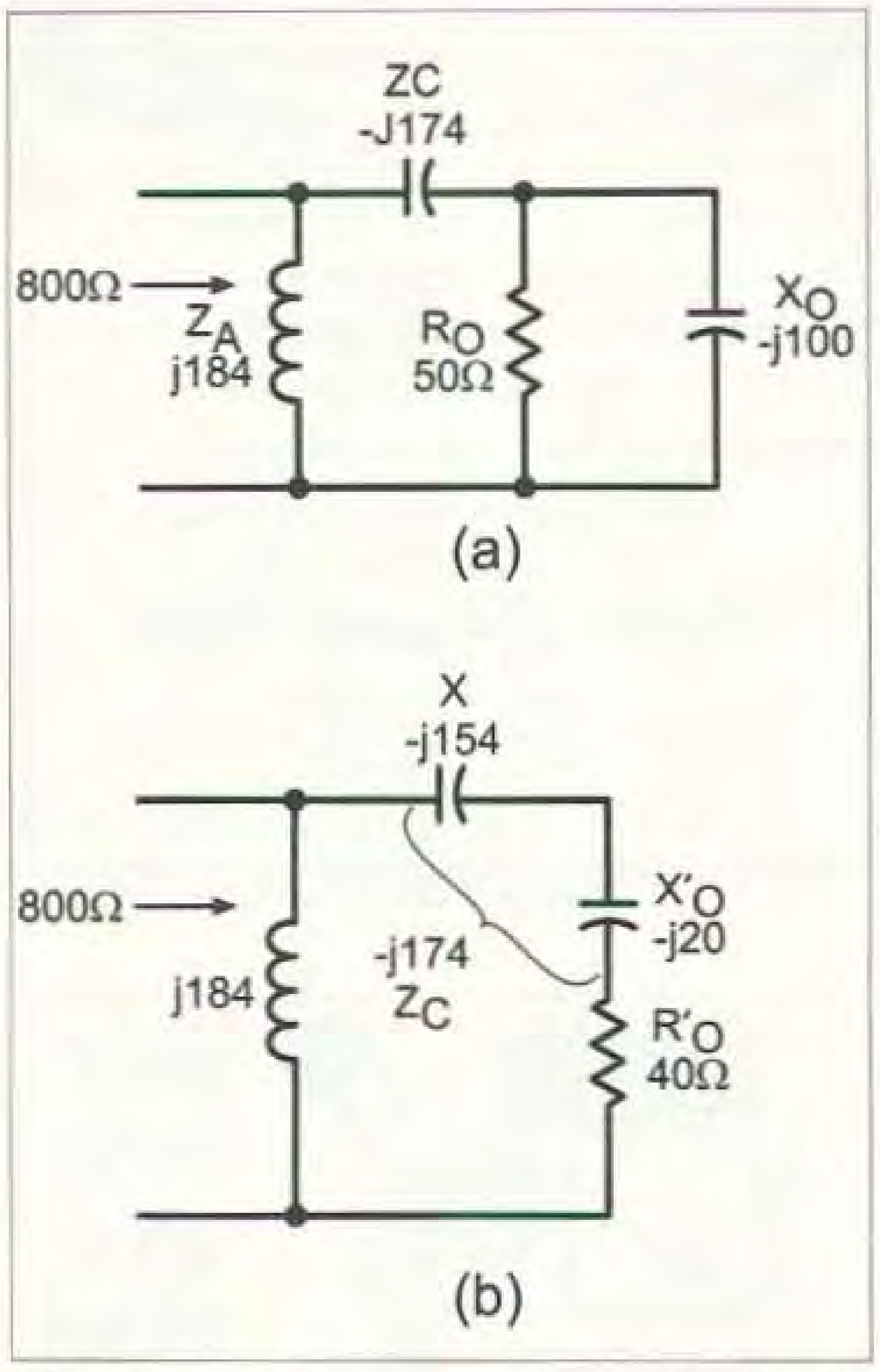

Fig. 5. An "L" can match reactive loads. (a) A load with a shunting capacitor. (b) The equivalent series circuit.

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blocking to the load and a DC feedpath to the amplifier.

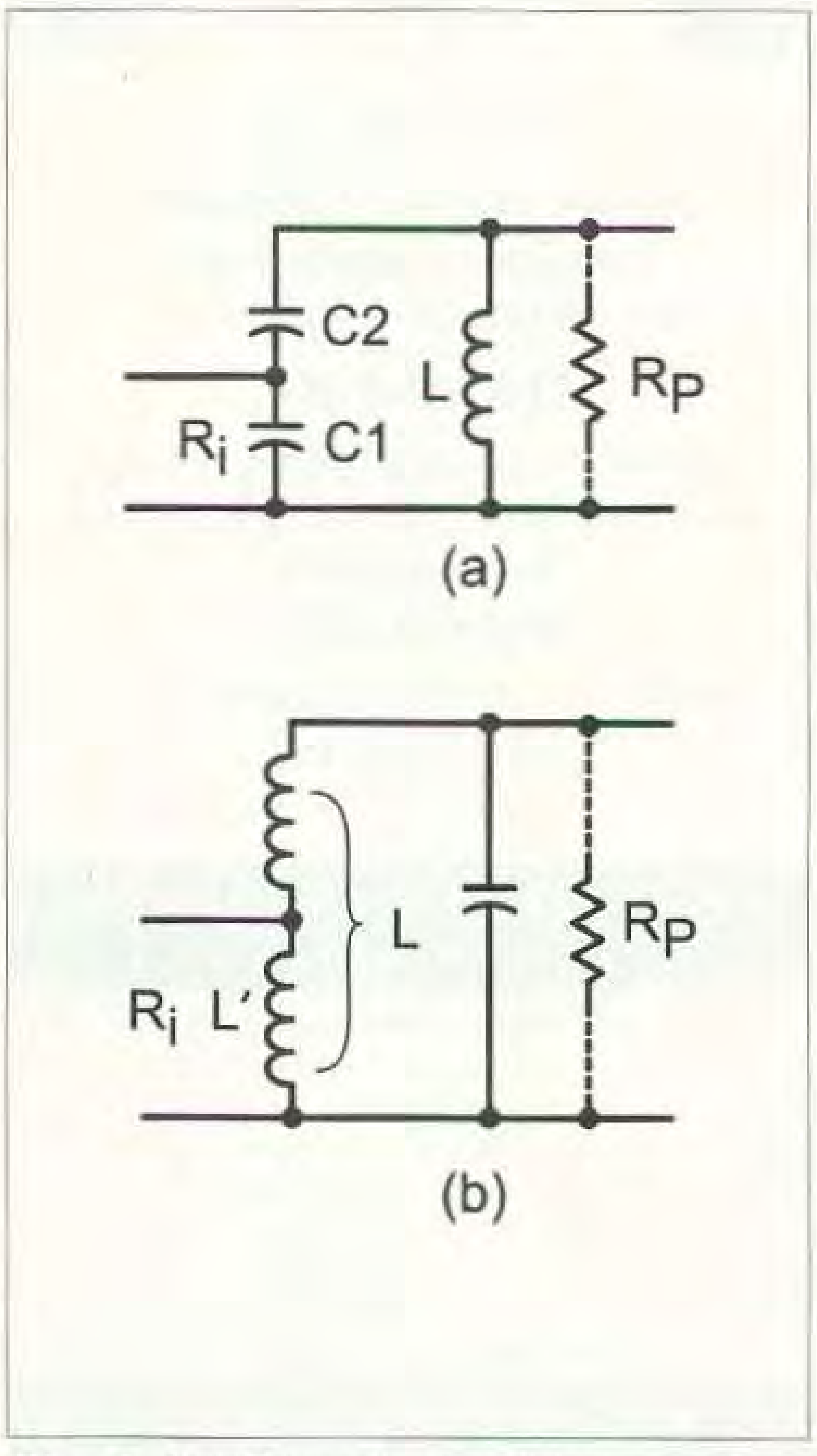

Fig. 6. (a) Tapped capacitors transform resistances. (b) Tapped inductors or an autotransformer can transform resistances.

The "π" matching network shown in **Fig. 2a** is commonly found as the plate or collector load of an amplifier. Z_b is often called the "loading" capacitor and Z_a the "tuning" capacitor. As in the "L", here the selectivity can be increased by using a series-tuned circuit for Z_c.

Occasionally both the phase shift and impedance transformation are important characteristics of the matching network as in the Wilkenson hybrid shown in **Fig. 4**. Like any hybrid, the Wilkenson combines two inputs into one output and isolates the inputs from each other. Isolation means that input #1 does not appear at input #2 and vice versa. Without isolation, some of the signal #1 at point A will appear at point C and may modulate or pull signal source #2.

The "π" networks transform the 50 Ω source to 100 Ω. The two 100 Ω outputs are paralleled to present a 50 Ω source to the 50 Ω output. The phase shift from point "A" to point "B" is 90°, and from point "B" back to point "C" is another 90° for a total of 180°. Therefore, a non-phase shifted portion of input #1 applied to input #2 through R_b cancels #1's signal coming back from the output. The two output capacitors can be combined into one with a reactance of 35.35.

In passing, the resonator of the Colpitts oscillator can be viewed as a "π" with 180° of phase shift. **Fig. 4b** shows the RF circuit of the Colpitts; the DC circuit is not shown. The input to the "π" is fed from the output of the amplifier and the output of the "π" is shifted 180° to drive the input of the

Impedance and Power

Fig. 7 shows the equivalent circuit of a generator and load. When the load is a resistor R_L, the power dissipated in the load is $I_L^2R_L$ or E_L^2/R_L, where I_L is the current in the load and E_L is the voltage across the load. The generator is matched when R_L equals R_g, and the voltage across a matched load is $E_g/2$. When the load is an impedance Z_L, the voltage across the load is:

$$E_L = \frac{E_g Z_L}{(Z_L + R_g)}$$

but only the real (resistive) part of Z_L dissipates power; the reactive part stores energy, then gives it back. The resistive part of an impedance Z_L is:

$$R = Z_L \cos\theta$$

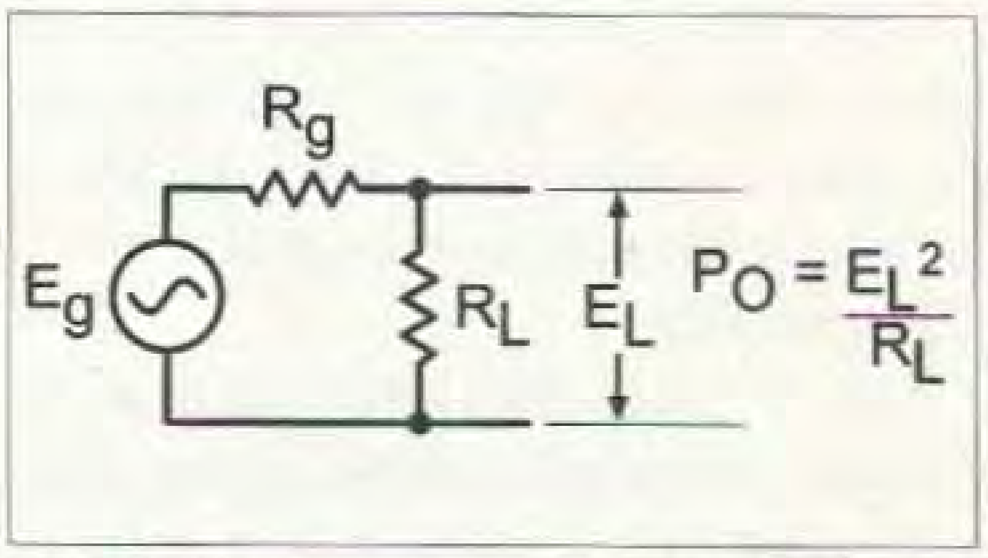

Fig. 7. The power output available from a generator depends on the load resistance compared to the generator.

The relationship of power in the load to the generator's internal resistance is given in **Table 1**. From the values in the table, you can see that maximum power is obtained when the load resistance is equal to the generator resistance.

R_L / R_g	E_L	P_L
6	1.714	0.490
5	1.666	0.555
4	1.60	0.640
3	1.50	0.75
2	1.333	0.888
1	1	1.0
1/2	0.667	0.888
1/3	0.5	0.75
1/4	0.400	0.64
1/5	0.333	0.555
1/6	0.284	0.490

***Table 1.** Relationship of power in the load to the generator's internal resistance.*

amplifier. If the shunt arms of the "π" are inductors and the series arm a capacitor, the oscillator is seen to be the Hartley oscillator shown in **Fig. 4c**.

When the load R_o is in parallel with a capacitive reactance $-jX_o$, as shown in **Fig. 5a**, the equivalent series circuit is shown in **Fig. 5b**. X_o and R_o are transformed to their equivalent series components X_o' and R_o'. 50 Ω in parallel with –j100 is equivalent to 40 Ω in series with –j20.

"Conversions" describes the process of converting parallel and series Rs and jXs to their series equivalents. These calculations are rather time-consuming with a four-function calculator, but tolerable with a "scientific" calculator that has trig functions, squares, and square roots. A calculator that can compute polar and rectangular coordinates makes the calculations a snap.

For example, when $R_o = 50$ and $X_o = -j100$, the equivalent series circuit shown in **Fig. 5b** is $R_o' = 40$ and $X_o' = -j20$. The "L" network that transforms 40 Ω to 800 Ω is found with equations 1 and 2 to have $Z_c = -j174$ and $Z_A = +j184$. Since X_o' will be part of Z_c, the series arm of the network Z_c is:

$$Z_c = -jX_1 - jX_o' = -j174$$

When $-jX_o'$ is –j20, then $-jX_1 = -j154$. The final "L" has $Z_c = -j154$ and $Z_a = +j184$.

Fig. 6a is **Fig. 5** redrawn as the tapped capacitor tuned circuit often used in receivers to match the antenna to the input and reduce the loading on the input circuit. R_i is the input resistance, the antenna resistance, and R_p the transformed output resistance. Near resonance, the effect is to make the impedance offered to the input terminals less in magnitude than the parallel impedance of the entire circuit without changing the character of the impedance curve as far as shape or equivalent Q is concerned. Tapping a parallel resonant circuit accordingly offers a means of adjusting the magnitude of impedance obtained without changing the characteristics of the circuit itself. The ratio of impedance offered by the input R_i to the parallel impedance of the circuit R_p is:

$$\frac{R_i}{R_p} = \left[\frac{C_1}{(C_1 + C_2)}\right]^2$$

For example, if R_i is 50 and X_1 is –j5, then their equivalent series values are $R_i' = 0.49$ and $X_1' = j4.9$. If X_2 is –j50, then the total capacitive reactance across L is –j55. The equivalent Q is:

$$Q = \frac{X}{R} = \frac{55}{0.49} \approx 112$$

A tapped inductor follows the same procedure as the tapped capacitor network when there is no mutual coupling between the inductors. But, when there is mutual coupling as shown in **Fig. 6b**, $R_i/R_p = (M_{eq}/L)^2$, where M_{eq} is the total equivalent mutual impedance between L' and the entire coil L, including both common and inductive coupling.

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Conversions

The impedance of the series circuit shown in **Fig. 8** is the vector sum of a resistance R and a reactance jX. The voltage across a resistor is in phase with the current in it and is shown on the horizontal axis. The voltage across a reactance is 90° out of phase with the current in it and is shown on the vertical axis. In an inductor, the voltage leads the current by 90°, and in a capacitor the voltage lags the current by 90°. The voltage across an inductor is expressed as jX_L, while the voltage across a capacitor is $-jX_c$. The factor j rotates a vector 90° in a counterclockwise direction. **Fig. 9** shows the vector sum of R and +jX. The resistive voltage is on the horizontal axis and the reactive voltage on the vertical axis. Their sum Z is the vector addition of R and +jX. From the Pythagorean theorem,

$$Z = \sqrt{(R^2 + X^2)}$$

and from trigonometry, the angle θ is the angle whose tangent is X/R ($\tan^{-1}X/R$). Also from trigonometry, $X = Z\sin\theta$ and $R = Z\cos\theta$.

The horizontal axis represents the real component, resistance R or conductance G (G = 1/R), while the vertical axis represents the imaginary component, the reactance X or susceptance ß (-jß = 1/jX).

$R_1 + jX_1$ can be added to $R_2 + jX_2$. Reals are added to reals and imaginaries added to imaginaries: $(R_1 + R_2) + (jX_1 + jX_2)$. Of course, $R_1 + jX_1$ can be multiplied with $R_2 + jX_2$, but it's much easier to do so in polar form:

$$Z_1\angle\theta(Z_2\angle\phi) = Z_1Z_2\angle(\theta + \phi)$$

Treat the angles as exponents.

Dividing rectangular forms is also much easier in polar form:

$$\frac{Z_1\angle\theta}{Z_2\angle\phi} = \left(\frac{Z_1}{Z_2}\right)\angle(\theta - \phi)$$

Again, treat the angles as exponents.

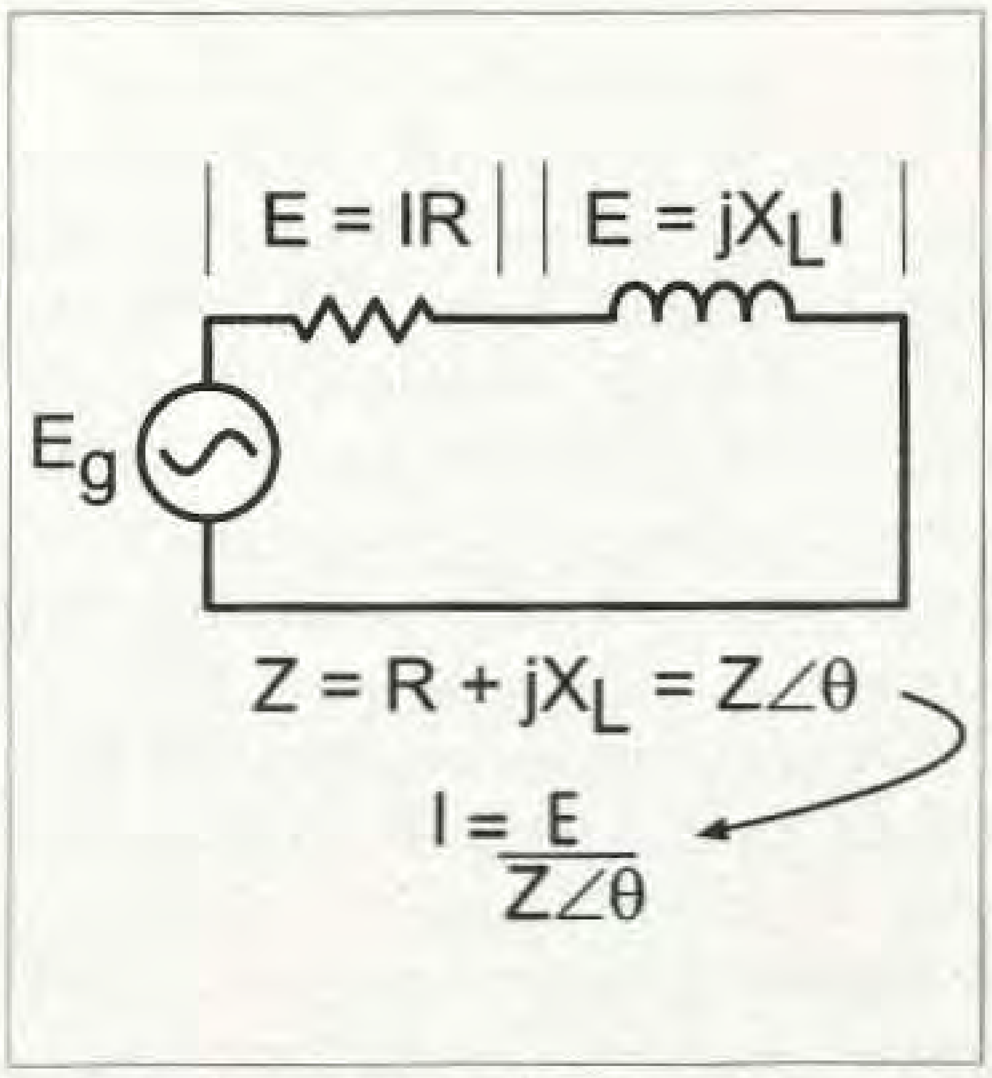

Fig. 8. A resistor and inductor in series add vectorially.

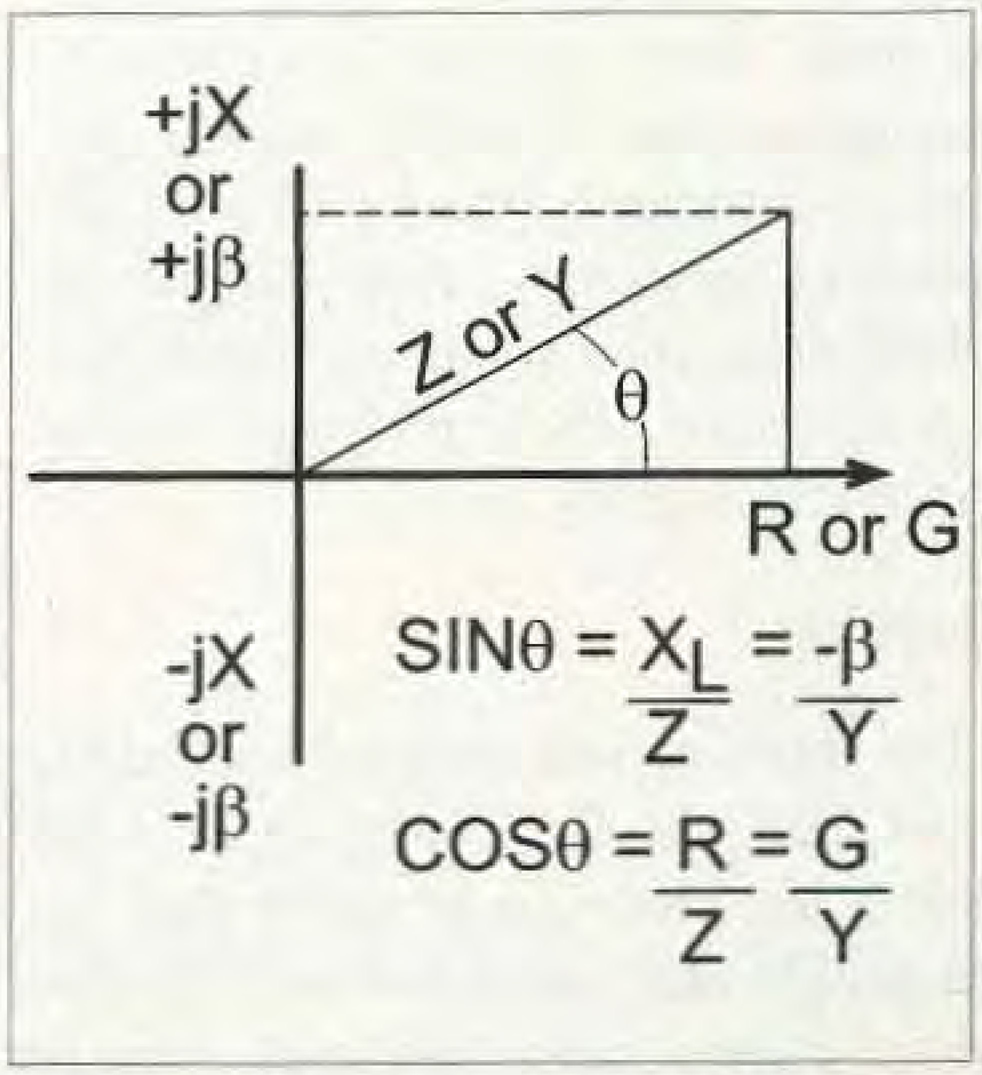

Fig. 9. A circuit containing R and X is depicted as a right triangle.

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However, if a tapped coil is wound on a ferrite core so that the mutual coupling is high, the two coils behave like an ideal transformer with predictable results (the impedance is transformed by the turns ratio squared). With air-cored coils, the coupling coefficient is small and the ideal transformer approximation does not hold in all cases. However, if $R_p/w_0L \geq 10$, and $R_i \geq w_0L$, the circuit still behaves like an ideal transformer at the resonant frequency. In passing, I should mention that you should be aware that when the turns ratio is large and the secondary is a few at the end of the primary, the coupling may be low, even with a core. To achieve tighter coupling, distribute the secondary over the entire primary; the distributed capacity will increase slightly but the coupling will be much tighter.

Matching networks can be used to match an arbitrary load to an arbitrary generator for maximum power output, maximum voltage, or a particular phase shift. Calculating the values of reactances for a match may seem intimidating, but it will give you a better feel for what the parts do than using nomographs or charts. Matching networks made with discrete reactive elements are flexible and can be adjustable—and that's something wound transformers can't easily offer. 73