Tuning of a resonant loop with stubs RDL Tuning-8
7/3/98

•Model of R&D ITER antenna, with improved loss formulas in Ytransf function, etc.
•Solves for d1 and d2.
•Added stub feeder sections, length df and impedance Zf between strap ends and tuning admittance.
•Added line feeder sections between tap point and the coax line.
•Changed to include losses in stub tuners
•Changed definition forf Q calculation 7/3/98

Define constants for use in calculations

$f := 30$ Frequency (MHz)

$R := .04$ R= plasma loading resistance (ohms/m)

$Z_{svac} := 58$ Strap impedance (ohms)

$\beta_{vac} := 1$ phase velocity on strap with no Faraday shield

$C_{FS} := 55 \cdot 10^{-12}$ Additional capacitance added by Faraday shield (F)

$h := 1.15$ full strap height (m)

$\alpha := .5$ Fraction of distance along line that tap is

$d_{s1} := .297$	$Z_{s1} := 26$	$R_{s1} := R$	Section between strap and tuning stub
	$Z_{s3} := 30$	$R_{s3} := .08$	Shorted stubs (lengths individually adjustable)
$d_{lf} := .25$	$Z_{lf} := 58$	$R_{lf} := .0$	Feeder between strap and start of coax
$d_{30} := .21$	$Z_{30} := 28$	$R_{30} := .0$	30-ohm section of feedline
$d_{ft} := 0.$	$Z_{ft} := 32$	$R_{ft} := .0$	Vacuum feedthrough (if present)

$Z_{line} := 50$ Char. impedance of transmission line

Calculate effect of adding capacitance of Faraday shield to strap impedance and phase velocity

$L_{vac} := \frac{Z_{svac}}{3 \cdot 10^8}$ $C_{vac} := \frac{1}{3 \cdot 10^8 \cdot Z_{svac}}$ $C := C_{vac} + C_{FS}$ $Z_s := \sqrt{\frac{L_{vac}}{C}}$ $\beta := \frac{1}{3 \cdot 10^8 \cdot \sqrt{L_{vac} \cdot C}}$

$L_{vac} = 1.9333 \cdot 10^{-7}$ $C_{vac} = 5.7471 \cdot 10^{-11}$ $C = 1.1247 \cdot 10^{-10}$ $Z_s = 41.4603$ $\beta = 0.7148$

$k_0(f) := \frac{f}{47.75}$ Free-space wavenumber in m for freq in MHz

$k_s(f) := \frac{k_0(f)}{\beta}$ Wavenumber on strap

Define Functions

$\gamma(k, R, Z) := i \cdot k \cdot \sqrt{1 - \frac{i \cdot R}{k \cdot Z}}$ Complex prop constant for lossy line with wavenumber k and loss R

$Z_0(k, R, Z) := Z \cdot \sqrt{1 - \frac{i \cdot R}{k \cdot Z}}$ Impedance of line with losses

$Chf(f, R, x, Z) := \cosh(\gamma(k_0(f), R, Z_0(k_0(f), R, Z)) \cdot x)$

$Shf(f, R, x, Z) := \sinh(\gamma(k_0(f), R, Z_0(k_0(f), R, Z)) \cdot x)$

$Chs(f, R, x) := \cosh(\gamma(k_s(f), R, Z_0(k_s(f), R, Z_s)) \cdot x)$

$Shs(f, R, x) := \sinh(\gamma(k_s(f), R, Z_0(k_s(f), R, Z_s)) \cdot x)$

General transformation of termination impedance distance x along line w. char. imp. Z l and losses R (ohms/m)

$$Ytransf(f, R, x, Yin, Zl) := \frac{1}{Z_0(k_0(f), R, Zl)} \cdot \frac{Z_0(k_0(f), R, Zl) \cdot Yin \cdot Chf(f, R, x, Zl) + Shf(f, R, x, Zl)}{(Chf(f, R, x, Zl) + Z_0(k_0(f), R, Zl) \cdot Yin \cdot Shf(f, R, x, Zl))}$$

$Ys(f, R) := \frac{1}{Z_0(k_s(f), R, Z_s)}$ Complex char. admittance of strap

$Ystbf(f, x) := \frac{1}{Z_0(k_0(f), R_{s3}, Z_{s3})} \cdot \frac{\cosh(\gamma(k_0(f), R_{s3}, Z_0(k_0(f), R_{s3}, Z_{s3})) \cdot x)}{\sinh(\gamma(k_0(f), R_{s3}, Z_0(k_0(f), R_{s3}, Z_{s3})) \cdot x)}$ Admittance of top and bottom stubs

$h1 := \alpha \cdot h$ $h2 := h - h1$

$Y1af(f, d1) := Ytransf(f, R_{s1}, d_{s1}, Ystbf(f, d1), Z_{s1})$ Admittance at top of strap

$Y2af(f, d2) := Ytransf(f, R_{s1}, d_{s1}, Ystbf(f, d2), Z_{s1})$ Admittance at bottom of strap

$Y3f(f, R, d1) := Ys(f, R) \cdot \left[\frac{Y1af(f, d1) \cdot Chs(f, R, h1) + Ys(f, R) \cdot Shs(f, R, h1)}{Ys(f, R) \cdot Chs(f, R, h1) + Y1af(f, d1) \cdot Shs(f, R, h1)}\right]$ Admittance of top line at T

$Y4f(f, R, d2) := Ys(f, R) \cdot \left[\frac{Y2af(f, d2) \cdot Chs(f, R, h2) + Ys(f, R) \cdot Shs(f, R, h2)}{Ys(f, R) \cdot Chs(f, R, h2) + Y2af(f, d2) \cdot Shs(f, R, h2)}\right]$ Admittance of bottom line at T

$YTf(f, R, d1, d2) := Y3f(f, R, d1) + Y4f(f, R, d2)$ Admittance at T

$Ylf(f, R, d1, d2) := Ytransf(f, R_{lf}, d_{lf}, YTf(f, R, d1, d2), Z_{lf})$

$Y30f(f, R, d1, d2) := Ytransf(f, R_{30}, d_{30}, Ylf(f, R, d1, d2), Z_{30})$

$Yinf(f, R, d1, d2) := Ytransf(f, R_{ft}, d_{ft}, Y30f(f, R, d1, d2), Z_{ft})$ Admittance at coax line

$ZTf(f, R, d1, d2) := \frac{1}{YTf(f, R, d1, d2)}$

$Zinf(f, R, d1, d2) := \frac{1}{Yinf(f, R, d1, d2)}$

Do fast freq sweep to get first guesses for d1 and d2

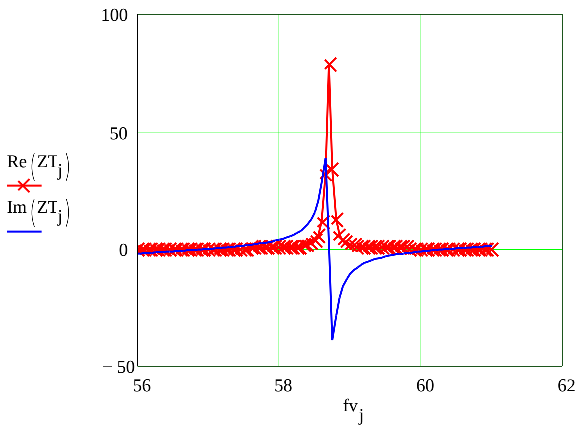
$fwant := 58.5$ Input frequency for antenna resonance

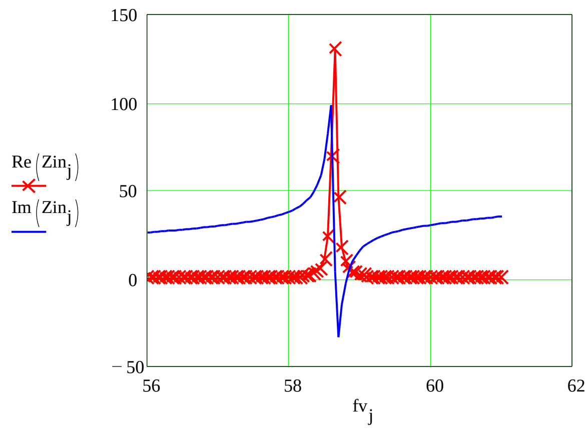
$a1 := .36$ $a2 := .23$ $d1 := 1.596 - a1$ $d2 := 1.603 - a2$

$d1 = 1.236$ $d2 = 1.373$

$Nj := 100$ $j := 0 .. Nj$ $a1 + a2 = 0.59$

$fv_j := 56 + 5 \cdot \frac{j}{Nj}$ $ZT := \overrightarrow{ZTf(fv, R, d1, d2)}$ $Zin := \overrightarrow{Zinf(fv, R, d1, d2)}$





List parameters for given frequency

$ft := fwant$

$\gamma(k_0(ft), R_{s3}, Z_0(k_0(ft), R_{s3}, Z_{s3})) = 1.3333 \cdot 10^{-3} + 1.2251i$

$Z_0(k_0(ft), R_{s3}, Z_{s3}) = 30 - 0.0326i$

$Ystbf(ft, d1) = 5.7162 \cdot 10^{-5} - 1.8863 \cdot 10^{-3} i$ $Ystbf(ft, d2) = 5.7729 \cdot 10^{-5} + 3.7258 \cdot 10^{-3} i$

$Y1af(ft, d1) = 6.3621 \cdot 10^{-5} + 0.0125i$ $Y2af(ft, d2) = 7.3121 \cdot 10^{-5} + 0.0191i$

$Y3f(ft, R, d1) = 5.2249 \cdot 10^{-3} + 0.2259i$ $Y4f(ft, R, d2) = 7.9108 \cdot 10^{-3} - 0.2867i$

$Y3f(ft, R, d1) + Y4f(ft, R, d2) = 0.0131 - 0.0608i$

$ZTf(ft, R, d1, d2) = 3.393 + 15.7096i$

$Zinf(ft, R, d1, d2) = 11.2154 + 67.671i$

Find d1 and d2 to match. given f and R

$d2rootf(d1g, d2g, fwant, Rwant) := root(10^4 \cdot Im(YTf(fwant, Rwant, d1g, d2g)), d2g)$

Given

$10^4 \cdot Re(Yinf(fwant, Rwant, d1g, d2g)) = \frac{10^4}{Z_{line}}$ $10^4 \cdot Im(Yinf(fwant, Rwant, d1g, d2g)) = 0$

$Reslengths(d1g, d2g, fwant, Rwant) := Find(d1g, d2g)$

$\begin{bmatrix} d1 \\ d2 \end{bmatrix} := Reslengths(d1, d2, fwant, R)$ Make this active to find d1 and d2 that make resonant AND match to freq fwant

$d2 := d2rootf(d1, d2, fwant, R)$ ▫   Make THIS active to find d2 that makes resonant for specified value of d1, but doesn't match

$Zinf(fwant, R, d1, d2) = 50$ Check to see that solved d1 and d2 give correct result

$a1 := 1.596 - d1$ $a2 := 1.603 - d2$ $d1 = 1.2622$ $d2 = 1.3606$ $fwant = 58.5$

$a1 = 0.3338$ $a2 = 0.2424$ $a1 + a2 = 0.5762$ $a1 - a2 = 0.0914$

Do tighter frequency scan to determine freq. width and Q –

$fresf(fg) := root(10^4 \cdot Im(Yinf(fg, R, d1, d2)), fg)$

$fres := fresf(fwant)$ $fres = 58.5$

$fv_j := fres - 1 + 2 \cdot \frac{j}{Nj}$ Do freq scan $Y_1 := \frac{1}{Z_{line}}$

$Rv_j := R$

$Y3v := \overrightarrow{Y3f(fv, Rv, d1)}$ $Y4v := \overrightarrow{Y4f(fv, Rv, d2)}$ $YT := Y3v + Y4v$

$Iangle := \overrightarrow{\left[\frac{180}{\pi} \cdot (arg(Y3v) - arg(-Y4v))\right]}$ $Iratio := \overrightarrow{\frac{|Y3v|}{|Y4v|}}$ $ZT := \overrightarrow{\frac{1}{YT}}$

$Yin := \overrightarrow{Yinf(fv, Rv, d1, d2)}$ $Zin := \overrightarrow{\frac{1}{Yin}}$

$\rho in := \overrightarrow{\frac{Y_1 - Yin}{Y_1 + Yin}}$ $\rho T := \overrightarrow{\frac{Y_1 - YT}{Y_1 + YT}}$ $rhoabs := \overrightarrow{|\rho in|}$ $rhoTabs := \overrightarrow{|\rho T|}$

Calculate frequency half-width of resonance

$\rho min := min(rhoabs)$ $\rho min = 0$ $\rho half := .707 \cdot (1 + \rho min)$ Changed definition 7/3/98; coeff was 0.5

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jmin := | err ← 1
        | jmin ← 0
        | for j ∈ 0 .. Nj
        |   | errtst ← |ρmin − rhoabs_j|
        |   | jmin ← j  if errtst < err
        |   | err ← errtst  if errtst < err
        | jmin

jh1 := | for j ∈ 0 .. jmin
       |   (ans ← j)  if rhoabs_j ≥ ρhalf
       | ans

jh2 := | for j ∈ jmin .. Nj
       |   (ans ← j)  if rhoabs_j ≤ ρhalf
       | ans
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$\rho half = 0.707$

$jmin = 50$ $rhoabs_{jmin} = 0$

$jh1 = 45$ $rhoabs_{jh1} = 0.7613$

$jh2 = 54$ $rhoabs_{jh2} = 0.6845$

$\delta f := fv_{jh2} - fv_{jh1}$ $\delta f = 0.18$ $Q := \frac{fv_{jmin}}{\delta f}$

Current strap/Far. shield

- $R = 0.04$
- $Z_{svac} = 58$
- $\beta_{vac} = 1$
- $C_{FS} = 5.5 \cdot 10^{-}$
- $Z_s = 41.4603$
- $\beta = 0.7148$
- $h = 1.15$
- $\alpha = 0.5$
- $h1 = 0.575$
- $h2 = 0.575$

Stub feeders

- $Z_{s1} = 26$
- $d_{s1} = 0.297$
- $R_{s1} = 0.04$

Main line

- $Z_{line} = 50$

Feedline

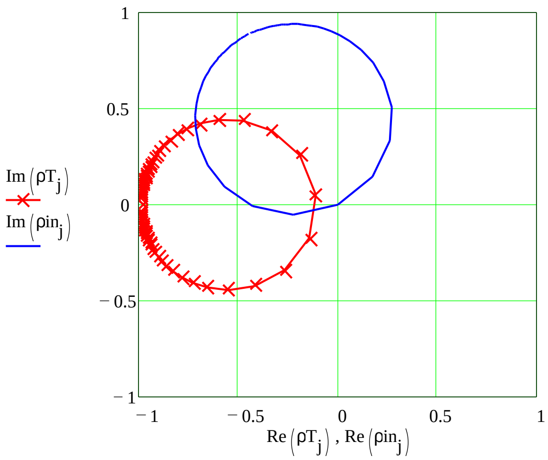
- $Z_{lf} = 58$
- $d_{lf} = 0.25$
- $d_{30} = 0.21$
- $Z_{30} = 28$
- $d_{ft} = 0$
- $Z_{ft} = 32$

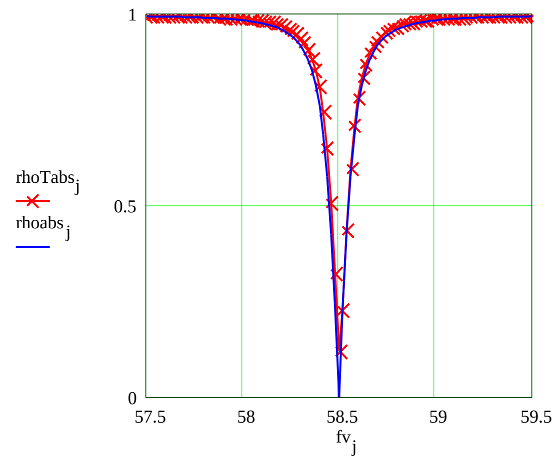
Stubs

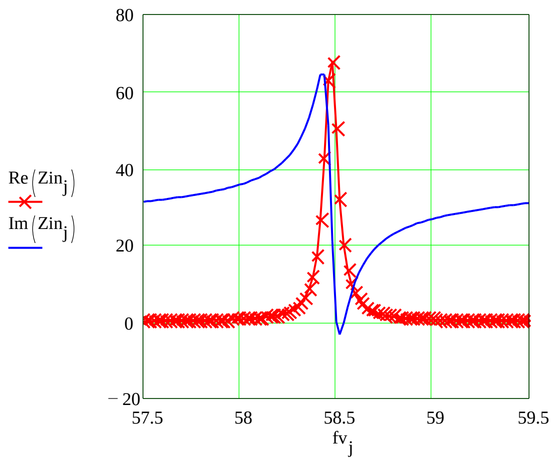
- $Z_{s3} = 30$
- $d1 = 1.2622$
- $d2 = 1.3606$
- $d1 + d2 = 2.6228$
- $d2 - d1 = 0.0984$
- $a1 = 0.3338$
- $a2 = 0.2424$
- $a1 + a2 = 0.5762$
- $a1 - a2 = 0.0914$

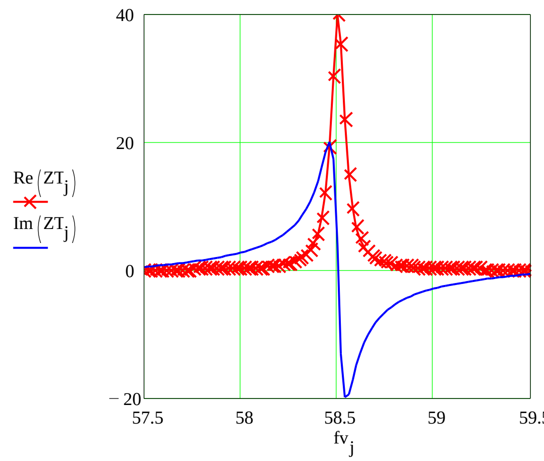
Output

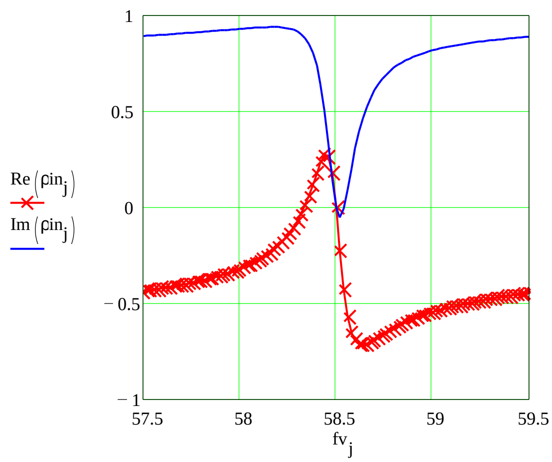
- $fres = 58.5$
- $\delta f = 0.18$
- $Q = 325$
- $\rho T_{jmin} = -0.11 + 0.051i$
- $Zin_{jmin} = 50$

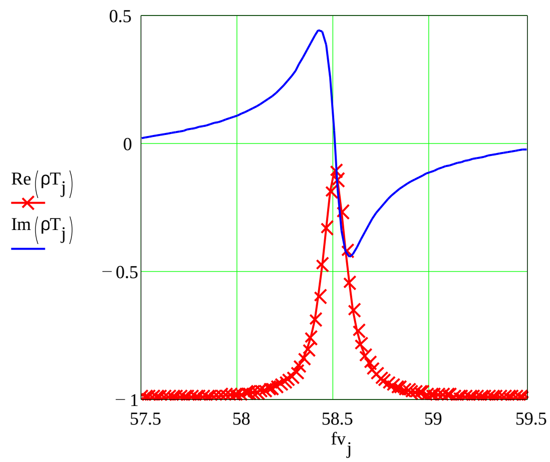












Calc. current and voltage along current strap, for specified f and R -

$fc := fres$ $VT := 1$ $fres = 58.5$ Input frequency for calculation V and I curves. Put 1 kV at T $d1 = 1.2622$ $d2 = 1.3606$

$Y3c := Y3f(fc, R, d1)$ $Y3c = 0.0126 + 0.3532i$

$Y4c := Y4f(fc, R, d2)$ $Y4c = 0.0122 - 0.3558i$

$I3ctr := Y3c \cdot VT$ $I3ctr = 0.0126 + 0.3532i$

$I4ctr := Y4c \cdot VT$ $I4ctr = 0.0122 - 0.3558i$

$IT := I3ctr + I4ctr$ $IT = 0.0248 - 2.5484 \cdot 10^{-3} i$

$PT := \frac{1}{2} \cdot Re(VT \cdot IT)$ $PT = 0.0124$ Power input to T

$V3f(x) := VT \cdot Chs(fc, R, x) - Z_s \cdot I3ctr \cdot Shs(fc, R, x)$ V and I along top half of strap

$I3f(x) := I3ctr \cdot Chs(fc, R, x) - \frac{VT}{Z_s} \cdot Shs(fc, R, x)$

$V1a := V3f(h1)$ $I1a := I3f(h1)$ V and I at top end of strap

$V1af(x) := V1a \cdot Chf(fc, R_{s1}, x - h1, Z_{s1}) - Z_{s1} \cdot I1a \cdot Shf(fc, R_{s1}, x - h1, Z_{s1})$ V and I along top feeder

$I1af(x) := I1a \cdot Chf(fc, R_{s1}, x - h1, Z_{s1}) - \frac{V1a}{Z_{s1}} \cdot Shf(fc, R_{s1}, x - h1, Z_{s1})$

$V1s := V1af(h1 + d_{s1})$ $I1s := I1af(h1 + d_{s1})$ V and I at end of feeder

$V1f(x) := V1s \cdot Chf(fc, R_{s3}, x - h1 - d_{s1}, Z_{s3}) - Z_{s3} \cdot I1s \cdot Shf(fc, R_{s3}, x - h1 - d_{s1}, Z_{s3})$ V and I on top stub

$I1f(x) := I1s \cdot Chf(fc, R_{s3}, x - h1 - d_{s1}, Z_{s3}) - \frac{V1s}{Z_{s3}} \cdot Shf(fc, R_{s3}, x - h1 - d_{s1}, Z_{s3})$

$Vtopf(x) := if(x < h1, V3f(x), if(x < h1 + d_{s1}, V1af(x), V1f(x)))$ V and I along top part of strap, feeder, and stub

$Itopf(x) := if(x < h1, I3f(x), if(x < h1 + d_{s1}, I1af(x), I1f(x)))$

$Rtopf(x) := if(x < h1, R, if(x < h1 + d_{s1}, R_{s1}, R_{s3}))$

$xv1_j := (h1 + d_{s1} + d1) \cdot \frac{j}{Nj}$

$Vtop := \overrightarrow{Vtopf(xv1)}$ $Itop := \overrightarrow{Itopf(xv1)}$ $Rtop := \overrightarrow{Rtopf(xv1)}$

$V4f(x) := VT \cdot Chs(fc, R, x) - Z_s \cdot I4ctr \cdot Shs(fc, R, x)$ V and I along bottom half of strap

$I4f(x) := I4ctr \cdot Chs(fc, R, x) - \frac{VT}{Z_s} \cdot Shs(fc, R, x)$

$V2a := V4f(h2)$ $I2a := I4f(h2)$ V and I at bottom end of strap

$V2af(x) := V2a \cdot Chf(fc, R_{s1}, x - h2, Z_{s1}) - Z_{s1} \cdot I2a \cdot Shf(fc, R_{s1}, x - h2, Z_{s1})$ V and I along bottom feeder

$I2af(x) := I2a \cdot Chf(fc, R_{s1}, x - h2, Z_{s1}) - \frac{V2a}{Z_{s1}} \cdot Shf(fc, R_{s1}, x - h2, Z_{s1})$

$V2s := V2af(h2 + d_{s1})$ $I2s := I2af(h2 + d_{s1})$ V and I at end of feeder

$V2f(x) := V2s \cdot Chf(fc, R_{s3}, x - h2 - d_{s1}, Z_{s3}) - Z_{s3} \cdot I2s \cdot Shf(fc, R_{s3}, x - h2 - d_{s1}, Z_{s3})$ V and I on bottom stub

$I2f(x) := I2s \cdot Chf(fc, R_{s3}, x - h2 - d_{s1}, Z_{s3}) - \frac{V2s}{Z_{s3}} \cdot Shf(fc, R_{s3}, x - h2 - d_{s1}, Z_{s3})$

$Vbotf(x) := if(x < h2, V4f(x), if(x < h2 + d_{s1}, V2af(x), V2f(x)))$ V and I along bottom part of strap, feeder, and stub

$Ibotf(x) := if(x < h2, I4f(x), if(x < h2 + d_{s1}, I2af(x), I2f(x)))$

$Rbotf(x) := if(x < h2, R, if(x < h2 + d_{s1}, R_{s1}, R_{s3}))$

$xv2_j := (h2 + d_{s1} + d2) \cdot \frac{j}{Nj}$

$h1 + d_{s1} + d1 = 2.1342$ Total length of top

$Vbot := \overrightarrow{Vbotf(xv2)}$ $Ibot := \overrightarrow{Ibotf(xv2)}$ $Rbot := \overrightarrow{Rbotf(xv2)}$

$h2 + d_{s1} + d2 = 2.2326$ Total length of bottom

$jp := 0 .. 2 \cdot Nj$

Voltage and current along RDL

$xp_{jp} := if(jp < Nj, -xv2_{Nj-jp}, xv1_{jp-Nj}) + \frac{h2 - h1}{2}$

$Vp_{jp} := if(jp < Nj, Vbot_{Nj-jp}, Vtop_{jp-Nj})$

$Ip_{jp} := if(jp < Nj, -Ibot_{Nj-jp}, Itop_{jp-Nj})$

$Rp_{jp} := if(jp < Nj, Rbot_{Nj-jp}, Rtop_{jp-Nj})$

$feedpt := \frac{h}{2} - h1$

$Ia := \overrightarrow{|Ip|}$ $Va := \overrightarrow{|Vp|}$

$feedpt = 0$

$jimax := Nj - ceil(.3 \cdot Nj) .. Nj + ceil(.3 \cdot Nj)$

$Iaimax_{jimax} := Ia_{jimax}$

$Imaxstrap := max(Iaimax)$ Maximum current on strap (NOT max anywhere)

$Vmax := max(Va)$ Maximum voltage anywhere

$Vmax = 13.5545$ $Imaxstrap = 0.3568$

Normalize to 1 kA strap peak current –

$Vtop := \frac{Vtop}{Imaxstrap}$ $Vbot := \frac{Vbot}{Imaxstrap}$ $Itop := \frac{Itop}{Imaxstrap}$ $Ibot := \frac{Ibot}{Imaxstrap}$

$Vp := \frac{Vp}{Imaxstrap}$ $Ip := \frac{Ip}{Imaxstrap}$

$dPhi := \frac{180}{\pi} \cdot (arg(I3ctr) - arg(-I4ctr))$ $Iratio := \frac{|I3ctr|}{|I4ctr|}$ $PT := \frac{PT}{Imaxstrap^2}$

$I3ctr = 0.0126 + 0.3532i$ $|I3ctr| = 0.3534$

$Vmax := \frac{Vmax}{Imaxstrap}$ $Vmax = 37.9919$

$I4ctr = 0.0122 - 0.3558i$ $|I4ctr| = 0.356$

$arg(I3ctr) \cdot \frac{180}{\pi} = 87.9634$ $arg(-I4ctr) \cdot \frac{180}{\pi} = 91.9668$ $x_{Vt} := h1 + d_{s1} + .2$ $x_{Vb} := h2 + d_{s1} + .19$

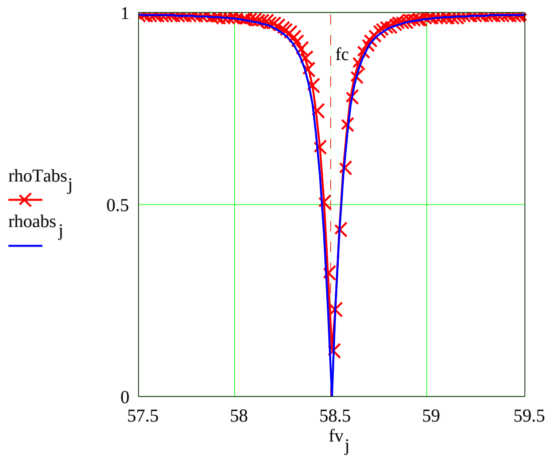
Calculate power dissipated in circuit –

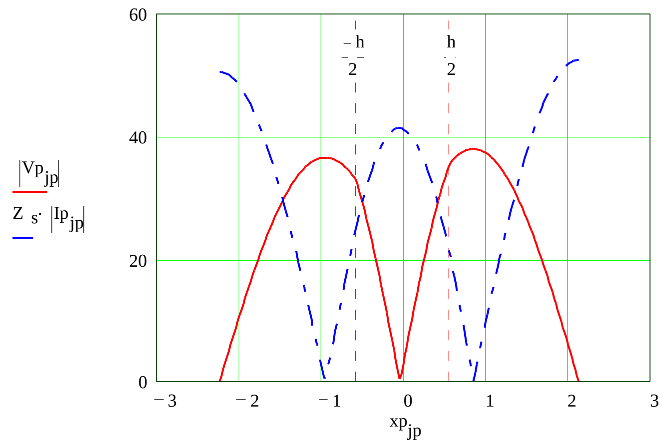
$jp1 := 1 .. 2 \cdot Nj$ $dx_{jp1} := |xp_{jp1} - xp_{jp1-1}|$ $Idiss_{jp1} := \frac{Ip_{jp1} + Ip_{jp1-1}}{2}$

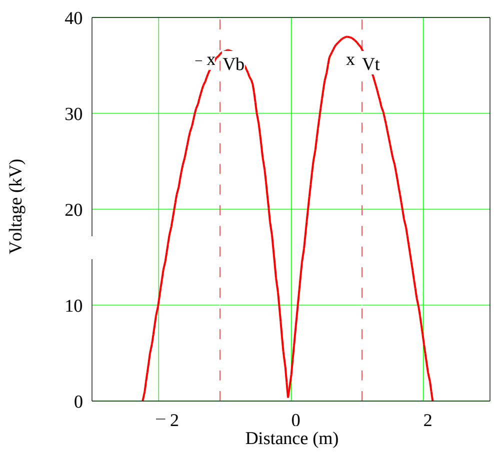
$dPdx_{jp1} := Rp_{jp1} \cdot (|Idiss_{jp1}|)^2$ $dP_{jp1} := dPdx_{jp1} \cdot dx_{jp1}$

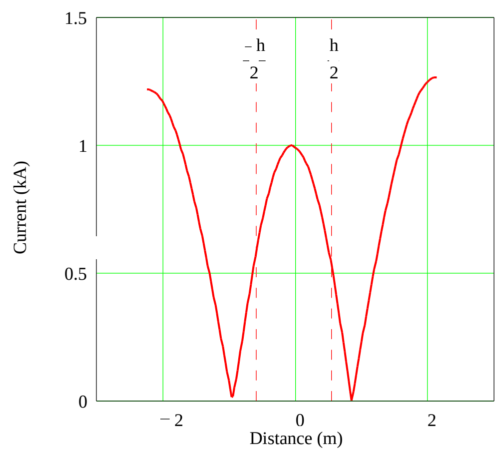
$$Pdiss := \frac{1}{2} \cdot \sum_{jp1=1}^{2 \cdot Nj} dP_{jp1}$$

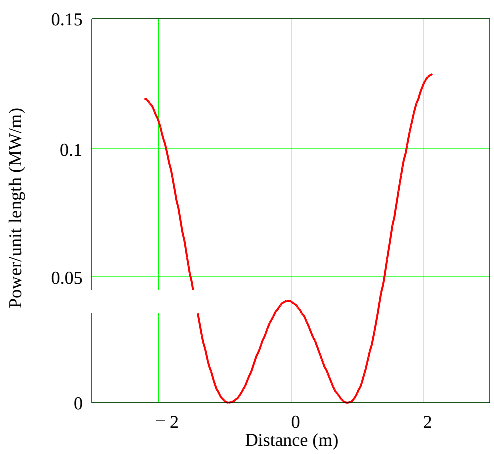
$R = 0.04$	$Z_{s1} = 26$	$fres = 58.5$	$Z_{s3} = 30$	$fc = 58.5$	$\left\lvert \frac{Vtopf(x_{Vt})}{Imaxstrap} \right\rvert = 36.6235$
$Z_s = 41.4603$	$d_{s1} = 0.297$	$\delta f = 0.18$	$d_{lf} = 0.25$	$dPhi = -4.0034$	
$h = 1.15$	$d1 = 1.2622$	$d1 + d2 = 2.6228$	$Z_{lf} = 58$	$Iratio = 0.9929$	$\left\lvert \frac{Vbotf(x_{Vt})}{Imaxstrap} \right\rvert = 36.1629$
$\beta = 0.7148$	$d2 = 1.3606$	$\rho min = 0$	$Z_{line} = 50$	$PT = 0.0973$	
$\alpha = 0.5$				$Q = 325$	$Pdiss = 0.0973$











Calc max. voltage and current as function of input power

$Vmax1kA := |max(Vp)|$ $Vmax1kA = 37.969$

$PkW1kA := 1000 \cdot PT$ $PkW1kA = 97.3284$

$Ipa := \overrightarrow{|Ip|}$ $Imax1kA := max(Ipa)$ $Imax1kA = 1.2665$

$Pp_j := \frac{j}{Nj} \cdot 400$ $Vp_j := Vmax1kA \cdot \sqrt{\frac{Pp_j}{PkW1kA}}$

$Ip_j := 1.0 \cdot \sqrt{\frac{Pp_j}{PkW1kA}}$ $Ipmax_j := Imax1kA \cdot \sqrt{\frac{Pp_j}{PkW1kA}}$