The Terminal Impedance Of An Attenuator

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Part II. Continuing the presentation of simplified methods for determining characteristics of attenuators in audio circuits.

A NOMOGRAPH was given in Part I which relates the terminal impedance of an attenuator to the terminating impedance, image impedance, and decibel loss. The nomograph was restricted in use to resistive impedances. A large class of impedances encountered in audio work, particularly those associated with electro-mechanical transducers such as speakers and phonograph pickups, vary over wide limits within their rated frequency range. A graphical means in the form of a circle diagram, is now given to assist in the calculation of an attenuator terminal impedance when the terminating impedance is appreciably reactive.

The circle diagram is applicable to the general case of finding the input impedance of any four-terminal network with specified attenuation and phase constants, and with any type of load.

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As such it may also be used to compute with good accuracy the terminal impedances of filters and equalizers.

Use Of The Chart

The chart given in *Fig.* 1 is composed of two families of orthogonal circles, one of constant magnitude and the other of constant phase. The chart is used in much the same way as a table of logarithms, the terminating impedance data being applied to the chart and values read from the circular coordinates. Operations are performed on these values, and the results are again referred to the circular coordinates. The final values of terminal impedance are then read from the real and imaginary axis of the chart.

To use the chart proceed as follows:

1. Divide the terminating impedance by the rated impedance of the attenuator, and locate the point on the chart at which these values occur.
2. Read from the circular coordinates the magnitude of the quantity W and its phase angle.
3. Divide the magnitude by W by the power ratio of the attenuator and call this W'. The power ratio of the attenuator is given by the expression *antilog db/10*.
4. Locate the point W' at the original phase angle on the chart. Read off the terminal impedance ratio from the real and imaginary axis of the chart. The numerical value of the terminal impedance can now readily be found by multiplying the ratio by the rated impedance of the attenuator.
5. If the ratio of terminating impedance to rated attenuator impedance is less than one, take the reciprocal and proceed as above. The result in step 4 will then be the ratio of attenuator impedance to terminal impedance.

Approximate Solution

When the magnitude of mismatch between the terminating impedance and the rated attenuator impedance is small, a solution by means of the chart will be subject to considerable inaccuracy, since the readability is poor in the region of unity impedance ratio. If the mismatch is no greater in magnitude than about 20 per cent a good approximation may be made by direct calculation in the following equation:

$$\frac{Z_1 Z_K - 1}{\epsilon^{2\theta}} + 1 = Z_2/Z_K$$

Z_1 = terminating impedance
Z_2 = terminal impedance
Z_K = rated impedance of the attenuator
$\epsilon^{2\theta}$ = Antilog db/10

Application

Let us consider the use of the diagram in the following example. It is well known that a beam tetrode amplifier without feedback and operating at a high level may distort badly when mismatched into a load. There are two reasons for this. First, the condition for maximum power with minimum distortion is determined by locating the load line so that it intersects the knee of the tube characteristic curve. In order to do this the load can vary only a small amount from its nominal value. Secondly, if the load is reactive, the load line becomes an eclipse which will carry operation into the nonlinear region of

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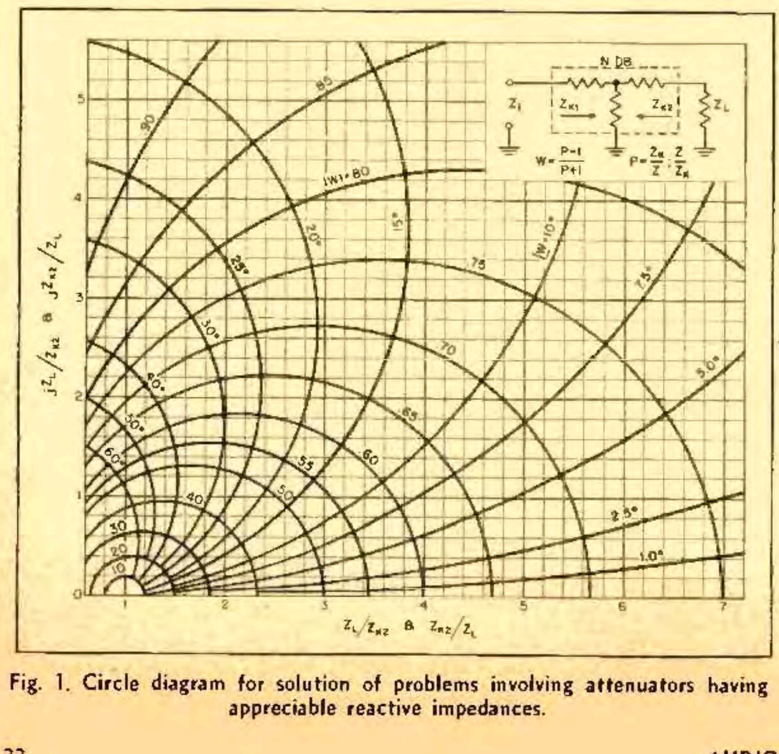

Fig. 1. Circle diagram for solution of problems involving attenuators having appreciable reactive impedances.

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the tube characteristics. This condition is sometimes corrected by placing a resistor in parallel with the load. It can be corrected equally well by using an attenuation pad between the amplifier and the load. Let it be required to find the amount of reduction of impedance variation that can be effected by means of a 3 db pad between an amplifier and a speaker load. The impedance of the speaker used rises from a nominal value of 8 ohms at 400 cps to 20 ohms with a phase angle of 45 deg. at the high-frequency end of its range. If an 8-ohm pad is used, the terminating impedance ratio is $1.77 + j1.77$. From the chart this corresponds to a value of W equal to $0.59\underline{/33^\circ}$. Dividing the magnitude by 2, the power ratio of the pad, W'' equals $0.295/33^\circ$. From the chart the terminal impedance ratio is $1.5 + j0.55$, and the terminal impedance is $12.8\underline{/20^\circ}$. This impedance more closely matches that of the amplifier, and the reactive component of the load is greatly reduced.

It is of interest to note that the diagram may be used to compute the input impedance of a wave filter if the attenuation and phase constants of the filter are known. For this calculation an additional operation is performed in step 3. The angle of the phase constant is doubled and subtracted from the angle of W' when the value of W'' is obtained.

Appendix

The expression relating the terminal impedance, terminating impedance, and attenuator constants is:

$$e^{2\theta} = \frac{Z_{K2}/Z_L - 1}{Z_{K2}/Z_L + 1} \times \frac{Z_{K1}/Z_1 + 1}{Z_{K1}/Z_1 - 1} \qquad (1)$$

Let us define a quantity, W', according to the following identity:

$$W' = \frac{P_1 - 1}{P_1 + 1}\ ;\ W'' = \frac{P_L - 1}{P_L + 1} \qquad (2)$$

Upon substituting (2) in equation (1), and using appropriate subscripts, we obtain:

$$\frac{W'}{e^{2\theta}} = W'' \qquad (3)$$

In the above equations, the quantities are defined as follows:

P_1 $= Z_1/Z_{K1}$ or Z_{K1}/Z_1
P_L $= Z_L/Z_{K2}$ or Z_{K2}/Z_L
Z_1 = terminal impedance
Z_L = terminating impedance
Z_{K1} = image impedance of the attenuator on the terminal impedance side
Z_{K2} = image impedance of the attenuator on the terminated side
$e^{2\theta}$ = antilog $db/10$, the power ratio of the attenuator

An examination of equation (3) shows each side to be of the same form, but with the left side divided by the power ratio of the attenuator. If then a graphical representation is made of W, the complex transformation of equations (2) and (4), the resulting chart may be used to solve for the impedance ratios P_r and P_L.

The representation of W on the complex P plane is, for this application, best accomplished by two families of orthogonal circles, one of constant magnitude and the other of constant phase. The equation of the circles is obtained by substituting for P in equation (2) the complex equivalent $X+iY$.

With this substitution equation (2) becomes:

$$W=\frac{(X-1)+iY}{(X+1)+iY} \qquad (4)$$

$$W^2=\frac{(X-1)^2+Y^2}{(X+1)^2+Y^2} \qquad (5)$$

Equation (5) may be arranged in the following form:

$$\left[X-\frac{1+W^2}{1-W^2}\right]^2+Y^2=\frac{4W^2}{(1-W^2)^2} \qquad (6)$$

This is the equation of the constant magnitude circles. The circles are centered on the real axis with a displacement from the imaginary axis by a distance equal to $(1+W^2)/(1-W^2)$. The radius is equal to $2W/(1-W^2)$.

By rationalizing the denominator of equation (4) we obtain:

$$Tan\,\theta=\frac{2Y}{(X^2+Y^2-1)} \qquad (7)$$

This, when rearranged as follows, gives the equation for the constant phase angle circles.

$$X^2+(Y-Cot\,\theta)^2=Csec^2\,\theta \qquad (8)$$

The centers of these circles are located on the imaginary axis and are displaced from the real axis by a distance equal to $Cot\,\theta$. The radius is equal to $Csec\,\theta$. The complete circle diagram is shown in Fig. 1.

The approximate solution results from the following consideration. If Z_R/Z_L approaches one, then Z_R/Z also approaches one, and equation (1) reduces to the expression:

$$\frac{Z_s/Z_R-1}{e^{-i\theta}}+1=Z_s/Z_R \qquad (9)$$