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# FRACTAL ANTENNAS PART 1

## *Introduction and the Fractal Quad*

The objective of an antenna is to efficiently radiate or receive a signal, presumably with significant directivity and gain. That goal has always been at odds with the physical restrictions of the design, especially at HF and MF. In effect, every antenna incorporates some compromise—playing off gain with size, efficiency with bandwidth, and so on. Through the years, the exploration of these compromises has produced some useful design solutions. However, as antenna design has become a mature field, it's rare that a new approach comes to light. Here I describe a new approach that uses fractal geometry to produce very small antennas of high efficiency, with other useful attributes. Some shrunken single-element fractal antennas appear to have gain over their classic full-sized counterparts. These antennas demonstrate that a deeper investigation of electromagnetics and antennas is necessary in the context of simple and complex fractal structures and arrays.

In Part 1, I'll present a practical design for very small area single-element cubical "quads," along with their comparative results. In Part 2, I'll elaborate upon several more examples of fractal antennas and their expected applications.

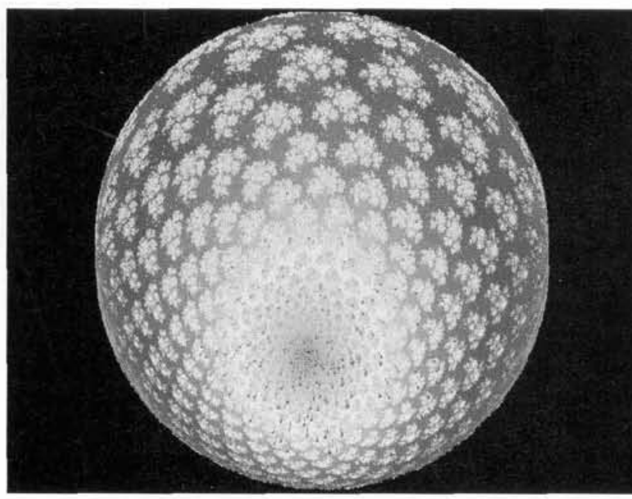

**Figure 1. A fractal taken from Walnut Creek's CD-ROM *Fractal Frenzy*, "Visions of Chaos: Volume 1," by Lee H. Skinner.**

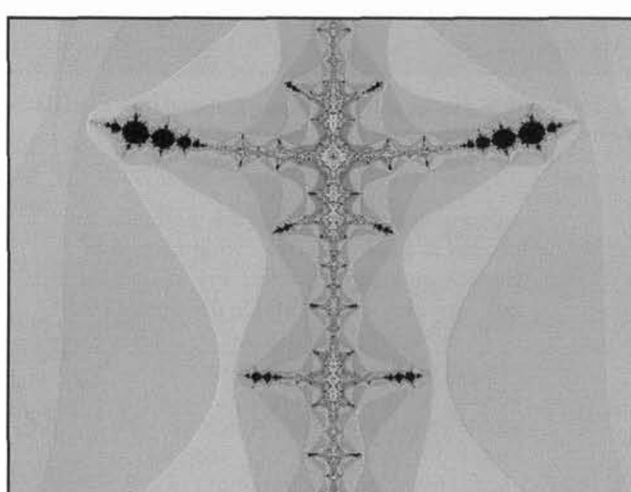

**Figure 2. A second fractal from *Fractal Frenzy*.**

### Antennas and geometry

Antenna users place great emphasis on resonance and power patterns. The philosophy has been to pick a geometric construction and explore its radiation characteristics, rather than to shape an antenna around certain radiation and/or physical characteristics. Thus structures, almost all simple in design, are made and studied. Simple is a key word here: constructions of "classic" (Euclidean) geometry have dominated

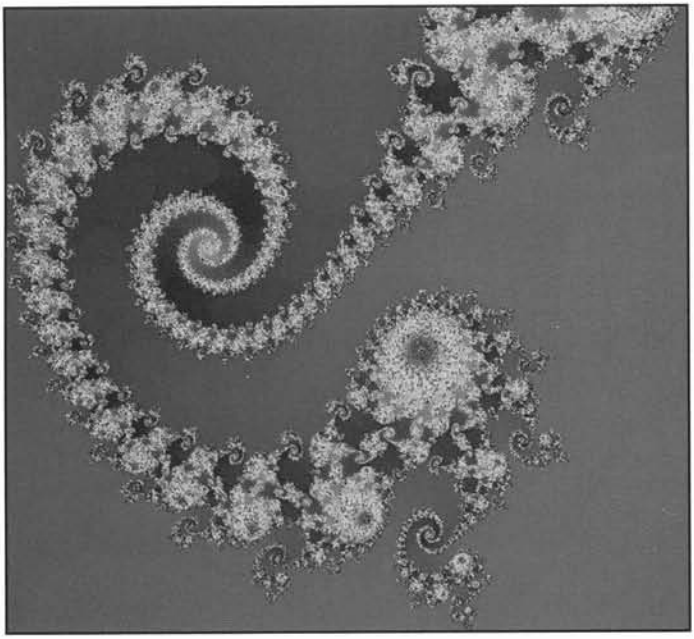

**Figure 3. Another fractal taken from *Fractal Frenzy*.**

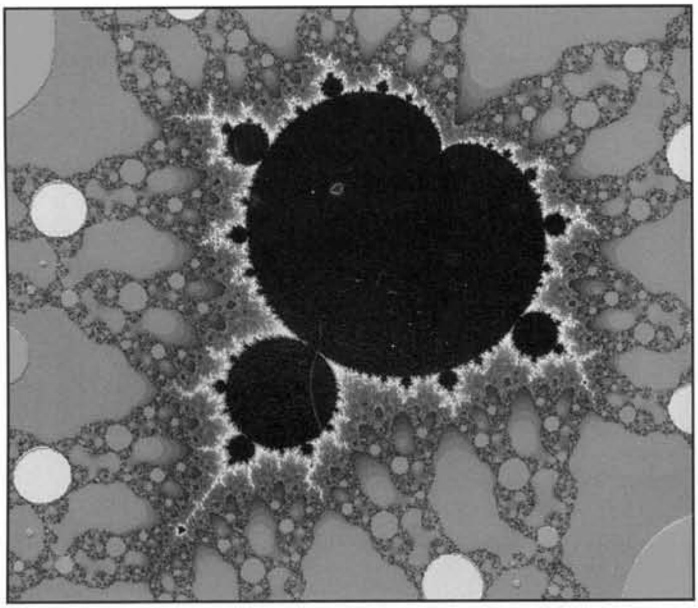

**Figure 4. The fourth in a series of fractal images found on the CD-ROM *Fractal Frenzy*, which illustrate the different types of fractals.**

antenna design. Note that the mathematics needed to explore the radiation pattern of Euclidean designs is relatively straightforward, and provides a comfortable predictive tool.

In increasing sophistication, we build antennas out of: lines, planes, circles, triangles, squares, ellipses, rectangles, hemispheres, paraboloids, and so on. Antennas look like projects from an introductory Euclidean geometry class. However, it is important to remember that Maxwell's equations, whose forms ultimately determine the radiation characteristics of the antenna, aren't limited to simple structures. It has been our desire for simplicity that has dominated antenna construction, and not the electromagnetics.

In 1985, Landstorfer and Sacher[1] published a monograph on antennas that demonstrated this point about Maxwell's equations and questioned the assumptions about simple geometry and antenna resonance. They found that if one reversed the process and looked at what shapes give dipoles and verticals higher gain, the results look far from Euclidean. Instead, randomly bent wires and waved crimps produce far better results. The clear implication is that simple geometric shapes don't necessarily produce the best antennas. Thus, there appears to be, a defined advantage to exploring non-classic geometric designs and patterns in making antennas. This provided motivation, in part, to use a branch of geometry that has been virtually unexploited in antenna design.

## Fractal geometry

Geometry branches far beyond its Euclidean, classic roots. For example differential (Riemann) geometry, a nonlinear variant, actually describes the fabric of the Universe, and it is a bent, distorted one with nary a sphere or a straight line to describe it.[2] Following in that tradition of nonlinearity, the newest branch of geometry is called fractal geometry. Here, a complex structure is built up through the repetition of a design or motif (sometimes called a generator) on a series of different size scales. Such structures are called self-similar. Mathematically, they can comprise very complex constructs, while being described by very simple equations that "nest" or iterate upon themselves. They are the mathematical equivalent of the Russian wooden dolls, each of which contains another doll, and so on.

Examples of fractals abound. Artist M.C. Escher used self-similar designs to produce many of his intriguing pictures and drawings. Benoit Mandlebrot, the scientist who defined the field, found examples of fractals over 100 years old in the mathematical literature. Their iterative feature makes them especially conducive to computation, since computers do simple math tasks over and over again very effectively. Fractals now dominate screen savers, graphic and art programs, and data compression methods. In nature, many dozens of objects, from ferns to clouds to mountain ranges have been found to be fractal. **Figures 1** through **4** are examples of several fractals.

To illustrate how a fractal is constructed we'll use the simple motif, built on a line segment, given in **Figure 5**. This is called a von Koch fractal.[3] Placing this triangle on a line segment seems simple enough; but after doing this, we can continue with another iteration and put a triangle on each one of those line segments. For the third iteration we put triangles on top of each of those line segments, and so on, ad infinitum. The result is a structure that on every scale has triangles, and looks the same at all magnifications.

One can construct a star-like structure by attaching several iterated Koch fractal ends into a closed unit, which for obvious reasons mathematicians call "islands." **Figure 6** is a Koch island star, the result of this amusing labor.

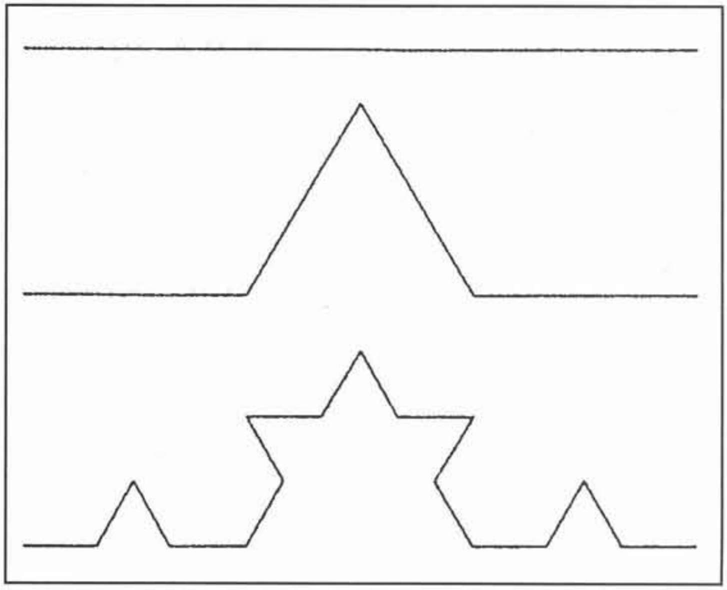

**Figure 5. A von Koch fractal for iteration 0, 1, 2, and 3.**

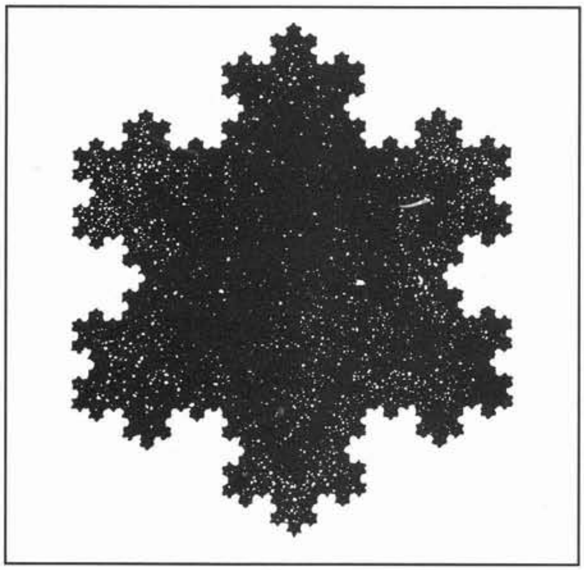

**Figure 6. A Koch Star is an enclosed island for some real stars.**

Now let's consider a strange trick of fractals. Looking at the star reveals that the *perimeter* is unrelated to the *area* of the island. In fact, as the number of iterations becomes large, the perimeter of the star has triangles on triangles down to an infinitesimal scale, and the perimeter goes on to infinity! Contrast this to a circle, square, or other closed Euclidean shape: the *area* and *perimeter* are intimately related, and it's impossible to have a large perimeter without a large area. This distinguishing characteristic of a fractal is one of many that I'll describe as "commandments" relevant to fractals and antennas. Thus we arrive at "Commandment 1" for fractals:

"Thou shall not relate perimeter as proportional to area."

An important corollary to Commandment 1 (which I'll call Commandment 2) occurs when the argument is reversed:

" For a given perimeter, the enclosed area of a multi-iteration fractal island will always be smaller or equal to that of any Euclidean Island."

Both of these Commandments are key to the use of fractals as antennas.

An additional fractal attribute is that it has a dimension, much like a Euclidean structure. However, mathematicians characterize their fractals by the "fractal dimension."[4] This is given by a simple ratio (actually a limit to a ratio), representing the one-dimensional initial length of a structure compared to its length in one dimension after the fractal is applied and is obtained by:

$$D = \log(L)/\log(r) \qquad \textbf{(1)}$$

where L is the total length before applying the motif and r is the one-dimensional length after applying it. To simplify: if you take a line or curve and scrunch it by shaping it into a given fractal, it will be compressed in the dimension you measured it in at first. It will be "folded" or "accordioned," and D gives you a measure of this. We'll see that fractal antennas, unlike mathematical fractals, are *not* characterized solely by D. Hence, the fractal dimension is a useful measure of the compactness of a fractal, but not of a fractal antenna.

One final aspect to note in this brief description of fractals is that they break down into two general types: *deterministic* and *chaotic* fractals. Deterministic fractals are those where the motif replicates at the 100 percent level on all size scales, while chaotic fractals have a random, noise component thrown in. Thus, while the Koch star is a good example of a deterministic fractal, a good example of a chaotic one

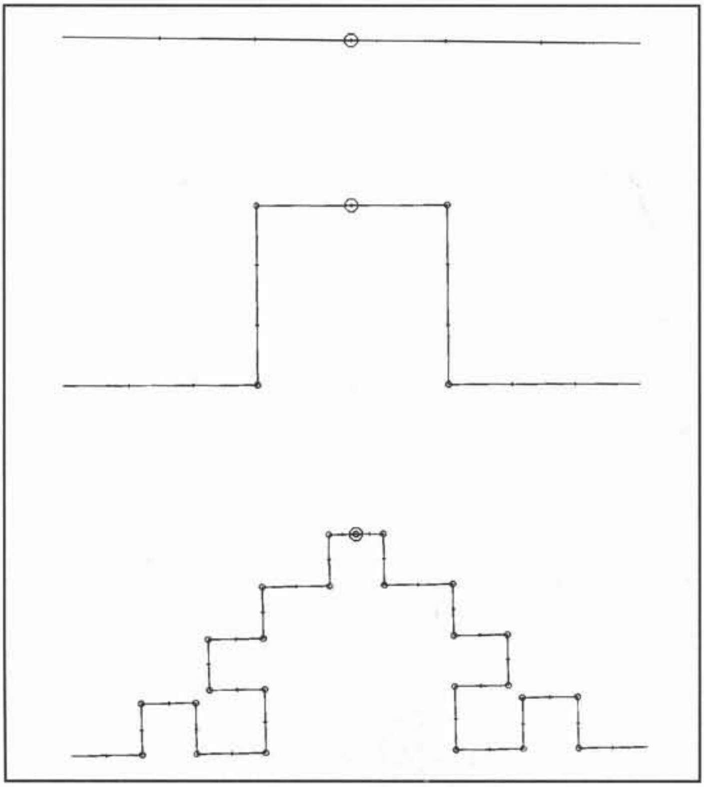

**Figure 7. Minkowski (box) fractal for iterations 0, 1, and 2.**

would be the fluctuating price of the Dow Jones Industrial Average, or a drunkard's "random walk." There's an error of uncertainty in the chaotic fractal that can make it wildly changeable in its shape as it's constructed on different scales.

## Fractal antenna prehistory

Why should antennas—and more generally resonant structures—be limited to Euclidean designs? Oddly, this question has seldom been posed, nor has its answer been found. Certainly, as an example, one could work very hard to build a drum shaped like the Koch star—and then see if beating on it produced sounds. You could then relate its resonance nodes to those of a circular drum. Does it "beat?" Is it louder or softer than the round drum? How are its pitches of resonance related? It's an interesting question; however, consider the work needed to make that star-shaped drum, with its huge perimeter.

Mathematically, the same problem arises. M.V. Berry[5] considered the calculation of fractal resonances so difficult that he attacked a similar, but simpler, problem—how fractals change waves that hit them. This process of diffraction (and what Berry called "diffractals") upon a fractal structure *does* bend waves, much as a Euclidean shape does. In the late 1980s, this simulation manifested as the first fractal product: a series of block sound diffusers for recording studios and auditoriums.[4]

In the 1960s, a European ham made the first clear use of self-similarity for an antenna array. I've been unable to find the article that describes his work, but my memory (from age 12) clearly recalls a huge 2-meter "quad," where each end of the spreaders supported another quad, which supported another quad, for a total of at least 48 quad elements. There was no electromagnetic motivation in choosing one geometry over another, simply a desire to find an efficient way of distributing the weight of all these antennas. Any way it's analyzed, this was an array that was fractally filled—and huge!

D.L. Jaggard was the first to deliberately apply the concept of fractals to antennas.[6] His objective, like that of the anonymous ham mentioned above, was to see if one could use fractals to spread out elements in a sparse microwave array (in synthetic aperture radar, for example). This means that the sidelobes of the array could be kept small without filling the entire array up with elements. His results were promising, but not necessarily better than other techniques, such as a totally random spreading of elements. Jaggard didn't apply the fractal condition to the elements themselves; his array wasn't any smaller, only differently patterned. No attempt was made to minimize the total area of the array.

Perhaps this wasn't too surprising. Fractals had been used before for purposes that exclude shrinking down the antenna size. We now know, for example, that a spiral is a type of continuous, deterministic fractal, whose motif keeps expanding continuously as the distance increases from a central point. Cones, Vs, and so on, meet the same criteria. A log-periodic antenna is a type of continuous fractal (see second sidebar) because it's built from a radially expanding structure. Its broadbandedness is a result of its continuous expansion and therefore its fractality. However, a log periodic isn't necessarily smaller than a Yagi of similar gain. Landstorfer and Sacher used another unintentional fractal approach (see third sidebar). By letting dipoles and verticals distort in shape to produce higher gain, they obtained some bizarre designs. Most notably, the optimum shape they found for their vertical antenna resembles what happens to towers that snap their guys in a tornado. Such shapes are defined as "Brownian fractals,"[4] a specific form of chaotic fractal. Here, finally, is a resonant fractal that actually affords higher gain than a straight-line design (its Euclidean counterpart).

Of course, fractals have been used, unintentionally, to shrink antennas down in size. Horn antennas use "double ridges" to decrease the resonant frequency.[7] A variety of shrunken quads (the Maltese quad, for example), delta loops, and other useful antennas all incorporate some type of loading using rectangles, boxes, and triangles to shorten the element dimensions. Pfeiffer brought this to a logical limit recently with a fan-shaped quad design of unreported gain and impedance.[8] Yet all of these designs use a fractal motif of the *first* iteration; they basically load Euclidean structures with another Euclidean structure in a repetitive fashion, using the same size in the repetition. They do not exploit the multiple scale self-similarity of real fractals. They could just be an exotic loading scheme and, therefore, of no particular importance when compared to other (lossy) methods of shrinking antennas, such as loading with LC circuits, capacitive hats, dielectrics, and so on. *Thus, there has never been a convincing case of a deliberately multi-iterative single-element fractal structure—that is, a structure immediately and uniquely identifiable as a fractal—being resonant, nor has any such structure been explored from the point of view of impedance, gain, frequency resonance nodes, shrinking ability, and other radiation characteristics.* It may be a quirk of these previous structures that allows them to radiate, as opposed to a useful applied property of fractals. Which is it?

## Resonant fractal structures

My objective in studying fractals as antennas was three-fold. First, I wanted to show that wire, slot, and other types of antennas shaped like multi-iteration fractals resonate. Second, I wanted to show that these structures, or at least a class of these structures, radiate. Third, I wanted to identify which fractals will shrink the size of an antenna and determine by how much. To a certain degree, this invites an infinitely difficult task. Fractals, unlike Euclidean structures, come in an infinite variety. No one will ever explore every fractal from the point of view of radiation characteristics. The hope of finding the "ultimate" fractal antenna is also a false one, just because of this complexity. It seemed most prudent to take a variety of multi-iteration fractals of simple and/or well-known motifs and investigate these.

My initial goal was simple (albeit tedious). I built several fractals of wire and/or made them into slot antennas, attached to 50-ohm coax, and explored them with an SWR/resistance analyzer. Surprisingly, I found that virtually all of the fractals resonate in at least one position of the feedpoint. All of the multi-iteration fractals resonate at multiple frequencies, in a non-harmonic fashion.

| Fractal | Maximum Iteration |
|---|---|
| Koch | 5 |
| Torn Square | 4 |
| Minkowski | 3 |
| Mandlebrot | 4 |
| Caley Tree | 4 |
| Monkey's Swing | 3 |
| Sierpinski Gasket | 3 |
| Cantor Gasket | 3 |

**Table 1. Some resonant fractals.**

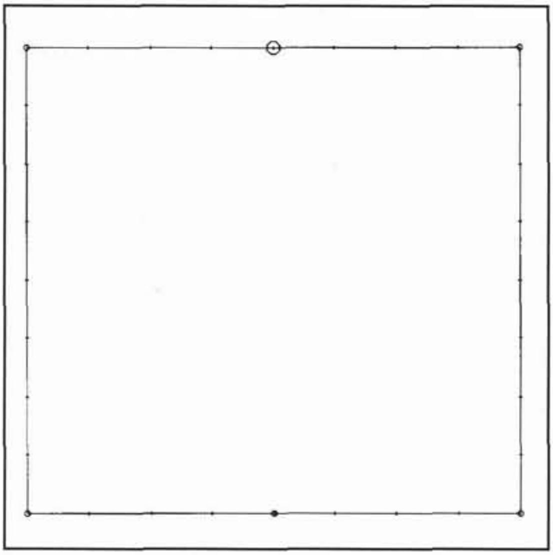

**Figure 8. Zero iteration Minkowski Island—a quad.**

For the sake of this analysis, resonance was defined as a total impedance that fell between 20 and 200 ohms, with medium, to high Q. In other words, the fractal had to manifest a radiative impedance that was frequency dependent. Ohmic resistances were measured to be a few ohms or less, confirming that the resonance nodes were not of a "dummy load." Of course, one could define resonance with an arbitrary impedance (75 or 600 ohms, for example). The consideration was to look for something that would be easy to match to 50-ohm coax. **Table 1** lists several well-known fractal types, that were found to resonate on at least one frequency, among many others.

There were two outstanding characteristics of fractals that emerged from my work. First, I found that fractals, when shaped into islands, *do not* experience significant drops in radiation resistance for their size (area). Many of these fractals were quite tiny (less than a foot across), but resonated at 60 to 100 MHz. Point 3 (shrinkage) was justly demonstrated. I'll discuss this paradox of a medium impedance in context to a small loop later.

# The First Fractal Antenna

In 1987, I gave a talk at a scientific meeting in Hungary. My topic was something exotic (gravity lenses), but the *most* exotic subject discussed was fractal geometry, presented by Benoit Mandlebrot. At that time fractals were still considered weird and only slowly gaining their now well-established notoriety. No one regarded them as a practical engineering tool (in fact, fractal antennas may be the first true example of "fractal engineering"); but, they were certainly pretty to look at.

A year later, I got back on 2-meter FM and wanted a continuous link to my DX packet-cluster to my 1-watt handie-talkie. K1EA was 35 miles away from my downtown Boston apartment, so I needed a reasonable antenna (no duckies) that would be augmented by my 200+ foot elevation. More importantly, the antenna could not attract my landlord and his grotesque obsession with the "no antenna clause" in my lease.

## Why not a fractal?

Using glue, construction paper, and aluminum foil, I made a complex-looking fractal microstrip antenna, with a "pagoda" motif. The experience was not unlike cutting out a paper doll (or so I imagine). The antenna was about six inches on a side. Lacking even a SWR meter, I had no idea if it would resonate. However, stuck to the railing of my 27th floor patio, it got into K1EA full-quieting. Perhaps a 1/4-wave vertical would have done much better; I had no way of knowing. Certainly, I had no theoretical motivation to assume it would resonate. It didn't seem very revolutionary at the time. If anything, it seemed an amusing way of beating a bad rule.

One winter day, I noticed that the PC was disconnected from packet. The antenna had been cut squarely from the thin coax, as if it had never been there. While I was at work, lecturing my students in continuous math, my landlord had cut the discrete math construction down. All I can say is he must have excellent eyesight to see the tiny patch on the balcony. However, at least he was convinced that this doily-like square was an antenna.

A week later, by chance, I discovered the remnant of the little antenna near the tennis courts below. It was a snow-soaked piece of detritus with little bits of aluminum foil still glued to it. A sad fate for a new idea.

Like all new ideas, this false start took some time to bear fruit. First, I had to get some real estate, some precision attenuators, an SWR analyzer, ELNEC, and some free time. I also needed some reassurance that weirdly shaped antennas were possible. In 1990, Landstorfer and Sachers' 1985 monograph on shaped antennas convinced me that perhaps we knew less about antenna design than we thought we did. Fractal antennas continue to point out this notion.

I also noted that the perimeters *do not* correspond to the lengths expected from the measured resonant frequencies. Instead, the lengths were *always longer*. There appears to be some effective velocity change of the wave caused by the fractality of the antenna. Pfeiffer[8] also noted this increased length when he shrank a full-sized quad using a first-order fractal. However, this effect is a property of fractals as radiators and not as geometric constructions. It seems that the fractal dimension, which otherwise might indicate a sense of how the shrunken the "side" of a fractal becomes (as with Commandment 2), isn't a good indicator of how much smaller an antenna may be made by using a fractal design, because it doesn't incorporate the perimeter lengthening of the radiator.

Instead, I define a new quantity called "perimeter compression," or PC. PC is, literally, a ratio of a full-sized antenna's side to that of the shrunken version's side. If you measure the physical size and compare it to the size expected for the fractal resonator's lowest resonant frequency, you'll measure the PC. A PC of 1, for example, represents the full-sized antenna, while a PC of 3 represents one shrunken by a factor of three on a side. It may be empirically represented by the fractal dimension with the equation:

$$PC = A \log[N (D+C)] \qquad \textbf{(2)}$$

where A and C are constant coefficients for that given fractal motif, N is the iteration number, and D is the fractal dimension. Thus, the PC becomes asymptotic to a real number for each fractal and *does not approach infinity* as the number of iterations becomes very large. This result is not a representation of a purely geometric fractal. Hence another commandment, Commandment 3, comes to light:

"The PC of a fractal radiator will asymptotically approach a noninfinite limit in a finite number of fractal iterations."

This immediately helps define which fractals are better resonators than others through Commandment 4:

"Optimized fractal antennas approach their asymptotic PCs in fewer iterations than nonoptimized ones."

In other words, the "best" fractals for antennas have large values of A and C. They shrink the most and the fastest.

Finally, the odd property of non-small radiation resistance invites yet another commandment, Commandment 5:

" The radiation resistance of a fractal antenna drops as some small power of the perimeter compression, PC. A fractal island always has a substantially higher radiation resistance than a small Euclidean loop of equal size."

Also a Commandment 6:

"The number of resonant nodes of a fractal island increases with the iteration number and is always equal to or greater than the number of resonant nodes of a Euclidean Island with the same area."

Finally, the change in perimeter leads to Commandment 7:

" A fractal resonator has an increased effective wavelength."

Because several of the attributes of fractal resonators conflict with accepted behaviors of antennas, I decided to illustrate a specific fractal antenna and elaborate upon both the conflict and the true fractal antenna characteristics. I compared Euclidean and fractal quad loop(s) to demonstrate my points.

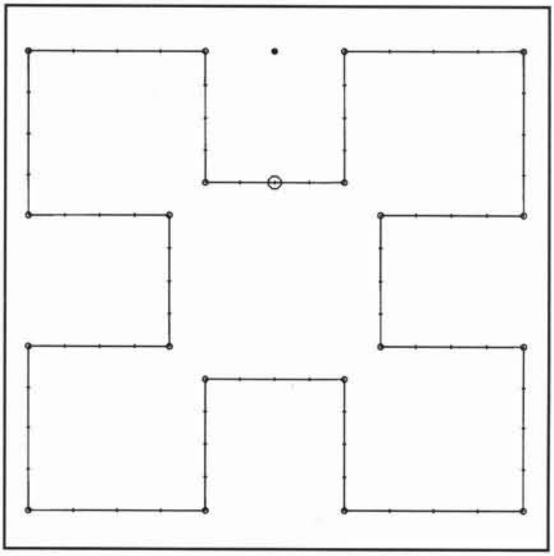

**Figure 9. First iteration Minkowski Island (MI1).**

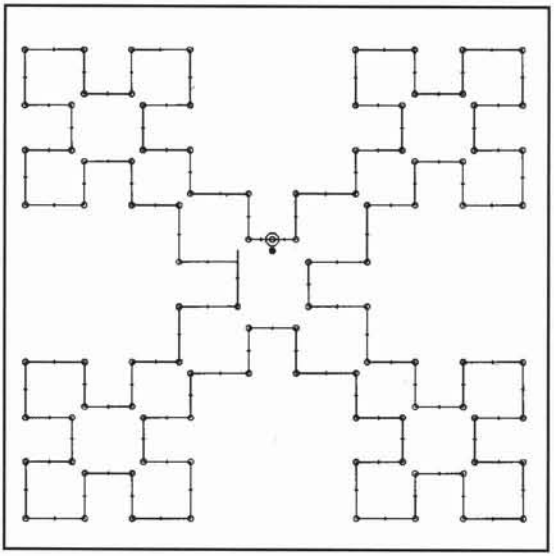

**Figure 10. Second iteration Minkowski Island (MI2).**

## The problem of loop antennas

Most of the fractal resonators I tested were shaped into closed loops (islands). My previous knowledge of these loops as antennas indicated that they shouldn't have produced resonance points at all.

It has long been known that small antennas don't work well. A compelling reason for this is that their radiation resistance, R, drops dramatically when shortened. A short dipole or tiny loop will experience the radiation pattern of a 1/2 wave and quad, respectively, if the impedance isn't swamped by the ohmic losses. Minimization of these losses is very difficult and requires the use of expensive matching networks. Also, these matched mini-loops are inherently very high Q (greater than 50). Commercial versions of the G3KPV (and others) loop[9] are limited examples of tiny loops of minimal ohmic losses, allowing great efficiency (50 to 85 percent). While these antennas have found a limited niche, they don't apply to the vast majority of antenna applications, because of their cost, complexity, and very limited effective radiated power.

While the trend for dramatically reducing radiation resistances with small area Euclidean loops is well-known experimentally, few realize the theoretical basis for it. Foster conducted the analysis in 1944 and it has been used as a basis by Kraus.[10] Its generalizations have become "laws" by which all small antennas designs have been defined.

Foster and Kraus looked at a circular loop with uniform current. This simple structure afforded simple solutions to radiation equa-

| Antenna | Res. Freq. (MHz) | Gain (dBi) | SWR (:1) | PC (for first) | Direction |
|---|---|---|---|---|---|
| **Ref. Quad** | 76 | 3.3 | 2.5 | 1 | Broadside |
| | 144 | 2.8 | 5.3 | — | Endfire |
| | 220 | 3.1 | 5.2 | — | Endfire |
| | 294 | 5.4 | 4.5 | — | Endfire |
| **MI1** | 55 | 2.6 | 1.1 | 1.38 | Broadside |
| | 101 | 3.7 | 1.4 | — | Endfire |
| | 142 | 3.5 | 5.5 | — | Endfire |
| | 198 | 2.7 | 3.3 | — | Broadside |
| **MI2** | 42.3 | 2.1 | 1.5 | 1.79 | Broadside |
| | 85.5 | 4.3 | 1.8 | — | Endfire |
| | 102 | 2.7 | 4.0 | — | Endfire |
| | 116 | 1.4 | 5.4 | — | Broadside |

**Table 2. ELNEC derived resonances for Minkowski Island Quads.**

| Antenna | Ratios |
|---|---|
| Ref. Quad | 1: 1.89: 2.89: 3.86 |
| MI1 | 1: 1.83: 2.58: 3.6 |
| MI2 | 1: 2.02: 2.41: 2.74 |

**Table 3. Ratio of resonant frequencies from ELNEC.**

tions. The gain of such a single loop makes a surprising limit of 1.8 dB over an isotropic radiator, as the area falls below that of a loop of 1 wavelength-squared aperture. However, the radiation resistance is very small for a small-area loop ($A<\lambda^2/100$), and is given by:

$$R = K(A/\lambda^2)^2 \quad (3)$$

where K is a constant, A is the loop's enclosed area, and $\lambda$ is wavelength. The radiation resistance can easily be less than an ohm for a small loop antenna.

While this analysis is, strictly speaking, for circular loops, Kraus made two generalizations: 1) the calculations can be defined by area, rather than perimeter and; 2) the analysis is correct for *any* geometric shape for small loops. Hence, we have the *predicted* death knell for small loops of other shapes—including fractal ones—because these antennas will supposedly have such a small radiation resistance that their ohmic resistances will be larger, and the efficiency will be very small. A bizarre spin-off of this analysis is that some have suggested building small antennas out of superconducting material, to drop the ohmic losses to zero.

Fractal antennas, however, don't obey these generalizations of small loops. To demonstrate this, and explore a useful fractal antenna, I'll describe an in-depth summary of work on simple, but real fractal antennas.

## Practical fractal: The Minkowski Island

Compelling evidence for radiation from a fractal antenna must start with a simple fractal motif and demonstrate the effects caused by successive iterations. This is actually a philosophy of approach dictated by Occam's Razor. Here, simplicity will make the case for a fractal antenna. I've chosen the Minkowski motif for this reason. It's also believed that exact analytical solutions for Maxwell's equations will be derived easily for this fractal motif, and, as we shall see, its orthogonal line segment design made it readily acceptable to numerical study with ELNEC and other modeling schemes.

The Minkowski motif I've chosen is a three-sided box placed on top of a line segment.[11]

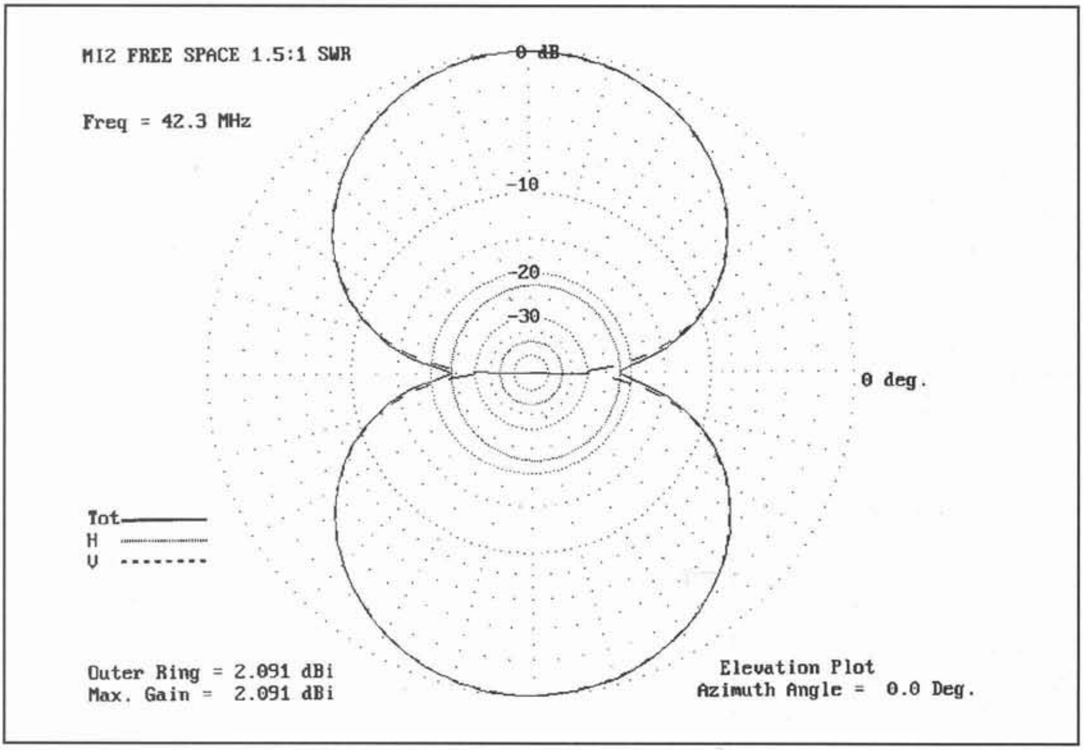


**Figure 11. Free-space pattern for MI2 from ELNEC. This pattern is very similar to that of a quad, except with 2 dB less front to side and 1 dB less forward gain; a little lower than measurements show.**

The box sides can be any arbitrary length; for the sake of this example, I'll use side lengths giving the box a height and width of 2, and the two remaining sides a length of 3. This Minkowski fractal has a fractal dimension of 1.2—not very high compared to other deterministic fractals, but useful in making a case for a simple fractal antenna.

Application of the motif to the line segment can be most simply expressed by the piecewise function:

$$f(x) = \begin{cases} 0 & 0 \geq x \geq 3x_{max}/8 \\ 1/4x_{max} & 3x_{max}/8 \geq x \geq 5x_{max}/8 \\ 0 & 5x_{max}/8 \geq x \geq x_{max} \end{cases} \quad \textbf{(4)}$$

where $x_{max}$ is the largest continuous value of x on that line segment. The second iteration can be expressed as relative to the first by:

$$f(x)_2 = f(x)_1 + f(x) \quad \textbf{(5)}$$

where $x_{max}$ is defined by **Equation 4**. Note that each separate horizontal line segment has a different lower value of x and $x_{max}$. Relevant offsets from zero may be entered as needed. The vertical segments may be boxed by rotating them 90° and applying the methodology above. Further iterations extend from **Equation 5**.

The Minkowski fractal quickly begins to look like a Moorish design pattern (see **Figure 7**). However, each successive iteration eats up more and more perimeter, thus squashing the overall length of the original line segment. Combining four of these fractals of the same

**Photo A. A picture of MI3 for 2 meters—about 8 inches on a side. Now to stick it out the window!**

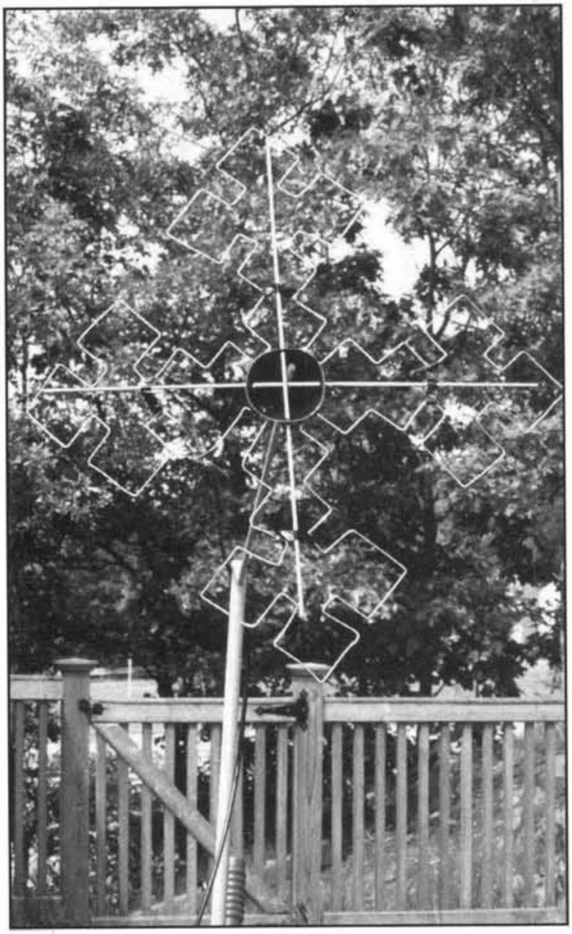

**Photo B. A picture of MI2 for 6 meters—about 30 inches on a side. Ready for an efficiency measurement. Note easy support to spreaders at many points.**

iteration number makes a Minkowski fractal island and a bona fide, "fractalized" cubical quad that can be built, tested, and simulated.

## Minkowski Island Quads: ELNEC

ELNEC is a graphics/PC version of MININEC, which is itself a PC version of NEC. It's a numerical tool using the method of moments for estimating power patterns, and has demonstrated success with simple wire geometries. Because ELNEC wasn't designed for complex antenna geometries, per se, I used it here only as a guide to far-field power patterns, resonant frequencies, and SWRs of Minkowski Island fractal antennas up to iteration 2. Since I ran out of line segments and pulses after a second iteration Minkowski fractal, a NEC-based comparison isn't available for the third or higher iteration at this time. Because the emphasis in this study was to demonstrate *working* fractal antennas, ELNEC results are viewed here as a general corroboration. Note that measurements of the Minkowski fractal antennas were in hand before ELNEC models were run.

I ran ELNEC in a straightforward fashion. One source was attached and each line segment was divided into at least two pulses. Higher pulse densities couldn't be achieved for the second iteration Minkowski fractal because of memory and software limitations. The goal was to keep the pulse densities consistent among the various iterations. So, while a second iteration Minkowski Island had only two pulses per segment, each segment occupied less than 2 percent of a wavelength. The number of total pulses was thus very high. In **Figures 8** through **10**, I show the antenna designs used in ELNEC to derive the lowest resonances and power patterns, up to and including iteration 2. All designs were constructed on the x,y axis. For convenience, each iteration will be designated MI (for Minkowski Island) followed by an iteration number; MI1, for example (Minkowski Island iteration 1). Also, the outer length was kept the same (42 inches) for each iteration. Thus, the frequency of lowest resonance *decreased* with the MI models compared to the quad. Put the opposite way, for a particular frequency of resonance, the quad size shrinks with the MI antennas.

The results of ELNEC demonstrate that far-field patterns of Minkowski Island Quads differ little from cubical quads, at their lowest resonant frequency. **Table 2** summarizes the radiation characteristics for each iteration, for the first four resonances. **Figure 11** contains representative free-space power patterns of MI fractals, in this case for MI2's lowest resonance.

**Table 2** reveals that Minkowski Islands are multi-resonant structures with virtually the same gain as full-sized quads.

Although I must caution that these are "free-space" values of gain, in the absence of any ground plane, the simulations over a perfect ground at 1 wavelength provided similar relative gain results. Keep in mind that discrepancies in frequencies and SWRs—as well as gain—were found when comparing the actual measurements. Roundoff, undersampling of pulses, and any of a variety of other reasons may contribute to the inaccuracy of the ELNEC results. Roughly, they are a good guide to demonstrate the fractal antenna phenomenon. You should also keep in mind the ratio of resonant frequencies, which for these first four resonance nodes are given in **Table 3**.

Thus, ELNEC *confirms* two commandments: the shrinking of the antenna and an increase in its number of resonance points. Indeed, MI2 had four resonance nodes before the reference quad experienced its second resonance!

The near-fields of these antennas are also

| Antenna | PC | PL | SWR | Corrected Gain (dB) | Side Length (wave) |
|---|---|---|---|---|---|
| Quad | 1 | 1 | 2.4:1 | 0 | 0.25 |
| 1/4 wave | 1 | — | 1.5:1 | –1.5 | 0.25 |
| MI1 | 1.3 | 1.2 | 1.3:1 | 0.5 | 0.2 |
| MI2 | 1.9 | 1.4 | 1.3:1 | 1.5 | 0.13 |
| MI3 | 2.4 | 1.7 | 1:1 | –1.2 | 0.10 |

**Table 4. 2-meter antenna measurements.**

very important, for it is the near fields by which multiple element antennas add to achieve high gain arrays. ELNEC was of little help in this regard because of the aforementioned programming limitations. However, I have built several different high-gain fractal arrays that exploit the near field and will discuss them in Part 2.

## Minkowski Island Quads: measurements

I built three Minkowski Island fractal antennas to study the gain of the Minkowski motif. These antennas were made of aluminum groundwire (no. 8) and/or thinner galvanized groundwire (no. 12). The antennas were cut for 2 meters to facilitate relative gain measurements of 2-meter FM repeaters. All measurements were performed in receive mode.

All of the antennas were cut so their lowest operating frequency fell close to the operating frequency. They were mounted for vertical polarization and placed so their center points were the highest practical point above the mounting platform (see **Figure 12**). I also built a vertical ground plane with three radials, and a reference quad, for gain comparisons, of the same size wire as the particular fractal being tested.

Few radio amateurs appreciate the difficulties of measuring gain properly. For example, there is virtually no way of avoiding multi-path reception. I chose my location specifically to minimize the effect from skewed angles. Located in Belmont, Massachusetts, on top of the highest hill near Boston, I had a clear line of sight to the Derry repeater K1MNS (146.85 MHz) in southern New Hampshire, about 45 miles. Still, multipath was evident in the form of "stationary picket fencing" with the gain changing as a function of vertical and horizontal position. By placing a vertical ground plane (1/4 wave) on a pole, I was able to locate positions for the measurements where these fluctuations were changing more slowly.

I experienced great difficulty making measurements with a low (less than 1 wavelength) test platform above a ground plane, on the ground. Measurements were especially sensitive to height and position above the plane (by values of 0.1 wavelengths). Indeed, because the bottoms of the fractal antennas were higher than that of the reference quad, there were clearly height-dependent effects being manifested, perhaps (favorably) biasing the fractal antennas in some way. Instead, I opted to remove the low-height effects by mounting the test platform at the edge of a third-story window, affording 3.5 wavelength height above ground and line of sight to the Derry repeater. This more closely replicates the results for free space. I stuck the antennas out the window about 0.8 wavelengths away from the house and any metallic material (such as window frames). I performed measurements on 5 different occasions, from various windows on the same floor, obtaining results within 1/2 dB of each trial. On two occasions, I made confirming measurements using the Waltham, Massachusetts repeater (146.64).

This type of procedure, and its caveats, is discussed at length in Kraus.[7] It will definitely be worthwhile to retest these antennas in an anechoic chamber, should one ever be available. However, real communication requires real results, which these findings present.

I attached each antenna to a short piece of 9913 coax (50 ohms) fed at a right angle to the antenna. I used two precision attenuators in the circuit: 1) 6 dB steps from 0 to 30 dB and, 2) 1 dB steps from 0 to 10 dB. Then I fed the signal with more 9913 to a Drake TR-22C transceiver. I tapped the meter circuit from the receiver and attached it to a VOM reading as an ammeter.

S-meter measurements are notoriously inac-

| Antenna | SWR | Z(ohms) | O(ohms) | E (%) |
|---|---|---|---|---|
| MI2 | 1.2:1 | 60 | ≤5 | ≥92 |
| MI3 | 1.1:1 | 55 | ≤5 | ≥91 |
| Quad | 2.4:1 | 120 | 5–10 | 92–96 |

Table 5. MI2, MI3 efficiencies.

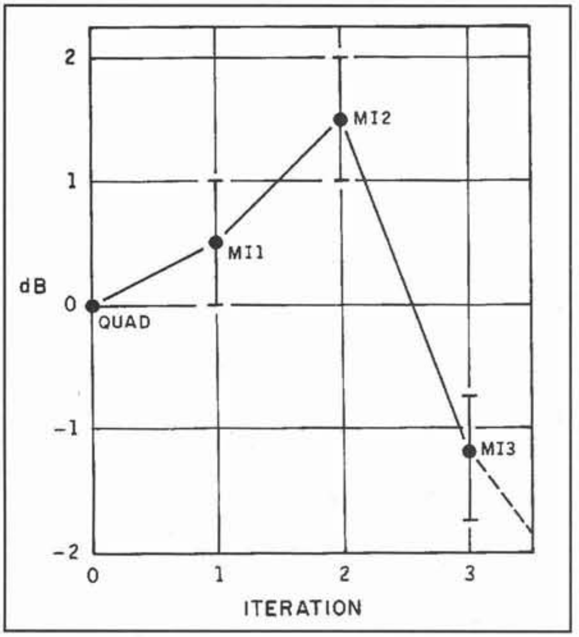


Figure 12. Strange gain changes with iteration for lowest resonance of the MI fractal antennas. This may be a diffraction phenomenon and is not due to losses from inefficiency.

curate. The problem can be especially prevalent on 2-meter FM, because the signal saturates the meter as full quieting is achieved. Even when signals aren't full quieting, the S-meter readings are highly nonlinear. I avoided these problems by inserting some initial attenuation with the rotary attenuators. Then I quickly switched in each test antenna for an ammeter measurement, and added or subtracted attenuation to obtain the same meter reading found with the reference quad. Every test antenna measurement had a successive comparison to the reference quad. I corrected all readings for SWR attenuation, which was 2.4:1 (120 ohms impedance) for the reference quad, but less than 1.5:1 at resonance for the fractal quads. I made no attempt to measure the efficiency of these particular antennas, because of the unavailability of a noise bridge that worked on 2 meters.

With this process, 1/2 dB differences produced quite noticeable meter deflections, while several dB differences produced substantial ones. Removal of the antenna demonstrated in excess of a 20 dB drop in received signal. In this way, I was able to cancel out systemic distortions in my readings and compare the results in a meaningful way.

For each antenna, I measured relative forward gain and optimized physical orientation. I made no attempt to correct for launch-angle, nor to measure power patterns other than to demonstrate the broadside nature of the gain. The results of the measurements are summarized in **Table 4**. Note that the SWRs at the resonant frequency weren't corrected with a balun, but gain measurement values were corrected for the loss. All dB values have a conservative uncertainty of 0.5 dB RMS. Perimeter compression, PC, indicates the amount of side "squashing," while PL represents the perimeter length normalized to a full-sized cubical quad (1/4 wave on a side). MI3, as shown by the side length, is 1/10 of a wavelength on a side.

## Discussion

There are clearly some surprising and important results shown in **Table 4**. First, note that for the vertical configuration being tested, the fractal quad versions either exceeded the gain of the cubical quad or fell within about 1 dB of it. This points out a key fact: a cubical (square) quad is not an optimized antenna for gain. In other words, you can get higher gain out of a quad by fractally shrinking it by a factor of two. If you need to shrink it further, you may still experience marginal (1 to 2 dB) losses. These results may actually be more favorable as other fractal motifs are explored for quads. Yet even MI3 generates an impressive quad. A 20-meter version about 7.5 feet

| Antenna | Freq. (MHz) | Freq. Ratio | SWR | 3:1 Bandwidth | Q |
|---|---|---|---|---|---|
| **MI3** | 53.0 | 1 | 1:1 | 6.4 | 8.3 |
| | 80.1 | 1.5:1 | 1.1:1 | 4.5 | 17.8 |
| | 121.0 | 2.3:1 | 2.4:1 | 6.8 | 17.7 |
| **MI2** | 54.0 | 1 | 1:1 | 3.6 | 15.0 |
| | 95.8 | 1.8:1 | 1.1:1 | 7.3 | 13.1 |
| | 126.5 | 2.3:1 | 2.4:1 | 9.4 | 13.4 |

Table 6. MI2, MI3 resonances and bandwidths.

on a side will be discussed in Part 2.

At this point, it's important to know the efficiencies of MI2 and MI3, because these compact designs offer real advantages over a full-sized quad element. As I stated, it wasn't possible to attempt this for the 2-meter versions, but I built and tested 6-meter versions of MI2 and MI3 (see **Figure 13**). I attached an RX-noise bridge between these antennas and a JST 245 transceiver. By nulling the receiver at about 54 MHz, and calibrating the 50-ohm resistance bridge with 5 and 10-ohm resistors, I obtained the results of **Table 5**.

Efficiency was defined as:

$$E = R/(Z) \times 100\% \qquad \textbf{(6)}$$

where Z is the measured impedance. R was obtained by subtracting the ohmic and reactive impedances (O) from the measured impedance.

Apparently, with their 1.2:1 SWRs and low ohmic and reactive impedances, these antennas are extremely efficient—*in excess of 90 percent*. Again, this goes against common sense arguments inculcated through the modeling of small loop antennas. In fact, this is compelling evidence that the "law" of low radiation impedances for small loops must be abandoned and only be invoked when discussing small Euclidean loops. It also advances the exciting case for highly efficient fractal loops of "micro-sized" area. MI3, at a tenth of a wavelength per side, and an area of about $\lambda^2/1000$, has not signaled the onset of inefficiency with smaller size.

If these antennas are efficient, how is it possible to explain the results of **Table 4**? Since MI3 has a drop of almost 3 dB from MI2 in gain, it's reasonable to assume that MI3 is (finally) manifesting some inefficiency due to its midget size. This explanation must be abandoned given the low ohmic loss of MI3. Although no unique explanation can be presented at this time, **Figure 14** plots the gain figures as a function of iteration. It wasn't physically possible to bend wire for a 4th or 5th iteration 2-meter Minkowski fractal (note that at lower frequencies this isn't a physical problem), but machines and printed circuit etchings can accomplish this to extend the graph. **Figure 14** reveals that the gain undergoes a possible sinusoidal beating characteristic of a diffraction process. This effect is seen in Foster's analysis for loops of increasing area. Perhaps this diffraction phenomenon has been pushed forward to far smaller areas by using a Minkowski fractal. I predict that a Minkowski Island *will*, in fact, reach the theoretical gain limit of 1.8 dB[10] seen for subwavelength loops, but not until later iterations. At that point, its efficiency may drop below 3dB. Conservatively, a 4th iteration

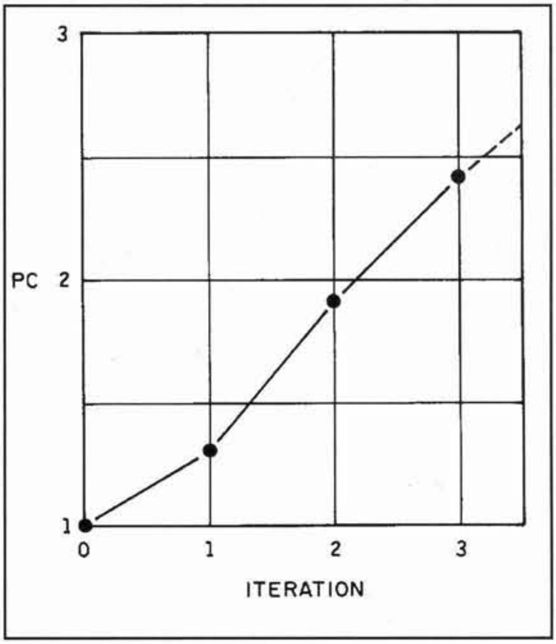


**Figure 13. PC has not topped out with iteration 3 of the MI fractal antennas. There's further shrinkage to be had!**

Minkowski Island quad should provide a factor of about 3 value of PC without suffering substantial inefficiency.

Is there any point to going to higher order iterations with the Minkowski Island fractal for greater shrinkage? In **Figure 15**, I've plotted the PC value for Mikowski Island fractal antennas of 0 to 3 iterations. As Commandment 3 states, this PC should approach an asymptotic limit that is not infinite. **Figure 15** roughly projects that any advantages beyond iteration 6 or so will be modest to the shrinking amount. Certainly the PC won't become asymptotic for iterations 4 or 5. Of course, further iterations will lead to a 2-meter quad smaller than 3 inches on a side and a 20-meter quad less than a yard on a side.

It's important to keep in mind that the Minkowski motif isn't demonstrated here as the *optimized* fractal for a fractal island quad, and that other motifs are more likely to keep high efficiencies while collapsing even more with each iteration.

The bandwidths and the multi-frequency resonances of these antennas are especially interesting. In **Table 4,** I've listed the resonant frequencies, and bandwidths and Qs for each node found between 30 to 175 MHz for 6-meter versions of MI2 and MI3. The bandwidths are SWR 3:1 bandwidths in MHz, irrespective of the resonant frequency SWR. Qs are estimated by dividing the resonant frequency by the 3:1 SWR bandwidth. Frequency ratio is the relative scaling of the resonance nodes.

## Log Period—the 40-year Fractal

If your log-periodic dipole array seems a little hard to figure out, it's because it is a highly modified version of the first, simple, log-periodic antenna, built by DuHamel and Isbell in the mid-1950s. It was a toothed spiral affair (see **Figure 16**). *A fractal!* Today, we know such structures are a type of continuous deterministic fractal.[4] The motif is a spiral that gets bigger and bigger as the distance from the center increases, much like a Nautilus shell.

Because a spiral motif increases in size continuously, its resonant points meld with one another—continuously. The use of toothed notches favors specific resonances, as shown in this diagram of the first log periodic. Thus a LP is broadband because it can have an arbitrary number of notches as the radius gets bigger.

Ironically, the pioneers of the log periodic were probably a step away from generalizing to *all* fractal antennas. However, some of the key points of fractal antennas—such as their fractal dimension, anharmonic resonances, PC (shrinking ability), and so on—were not part of their approach. They did not seem to anticipate self-similar structures that didn't require an opening angle. How ironic that the beautiful discovery of this unintentionally fractal antenna would need another 40 years to gestate into its own emerging field!

Incidentally, the reason these antennas are "log periodic" is because the ratio of spacings and element (teeth) sizes are periodically distributed so the comparison of any one with the next biggest one always provides a constant. Because the *ratios* are constant, the log of the ratios gives a constant *difference*; hence, the logarithmic part.

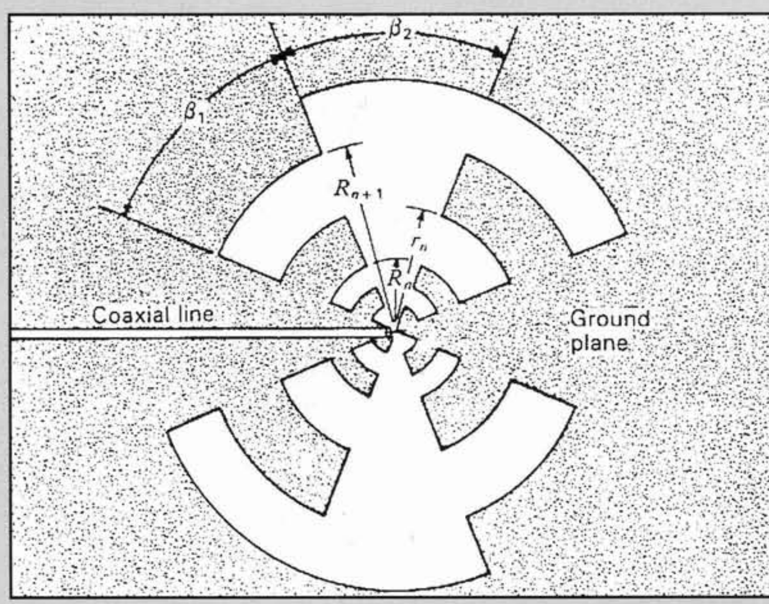


**Figure 14. Adapted from Kraus, 1985. Here's the first log periodic—a spiral fractal.**

The Qs of these nodes clearly show that MI2 and MI3 are viable multiband antennas. They don't display the very high Qs seen in small, tuned loops. A mathematical application to electromagnetics doesn't exist that can predict these resonances or Qs. One way to approach the problem is to estimate the scalar and vector potentials in Maxwell's equations by regarding each Minkowski Island iteration as series of vertical and horizontal line segments with offset positions. Summing them up leads to the Poynting vector calculation and power pattern.[7] My colleagues and I are performing this calculation as one of several methods to model fractal antennas, in order to predict their characteristics and achieve optimized shapes. Perhaps the best "basis functions" in these calculations are themselves fractal.

The discrepancies between ELNEC and measurements must finally be noted. A comparison of **Table 6** with **Table 2** demonstrates minor inconsistencies in modeling of ratios of resonant frequencies, PCs, SWRs, and gains. Further work must be done on both to fine-tune their

# Landstorfer's Weird Wires

Straight as an arrow. Hams take every last ounce of effort to get their Yagis perfectly aligned, and their verticals pointing in line with the Andromeda galaxy. Sure, they look nice, but how do they perform? For fifty years the gospel of wire/tube antennas dictated that straight shapes are best—save for a few exceptions, like the circular loop.

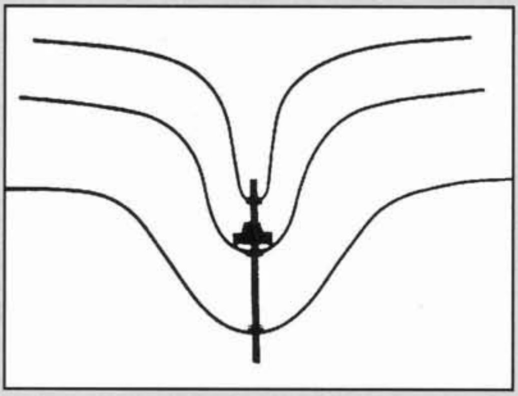

**Figure 15. Adapted from Landstorfer and Sacher, 1985. Oddly shaped Yagi, but it has 3 dB more than a regular one.**

In 1985, F.M. Landstorfer and R.R. Sacher summarized a decade of work using bent antennas. These bends, however, aren't the little bulges that develop in an 80-meter vertical or when a crow lands on your 20-meter director. These were gross unæsthetic bends and crimps; and oddly, they produced antennas with higher gain!

None of the "Landstorfer dipoles" looks particularly pretty, and all are hard to support mechanically. The Landstorfer Yagi, shown in **Figure 17**, is an antenna of elements with Bell Curves at their centers. The Landstorfer verticals are scrunched affairs. Some almost look like segments shaped into question marks (see **Figure 18**). You may recall seeing some of the verticals from the 3Y0PI DXpedition getting scrunched in this way by an Antarctic blizzard. According to Landstorfer, they must have worked better! How much? Perhaps 3 to 4 dB better. Time to get the bends, vertical fans!

Will your next antenna look like these chaotic fractals? Perhaps not. Innovation always takes time. Hopefully, in deference to F.M. Landstorfer, we shall see the first ham Landstorfer antenna within the next five years.

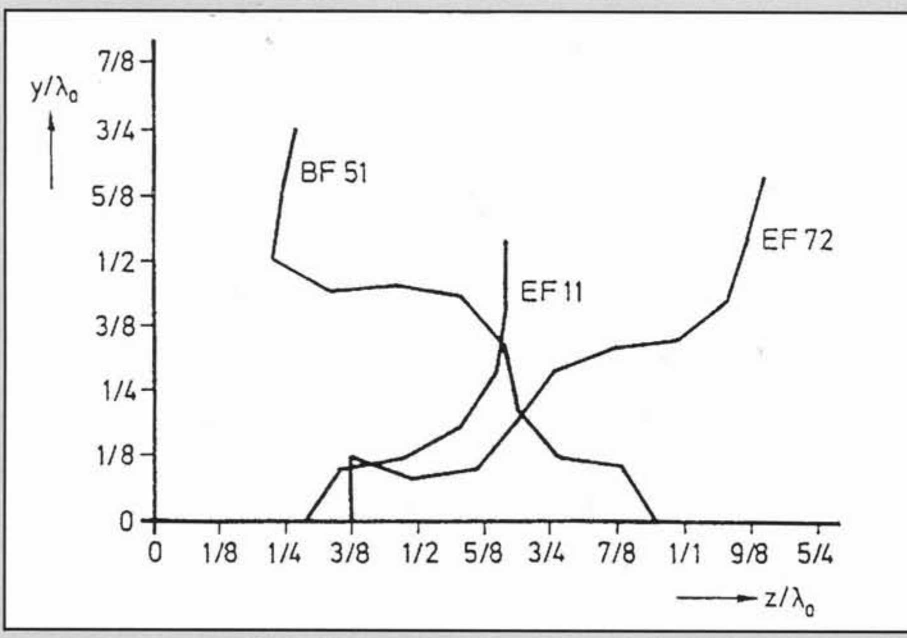


**Figure 16. Adapted from Landstorfer and Sacher, 1985. These bent lines represent optimum shapes for a vertical element. Who needs a straight line?**

accuracy; however, it appears that, from the point of view of SWR, PC, gain, and frequency resonance ratios, actual MI antennas work slightly better than these ELNEC predictions. In any case, these Minkowski Island fractal antenna efforts convincingly assert that fractals offer a profound new approach to antenna design with unusual practical value in shrinking antennas.

Part 2 will continue this intriguing saga of tiny, multiband fractal antennas and arrays.

## Acknowledgments

Paradigm shifts[12] are challenging and sometimes pleasurable. This one had its share of both. My thinking and resolve were sharpened by comments from individuals in many different fields. I would like to thank them all, in a "random fractal" order: Alexander Filippov, Frank Drake, Benoit Mandlebrot, Bruce Tis, Carl Helmers, Andrew Pfeiffer, Paul Pagel, Paul Horowitz, Bernard Steinberg, Seth Shostak, Darren Leigh, William Vetterling, Maury Peiperl, Robert Hohlfeld, D.L. Jaggard, Eugene Hastings, Enders Robinson. For obvious reasons, I acknowledge the management of Charles River Park Apartments for their early commentary on fractal antennas. ■

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