The role of scientific models

AM transmissions and the time domain

This article was prompted by the recent discussion in "Electronics Australia" concerning the nature of amplitude modulation. It seeks to show that the argument between protagonists concerning the frequency and time domain concepts is really no argument at all; simply a different way of looking at the same thing.

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The universe is a complex of matter and energy with the most intricate and subtle interactions underlying its fabric. In our attempts to understand this structure we devise models. The purpose of these models is to help us understand the particular aspect of nature we are studying. The models may be physical, mathematical or otherwise, but whatever their form, they are always analogs only of the fundamental object of our study. Sometimes the complexity of an object requires more than one model (or way of thought) to describe it fully.

Invariably, in such cases, one is led to ask which model more correctly describes the object (or process) we are trying to understand. The answer, of course, is that all models are required for a full understanding of the phenomenon; sometimes one model has precedence, and sometimes another, according to the situation. None is complete in itself.

One practical example of the above which has been well known to physicists since the early part of this century is the wave-particle duality which appears to be expressed by very small elements of matter. In some experiments, electrons appear to resemble bullet-like particles whereas in others they show properties just like water waves. What is the resolution of this apparent dilemma? Are electrons waves or are they particles? Or do they change, like Dr Jekyll and Mr Hyde, from one moment to the next?

The answer to the last question is no, they don't change, and the answer to the previous question is that electrons are neither waves nor particles. The resolution to the dilemma is that electrons are electrons. They are totally different to anything in our macroscopic world of which we have experience – such as water waves or bullets. Thus, it is impossible for us to construct one physical model that totally describes or is analagous to the behaviour of real electrons.

Another example, which is the prime concern of this note, is the understanding of time-varying or frequency-spread electrical signals. And already, in describing the problem, I have introduced the duality of understanding that is necessary for a complete description. The concepts of time and frequency are well known to all, although the expression of processes in the time domain is more comfortable to most of us. In the case of electrical signals, their variation in the time domain can be readily seen on an oscilloscope screen.

Behaviour in the frequency domain usually requires some form of spectrum analyser for its display. The word domain is used here to mean that set (or group) of possible values over which a signal may vary. In the time domain, a sine wave is described by specifying amplitude values for each instant of time that the sine wave exists. In the frequency domain the power of the sine wave at every possible frequency could be specified. For a theoretically pure oscillation, that has and always continues to exist, this specification would be a single non-zero value at the frequency of oscillation.

An experimental setup to illustrate the above discussion is shown in Fig. (1) where an oscilloscope and a spectrum analyser are used to display the output of a sinusoidal generator (f_1) in the time domain and frequency domain respectively. A similar analysis can also be done mathematically (see box).

Up to this point, everybody is probably still quite happy. The problems start to arise when we modulate f_1 with a much

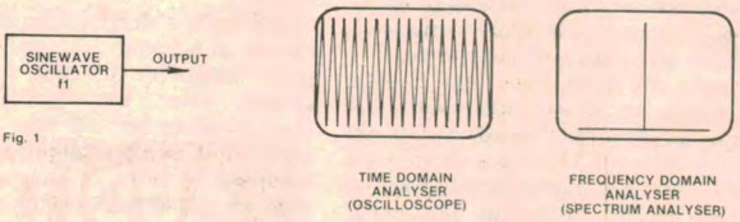

Fig. 1

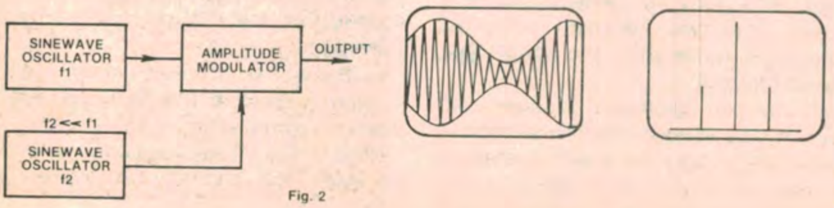

Fig. 2

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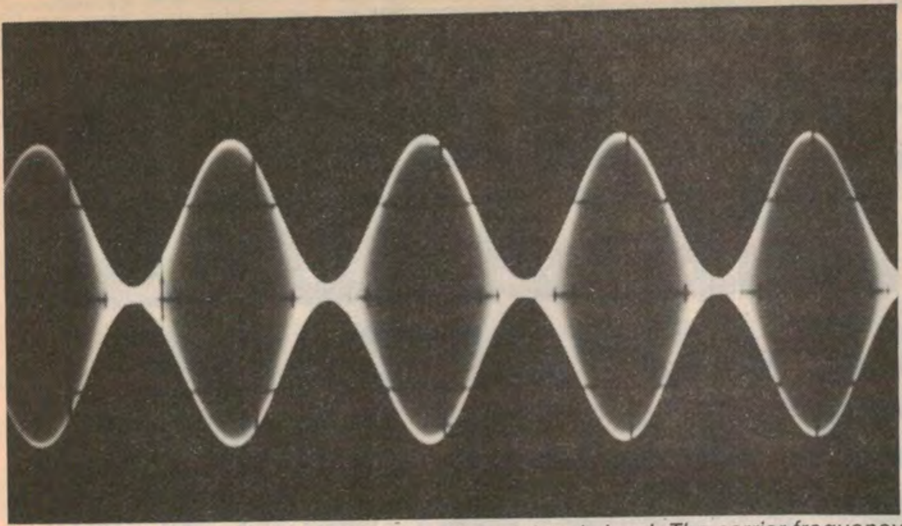
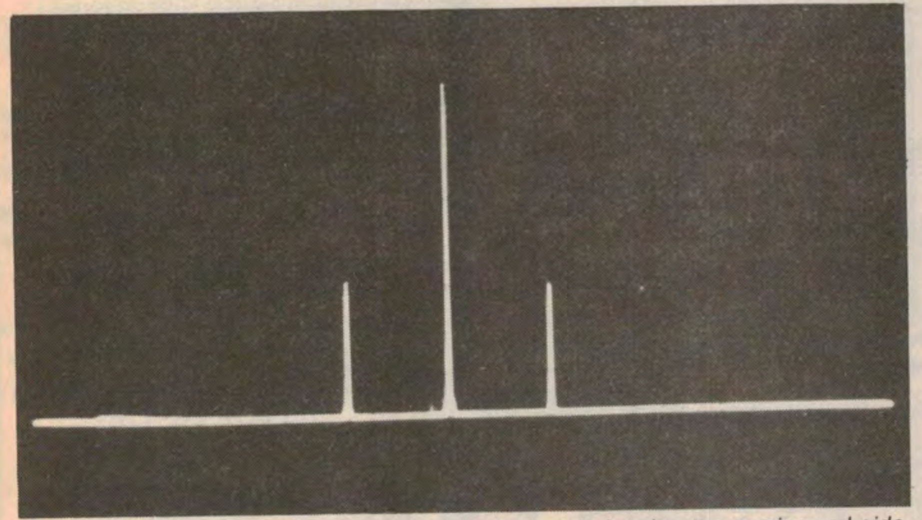


Time domain presentation of an amplitude modulated signal. The carrier frequency f_1 is 33.43MHz and the modulation frequency f_2 is 24kHz. Percentage modulation is just under 100%.



Frequency domain presentation of the same signal showing the carrier and sideband components. The spectrum analyser bandwidth was 300Hz.

lower frequency f_2. Fig. 2 shows the resulting time and frequency domain displays. The problems arise because some people insist that the time-domain display is maybe not a valid way of viewing the situation. To quote from *Electronics Australia,* January 1983 (p.99), "When an AM signal is viewed on an oscilloscope, it appears as if the carrier signal is varying in amplitude. The reason this occurs is because the oscilloscope receives three different frequencies and algebraically adds them together."

This reasoning, however, is incompatible with the domain of interest – that of time. The oscilloscope is a time domain display device and at each instant of time has a voltage presented to its Y-terminals which it, within certain conditions, displays faithfully along the time or X-axis on its CRT. An oscilloscope does not receive frequencies – it receives and displays voltages as a function of time. When we look at this display it does indeed appear as if the amplitude of the carrier is varying, and we should have no compunction about applying the principle of parsimony and say that, *in the time domain,* this is in fact what is occurring.

At this stage the antagonists are jumping angrily out of their seats and pointing to the frequency domain display. Here we see three spikes. The middle and largest one is centred at f_1, and a smaller one appears on each side. All three spikes show a rock steady amplitude. There, you say, that proves that the amplitude of the carrier signal does not change! But, in fact, we are now looking at the situation using different constructs (if you like, a different model or way of thinking), and how we interpret this display cannot really change the first display (as long as there is no fault in the first display). This is not simply philosophical goobledegook as I shall try to show.

The frequency and the time domain in mathematics

Constructs of reality can be made in physical models or in mathematical models. The latter are usually economical and are frequently easily manipulated to yield information about the modelled system. In the time domain, an unmodulated RF carrier is modelled by the mathematical function:

$$h(t) = A \sin(2\pi f_1 t)$$

When amplitude modulation is introduced the modelling function becomes:

$$g(t) = A(1 + m\cos(2\pi f_2 t))\sin(2\pi f_1 t)$$

where 100m is the percentage modulation. Plotting this function graphically is equivalent to displaying the respective voltage on an oscilloscope screen. To examine the frequency domain of this voltage a spectrum analyser is used. An equivalent operation on g(t) can be done mathematically using a device known as a Fourier transform, such that the frequency domain is given by:

$$G(f) = \int_{-\infty}^{\infty} e^{-j2\pi ft}\, g(t)\, dt$$

There also exists an inverse Fourier transform which allows one to go from the frequency to the time domain. This is given by the formula:

$$g(t) = \int_{-\infty}^{\infty} e^{j2\pi ft}\, G(f) df$$

Evaluating the transform integral to find G(f) for a particular g(t) is sometimes quite difficult and requires computer numerical assistance. For the AM g(t) function given above, however, we do not need to evaluate the integral, as a simple trigonometric identity can be used to rewrite the model as:

$$g(t) = A\sin(2\pi f_1 t) + \frac{Am}{2}\sin(2\pi\{f_1+f_2\}t) + \frac{Am}{2}\sin(2\pi\{f_1-f_2\}t)$$

which clearly shows the three frequency component amplitudes present. By inspection we can write the frequency domain function as:

$$G(f) = A\delta_{f,f_1} + \frac{Am}{2}\delta_{f,(f_1+f_2)} + \frac{Am}{2}\delta_{f,(f_1-f_2)}$$

where the symbol δ_{AB} is called a Kronecker delta function and has the value of one when A = B and zero elsewhere.

In the experimental setup of Fig. 2 I would now like to consider what happens when the modulating frequency f_2 is reduced to a very low value say 1Hz. In most real spectrum analysers we see a very interesting phenomenon. The central spike is no longer constant but starts bouncing up and down once per second. That is, the amplitude of the carrier appears no longer to be constant. At this point some astute people will again start yelling at me for being unfair; because something else has happened that has not been mentioned.

As we reduced the frequency f_2 down to 1Hz the sideband peaks drew closer and closer to the central peak until they became indistinguishable from one another. If we had a very expensive spectrum analyser we might be able to increase the frequency resolution until the two sideband peaks again become separately visible. Upon doing so we would observe that the central peak was again constant in amplitude. We would also observe that when initially switching on oscillator f_1, it took some time for this central peak to rise to its full amplitude. All these observations should give us a clue as to what this type of frequency domain display is actually showing us.

It is the integration time of the spectrum analyser that provides us with the answer. When the display is showing resolvable sidebands, it is of necessity integrating each frequency along the X-axis for a time equal to at least one complete cycle of the modulation frequency. Thus, this display is showing us the average power of the carrier frequency. And the average is computed over a full modulation period.

This concept of average power has no direct meaning to the time domain display (which is concerned with instantaneous amplitudes) and thus, as previously stated, an interpretation in the frequency domain cannot contradict the interpretation in the time domain. The two views are complementary to one another.

As with the wave-particle duality in matter, both the time domain and frequency domain viewpoints are necessary to fully understand the nature of varying electrical signals and the physical processes, such as modulation and demodulation, that they may undergo. Sometimes it may be possible to describe a process from both viewpoints. Many times it will be found that one viewpoint is superior to the other in providing a clear or easily understood explanation of what is happening. Two examples come readily to mind. When designing filters (eg, in IF amplifiers), the frequency domain viewpoint is paramount in matching the filter bandwidth to the signal components present.

On the other hand, the time domain viewpoint of demodulation (rectification and capacitor filling in the gaps) is usually the easiest explanation for most people to understand. It is also not incompatible with the frequency domain viewpoint. It is simply different. One cannot say that the three RF components would be shunted to earth in this description, because in the time domain description there are no three RF components — there is only one time-varying signal.

Conclusion

The basic argument presented above relies on the axiom that in our attempt to understand nature we build constructs that simplify a particular phenomenon to the point where we can perform useful operations (physically or mathematically) on selected aspects of the phenomenon. How useful our constructs are is dependent on how closely they model the real world. Some phenomena require multiple constructs to appreciate fully the nature of the beast in question. Each construct then gives us a different way of looking at the whole.

We should not lightly discard constructs that appear simplistic in the presence of what may at first appear to be more elegant and elaborate constructs. The former may serve us well without sacrificing the ultimate reality (which is probably mortally unknowable anyway). Yes, there really is a time domain!

Additional Reading:

A very interesting text which studies how completely science may describe the universe is: "Completeness in Science", by Richard Schlegel (Appleton-Century-Crofts, New York 1967).

A detailed description of the mathematical modelling and analysis of communication processes is given in: "Frequency Analysis, Modulation and Noise", by Stanford Goldman (McGraw-Hill, 1948/Dover, 1967).