Potential dividers and power supplies

RECENTLY on the *EPE Chat Zone* there has been some discussion on provision of multiple power supply voltages, power supply switching and regulator circuits, started by user ***Tuurbo46***. The discussion continued (under username ***Rocket Ron***) on the new *EPE* forum hosted by EEWeb (see this month's *Net Work*). This discussion has ranged over a number of related issues. Initially ***Tuurbo46*** posted on *EPE Chat Zone*, asking about obtaining a 2.5V supply from a 5V supply.

'I just need to confirm my circuit current understanding. Am I correct in thinking a 5V, 2A supply, with two series 10kΩ resistors to GND, can only supply 0.6mA from the potential divider? I would like to supply 2.5V and the possibility to source 100mA.'

After discussion moved to EEWeb and after ***Tuurbo46/Rocket Ron*** had received a number of helpful comments from other users on both forums, he posted more details of his requirements and updated his design ideas. In responses to ***Tuurbo46's*** initial question we will look at the nature of potential dividers and why they are generally unsuitable for directly providing supply voltages. This forum thread also raises other issues, which we may look at in a future article.

Fundamentals

The basic potential divider (Fig.1) comprises two resistors in series connected across a voltage source. Potential dividers can also be made using combinations of other components such as capacitors and inductors. The potential divider is a fundamental concept and basic 'building block' of electronic design. Therefore, discussion of the question posed by ***Tuurbo46*** leads us to introduce some basic circuit theory, which is useful to anyone designing, or endeavouring to understand, the operation of electronic circuits.

As the name suggests, the potential divider is a means of providing a lower (divided) potential (voltage) from a higher one. Thus, they act as attenuators – reducing the magnitude of a signal. Attenuation is commonly required in measurement, for example in high-voltage probes, which are used in conjunction with voltmeters or multimeters. Basic volume controls in analogue audio circuit use variable potential dividers. Other uses include biasing transistors, obtaining a voltage signal from a resistive sensor, such a thermistor, obtaining a reference voltage at exactly half the supply voltage (divide potential by two) for use as the mid-point for AC signals and setting the gain in op amp circuits. In the latter case, the potential divider sets the fraction (division) of the output fed back to the input. However, potential dividers are not suitable for some applications. In particular, they are generally unsuitable for providing power to circuits connected to their outputs – you *could* do it, but it is so wasteful of power that it is not worth doing. We will look the basic circuit theory behind potential dividers to explain why this is so.

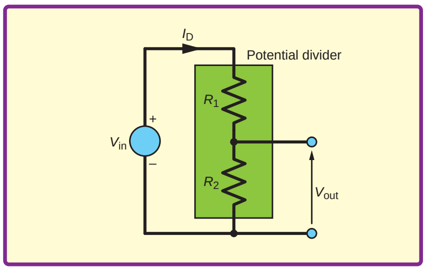

Fig.1. Potential divider

Equation

Refer to Fig.1, the current (I_D) in the potential divider resistors (R_1 and R_2) can be found using Ohm's law. Ohm's law is the relationship between the voltage (V) across a resistor (R) and the current (I) through it; specifically $I=V/R$. In this case, because the resistors are connected in series, the total resistance is the sum of the two, so Ohm's law gives the current in the divider as:

$$I_D = \frac{V_{in}}{(R_1 + R_2)}$$

The voltage dropped across resistor R_2 is the output voltage; again, by Ohm's law:

$$V_{out} = I_D R_2$$

Using the first equation for I_D, and substituting this into the equation above, we get the output voltage in terms of just the input voltage and resistor values

$$V_{out} = \left(\frac{R_2}{R_1 + R_2}\right) V_{in}$$

This is referred to as the 'potential divider equation'. From this equation, we see that because $R_1 + R_2$ must always be larger than R_2, V_{out} is always less than V_{in} by a factor determined by the resistor values. If we assume R_2 is n times the value of R_1, that is, $R_2 = nR_1$, the potential divider equation becomes:

$$V_{out} = \left(\frac{R_2}{nR_2 + R_2}\right) V_{in}$$

We can cancel the R_2 terms to get:

$$V_{out} = \left(\frac{1}{n+1}\right) V_{in}$$

The output voltage depends only on the relative values of the resistors, not on their absolute values. The current flowing through the divider does of course depend on the absolute resistor values, as does the power dissipation in the divider. If both resistors are the same value (so n = 1) the output voltage is half the input voltage. For example, if V_{in} = 5V and $R_1 = R_2$ = 10kΩ (as specified by ***Tuurbo46***) then V_{out} = 2.5V. With these resistors the current in the divider is (by Ohm's law) 5V/20kΩ = 0.25mA.

The current flowing through a potential divider will cause it to dissipate power. Electrical power is given by voltage times current (IV), which on applying Ohm's law can also be written as I^2R or V^2/R. The latter equation is generally most useful for finding the power in a potential divider because generally we know the applied voltage and resistor values. The 2 × 10kΩ divider across 5V discussed above consumes $(5V)^2$/20kΩ = 1.25mW of power.

Loaded

If we connect another circuit (a load) to the output of the potential divider the output voltage will change to some extent. This is an important consideration when deciding if a potential divider is appropriate, or when checking if we have chosen the most appropriate resistor values.

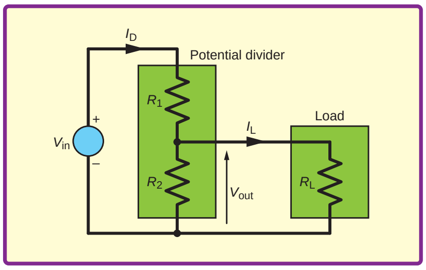

Fig.2. Loaded potential divider

The schematic of the loaded potential divider is shown in Fig.2. If we have a known load resistance we can work out the voltage across the load by finding the parallel combination of R_2 and R_L in Fig.2. We can then use this value in the potential divider formula to calculate a revised value for V_{out}. For two parallel resistors, R_2 and R_L, we have a combined resistance (R_p) given by:

$$\frac{1}{R_P} = \frac{1}{R_2} + \frac{1}{R_L}$$

So

$$R_p = \frac{R_L R_2}{(R_L + R_2)}$$

If we take our example potential divider (Fig.1 with two 10kΩ resistors) and connect a 10kΩ load, the effective value of the R_2 resistor in Fig.1 becomes 5kΩ (the parallel combination of two 10kΩ resistors). Using the potential divider formula, this gives an output voltage of 5 × 5/(10+5) =1.67V, a significant deviation (–33%) from the open-circuit output voltage of 2.5V. Adding the 10kΩ load has significantly reduced the output voltage. If the load is increased to 100kΩ then the voltage drops to 2.38V (–4.8% shift); and with a 1MΩ load the output is 2.49V (–0.5% shift). As a rule of thumb, a load resistance connected to a potential divider should be at least ten to one hundred-times larger than the resistor across which the load is connected to avoid the voltage shift from being too large.

Regulation

In the context of power supplies (***Tuurbo46's*** application) a shift in voltage under load is characterised by a parameter called 'load regulation'. A schematic of a generic regulator or power supply circuit is shown in Fig.3. Here, if the load current (I_o) varies then V_o should remain constant, but in practice it will vary to some extent.

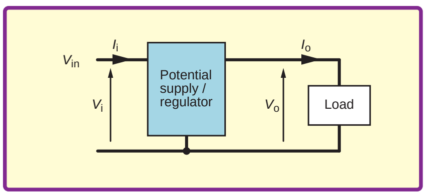

Fig.3. Power supply/regulator with load

The load regulation can be defined in terms of the ratio of change in V_o (written ΔV_o and pronounced 'delta V_o') produced by a change in I_o (ΔI_o)

$$\text{Load regulation} = \frac{\Delta V_o}{\Delta I_o}$$

This value is actually a resistance (voltage divided by current), which is the effective output resistance of the regulator (measured in ohms). We will discuss the concept of output, source or internal resistance shortly. Load regulation can also be expressed as the change in output voltage (ΔV_o) for some specified change in load current (typically a change from no load to maximum load) divided by the nominal or mean regulated output voltage (V_o).

$$\text{Load regulation} = \frac{\Delta V_o}{V_o}$$

This can be stated as a fractional value (eg, 0.002) or multiplied by 100 to give a % (eg, 0.2%).

Load regulation is usually specified on the datasheets of regulator IC and power supply units. Taking the LM317 regulator as an example (mentioned in ***Tuurbo46's*** discussion thread), the datasheet states a voltage change of typically 0.1% for a load current change of 10mA to 1500mA (low to full load). This regulation is much better than that provided by the 2 × 10kΩ potential divider, even with a 1MΩ load, where the load currents is in microamps.

Source

When considering power supplies, we do not always know the exact load resistance, but we may know the range of current that has to be supplied (***Tuurbo46*** quotes 100mA). When using a potential divider for anything, a useful reality check is to calculate the load current for a short-circuit load, that is R_L=0Ω in Fig.2. This is simply the current through R_1 if it is connected directly across V_{in}. In this case, we have 5V/10kΩ = 0.5mA.

As ***Tuurbo46*** indicated in his question, the potential divider cannot supply a higher current than this into a grounded resistive load; and at this current the voltage across the load is zero. This immediately proves that the potential divider in Fig.1 is unsuitable for the specified power supply application, which requires 100mA at 2.5V.

Calculating the short-circuit output current of the potential divider is useful because it helps us find the effective internal resistance, or source resistance, of the voltage source formed by the potential divider. We also need the open-circuit voltage (the unloaded potential divider output voltage), which we already know is

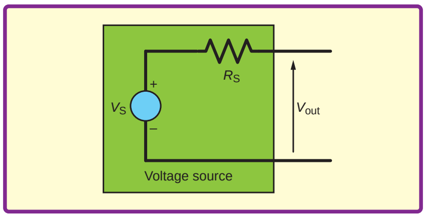

Fig.4. Thévenin equivalent circuit

2.5V for our example. From this we can find our source resistance from 2.5V/0.5mA = 5kΩ. This is equal to the parallel combination of the two potential divider resistors.

We can then replace the potential divider with the equivalent circuit shown in Fig.4, in which for our case the source voltage (V_S) is 2.5V and the source resistance (R_S) is 5kΩ. This circuit behaves exactly like the potential divider in terms of output voltages and currents.

This idea is not just applicable to potential dividers; any combination of independent voltage sources, current sources and resistors, for which we can designate two output terminals, is equivalent to a single voltage source and series resistor at those terminals.

Thévenin

This is an important piece of circuit theory known as 'Thévenin's theorem' – the circuit in Fig.4 is called the 'Thévenin equivalent circuit'. The same approach can be used with AC circuits where all the sources in the original circuit are at the same frequency, in which case we work in terms of impedance and can include capacitors and inductors in the original network. One limitation of this approach is that the total power dissipation in the equivalent circuit is *not* the same as in the original.

To obtain the Thévenin equivalent for any suitable circuit we find the open-circuit output voltage and then the short-circuit output current. The open-circuit voltage is used as the equivalent source voltage and the open-circuit voltage divided by the short-circuit current gives the source resistance. This and other types of equivalent circuit are widely used in circuit analysis and design, and are the underlying theory behind well-known concepts such as input and output impedance.

Fig.5 shows the Thévenin equivalent circuit of a potential divider connected to a load resistance. The load and source resistance form another

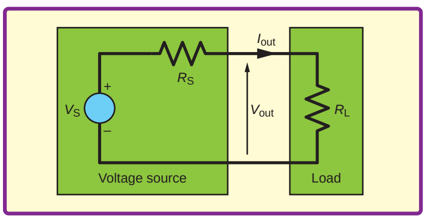

Fig.5. Equivalent circuit for the potential divider with load

potential divider, which determines the load voltage. If we again look at a load of 10kΩ, the potential divider equation for Fig.5 becomes:

$$V_{out} = \left(\frac{10000}{10000+5000}\right)\times 2.5 = 1.67\text{V}$$

As expected, this gives the same output voltage as calculated above using the parallel resistors approach.

Variation

Use of the equivalent circuit makes it easy to calculate the variation in output voltage as load current varies. We have already noted that obtaining 100mA is impossible with our example circuit, so we will assume a more realistic requirement of current from 50μA to 100μA. For I_L = 50μA, R_S will drop 50μA × 5kΩ = 0.25V, so the output will be about 2.25V. At 100μA the output will be 2.0V. This level of voltage change (20%) is much larger than would be generally acceptable for supply voltages.

Potential dividers can provide very accurate division of an input voltage – depending on the tolerance of the resistors or other components used to form the divider. However, the divider should be connected to a circuit, that takes as little current as possible from the divider to prevent voltage shifts occurring due to loading. Typically, in many circuit designs, these are inputs to op amps, ADCs or other devices with very high input impedances, which present very little load to the divider.

In some cases, such as bipolar transistor biasing, the load currents may be higher, in which case more care has to be taken to ensure the divider is not loaded. The rule of thumb mentioned earlier (ten to hundred times the current in the divider compared to the load) provides a useful starting point. The actual voltage shifts due to loading in any application can be calculated or estimated and used to determine appropriate resistor values. The lower in value the resistors used, the 'stiffer' the voltage output will be (lower source resistance) but more power will be consumed in the divider. This may be an important consideration in some applications where low power consumption is of particular importance.

Efficiency

We can use the equivalent circuit in Fig.5 to work back to a potential divider that might be able to deliver the required current in the original power supply scenario – and then look at the power consumed by the divider. For the sake of an example, we will assume that R_S must drop no more than 2% of 2.5V at 100mA (we do not know ***Tuurbo46's*** requirements in this respect, but 2% is much worse load regulation than that provided by a typical voltage regulator IC, such as the LM317 mentioned above). 2% of 2.5V is 0.05V, so the maximum value of R_S is 0.05V/100mA = 0.5Ω. As the potential divider suggested by ***Tuurbo46*** has equal resistors we can use twice the R_S value (1.0Ω) for both resistors. This is not practical due to the current and power dissipation in the divider. With no load, the divider current is 2.5A (5V/2Ω) and the power dissipated by the divider with no load is $V^2/R = 5^2/2.0$ = 12.5W – extremely wasteful, given that at 2.5V a 100mA load consumes 0.25W (2.5V × 0.1A). We can calculate the efficiency of power supply as:

$$\text{Efficiency} = \frac{P_O}{P_O + P_L},$$

Here, P_O is the output power and P_L is the power loss (power consumed by the supply circuit). For the potential example just given, ignoring voltage shifts, we have approximately P_O = 0.25W and P_L = 12.5W. So this 'potential divider power supply' has about 2% efficiency.

For a linear regulator IC, wired as shown in Fig.3, we can approximate $P_O = I_O V_O$ and $P_L = I_O(V_I - V_O)$, so:

$$\text{Efficiency} = \frac{P_O}{P_O+P_L} = \frac{I_O V_O}{I_O V_O + I_O\left(V_I - V_O\right)} = \frac{V_O}{V_I}$$

For a linear regulator supplied from 5V and outputting 2.5V, the efficiency is 50%, very much better than the potential divider and with a far better load regulation. The LM317 linear regulator mentioned above has a minimum $V_I - V_O$ of 3V, so it would need a larger input voltage and would consequently have a lower efficiency. Standard linear regulators are not very efficient, but are still much better than potential dividers. Greater efficiency can be achieved using 'Low Drop Out' (LDO) linear regulators, which require much lower $V_I - V_O$ to operate (for example, in the 100mV range, rather than several volts). Still higher efficiency can be obtained using switching regulators (often around 80% or more).

Configurations

Tuurbo46's requirement is for a system with multiple power supply voltages. This requires two regulators. The regulators could be connected in parallel (as in Fig.6) or in series (as in Fig.7). In both cases V_{O1} is higher than V_{O2}. Which is the most efficient? With linear regulators, at first it might seem like the series configuration is more efficient because V_O/V_I is smaller for the second regulator, so its own efficiency is better. However this power is supplied by the first regulator, so we have to multiply the efficiency of the second regulator by that of the first to get the overall value. With linear regulators the efficiency of both configurations is the same. For switching regulators their efficiency does not vary much with input voltage, so the series configuration is less efficient because of the multiplying effect of efficiency when one regulator supplies another.

For both switching and linear regulators the series configuration has the disadvantage that the first regulator has to handle all the current for both power supplies, whereas for the parallel version each regulator only has to handle the current for its individual output. However, if regulator 1 is a switching regulator and regulator 2 is linear, the series configuration may be a good option due to the improved efficiency of operating the linear regulator with a lower input voltage.

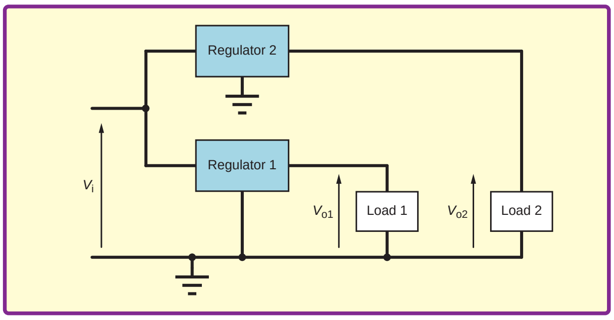

Fig.6. Two regulators in parallel for a system requiring two supply voltages

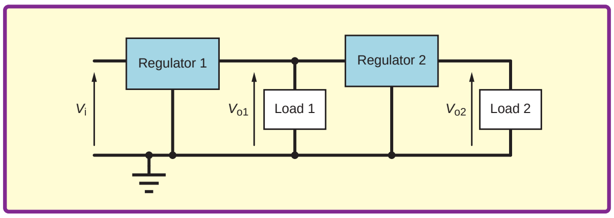

Fig.7. Two regulators in series for a system requiring two supply voltages