ELEMENTARY THEORY

OF

ALTERNATE CURRENT WORKING

BY

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FULLY ILLUSTRATED

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PREFACE.

IN view of the large number of books which have been written both on the theory and practice of alternating-current work, I feel that some apology is due for adding to their number.

It has been my own experience, and I think also that of others, that the larger and standard works on this subject can be read with more profit when a working knowledge of the very elementary theory has already been acquired. The aim of the present book is to put before the reader these elementary principles in as simple a form as possible, and to endeavour to rob electrical phenomena of the mystery with which they are often associated in the mind of the beginner, by reducing them, as far as possible, to every-day mechanical equivalents.

The latter part of the book deals with the simple theory of the action of alternating-current machinery, all reference to the construction, testing and design of actual machines being omitted.

My object has been to introduce the subject to the reader, who has already some knowledge of the simpler electro-magnetic phenomena and of continuous-current working, by dealing only with such elementary points as he may find somewhat difficult to extract from a book which makes any pretensions to being a complete treatise.

In conclusion, I wish to acknowledge the valuable assistance given by Q.M.S. Morecombe, R.E., B.Sc., in reading the MS. and making many most helpful suggestions.

G. L. H.

October, 1913.

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CHAPTER I.

ALTERNATING CURRENTS.

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1. Alternating Currents and Pressures.

If a steady uni-directional electric pressure is applied to a closed circuit a current will flow through that circuit in a steady direction and will be unaltered in magnitude so long as the work which it is doing remains a constant quantity, whether that work is the driving of a motor or the lighting of lamps. The flow of such a current may be likened to the movement of an engine flywheel, which will remain constant in speed and direction so long as the pressure of the steam and the work which the engine is doing also remain constant. It is important to remember that when once the flywheel has been run up to speed its mass has no effect, provided that the work done by the steam is perfectly uniform, a state of affairs which is approximately true in the case of a steam turbine, for example. The case of the continuous current is similar. Any magnetic effect which is produced by the current remains unaltered, and the presence of "inductance," which may be taken as the electrical equivalent of inertia or mass, will not in any way affect the problem.

If, however, we apply to the closed circuit a pressure which is neither steady nor uni-directional, but is varying from a maximum in one sense, through zero to a maximum in the opposite sense, a very different state of affairs will result. Under the influence of this varying pressure a current will flow in the circuit which will alter in value and direction, as does the pressure which causes it to flow. Such a current is known as a periodic, harmonic, or most generally as an **alternating current.**

The flow of such a current may be compared to the movement of the balance wheel of a watch, which oscillates backwards and forwards instead of rotating steadily in a uniform direction. It is obvious that the mass of such a wheel will now have a considerable effect, since all momentum must be destroyed and the wheel brought to rest before the direction of its motion can be reversed. The heavier is this wheel the greater will be its momentum in any direction. Similar problems are frequently met with in all branches of engineering. For example, the motion of a piston rod is oscillatory, just as is the flow of an alternating current, and upon the mass of the reciprocating parts will depend the amount of cushioning which is necessary in order to bring them quietly to rest before their motion is reversed. The effect of momentum will be, in fact, to oppose any change of motion, and the greater is the mass of any reciprocating body the more force will have to be expended in destroying its momentum before each reversal of motion.

In the case of alternating currents, then, the electrical equivalent of mass, that is to say, inductance, is no longer to be ignored. The value of the current will now be affected not only by the work which is being done, but also by the inductance which is present in the circuit; and we shall see that Ohm's law, in its simple form, no longer holds good, and that we are confronted by a problem which depends for its solution not only on the three variants, pressure, current and resistance, but on at least one additional factor, which is known as inductance, and will be dealt with more fully later on. We shall see also that there is a close electrical analogy to mechanical elasticity which will exercise a modifying influence on our calculations and which will be dealt with in its turn.

A very natural question will suggest itself to the reader who has been unceremoniously confronted with these complications at the outset. Since it is a comparatively simple matter, by the addition of a commutator to any form of electric generator, to convert an alternating current to one which flows in a steady direction, why not always do so? Particularly since practically all consuming devices, notably arc lamps and motors, operate much more satisfactorily with continuous than with alternating currents.

The answer is simple. Alternating currents can be far more economically and safely generated in bulk and at very high pressures than can continuous currents. And since a high pressure naturally means a (comparatively) small current for the supply of a given amount of power, the cables which convey the power from generating station to consumer can be made very much smaller than would otherwise be the case. Furthermore, the concentration of the power plant into units of sizes which are at present altogether beyond the practical limit for continuous-current machines tends towards the same end, and the reason for the employment of alternating currents, at any rate for power transmission purposes, may be summed up in the single word "economy."

2. Harmonic Motion : The Sine Law.

On reference to Fig. 1, which shows the elementary generator in its simplest form, it will be apparent that the pressure

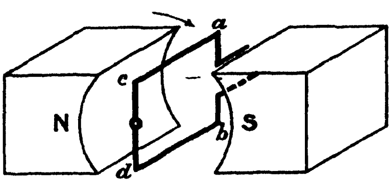

FIG. 1.

between the two ends *ab* of the rectangular coil will be constantly varying as the coil rotates in the magnetic field lying between the poles NS. Suppose this coil to be rotated in a clockwise direction. At the instant shown in Fig. 1 there will be no pressure between the points *a* and *b*, since both the active conductors—namely, *ac* and *bd*—are lying parallel to the lines of magnetic force which connect the poles N and S. An instant later, as the coil rotates, both *ac* and *bd* begin to cut these lines of force, *ac* in a downward direction and *bd* in an upward direction, with the result that a difference of pressure is produced between the points *a* and *b*. Under ordinary generator laws this will produce a positive polarity at *b* and a negative polarity at *a*; in other words, if *a* and *b* were connected a current would flow *from b towards a*. This state of affairs will continue until the coil has revolved through 180°, when the pressure will again be zero, just as it was in Fig. 1. The next instant, however, *bd*, which is now at the top (*see* Fig. 2), will begin to cut the lines of magnetic force in a *downward* direction, and *ac* will begin to cut them in an *upward* direction, so that,

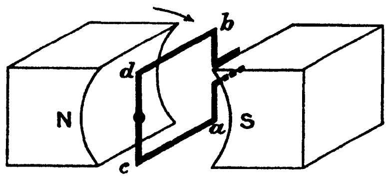

FIG. 2.

for the remaining half of the complete revolution or cycle, *a* will exhibit a positive polarity and *b* a negative polarity. The final result of one complete revolution will then be a pressure between the points *a* and *b*, which, starting at zero, gradually increases to a **maximum** in one direction, dies down again to zero, rises to a maximum in the *opposite* direction, and finally returns again to zero. If the two ends of this coil are connected to a pair of **slip rings** (*see* Fig. 3), which are electrically insulated from one another and the shaft which carries them,

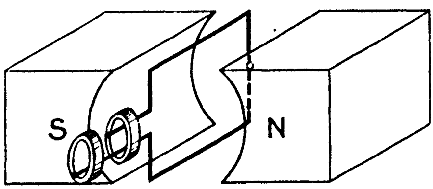

FIG. 3.

and these slip rings are connected by way of rubbing contacts or **brushes** to two stationary terminals on the machine, we shall have at these terminals, when the machine is rotating, an **alternating E.M.F.**, and the machine will be an alternating current generator, or **alternator.**

The number of complete periods or cycles made by the revolving conductor, in unit time—that is to say, the number of times it completes its journey from N pole via S pole back to N pole again in one second—is known as the periodicity or **frequency** of the machine, and is spoken of as a frequency of so many cycles per second. A complete cycle is conveniently regarded as 360 electrical degrees, which may or may not be the same as 360 mechanical degrees. For example, if the conductor rotates in a two-pole field (Fig. 1), the frequency of the machine will be the same as the number of revolutions made by the conductor per second, and a full cycle of 360° (electrical) will correspond to one complete revolution—that is, to 360° (mechanical). If, however, the conductor rotates in a four-pole field, it will complete *two* electrical cycles for each revolution, since there are two north and two south poles to be passed in so doing, and 360° (electrical) will correspond to one-half a complete revolution—*i.e.*, to 180° (mechanical).

Generally speaking, then, we may write

$$f = \frac{p \times \text{revolutions per minute}}{60},$$

where f = frequency in cycles per second,
p = number of *pairs* of poles.

A frequency of 50 cycles per second is conveniently expressed as 50 $\backsim$, the symbol $\backsim$ representing, more or less graphically, one complete cycle, and the unit of time—namely, the second—being understood.

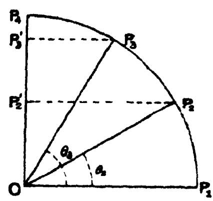

FIG. 4.

We have next to consider the law which governs the instantaneous values of an alternating E.M.F. as it varies from zero to a maximum. Let us take any point, P, on either of the conductors *ac* or *bd* in Fig. 1. The motion of the point P may be represented by the rotation of a line, OP, about a point, O, at a uniform angular velocity, varying positions of this point being shown at P_1, P_2, P_3 and P_4 in Fig. 4. P_1 is the position of this point when the conductor is lying along the lines of magnetic force—that is, midway between the poles—under which circumstances the E.M.F. is zero. P_4 will then represent the position of P after the conductor has revolved through 90° (electrical)—that is, when the E.M.F. is a maximum. OP'_2 and OP'_3 represent the instantaneous values of the E.M.F. when the line has rotated through the angles θ_2 and θ_3 respectively, just as they also represent the speed of a reciprocating body, as, for instance, a steam-engine piston which is imparting a uniform angular velocity to a revolving flywheel.* The point P′ then moves up and down the line OP_4 with **simple harmonic motion**—that is to say, the instantaneous values of an alternating E.M.F. induced in a conductor are **proportional to the sine of the angle through which the conductor has moved from its zero position.** The acceleration of the point P′, or, in other words, the rate of change of E.M.F., will be proportional to the cosine of the angle through which the conductor has

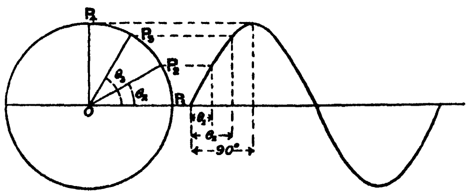

FIG. 5.

moved, and will, therefore, be a maximum when the E.M.F. is zero and a minimum when the E.M.F. is at its maximum value in either sense.

The **amplitude** of the E.M.F., or its maximum value, may be represented by the line OP_4.

We may then represent the instantaneous values of an alternating E.M.F. by means of a curve, plotted with values of E.M.F. as ordinates, and electrical degrees as abscissæ. Taking, as before, any points on our circle P_1, P_2, P_3, P_4, to represent instantaneous positions of our revolving conductor when that conductor has rotated through 0°, θ_2, θ_3 and 90° (electrical) respectively, a curve of E.M.F. may be plotted to any convenient scale, as shown in Fig. 5.

* Neglecting the obliquity of the connecting rod.

This curve is known as the **wave form,** and its equation may be written

$$e = E_{max.} \sin \theta,$$

where e and $E_{max.}$ are the instantaneous and maximum values of the E.M.F. respectively.

There is, of course, a definite relation between these electrical degrees and time, the exact relation depending on the frequency of the machine. If f be the frequency, our revolving line or **vector** OP will revolve through an angle of 360° (electrical) in $\frac{1}{f}$ seconds, or, substituting 2π for 360°, we may write

Angular velocity $\omega = 2\pi f,$

and $\theta = \omega t$

where θ is the angle through which the conductor has moved in t seconds. This expression is important, and will be referred to later on.

Many mechanical examples of this simple harmonic motion might be quoted for the sake of comparison, two common ones being the swing of a simple pendulum and the motion of a steam-engine piston. The *speed* of a pendulum at any instant may be compared to the value of an alternating E.M.F. at the corresponding instant, and the *acceleration* of the pendulum to the rate of change in value of an alternating E.M.F.

In machines as actually constructed the wave form need not necessarily follow this fundamental sine law. A very large number of factors will affect the question in practice, the chief of which is the fact that the field in which the conductors rotate is not necessarily uniform (as has been hitherto assumed), this being particularly noticeable when the machine is giving a heavy current. The shape of the pole faces, the form of the armature coils, the shape and depth of the armature slots, armature reaction and other similar causes may all tend to differentiate the actual wave form from the true sine curve.

The full investigation of the laws which govern periodic functions other than those following the fundamental sine law is beyond the scope of this book. It is essential, however, that we should have some knowledge of these laws for the proper understanding of phenomena which occur in practice, and which will be referred to in due course under the heading of **resonance.**

Briefly, the fact, as stated in **Fourier's Theorem,** is as follows :—

Any periodic function, of whatever form, may be regarded as a simple or **fundamental** sine curve, to which has been added one or more other simple sine curves of varying frequencies.

These subsidiary sine curves are known as **harmonics,** and are numbered according to their frequency. For example, the

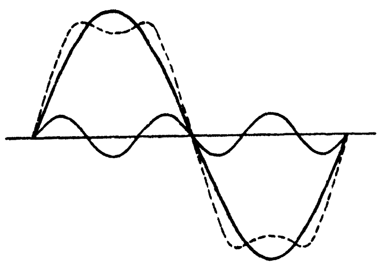

FIG. 6.

third harmonic is a simple sine curve whose periodic time is one-third of that of the fundamental sine curve. The result of the addition of the third harmonic to the fundamental is shown in Fig. 6.

A mechanical example of this compound harmonic motion may be of assistance. Imagine a pendulum with a very light bob and flexible cord oscillating as shown in Fig. 7 (*a*). The motion of this bob will be a simple harmonic one and will

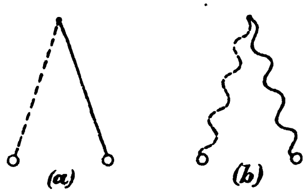

FIG. 7.

follow the simple sine law. If, however, the cord be touched with the hand while the pendulum is swinging it will be distorted from a straight line in a series of ripples (Fig. 7 (*b*)). The bob will now move not with a simple but a compound harmonic motion, the value of the harmonic impressed on the fundamental by touching the cord depending on the place where the cord was touched. The motion of the bob will now be jerky, and may be compared to the resultant wave form shown in Fig. 6.

Various additional factors considerably complicate the case. For instance, a number of harmonics may, in practice, be simultaneously impressed on the fundamental. Also, these harmonics may not be, and, in fact, generally *are* not, in phase * with the fundamental. Any number of examples could be given showing the form of the resultant of the fundamental and various harmonics; but the reader who wishes to pursue the subject further is referred to the very clear investigation of such periodic functions given in Chapter II. of Prof. S. P. Thompson's "Dynamo Electric Machinery, Vol. II., Alternating-Current Machinery."

It is sufficient for our purpose to realise that, if for any of the reasons given on p. 7 the form of the current wave given by an alternator is not a true sine curve, one or more of these harmonics must be present. For reasons which it is unnecessary to investigate, these harmonics must be *odd*—*i.e.*, their periodic time must be some *odd* sub-multiple of that of the fundamental.

We shall see later that the presence of these harmonics may lead to considerable trouble in practical work, and the object of the designer is therefore to approximate as closely as he can to the true sine wave form, and so nearly is this accomplished in up-to-date design that we need have little hesitation in assuming the fundamental equation for purposes of future calculation.

3. Periodic Currents.

If the terminals of our alternator are connected to some external circuit a current will flow by virtue of the E.M.F. of the machine. If this E.M.F. is alternating so also will the current be, and it will rise and fall under the influence of the alternating pressure, the number of cycles naturally corresponding, in a given time, to the number of cycles completed by the machine; that is to say, there will flow in the external circuit an **alternating current** whose frequency is the same as the frequency of the E.M.F. which produced it. The value of this current will also follow the simple harmonic law, and

* *See* page 26.

we may write, just as we did in the case of our alternating E.M.F.,

$$i = I_{max.} \sin \theta,$$

i and $I_{max.}$ being instantaneous and maximum values of the current respectively.

4. Measurement of Alternating Pressures and Currents: Average Values.

Our next step is to consider how these alternating pressures and currents are to be measured, and how an alternating current is to be compared to a corresponding continuous current. Clearly, the passage of any current, whether alternating or continuous, through a conductor will produce heat in that conductor, and so raise its temperature. An incandescent lamp, for example, can be made to glow just as efficiently by passing through its filament an alternating current as it will under the influence of a continuous current, provided that the frequency of the former is sufficiently high to maintain the temperature of the filament at a practically constant value. The fact that the *direction* of the current is constantly changing will have no effect on its heating properties, which will depend solely on some average value of the current taken over one or more complete cycles. These considerations naturally lead us to the assumption that the average value of an alternating current is such as will produce the same heating effect as a corresponding continuous current. Now, the power expended in heating any conductor varies, as we know from continuous-current working, as the **square of the current** passing through that conductor. If, then, we take instantaneous values of our alternating current in amperes, square those instantaneous values, and take the average of these squared values over one or more complete cycles, the result will be the square of the value of the continuous current, in amperes, which would produce the same heating effect. The square root of this value will then give us our required average value in amperes; that is to say, the effective or **virtual** value of an alternating current is expressed as—

The square root of the mean of the squares of instantaneous values, taken over one or more complete cycles.

In exactly the same way, the virtual value of an alternating E.M.F. is expressed as the square root of the mean of the squares, since the expression I^2R may be equally well written E^2/R.

Alternating-current instruments, whether ammeters or voltmeters, are calibrated to read these virtual values, so that the expression 100 amperes when used with reference to alternating currents will imply, not the maximum value of the periodic current wave, nor the arithmetical mean of its instantaneous values, but its **quadratic** mean, as explained above.

It remains now to determine the relation between the maximum value or **amplitude** of any alternating current (or E.M.F.) and this quadratic mean. A straightforward, if

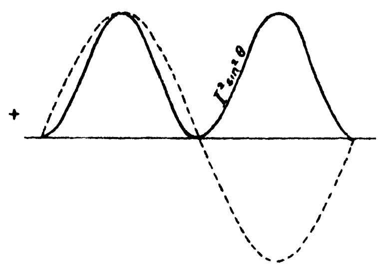

FIG. 8.

laborious, method would be to select a number of values of the angle θ between 0° and 360°, to look up in a table the value of their sines, to square these values, find the mean, and take the square root of the result. Such a method, to be accurate, would necessitate selecting a very large number of values; but is instructive as demonstrating clearly that the *square of any real function must always be positive.* If the fundamental sine curve be plotted between 0° and 360°, and the square of this function be plotted with it, as shown in Fig. 8, it will be apparent that the arithmetical mean height of the former is zero, since the positive portion above the zero line is balanced by the negative portion below the line; whereas the mean of the latter is *not* zero, the curve lying wholly on the positive side of the zero line.

The quadratic mean may be much more accurately determined by calculating the area enclosed by the curve of the square of the sine function, dividing this area by the length, and taking the square root of the result. The equation of the curve will be $i^2 = I^2_{max.} \sin^2 \theta$, and the area enclosed by this curve between the limits $\theta = 0°$ and $\theta = 360°$, or 2π, will be

$$\int_{0,}^{2\pi} I^2_{max.} \sin^2\theta \, . \, d\theta,$$

which is equal to $I^2_{max.} \times \pi$.

Dividing this area by the length 2π, we have

$$I^2 = \frac{1}{2\pi} \times I^2_{max.} \times \pi$$

$$= \frac{1}{2} I^2_{max.},$$

from which
$$I = \frac{I_{max.}}{\sqrt{2}},$$

or, in words, the virtual value of an alternating current or E.M.F. is equal to the **maximum value divided by $\sqrt{2}$.**

If, then, an alternating current ammeter is reading 100 amperes we know that the maximum value or amplitude of that current is $100 \times \sqrt{2}$, or approximately 141·2 amperes, and, conversely, if we know that the amplitude of an alternating current of another value is 100 amperes then its virtual value as read on an ammeter will be $\frac{100}{\sqrt{2}}$, or approximately 70·7 amperes.

This relation between virtual and maximum values is important in practice when dealing with alternating E.M.F.s, since danger to life and liability to break down insulation depends, not on the virtual, but the *maximum* value of the pressure. For example if an alternating current is to be transmitted along an insulated cable at a pressure of 10,000 virtual volts the insulation of that cable must be designed to withstand a pressure, not of 10,000, but of over 14,000 volts.

In practice the qualifying epithet " virtual " or " effective " is omitted, and the term is understood in referring to an alternating current of so many amperes or to a pressure of so many volts.

CHAPTER II.

INDUCTANCE.

5. Self-induction.—6. Choking Coils.—7. Phase Difference due to Inductance.—8. Reactance due to Inductance.

5. Self-induction.

Mention has already been made of the modifying influence exercised by inductance in an electric circuit on the elementary form of Ohm's law, when the current in that circuit is constantly varying in value. This inductance has been compared to mechanical mass, in that its presence tends to oppose any change in the current value, just as momentum or inertia tend to oppose any change in the motion of moving bodies.

In order to understand the effect of inductance it is necessary to keep very clearly in our minds the fundamental law of electric induction, which is as follows :—

Whenever relative motion takes place between a conductor and a magnetic field, so that the lines of force of that field are cut by or linked with the conductor, an E.M.F. is induced in the latter, the magnitude of this E.M.F. depending solely upon the time rate of cutting of the lines of force by the conductor ; or, in symbols,

$$e \propto \frac{dn}{dt},$$

e being the induced E.M.F., n being the total flux.

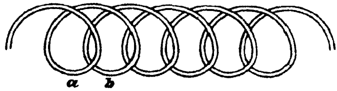

Fig. 9.

Let us now imagine an electric circuit, part of which consists of a number of loops formed by the conductor and arranged like an ordinary helical spring, such an arrangement being known as a **solenoid,** as shown in Fig. 9. If the ends of this circuit are attached to a source of E.M.F. a current will begin to flow, and will, of course, pass through the loops forming the solenoid. This current will produce, as we already know, a magnetic field round the conductor, the magnetic lines of force taking the form of concentric rings whose centre is the centre of the conductor in section. At the instant when the current begins to flow these lines of force begin to form round the conductor, and the magnetic field does not reach a steady value until the current has itself reached a steady value. Now, an alternating current is, unlike a continuous current, constantly changing in value, and, furthermore, as we have already seen, its rate of change is not uniform.

Let us now consider the effect produced in a solenoid by the flow of such a current, taking, for the sake of making the case as simple as possible, two adjacent loops only, *a* and *b* in Fig. 9. Imagine a current to begin to flow in these loops. Lines of magnetic force produced by this current will begin to grow round the conductor as the current begins to flow, and during the process of their growth they will be cut by any conductor which lies in their path ; that is to say, the lines of force forming round conductor *a* will be cut by conductor *b*, and *vice versa*. This motion of lines of force with respect to adjacent conductors will continue so long as the current is altering in value, and the more rapid is the alteration in the current value the greater will be this relative motion.

Now, we know that whenever and however such lines of force are cut by a conductor an **E.M.F. is induced in that conductor.** It follows, then, that the growth of the magnetic field round *a* must result in an E.M.F. being induced in *b*, and *vice versa*.

This E.M.F. is called the **E.M.F. of self-induction** in a circuit which is so arranged that the lines of force emanating from a conductor, forming part of the circuit, cut adjacent conductors forming part of the same circuit.*

* This self-induced E.M.F. is always, to a greater or less extent, in evidence in any circuit, even if it consists of perfectly straight conductors. The effect in this case is, however, so very slight that in low-frequency work we may consider such a circuit to be practically " non-inductive." With very high frequencies, such as are used in wireless telegraphy, the effect of self-inductance in straight conductors cannot be neglected. As a result the current in such cases is more or less confined to the skin of the conductor.

Such a circuit is said to be **inductive,** and the strength of the magnetic field, or, conventionally, the number of lines of magnetic force which are cut by the conductors, will give the degree of **inductance** of the circuit.

We will now consider the questions of the direction and magnitude of this E.M.F. of self-induction when an alternating current flows through such a circuit. Consider the periodic time *pt*, Fig. 10 (*a*), to be divided up into four quarter periods, each of 90° (electrical), and consider the mutual inductive action, during these quarter periods, of two adjacent conductors, *a* and *b*, forming part of the same circuit and shown in section in Fig. 10 (*b*).

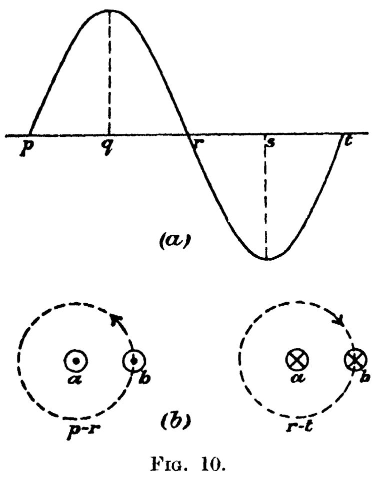

Fig. 10.

It will simplify the explanation if we consider only the inductive action of conductor *a* on conductor *b*, remembering that precisely the same effect will be produced by conductor *b* on conductor *a*, and, generally, that the action is mutual between all adjacent conductors forming the inductive winding.

During the quarter period *pq* the current in *a* is rising in a certain direction, which we will regard as positive, and imagine to be represented by the dot shown on the conductor in section, which is conventionally used to represent a current flowing out from the paper towards the observer.

As the current rises during the first quarter period *pq* the concentric magnetic field set up round the conductor *a* will be constantly varying in strength, rising in value as the current rises in value, the intensity of this field being a maximum at the point *q*. Conductor *b* during this time will then be situated in a varying magnetic field, which is equivalent to moving *b* so as to cut the lines of force, much as the conductors forming the windings of a dynamo armature are moved. An E.M.F. will, therefore, be induced in *b*, which, by a natural reactive tendency, will be in such a direction as will tend to *oppose the motion which produced it.*

This last fact is of fundamental importance, and it is essential that it should be clearly understood before proceeding further. The general fact is stated in **Lenz's** law, which says :

Any induced currents tend to oppose the motion which caused them.

The fact is, in reality, self-evident, being the electrical equivalent of Newton's second law of motion, which says that to every action there is an equal and opposite reaction. Currents induced in a dynamo armature, for example, oppose the motion which produced them, and this opposition is overcome by the pressure of the steam or other fluid in the engine which drives the machine. In what direction, then, will this E.M.F. of self-induction in conductor *b* try and send a current ? Clearly, in such a direction as will tend to prevent the growth of the current in conductor *a*, since it was the *growth* of the current which was responsible for the induction of this E.M.F. ; that is to say, during the quarter period *pq*, while the current is rising in value, the effect of inductance is to *retard its growth.*

Secondly, during the quarter period *qr* the current is *falling* in value, and the magnetic field is therefore *decreasing* in strength. The lines of force of this field then cut *b* in the opposite sense, and induce an E.M.F., which, again, will tend to oppose the motion which produced it ; that is to say, during this quarter period *qr* the E.M.F. of self-induction will endeavour to *prevent the fall of the current.*

Exactly similar actions will take place during the quarter periods *rs* and *st*, both the current and the direction of the E.M.F. of self-induction being in opposite senses to what they were during the quarter periods *pq* and *qr*.

We are now in a position to see clearly the analogy between this inductance and mechanical mass or inertia.

The effect of the latter is, as we already know, to oppose any change in the motion of matter. The effect of the former, as we have just seen, is to oppose any change in the current value; that is, inductance will retard the growth of the current and oppose its fall. The effect is to modify considerably the rate of change in the current value, and to cause it both to rise and to fall much more slowly than it would do in a non-inductive circuit. But since the current is bound to complete a cycle in its periodic time, the result will be, as shown in Fig. 11, that the **amplitude of the current will be reduced,** since the *slope* of the current curve—that is, its rate of change—will be considerably reduced by the presence of inductance in the circuit. The difference in the rates of change in current value at a given

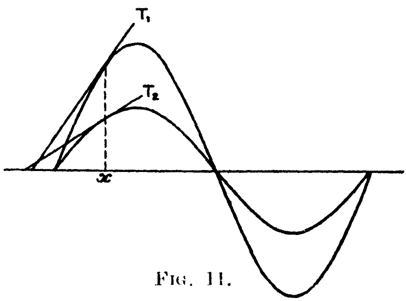

FIG. 11.

instant, x, is clearly shown in Fig. 11 by the relative slopes of the tangents to the curves at this point.

If we take two circuits, both of the same ohmic resistance, one possessing considerable inductance and the other being non-inductive, and apply to the ends of these circuits exactly the same alternating E.M.F., the currents will not be the same in the two circuits, but will, in fact, be considerably less in the inductive circuit than in the non-inductive, owing to the **choking** effect of the E.M.F. of self-induction.

Inductance, then, in an alternating-current circuit acts as a kind of resistance, and its effect must be considered together with the ohmic resistance before we are in a position to calculate the current which will flow by virtue of a given E.M.F.

6. Choking Coils.

The magnitude of the E.M.F. induced (or self-induced) in a conductor at any instant depends, as we have noted on p. 13, on the rate of cutting at that instant of the lines of force of the field by the conductor—*i.e.*,

$$e \propto \frac{dn}{dt}.$$

Now, if we take two solenoids, both having the same number of turns, cross-sectional area and length, and pass an alternating current of the same virtual value through each, we shall find that the self-induced E.M.F. will be greater in the case of the solenoid having the higher **permeability**; that is, in the case where the facilities offered to the flow of the magnetic flux are greater.

Taking the case of the first solenoid, the flux at any instant

$$n_1 \propto b_1 \text{A},$$

b_1 being the flux density at that instant, and A the cross-sectional area (of each solenoid).

$$\therefore \quad n_1 \propto \mu_1 h,$$

μ_1 being the permeability of the first solenoid, and h the instantaneous value of the magnetising force in the case of each solenoid. Since h varies as i (the current),

$$n_1 \propto i\mu_1$$

$$\therefore \frac{dn_1}{dt} \propto \frac{d(i\mu_1)}{dt},$$

$$\propto \mu_1 \frac{di}{dt}.$$

Similarly, in the case of the second solenoid,

$$\frac{dn_2}{dt} \propto \mu_2 \frac{di}{dt}.$$

That is to say, in a solenoid of given dimensions, through which is flowing an alternating current of a given value, the self-induced E.M.F. at any instant depends upon the product of the permeability * of the magnetic circuit, and the time rate of change of the current at that instant.

---

* As a matter of fact, the permeability of the magnetic circuit will not be strictly constant over a complete cycle (unless its value is unity), but may conveniently be considered constant for the present. It is very nearly so in the case of electromagnets having straight cores. More will be said on this subject under the head of Transformers.

Postponing for the moment the consideration of the second factor, and dealing only with the first, we know that the magnetism of any solenoid, and therefore the permeability of the magnetic circuit, is immensely increased by filling the interior with an iron core. The inductive value of a solenoid carrying an alternating current will therefore be greatly increased by so doing. The alternating-current electromagnet so formed is known as a **choking coil,** from the choking or throttling effect of inductance on the current wave, which we have already investigated.

The practical application of a choking coil is important, as it enables us to reduce the current readily and *economically* without altering the ohmic resistance of the circuit. Imagine two similar groups of lamps, as shown in Fig. 12, the upper

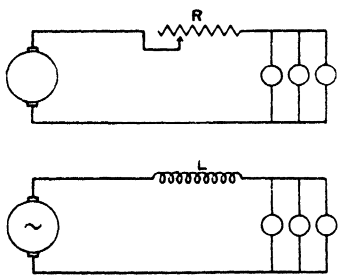

FIG. 12.

set supplied with continuous current and the lower with alternating current, and suppose the pressures of both generators to be 100 volts. For certain purposes, as, for instance, in theatre lighting, it is necessary to reduce the light given by the lamps by reducing the current through their filaments, and consequently the pressure across their terminals. In the case of the continuous-current system this can only be done by inserting *ohmic resistance* in series with the generator; and during the time that this resistance is in circuit there will be a constant loss of power in heating the resistance, equal to I^2R watts when I is the current in amperes and R the resistance in ohms. But with an alternating current the amplitude, and consequently the virtual value, of the current can be reduced by introducing inductance into the circuit by means of a choking coil, which involves no waste of power except the I^2R losses in the coil itself. These can be rendered negligible by making the turns of wire forming the coil of ample section. Instead, then, of introducing a variable *resistance*, R, as we did in our continuous system, we can obtain a similar result by introducing a variable *inductance*, L, which variation can be readily obtained by altering the amount of iron core introduced into the coil.

We may compare the difference between the wasteful and the economical methods of current reduction described above to two steam engines supplied by a boiler working at a given pressure. On the one engine there is no stop valve to regulate the steam supply, and the pressure on that engine can only be reduced by giving the steam useless work to do before it arrives at the engine. On the second engine a stop valve is fitted, by means of which the pressure may be indefinitely reduced with no loss apart from the slight increase in fluid friction due to the reduced passage through which the stream now passes.

There is one further point to notice before we leave this branch of the subject, and it is one of vital importance.

Iron is a conductor of electricity, and to bring such a conductor into the neighbourhood of a magnetic field which is continually varying in strength will infallibly mean that an E.M.F. will be induced in that conductor. Unless steps are taken to prevent it, this E.M.F. will set up local or **eddy** currents in the fibres of the iron. Now, the iron core of an alternating-current electromagnet or choking coil is not only introduced into the neighbourhood but into the very heart of a rapidly fluctuating magnetic field. If this core is left *solid*, very heavy eddy currents will circulate in it, caused by the E.M.F. induced by the repeated cutting of lines of magnetic force. What would be the result of these currents if they were allowed to flow?

Firstly, and most obviously, the iron would get extremely hot; even hot enough, if it touched the coils of the solenoid, to cause a fire by igniting the insulation of the leads forming the coil.

Secondly, and this is a fact of primary importance, these eddy currents would **reduce the choking effect of the coil.** The reason should not be difficult to discover. The generation of sufficient heat energy to raise the temperature of the iron necessarily implies extra power from somewhere. Now, the only source of power is the generator, which is supplying current to the coil at a certain virtual pressure. This extra power *must* then, come from the generator, and since the virtual pressure of the latter is practically independent of its load, *more current* * must flow from the generator, and therefore through the choking coil, in order to supply this extra power which is manifested as heat in the iron core. We shall meet this fact later on, and shall see how this **secondary E.M.F.** can be usefully employed to send a current through a wholly separate circuit.

For the above reasons, the iron core of any alternating-current electromagnetic apparatus must be **laminated** in the direction of the magnetic flux—that is, split up into a large number of thin plates which are insulated from one another. Its construction is, in fact, similar to that of the core of a dynamo armature, and for a precisely similar reason. This splitting up or laminating of the core will prevent any considerable eddy currents from flowing, though it is, in practice, impossible to get rid of them entirely. Such eddy currents will then constitute a loss of efficiency in all such apparatus, though, by careful design, this loss can be reduced to a very small value.

A further loss of power will also be apparent. If an electromagnet is energised by means of an alternating current, the iron core is continually reversing its polarity, the end which was a north pole during the first, or positive, half of the current period, being a south pole during the second, or negative half period. These reversals of polarity are taking place extremely rapidly, and there will be a natural tendency for one end of the core to retain its north polarity after the current has reversed in direction. This property, known as **residual magnetism,** is familiar to us from continuous-current work, and depends very largely upon the physical properties of the iron forming the core. Very soft and pure iron, for example, exhibits little residual magnetic properties, while hard steel retains a considerable amount of magnetism after the magnetising force has been removed. The lagging effect caused by residual

* This explanation is not complete, for reasons which we shall be in a position to appreciate when we are studying the action of transformers.

magnetism is known by the Greek synonym **hysteresis.** The effect in a choking coil is that the magnetic field is never quite so strong as it would otherwise be, since part of the rising current is employed in reversing the existing magnetism produced during the previous half period. This will naturally reduce the choking effect of the coil, just as do eddy currents, and therefore imply extra power from the generator, this power being dissipated in the form of heat in the core.

Cores for this class of apparatus must then be made of very soft iron which will exhibit as little as possible of this residual magnetic property.

In short, the problem of reducing to a minimum losses due to eddy currents and hysteresis is precisely similar to that with which we are already familiar in the case of the armature core of a dynamo.

7. Phase Difference due to Inductance.

So far we have considered only the effect on the E.M.F. of self-induction of the permeability of the magnetic circuit. The second factor on which its value depends is one of very great importance and far-reaching effects; that is, the rapidity with which the current is altering in value at any instant. Briefly, on **the rate of change of the current,** since the field strength is changing most rapidly in value when the current which produces it is varying most rapidly. The rate of change of motion, or acceleration, of any body which possesses simple harmonic motion, is proportional, as already shown, to the cosine of the angle through which that body has moved from rest. This is capable of very simple mathematical proof. As applied to an alternating current, we may write, as before,

$$i = \mathrm{I}_{\mathrm{max.}} \sin \theta.$$

The rate of change of the current I with respect to the angle θ is expressed mathematically, then, as

$$\frac{di}{d\theta}, \text{ which is equal to } \mathrm{I}_{\mathrm{max.}} \cos \theta.$$

Now, $\cos \theta$ is a maximum when

$$\theta = 0°, \text{ or } 180°, \text{ or } 360°,$$

when its value is unity.

That is, $\frac{di}{d\theta}$ is a maximum when $\theta = 0°, 180°, 360°$, &c.

In other words, the current is **changing in value most rapidly when its strength is zero**, that is, at the points *a a* in Fig. 13.

It follows, then, that in any choking coil of a given inductance the E.M.F. of self-induction is a maximum at the instant when the current through that coil is zero, and, similarly, that this E.M.F. is a minimum when the current has its maximum value. When $\theta = 90°$, for example, $\cos\theta = 0$—*i.e.*, $\frac{di}{d\theta} = 0$, or in other words, at this precise instant the current value is momentarily steady, so that the field produced by it is momentarily steady. There is, therefore, for this infinitely short space of

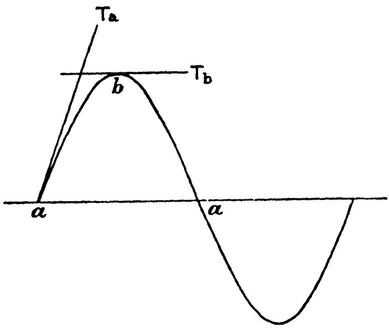

FIG. 13.

time, no E.M.F. of self-induction, since there is no relative motion between conductors and field.

These facts can be readily verified by inspection of the wave form of the current (Fig. 13). It is obvious that the current is rising (or falling) most rapidly when the *slope* of the current wave is steepest, and conversely. The slope of the tangent to the curve at *a* (T_a in Fig. 13) is obviously a maximum, and that of the tangent at *b* (T_b) a minimum.

Now, we have already seen that inductance in a circuit acts as a kind of resistance by reducing the amplitude of the current wave. Let us then imagine a circuit possessing a certain total resistance, R, and also inductance, through which it is desired to send a current, I. Now, part of the pressure which it will be necessary to apply to this circuit will be employed in overcoming the ohmic resistance R and part in overcoming the apparent resistance due to the inductance, that is, in balancing the E.M.F. of self-induction.

We may arrive at the necessary E.M.F. graphically by drawing the wave form of the two parts or **components** of this E.M.F. First let us draw, as shown in Fig. 14, the component necessary to overcome the ohmic resistance. This may be called, by Ohm's law, IR. Taking R as unity for the sake of simplifying the diagram, this curve will also represent the current value to the same scale.

Next, the wave form of the E.M.F. of self-induction may be plotted to the same scale as IR. Since this E.M.F. is always opposing any change in the current value, it will obviously

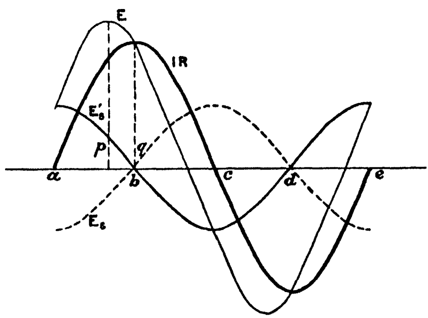

FIG. 14.

be opposing the current when it is rising and acting with it when it is falling; that is, during the quarter periods *ab* and *cd*, when the current is rising, the E.M.F. of self-induction will be acting against the current, *i.e.*, will be negative between *ab* and positive between *cd*. Furthermore, this E.M.F. has its maximum value, as we have just seen, at the points *a*, *c*, *e*, and is zero at the points *b* and *d*.

Between the points *bc* and *de*, when the current is *falling* in value, the E.M.F. of self-induction will be acting *with* the current, since it is now trying to prevent its fall; that is, it will be positive during the quarter period *bc* and negative during the quarter period *de*. We are now in a position to draw our wave form E_s in Fig. 14, having its maximum positive value at *c* and its maximum negative values at *a* and *e*.

The volts from our generator required to balance this

E.M.F. of self-induction must necessarily be at each instant equal in value, but opposite in sign to E_s. The wave form of this component of the applied volts will then be as shown by the curve E'_s in Fig. 14.

To find the total volts necessary to send a current, I, through this circuit we have only to add together the curves IR and E'_s. This curve is shown at E in Fig. 14.

Something entirely novel and unexpected is now apparent. The curves E and I(R) represent the wave forms of pressure and current respectively; but it will be at once obvious that the former is reaching its maximum value before the latter, that is, the current **lags behind the pressure.**

Now we are really perfectly familiar with the corresponding mechanical phenomenon. Suppose we wish to move a truck backwards and forwards by alternately pushing and pulling it. On first pushing it the truck will not move immediately. It will, in fact, not move until its inertia has been overcome by our push, that is, its forward motion takes place *some period of time* after the pressure was first applied to it. Similarly, when we wish to reverse its motion, we shall have to reverse our pressure, *i.e.*, start *pulling* the truck *some period of time* before the truck will reverse its motion, owing to the momentum which it has acquired in a forward direction. The greater is the *mass* of the truck, and therefore the more marked is the effect produced by inertia and momentum, the more will its motion lag behind the applied pressure.

We can then readily understand the reason for our current lagging behind the volts, as being caused by the electrical momentum in either direction. The more inductive is the circuit the greater will be the component E'_s of the volts, and the more will the current lag. The length *pq* in Fig. 14 will represent the amount of lag, and is conveniently expressed as an angle, whose value is readily seen by comparing the length *pq* with that of *ae*, which represents 360°, assuming, as before, that the conductors are moving through the field at a constant angular velocity.

This angle is called the **angle of lag,** and is generally known by the symbol φ.

This angle φ will vary according to the relative amplitudes of the curves E'_s and IR, that is, on the comparative effects of inductance and resistance in the circuit. If the resistance is infinitely small the component IR of the applied pressure disappears, and the angle ϕ is now a maximum, its value being obviously 90°. The angle of lag may, then, have any value between 0° in a non-inductive circuit and 90° in a circuit which possesses no ohmic resistance, the latter case being, of course, purely theoretical.

When $\phi = 0°$—*i.e.*, when the component E'_s of the volts is zero—pressure and current are said to be **in phase** with one another; the effect of inductance in causing this lag of current behind volts is known as **phase difference,** or the current and volts are said to be mutually **out of phase.**

Now, we have already regarded our E.M.F. as being made up of two components—namely, IR in phase with the current and E'_s 90° out of phase with the current. We may then

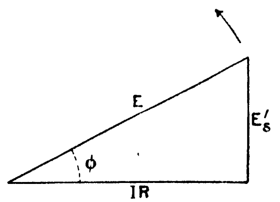

FIG. 15.

regard the total E.M.F. (E) as being made up of two components at right angles to one another, and knowing the value of any two of these three quantities we may graphically arrive at the third by constructing a "triangle of pressures" just as we construct a triangle of forces. Such a triangle is shown in Fig. 15, where E is the applied pressure, E'_s the component of that pressure necessary to balance E_s, and IR the component (at right angles to E'_s) necessary to overcome the ohmic resistance.

This fundamental "triangle of pressures" should be thoroughly understood, as it is of great importance for future calculation. It is of no consequence whether we take maximum or virtual values for its construction, provided that all three sides of the triangle are taken to represent values which correspond to one another. For example, if E'_s and IR are plotted to scale to represent the *virtual* values of the two components, then E will represent the *virtual* value of the applied volts. The triangle will be similar if we wish to represent maximum values, each side being multiplied by $\sqrt{2}$.

The triangle may be regarded from another point of view. Let us represent current and pressure by means of revolving lines or **vectors,** as we did in our early investigations (*see* p. 5). Suppose OE (Fig. 16) to represent the applied pressure to any convenient scale, and suppose this vector to have revolved in a counter-clockwise direction through 90°, so that the pressure has attained its maximum value. The current vector OI must be drawn *behind* the pressure vector OE, since the current has not yet attained its maximum value, φ being the angle of lag between current and pressure. From E draw ER perpendicular

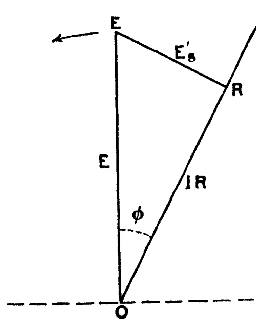

FIG. 16.

to OI. OR will now represent the component of OE in phase with the current, *i.e.*, it will be equal to the product of the current and the ohmic resistance. ER will represent the component E'_s at right angles to the current vector. This diagram should be compared to Fig. 14, which shows the wave forms of the three components.

A glance at the vector diagram in Fig. 16 will show that the value of the two components is given by the equations,

$$IR = E \cos \varphi.$$
$$E'_s = E \sin \varphi.$$

By taking OI as representing the current value to any convenient scale we can, in a similar way, regard the current as being made up of two components :

I cos φ in phase with the volts,

I sin φ at right angles to the volts, or, as it is sometimes expressed, **in quadrature** with the volts.

8. Reactance due to Inductance.

Our next step is to find an expression which will give us a value for the E.M.F. of self-induction in a circuit of given inductance through which is flowing a current of a given strength and at a given frequency. We have already seen that this E.M.F. varies in a coil of given permeability as the rate of change of the current, which we may mathematically express by the relation,

$$E_s \propto \frac{di}{d\theta} \text{ (\textit{see} p. 22)},$$

but, since we may substitute for the scale of electrical degrees a corresponding scale of time, for the reason that our revolving conductor is moving at a uniform angular velocity, we may write,

$$\therefore E_s \propto \frac{di}{dt},$$

that is, as the rate of change of current with respect to time.

We may write this relation as an equation by inserting a constant, which is known as the **coefficient of self-induction,** and is generally expressed by the letter L.

Our equation then becomes

$$E_s = L\frac{di}{dt}.$$

The value of this coefficient, the unit of which is known as the **henry,** is chosen to correspond with our practical units—the volt, ampere and second, so that the inductance of a circuit is 1 henry,* in which an E.M.F. of 1 volt is induced by a current which varies at the rate of 1 ampere in one second.

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* The inductance or self-induction of any electromagnet

$$= \frac{4\pi S^2 A\mu}{l},$$

where S is the number of turns,
A is the cross-sectional area,
μ the permeability of the magnetic circuit,
l the length,
all being in C.G.S. units. The result will give the C.G.S. unit of inductance known as a "centimetre of inductance."

Reducing this to practical units, to correspond with the volt and ampere, we have to divide by 10^9, since 1 ohm $= 10^9$ absolute units of resistance, so that our practical expression for the self-induction of the electromagnet reads :—

$$\frac{4\pi S^2 A\mu}{l \times 10^9} \text{ in \textbf{henrys},}$$

so that 1 henry $= 10^9$ cm. of inductance.

Now, we are already familiar with the equation

$$i = I_{max.} \sin \theta ;$$

and, assuming that the conductor has rotated through an angle, θ, in t seconds, we may write this equation, as explained on p. 7,

$$i = I_{max.} \sin \omega t,$$

from which $$\frac{di}{dt} = \omega I_{max.} \cos \omega t.$$

But $$E_s = L\frac{di}{dt};$$

$$\therefore \quad E_s = \omega L I_{max.} \cos \theta$$
$$= \omega L I_{max.} \sin (\theta - 90°)$$
$$= -\omega L I \text{ 90° out of phase with I.}$$

Drawing now our "triangle of pressures" we may write for E_s (or E'_s)

$$-\omega L I \text{ (see Fig. 17).}$$

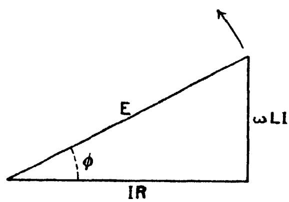

FIG. 17.

From this we arrive at the equation,

$$E = \sqrt{(-\omega L I)^2 + (IR)^2},$$
$$= I\sqrt{(-\omega L)^2 + R^2},$$

or $$I = \frac{E}{\sqrt{(-\omega L)^2 + R^2}}.$$

This will then give us the modified form of Ohm's law when inductance is present in the circuit, and the obstruction to the flow of the current is made up of ohmic resistance, and the apparent resistance due to inductance. This latter term—namely, ωL—is spoken of as **reactance** and the resultant of the two, or $\sqrt{(-\omega L)^2 + R^2}$, as the **impedance.**

The effect of the frequency on the reactance is very important, particularly in certain cases of oscillating currents when the frequency is extremely high. In such cases even a small amount of inductance offers almost a total obstruction to the current. A common practical example is the case of lightning, which may be regarded as an alternating current of very high frequency. Any inductance present in the path of such a discharge will, therefore, cause an enormous impedance.

The unit henry is inconveniently large, as will be readily understood when we realise that the number of turns and sectional area of an alternating-current electromagnet would have to be very great in order that 1 volt might be induced by a current which varied as slowly as 1 ampere per second. In practical work the whole cycle is completed in a space of time of $\frac{1}{25}$ of a second or less, so that the rate of change of the current is very much higher than 1 ampere per second.

A convenient practical sub-multiple is therefore used—namely, the **millihenry,** or $\frac{1}{1000}$ part of the henry.

EXAMPLE.—The inductance * of a circuit is 25 millihenries, its resistance being 20 ohms. If an alternating E.M.F. of 1,000 volts at a frequency of 100 cycles per second is applied to the circuit,

(1) What current will flow through the circuit ?

(2) What will be the angle of lag ?

(1) $$I = \frac{E}{\sqrt{(-\omega L)^2 + R^2}}$$

Since $$\omega = 2\pi f,$$

$$I = \frac{1{,}000}{\sqrt{\left(\frac{2\times 22}{7}\times 100\times\frac{25}{1{,}000}\right)^2 + 20^2}}$$

$$= \frac{1{,}000}{\sqrt{(-15{\cdot}7)^2 + 20^2}}$$

$$= 39{\cdot}3 \text{ amperes.}$$

(2) $$\tan\varphi = -\frac{\omega L}{R},$$

$$= \frac{-15{\cdot}7}{20},$$

$$= -0{\cdot}785,$$

$$= \tan^{-1} \doteq -38^\circ$$

$$\therefore \quad \varphi = 38^\circ, \text{ lagging.}$$

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* This is generally assumed to be constant, though it really varies, of course, with the permeability (*see* footnote on p. 28).

CHAPTER III.

CAPACITY.

9. Capacity.

In the introductory remarks at the beginning of Chapter I. mention was made of an electrical analogy to mechanical elasticity. A little more difficulty is usually experienced in grasping this question than in studying the effect of inductance in alternating-current work. This is due to the fact that magnetic phenomena are among the first to be studied by the beginner, and his earliest lessons in continuous-current work will include the laws of induction between conductors and magnetic fields.

We must now, however, be prepared to consider phenomena which have little weight in direct-current work, the elementary investigation of which really belongs to the branch of the science known as "statical electricity."

Briefly, the essential fact may be stated as follows :—

If two conducting substances, such as two plates of metal separated by air or some other insulator, be connected to a steady source of electric pressure, a current will flow for a very short space of time, the action of this current being to **charge** one of the plates, with respect to the other, with a certain definite **quantity** of electricity (expressed in coulombs), this quantity being dependent on the value of the charging pressure and the **capacity** of the arrangement. Any such arrangement of conducting surfaces which is capable of receiving a charge is known as a **condenser.**

A common practical example is a two-core cable, each core of the cable acting as one of the plates of a condenser. When a steady pressure is first applied to such a cable the two cores become charged with respect to one another, the pressure which exists between them being, of course, equal to the steady applied pressure. So long as this *pressure* is kept steady, the capacity effect of the cables will not cause complications, any more than, as we have already seen, will inductance in a circuit so long as the *current* remains steady.

But in alternating-current work our pressure is not steady. It will, in fact, be constantly varying both in degree and sense. Any capacity, then, in our circuit will have constantly varying effects, the cores of our cable or any condensers in the circuit constantly receiving and expelling charges of opposite signs.

It will be convenient, in studying the effects of capacity, to speak of a condenser consisting of two flat plates of conducting material close to one another, remembering that exactly the same rules apply to *any* conducting surfaces which are in proximity (as, for example, the two cores of a cable), including, among our conducting surfaces, that of the earth. The word "proximity" must also not be misunderstood. Just as the magnetic field surrounding a conductor may be considered as extending for an infinite distance round that conductor, so may the "electric" field lying between the plates of a condenser be considered to exist, however far apart those plates may be.

The first point to notice, before proceeding further with the subject, is this: The insulating material or **dielectric** (whether air or some other substance) between the plates is in a state of mechanical strain, this strain being caused by the electrical stress, which stress is due to the fact that when the condenser is in a charged condition the plates are at different potentials.

The steel plates forming the shell of a steam boiler are in a similar condition of strain, due to the difference of pressure between the steam inside the boiler and the atmosphere.

If, now, the two plates of a charged condenser are connected by means of a conductor, a current will flow for a short space of time until the plates are mutually discharged, or brought to the same state of pressure. Now the fact that it takes a definite *period of time*, however small, both to charge and discharge a condenser, suggests that such an apparatus exhibits **elastic** properties. A mechanical analogy may perhaps assist us in realising this point, which is of vital importance. Suppose a closed steel vessel to be filled with water, at a certain pressure above that of the atmosphere. If even a pinhole be now made in this vessel the pressure will *instantly* fall to that of the atmosphere, assuming, as is very nearly true, that water is incompressible. If, however, a closed vessel made of rubber, or other **elastic** material, be filled with water at a certain pressure above that of the atmosphere, and a pinhole be made in that vessel, water will flow for a certain *period of time*, just as did the current from the discharging condenser, before the pressure in the vessel falls to that of the atmosphere.

Capacity may then be compared to mechanical elasticity, and we shall find the analogy useful in our future investigations.

The second elementary point to notice is this: A condenser such as we have described offers a practically total obstruction to the flow of a continuous current, since the insulating material or **dielectric** shown at *d* in Fig. 18 interposes a nearly infinite ohmic resistance between the points *a* and *b*, at which the pressure is applied.

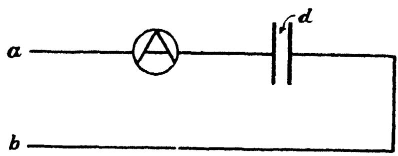

FIG. 18.

But it does *not* offer such obstruction to the flow of an alternating current. In order to understand the reason for this fact let us consider the effect on an ammeter connected in series with a condenser, as shown in Fig. 18. If, now, a steady pressure be applied to the circuit the needle of the ammeter will show a momentary deflection, indicating the flow of a charging current into the condenser. This throw will only be momentary, since the current will cease as soon as the condenser is fully charged—that is, when the potential between the plates, or the "back E.M.F." of the condenser, is equal to the applied pressure.

If, now, we apply an alternating E.M.F. to the circuit the condenser will be periodically charged in opposite senses, and the tendency for the ammeter needle would be to throw one way while a charging current was flowing in one sense, and the other way when the charging current was flowing in the opposite sense, and its amount of deflection would register at any instant the value of the charging current at that instant. But, as we already know, alternating-current instruments are designed to read not momentary, but virtual current value. If the ammeter depends for its action on the heating effect of the current, for example, as some types do, the passage of this charging current through it will produce a certain steady temperature in its windings, and therefore a steady deflection, in spite of the fact that no current is actually passing *through* the condenser.

In general terms we may say that the effect on an ammeter is the same as if this were so—that is, we may regard a condenser as not offering an obstruction to the flow of an alternating current which is at all comparable to the almost total obstruction it offers to a continuous current.

A certain reactance is, in point of fact, attributable to such a condenser ; but, if the capacity of the condenser has a reason-

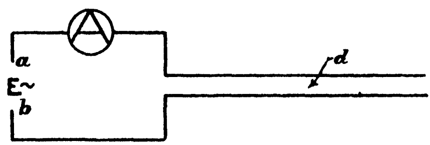

Fig. 19.

able value, the reactance is comparatively small. It will be our business to obtain a value for this reactance, as we did tor that due to inductance, in due course.

We have seen that under the influence of an alternating E.M.F., a current will flow in and out of the condenser in alternate directions, and will, therefore, be recorded on an ammeter placed in the circuit. This will account for the fact that when an alternator supplying current through a two-core cable is switched on to the mains the machine ammeter will record a current though the circuit is "open"—that is, the two cores of the cable are entirely unconnected. The circuit is shown in Fig. 19, which is similar electrically to Fig. 18, the two cores of the cable acting as the two plates of an elementary condenser, and a charging current, therefore, flowing in and out through the ammeter A under the influence of the pulsating pressure from the alternator.

10. Phase Difference due to Capacity.

We have now to consider the effect of an alternating pressure when applied to a condenser. To do so let us follow out one complete cycle and imagine the result produced.

As the pressure begins to rise in a positive sense the condenser will begin to charge, this process of charging continuing through the first 90° or quarter cycle between the points *a* and *b* in Fig. 20. Now, directly the condenser has received any charge it will, as already explained, be in a state of strain, and, owing to its elastic properties, it will constantly be endeavouring to *discharge* itself—that is, to equalise the pressure of its plates. We can then imagine the condenser to possess a back E.M.F.

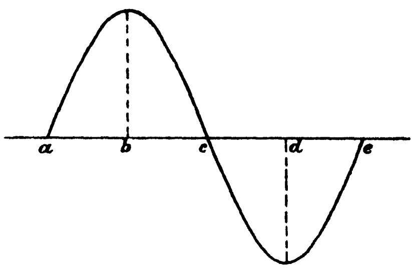

FIG. 20.

(E_c) which always opposes the charging pressure. Now, at the instant *b* the condenser is in a maximum state of charge, and the instant the charging pressure is *relaxed*, or reduced in value, the condenser will begin to discharge, and will continue doing so until the plates are reduced to the same potential, which will occur at the point *c*. The process of charge and discharge will be repeated in the same order but in the opposite sense during the quarter periods *cd* and *de*.

Our charging current will, therefore, be flowing in a positive sense during the quarter period *ab*, and will flow *with* the pressure while the latter is rising. During the quarter period *bc*, however, the charging current will flow in a *negative* sense, or *against* the pressure, while the latter is falling, since the condenser is, during this time, discharging itself.

A mechanical example may make the matter clear. Imagine a pair of pistons working in the cylinders P and Q, Fig. 21 (*a*), and connected by a rod. First suppose the cylinders to be filled with air * and the system to be in a state of balance—*i.e.*, the air in P and Q to be in the same state of compression. This will correspond to the moments *a*, *c* and *e* in Fig. 20, when the pressure is zero. Now let a periodic pressure be applied to the rod, so that it is given a reciprocating motion. As soon as an upward pressure is applied to the rod it will begin to move upwards, and the faster is the *rate of change* of this pressure the greater will be the rate of motion.

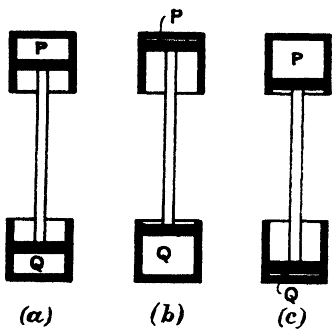

FIG. 21.

At the instant corresponding to *b* in Fig. 20 the upward pressure will be a maximum, and therefore the charge in P with respect to Q will be a maximum, and at this instant motion will cease, since the pressure is about to be relaxed. This state of affairs is shown in Fig. 21 (*b*). Now, directly this pressure is relaxed the rod and pistons will begin to move downwards, and, as before, the faster is the *rate of change* of the pressure the greater will be the rate of motion. The system will move *downwards*, although the pressure is still in an upward direction so long as that pressure is falling in value, until the states of compression in P and Q are again balanced, which will correspond to the point *c* in Fig. 20, at which point the pressure is again zero. The exact process will be repeated

* The analogy would really be closer if we considered elastic cylinders filled with incompressible fluid, which passed from one to the other in turn.

during the next half period, the state of affairs at the point d in Fig. 20 being shown in Fig. 21 (c).

In other words, the motion of the rod will be in the same direction as the pressure while the latter is *rising* (in either sense), and in the opposite direction while the pressure is falling. Furthermore, the motion of the rod will **lead** the pressure, since the rod will begin to move downwards one quarter of a period before the pressure begins to be exerted in a downward or negative sense.

We are now in a position to plot the wave forms of current and pressure in a corresponding electrical sense. Imagine, first of all, a circuit possessing no ohmic resistance to which is applied an alternating pressure whose wave form is plotted (E) in Fig. 22.

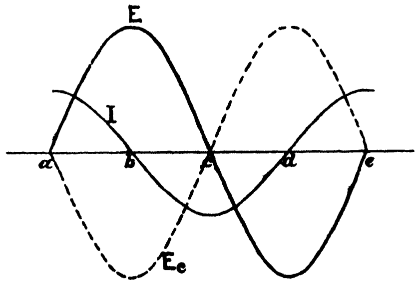

FIG. 22.

During the quarter period ab the condenser will be charging, and the amplitude of the charging current (just as the amplitude of motion of our piston rod) will be a maximum when the *rate of change* of the pressure is a maximum—*i.e.*, at the points a, c and e in Fig. 22. At b and d the current will be zero, since the charging pressure is momentarily steady, this instant corresponding to the end of the stroke of our pistons in either direction. And lastly: during the quarter periods ab and cd the current will be flowing *with* the pressure, since the latter is rising, and against the pressure during the quarter periods bc and de, since the latter is falling. Our current curve will then lie as shown at I in Fig. 22, and it will be seen that it will **lead** the pressure, the angle of lead being 90° when the ohmic resistance is zero, just as the angle of lag is 90° in an inductive circuit without resistance. The back E.M.F. of the condenser is, of course, equal and opposite to the applied E.M.F (since the latter has only this back E.M.F. to overcome), and is shown at E_c in Fig. 22.

The result of capacity and ohmic resistance in series may now readily be shown, as was done in the case of an inductive circuit. Let IR (Fig. 23) represent the volts necessary to overcome the ohmic resistance, the curve representing the current value by taking R as unity. E_c may be plotted as in Fig. 22. The volts necessary to balance E_c are plotted at E'_c, being, of course, equal and opposite. The resultant volts necessary to produce the charging current are found, as before, by adding the curves IR and E'_c, and are plotted at E. The current is now seen to *lead* the volts, the angle of lead (φ) being *pq*.

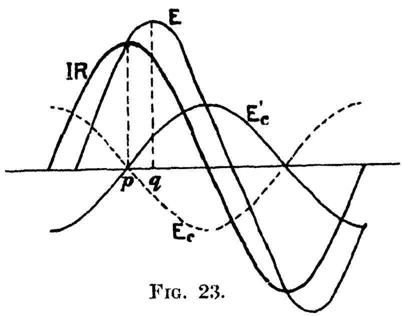

FIG. 23.

11. Reactance Due to Capacity.

We have already seen on p. 31 that the quantity of electricity, expressed in coulombs and denoted by the symbol q, which is stored at any instant in a condenser, depends upon the charging pressure at that instant.

We may then write

$$q \propto e,$$

or, by introducing a constant C,

$$q = Ce.$$

This constant is chosen to correspond with our practical volt-ampere units, and may therefore be defined as

The capacity of a condenser which is charged with one coulomb by a pressure of one volt.

The name given to this unit of capacity is the **farad**. In point of fact, just as the henry is an unduly large unit of inductance, so is the farad an unduly large unit of capacity, as it would take a condenser of very large dimensions to be charged with one coulomb by as low a pressure as one volt.

The submultiple **microfarad** is therefore used, which is equal to one-millionth part of a farad.

In order to obtain a value for the reactance due to any capacity in our circuit, we must find a relation between charging current and applied pressure.

Now the quantity of electricity stored in a condenser to which is applied an alternating pressure is continually varying. We may express this rate of change as so many units of quantity per unit time; that is to say, at any instant, the condenser is charging or discharging, as the case may be, at the rate of so many **coulombs per second.** But we have a name for the "coulomb per second," that is, the ampere, our unit of current. We may then express our rate of change of quantity at any instant in *amperes*, this number of amperes being a measure of the charge (or discharge) current at that instant.

Expressing this in symbols, we write

$$\frac{dq}{dt} = i.$$

But since $q = \mathrm{C}e$

$$\frac{dq}{dt} = \frac{d(\mathrm{C}e)}{dt},$$

$$= \mathrm{C}\frac{de}{dt}.$$

But $$e = \mathrm{E}_{\text{max.}} \sin \omega t,$$

$$\therefore \quad \frac{de}{dt} = \omega \mathrm{E}_{\text{max.}} \cos \omega t.$$

But $$i = \frac{dq}{dt} = \mathrm{C}\frac{de}{dt},$$

$$\therefore \quad i = \omega \mathrm{CE}_{\text{max.}} \cos \omega t$$

$$= \omega \mathrm{CE}_{\text{max.}} \cos \theta$$

$$= \omega \mathrm{CE}_{\text{max.}} \sin (\theta + 90).$$

$$\therefore \quad \mathrm{I}_{\text{max.}} = \omega \mathrm{CE}_{\text{max.}} \text{ } 90^\circ \text{ out of phase with E,}$$

i.e., $$\mathrm{I} = \omega \mathrm{CE}.$$

This expression should be compared with that found on p. 29, when investigating the relation between current and pressure in an inductive circuit, namely,

$$E = -\omega LI$$

or
$$I = \frac{E}{-\omega L}.$$

That is, with capacity in circuit the current varies directly as the capacity, while in an inductive circuit it varies inversely as the inductance, as we should naturally expect.

We may construct our triangle of pressures just as we did

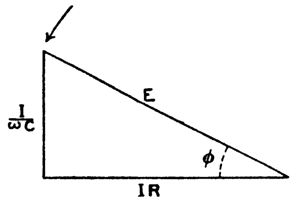

FIG. 24.

in the case of an inductive circuit; the two components of the volts being IR and $\frac{I}{\omega C}$ at right angles to one another, as shown in Fig. 24. The reactance due to capacity may be written then as
$$\frac{1}{\omega C}.$$

The expression for Ohm's law, when capacity is present in the circuit, therefore becomes

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

CHAPTER IV.

RESONANCE.

12. The Combined Effect of Inductance and Capacity.

It will be noticed on p. 29 that the expression for the reactance due to inductance has been written with the minus sign, since the current in this case lags behind the pressure.

It is, in practice, quite immaterial whether we write this expression with a plus or with a minus sign, so long as the

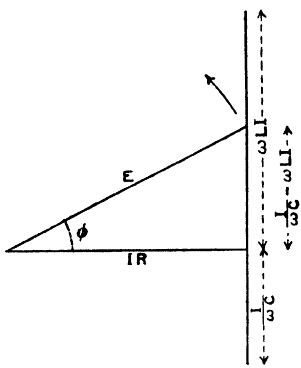

Fig. 25.

expression for the reactance due to capacity is written with the opposite sign.

The reason is that since the effect of inductance is to make the current *lag*, and that of capacity to make it *lead*, if both inductance and capacity are present in a circuit simultaneously they will tend to neutralise one another, as shown in Fig. 25, and to bring the current into phase with the volts.

To obtain the resultant reactance we must, therefore, take the difference between the two, and since this difference is squared in order to be added in quadrature with the resistance it is immaterial which we consider the minus quantity.

In an alternating-current circuit, then, which possesses ohmic resistance, capacity, and inductance in series with one another, the relation between pressure, current and impedance becomes

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}.$$

This may be clearly expressed graphically by means of a simple vector diagram.

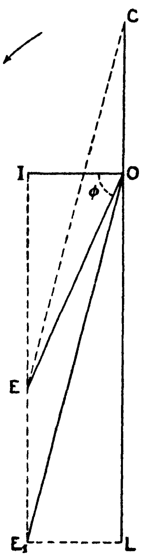

FIG. 26.

The pressure necessary to send a certain current through the circuit is the resultant of three components :—

(*a*) That necessary to overcome the ohmic resistance, which we will call IR, and which is, as we have already seen, in phase with the current.

(*b*) That necessary to overcome the E.M.F. of self-induction, which we will call E'_s.

(*c*) That necessary to balance the back E.M.F., due to capacity, which we will call E'_c.

The vector diagram shown in Fig. 26 may then be constructed. Let us suppose that the current is to be 10 amperes, the resistance of the circuit being 2 ohms, the frequency of the supply being 50 ~, the inductance 25 millihenries, and the capacity 1,000 microfarads.

(*a*) The IR component (OI) may be drawn first and will equal 10 × 2 or 20 volts to any convenient scale. This vector will be the direction of the current.

(*b*) The component E'_s can now be drawn, and since the effect of inductance is to make the current *lag*, this vector will be 90° in advance of OI in the direction of rotation.

Its value will be $I \times -\omega L$.

$$= -10 \times 2 \times \frac{22}{7} \times 50 \times \frac{25}{1{,}000}$$

$$= -78{\cdot}6 \text{ volts.}$$

This is shown at OL.

The resultant of these two is shown at OE_1, and is equal, of course, to

$$\sqrt{(IR)^2 + (-\omega LI)^2},$$

or

$$\sqrt{20^2 + 78{\cdot}6^2},$$

which is 81·1.

(*c*) The component E'_c will be drawn 90° behind OI, since the effect of capacity is to make the current *lead*, and its value will be

$$I \times \frac{1}{\omega C} = \frac{10}{2 \times \frac{22}{7} \times 50 \times \frac{1{,}000}{10^6}}$$

$$= 31{\cdot}82 \text{ volts.}$$

This is shown at OC.

The resultant of OC and OE_1 will then give us the required pressure vector in magnitude and direction, as shown at OE. The phase angle (φ) between pressure and current in the circuit is the angle IOE, and is in this case an angle of *lag*, since the effect of inductance in this particular circuit is more marked than that of the capacity.

The value of the resultant OE is given by the equation

$$E = \sqrt{(IR)^2 + \left(\frac{I}{\omega C} - \omega LI\right)^2}$$

$$= \sqrt{20^2 + (31{\cdot}82 - 78{\cdot}6)^2}$$

$$= 50{\cdot}87.$$

This resultant could, of course, have been obtained directly by subtracting ωLI from $\frac{I}{\omega C}$ and taking the resultant of IR and

$$\left(\frac{I}{\omega C} - \omega LI\right).$$

The value of the angle φ is

$$\tan^{-1}\frac{\frac{1}{\omega C}-\omega L}{R},$$

φ being an angle of lag when this expression is negative and of lead when the expression is positive.

We shall return to this vector diagram (Fig. 26) shortly, as demonstrating a fact of very great practical importance and far-reaching effects. But before we do so it will be necessary to say a few words on the subject of the impedance produced by inductance, resistance and capacity in parallel.

Now we are already in a position to calculate the resultant impedance of any arrangement of these three factors, on exactly the same lines as we calculate the resultant of ohmic resistances in parallel. It will only be necessary to work out a few cases, from which the reader will be able to solve for himself as many problems as he cares to.

(1) Let us take first the case of two inductances in parallel, the circuit being shown in Fig. 27. Let an alternating pressure,

FIG. 27.

E, be applied to the circuit, and let us suppose the ohmic resistance of the two coils L_1 and L_2 to be negligible, as would generally be the case in practice.

If E be the pressure across L_1 and L_2, then

$$I_1 = -\frac{E}{\omega L_1}$$

and

$$I_2 = -\frac{E}{\omega L_2}.$$

These two currents will be in phase with one another, since they both lag 90° behind the volts. Therefore, the resultant current as read on an ammeter in the main circuit will be

$$I = I_1 + I_2.$$

But $$I = \frac{E}{\text{Reactance}}.$$

$$\frac{1}{\text{Resultant reactance}} = \frac{1}{\omega L_1} + \frac{1}{\omega L_2}$$

or, in other words, the reciprocal of the resultant reactance of two inductances in parallel is equal to the sum of the reciprocals of the two reactances.

(2) Taking next the case of two capacities in parallel, as shown in Fig. 28, and again neglecting ohmic resistance.

As before, $$I_1 = \frac{E}{\frac{1}{\omega C_1}} = \omega C_1 E,$$

$$I_2 = \omega C_2 E.$$

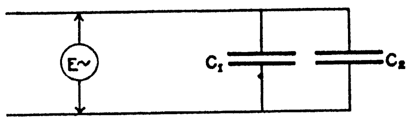

FIG. 28.

I_1 and I_2 are again in phase with one another, and therefore $I = I_1 + I_2$.

But $$I = \frac{E}{\text{Reactance}}.$$

$$\therefore \quad \frac{1}{\text{Reactance}} = \omega C_1 + \omega C_2$$

$$= \frac{1}{\frac{1}{\omega C_1}} + \frac{1}{\frac{1}{\omega C_2}}.$$

That is, as before, the reciprocal of the resultant reactance of two capacities in parallel is equal to the sum of the reciprocals of the two reactances.

This result is what we should naturally expect, since the effect of connecting C_1 and C_2 in parallel is to produce a condenser whose capacity is the sum of the capacities of the two, as shown in Fig. 29. This is equivalent to rewriting the equation above in the form

$$\text{Reactance} = \frac{1}{\omega(C_1 + C_2)}.$$

It follows, as a corollary to this, that the reciprocal of the equivalent *capacity* of two condensers connected in series is equal to the sum of the reciprocals of the two capacities. This should be worked out as a simple exercise, remembering the fundamental fact that in any simple circuit—*i.e.*, in any circuit where the various reactances are connected in *series*—the

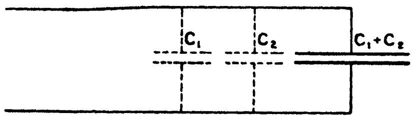

FIG. 29.

current is the same throughout the circuit. This law is always true, whether the current be continuous or alternating.

To sum up. If two condensers of capacities C_1, C_2 are connected

(i.) In series: the capacity of an equivalent single condenser (C_r) is given by the formula

$$\frac{1}{C_r} = \frac{1}{C_1} + \frac{1}{C_2}.$$

(ii.) In parallel: The capacity of an equivalent single condenser

$$C_r = C_1 + C_2.$$

(3) Lastly, taking the case of inductance + resistance, and capacity connected in parallel, as shown in Fig. 30.

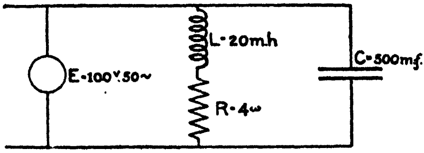

FIG. 30.

Suppose the value of E to be 100 volts, the frequency of supply 50~, the inductance 20 millihenries, resistance 4 ohms, and capacity 500 microfarads.

The current through L and R will be

$$I_1 = \frac{E}{\sqrt{R^2 + (-\omega L)^2}}$$

$$= \frac{100}{\sqrt{16 + \left(2 \times \frac{22}{7} \times 50 \times \frac{20}{1,000}\right)^2}}$$

$$= 13{\cdot}16 \text{ amperes.}$$

The current through C will be

$$I_2 = \omega CE$$

$$= 2 \times \frac{22}{7} \times 50 \times \frac{500}{10^6} \times 100$$

$$= 15{\cdot}7 \text{ amperes.}$$

In order to find the resultant current I which will be read on an ammeter placed in the main circuit, we will construct a

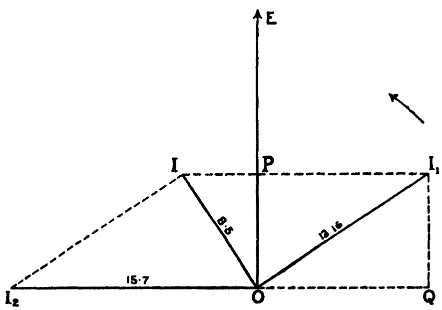

Fig. 31.

vector diagram as we did on p. 42, plotting *currents* to scale in this case, since there is only one pressure to be considered—namely, E—which is common to each branch of the circuit.

This current I is made up of two components, whose values have been calculated above—namely, $I_1 = 13{\cdot}16$ amperes lagging at an angle (φ_1) $\tan^{-1}\frac{\omega L}{R}$ behind the pressure (OI_1 in Fig. 31); and $I_2 = 15{\cdot}7$ amperes leading the vector OE by 90° as shown at OI_2.

The vector OI will then represent the current in the main circuit, both in direction and magnitude.

The algebraical expression for the value of OI, although quite simple, is very clumsy, and the reader can very readily find it for himself.

Geometrically, $OI = \sqrt{OP^2 + PI^2}$.

PI is, of course, equal to $OI_2 - OQ$.

We know the values of all these terms since

$$OQ = I_1 \sin \varphi_1,$$
$$OI_2 = I_2,$$
$$OP = I_1 \cos \varphi_1.$$

The full expression can then readily be found by expressing $I_1 \sin \varphi_1$ and $I_1 \cos \varphi_1$ in terms of volts and reactance.

For example,

$$\begin{aligned} OQ &= I_1 \sin \varphi_1 \\ &= \frac{E}{\sqrt{R^2 + (-\omega L)^2}} \times \frac{\omega L}{\sqrt{R^2 + (-\omega L)^2}} \\ &= \frac{E \times \omega L}{R^2 + (-\omega L)^2}. \end{aligned}$$

13. Voltage Resonance.

We have now to investigate a very important effect produced by the presence of inductance and capacity simultaneously in a circuit. This effect is one which is apt to cause considerable difficulty to the beginner, since, baldly stated from an electrical point of view, it appears to him at first sight impossible.

Before attempting to explain the cause we will note first the effect. Let us turn back to Fig. 26, on p. 42, which is a vector diagram of pressures in a circuit in which inductance and capacity are present in *series*. Fig. 32 shows the arrangement of the circuit consisting of an inductance L = 25 m.h., a resistance R = 2ω, and a capacity C = 1,000 m.f., the current throughout the circuit being 10 amperes.

Now we have already seen on p. 43 that the pressure necessary to send this current through the circuit (= OE in Fig. 26) is 50·87 volts.

A voltmeter, therefore, placed across the points AB will read this pressure.

But the pressure necessary to send 10 amperes through an inductance of 25 millihenries we found to be 78·6 volts (OL in Fig. 26). This will be the pressure read on V_l connected across the inductance.

Similarly, voltmeters V_r and V_c connected across R and C will read, as we have found on pp. 42—43, 20 and 31·82 respectively. Instead, then, of finding the applied volts to be equal to the sum of these three components, as we are accustomed to do in continuous-current work, we find these resultant applied volts to be actually less than *one* of the components. Now, by taking suitable values for L and C we can very readily arrange a circuit so that the resultant volts are less than *either* of the components

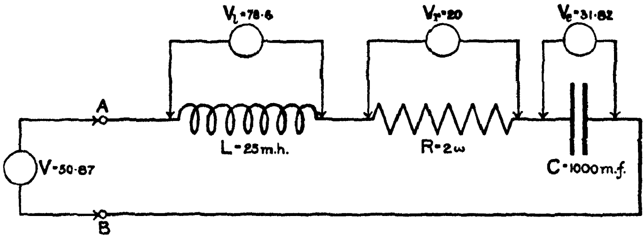

FIG. 32.

V_l and V_c, the smallest possible value of V (or the resultant volts) being equal to V_r, or in this case 20 volts.

Having already had some slight practice in constructing these diagrams we should begin to realise the cause of this novel phenomenon. To find the virtual value of the resultant volts necessary to send 10 amperes through this circuit we have added the components together **not arithmetically** but **vectorially.**

Provided that these various components are all in phase with one another, this vectorial sum will be the same as the algebraic sum; and, in general, whatever the phase difference between the components may be, the resultant pressure *at any given instant* is equal to the algebraic sum of the components *at that instant.*

Turning back to Fig. 26, we see that in order to arrive at OE we take the resultant of two quantities,

(1) The *difference* between ωLI and $\frac{I}{\omega C}$.

(2) The value of IR.

Now **if the difference between ωLI and $\frac{I}{\omega C}$ is zero,** the resultant of these two quantities (1) and (2) becomes obviously IR, which is the smallest possible value; ωLI and $\frac{I}{\omega C}$ may each be of *any value whatever* to produce this result, provided their values are equal. A short study of Fig. 26 will show that with a high value for L and a corresponding low one for C, enormous pressures may be registered across L and C respectively, with quite a low value for I.

In other words, if the effect of the inductance in a circuit

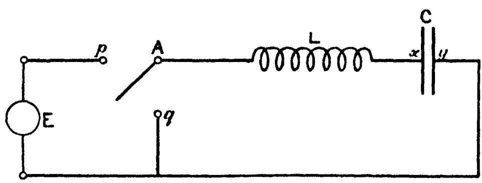

FIG. 33.

exactly balances that of the capacity in series with it, the circuit *as a whole* obeys the law $I = \frac{E}{R}$, and the effect on the circuit is known as **voltage resonance.** This resonance is complete if the effect of inductance and capacity exactly balance, but will be present in a greater or less degree whenever inductance and capacity are present.

Now what does the term "resonance" mean, and why is it so called?

Let us imagine what happens when a current flows into and out of a condenser through an inductance. Suppose the condenser to be charged with a maximum quantity of electricity in one sense, say, positive. At this point, immediately remove the applied pressure and connect the plates of the condenser together through an inductance, as could be done by means of a switch (shown in Fig. 33) which disconnects A from p and connects A to q, *i.e.*, connects the plates of the condenser C through the inductance L.

Now at this instant we have imagined C to have its maximum positive charge, so that one plate, say, x, is at a considerable potential above the other, y. Directly the points A and q are connected the condenser will begin to discharge and in doing so a current will flow from x through L to y. Now, if L were *non-inductive*, this current would flow for a very short period of time until the potentials of x and y were equal and then cease. But the effect of the inductance, or electrical mass, is to give the system *momentum*, so that the current from x *overshoots the zero point* and the condenser becomes charged in the opposite sense. It will now discharge in the other direction, and again overshoot the zero point owing to the electrical

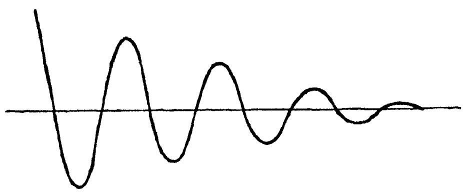

FIG. 34.

momentum caused by the inductance. This action causes an **oscillatory** discharge (as shown in Fig. 34) from the condenser, which continues for some time after the original charging pressure is removed. The oscillations gradually die down; or are **damped** by the ohmic resistance of the circuit, and the greater this ohmic resistance is the more quickly will they be damped.

A mechanical example may help us to understand this action. Imagine a weight of mass L attached through a spring, C, to the point P (Fig. 35). If a downward pressure is applied to L the spring will be elongated as shown in Fig. 35 (*b*), the system moving downwards under the influence of the downward pressure. If, now, this downward pressure be completely removed, the weight L will be pulled up again by the action of the spring; but, owing to the momentum which it acquires in an upward direction it will overshoot its natural position of rest, Fig. 35 (*a*), and rise to the position shown in Fig. 35 (*c*).

The system will therefore begin to oscillate, and will gradually come to rest owing to the frictional resistance of the air or other medium in which it is oscillating. The greater is the friction (which corresponds to electrical resistance) the more rapidly will these oscillations be damped out. If the medium is treacle, for instance, the damping effect would be so marked that the system would probably come to rest without oscillating.

Now each of these systems, whether electric or mechanical, has a definite **period of vibration** or natural frequency. We may make the experiment with our spring and weight, and, by taking a certain mass and elasticity, can actually count the

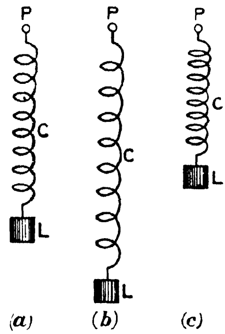

FIG. 35.

number of complete oscillations made in a certain time. Varying the mass of our weight and the elasticity of our spring will result in varying the natural frequency of the system. For example, a small mass and a stiff spring will mean a high natural frequency and *vice versâ*.

It is impossible, owing to the very rapid vibrations in an electrical circuit, actually to count these. If they were slow enough we could count the oscillations of the needle of an ammeter placed in the circuit and arrive at our natural frequency in this way. This, however, is out of the question, since the oscillations are, in practice, so extremely rapid that, as we have already noted, the ammeter needle remains perfectly steady, actually recording, of course, the virtual value of the current wave form.

But we may *calculate* the natural frequency in a very simple manner. If the value of the inductance is L, and that of the capacity C, the value for the natural frequency (f) may be found from the equation

$$\omega L = \frac{1}{\omega C},$$

or

$$2\pi f L = \frac{1}{2\pi f C},$$

since the natural frequency is such as will cause the reactances due to capacity and inductance to exactly balance one another.

That is

$$f = \frac{1}{2\pi \sqrt{LC}}.$$

Returning to the circuit shown on page 49, whose vector diagram was drawn on page 42.

Here L = 25 m h.
C = 1,000 m.f.

The natural frequency of this circuit is found by the equation

$$2\pi f \frac{25}{1,000} = \frac{1}{2\pi f \frac{1,000}{10^6}},$$

from which $f = 31{\cdot}8$.

If, now, the frequency of our supply had been *exactly* 31·8 (instead of being 50) we should have got *complete voltage resonance*, since the effect of inductance and capacity would have exactly balanced.

That is to say, there would have been a state of **resonance** between the forced vibration due to the applied pressure, and the natural vibration of the circuit itself. The term resonance is derived from that used to express the corresponding mechanical phenomenon. If, for instance, a certain note be struck on a piano, air waves of a certain frequency will be set in motion. If these air waves strike against any object which possesses a natural frequency of vibration exactly equal to that of the air waves set in motion by the striking of the note this object will vibrate in *resonance* with the note on the piano. If the note is struck with sufficient force these resonant vibrations set up in the object in question may be so violent as to break it.

Similar sympathetic vibrations can be produced in an electric circuit possessing inductance and capacity. This is the principle of wireless telegraphy. Oscillating currents are produced in a circuit, and these oscillations set waves in motion in the ether, just as mechanical vibrations cause waves in the air. If these ether waves strike an electric circuit which resonates or is "in tune" with the sending circuit, sympathetic current oscillations will be set up and signals may thus be given.

To return, now, to our circuit shown on page 49. This circuit only exhibits partial resonance, since, as we have found, the natural frequency of the circuit is 31·8, whereas the frequency of the applied pressure is 50. That is, vibrations are *forced* on

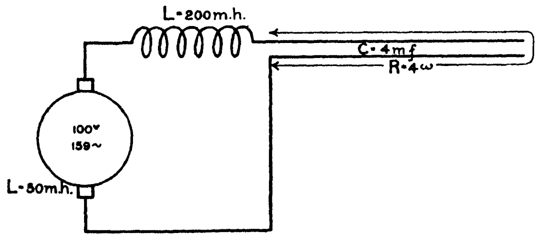

FIG. 36.

the circuit which are not in tune with its natural vibrations. *Complete* voltage resonance will be produced, as we have seen, in a circuit if the applied pressure has a frequency equal to the natural frequency of the circuit. In this case the reactances due to capacity and inductance will exactly balance, and the value of the applied volts will be IR. The pressure across the inductance and capacity will be equal to ωLI or $\frac{I}{\omega C}$, and may very possibly reach an extremely high figure. This high pressure will probably puncture the insulation of the conductors (or the dielectric of the condenser) and cause short-circuiting and breakdown.

Under these circumstances we are applying our periodic pressure to the circuit exactly in step with the natural periodicity. The result produced is the same as we should obtain with our weight and spring (Fig. 35) if we were to give it a push just as it was about to rise upwards and a pull just as it was about to move downwards—namely, an enormous increase in the amplitude of the wave form.

An example of complete voltage resonance is shown in Fig. 36.

Suppose an alternator giving an E.M.F. of 100 volts to be connected to a two-core cable (on *open* circuit), through an inductance of 0·2 henry.

Suppose the resistance of the cable to be 4 ohms, its capacity 4 m.f., and the inductance of the alternator ·05 henry. The frequency of supply is 159.

On closing the switch, which would produce no result in a continuous-current system (since the cable is on open circuit), what will happen ?

Here we have an inductance, resistance and capacity in series ; to which is applied an alternating pressure.

The capacity effect of the cable may be taken as approximately equivalent to that of a condenser of the same capacity at its central point ; that is, one half the resistance of the cable is in series with the condenser.

The value of the current

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

$$= \frac{100}{\sqrt{\left(\frac{4}{2}\right)^2 + \left(\frac{1}{2\pi \times 159 \times \frac{4}{10^6}} - 2\pi \times 159 \times \frac{1}{4}\right)^2}}$$

$$= \frac{100}{\sqrt{2^2 + (250 - 250)^2}}$$

$$= \frac{100}{2} = 50 \text{ amperes.}$$

A current, then, of 50 amperes will flow, and we have a condition of complete voltage resonance since the reactances due to capacity and inductance balance, that is to say 159, the frequency of supply, is exactly equal to the natural frequency of the circuit.

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What will be the pressure between the cores of the cable, *i.e.*, across the condenser ?

Obviously $$E = \frac{I}{\omega C}$$

$$= \frac{50}{2 \times \frac{22}{7} \times 159 \times \frac{4}{10^6}}$$

$$= 12{,}500 \text{ volts.}$$

Since the cable is only designed to work at 100 volts, immediate breakdown would result. It is obvious that, under these conditions, we must reduce the frequency of our generator to avoid this disastrous resonance.

For example, if we reduce our frequency of supply to $\frac{159}{2}$ we shall still get resonance, since the effect is now equivalent to giving our weight an impulse *every other* natural oscillation. But the amplitude of the wave is enormously reduced, since the damping effect of the resistance is very much more marked.

The value for our current now becomes

$$I = \frac{100}{\sqrt{2^2 + \left(250 \times 2 - \frac{250}{2}\right)^2}}$$

$$= \frac{100}{\sqrt{2^2 + (375)^2}},$$

or practically $\frac{100}{375}$, *i.e.*, 0·26 ampere instead of 50.

The pressure between the cores of the cable is now

$$E = \frac{I}{\omega C}$$

$$= \frac{100}{2 \times \frac{22}{7} \times \frac{159}{2} \times \frac{4}{10^6} \times 375}$$

$$= 135{\cdot}5 \text{ volts.}$$

This result, when compared with the pressure of 12,500 volts produced by complete resonance, is very striking.

14. Current Resonance.

Just as voltage resonance is produced in a circuit containing inductance and capacity in series, so is *current* resonance by the presence of these two factors in *parallel.*

This will be quite clear from Fig. 31 on p. 47, where it is seen that an ammeter in the main circuit reads a current (OI) which is not only less than the sum of the currents in the two branches (OI_1 and OI_2) but is actually less than either of them.

Here we have a very similar effect to the one which we have just been investigating, except that we are dealing with currents instead of pressures.

This is known as **current resonance.** It is not proposed to give detailed examples of this phenomenon, since the algebraical expressions are so extremely clumsy as to confuse rather than to help the reader. Complete resonance is manifested when the current in the main circuit is in phase with the volts, *i.e.*, when the values of OQ and OI_2 in Fig. 31 are equal.

15. Resonance with the Higher Harmonics.

We have now to consider resonance which may be produced by other causes; these causes being more complicated both to investigate and to guard against, although, unfortunately, much more common in practice than the simple cases with which we have hitherto dealt.

In order to understand what follows we must turn back to p. 8, where the question of wave forms other than the pure, or fundamental, sine curve was touched upon.

We remember, from what was said then, that whenever from whatever cause the wave form given by an alternator departs, however minutely, from the true sine curve *one or more of the higher harmonics must be present.*

Now, in practice, several of these are generally present simultaneously; but we will take as simple a case as we can—that is, the wave form already shown in Fig. 6 and repeated here in Fig. 37, which is made up of the fundamental and the third harmonic in phase.

Hitherto, in considering the question of resonance, we have only investigated the subject of *resonance with the fundamental,* that is to say, cases where the frequency of supply was equal to the natural frequency of the circuit.

But imagine a case where, for any reason, the wave form of the alternator includes the *third harmonic*, and suppose the natural frequency of the circuit to be three times that of

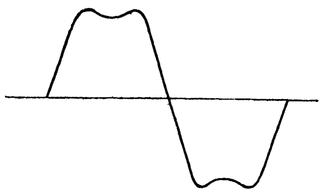

FIG. 37.

the supply. Since the frequency of the third harmonic is, of course, three times that of the fundamental, and therefore *also* three times the supply frequency, we shall get complete voltage resonance, not with the fundamental, but with the *third harmonic.* The result will be greatly to increase the minor ripples on the wave form and to produce a wave form somewhat as shown in Fig. 38.

Since it is the *peak value* of the E.M.F. wave which the insula-

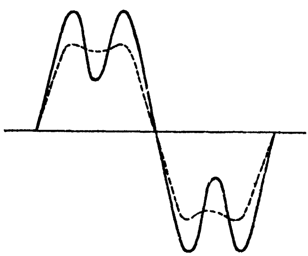

FIG. 38.

tion of the circuit is called upon to withstand, this state of resonance may be as disastrous in its effect as resonance with the fundamental, although its presence is not so readily detected on a voltmeter, for the reason that the virtual value may be hardly affected. This will be clear from an inspection of Fig. 38.

An actual example of voltage resonance with one of the higher harmonics may help in understanding this very important principle.

The case is that of a circuit supplied by an alternator at a frequency of 50 ∼, in whose E.M.F. wave is present the *thirteenth harmonic.*

On open circuit this was hardly noticeable, but on switching on to a circuit possessing inductance and capacity in series it was noticed that the amplitude of the ripples due to the presence of this harmonic was over four times the original value.

The inductance of the circuit was

6·7 millihenries.

The capacity

8·9 microfarads.

The resistance

0·5 ohms.

It remains to prove that this increase in the amplitude of the ripples was due to resonance with the thirteenth harmonic.

The natural frequency of the circuit is, as shown on p. 52,

$$f=\frac{1}{2\pi\sqrt{L\times C}}$$

$$=\frac{1}{\frac{44}{7}\times\sqrt{\frac{6\cdot7}{10^3}\times\frac{8\cdot9}{10^6}}}$$

$$=650.$$

Now, $\qquad 50\times13=650.$

That is, the natural frequency of the circuit is exactly equal to that of the 13th harmonic, so that resonance with this harmonic is certain.

If x volts is the virtual value of this 13th harmonic on open circuit, then the virtual value of the current due to this harmonic

$$I_{13}=\frac{E_{13}}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}.$$

But $\omega L = \frac{1}{\omega C}$, since we have complete resonance with this harmonic.

$$\therefore \quad I_{13} = \frac{E_{13}}{R}$$

$$= \frac{x}{0 \cdot 5} = 2x.$$

Reactance in circuit due to inductance

$$= \omega L$$

$$= \frac{44}{7} \times 50 \times 0 \cdot 0067$$

$$= 2 \cdot 1.$$

Amplitude of 13th harmonic across the inductance

$$= \omega L I$$

$$= 2 \cdot 1 \times 2x$$

$$= 4 \cdot 2x.$$

That is, its amplitude has been quadrupled by resonance with the circuit.

These considerations make it clear why great pains are taken by designers of machines to produce a wave form as nearly as possible a pure sine curve.

The wave form of machines which do not fulfil this condition is almost certain to contain not only one, but several of these higher harmonics, with the result that the chance of getting resonance in the circuit supplied by them is very much increased.

If, for example, the wave contains the 3rd, 9th, 13th and 21st harmonics, there are five chances of resonance as against only one when the wave form is a pure sine curve.

Little more has been attempted here than to touch upon this important subject, and the reader who wishes to investigate more fully is referred to the many standard works in which it is treated exhaustively. It is, in practice, one with which designers and engineers are very much concerned, and was the cause of many breakdowns in the early days when its importance was hardly realised.

CHAPTER V.

POWER MEASUREMENT.

16. The Measurement of Power in an Alternating-current Circuit.

In continuous-current work we have been accustomed to consider our power expressed in watts to be equal to the product of the pressure in volts and the current in amperes; and we have considered the expression,

$$E \times I = I^2R,$$

where the power is entirely absorbed in ohmic resistance.

Now this is *only* true provided that

$$I = \frac{E}{R},$$

when we may substitute IR for E in every case.

But, as we have already seen, in alternating-current work the expression $I = \frac{E}{R}$ is only true when **current and volts are in phase,** that is to say, when neither inductance nor capacity is present.

Now the expression I^2R for the value of the power in watts absorbed in a given resistance, R, is *always* true, whether the circuit be supplied with a continuous or an alternating pressure, and it will be better to so regard it, rather than as the power supplied to the *whole circuit*, since it will frequently happen that the bulk of the power supplied by the generator is not absorbed in ohmic resistance, but is being converted either to mechanical energy (in the case of a circuit supplying a motor load) or to chemical energy (in the case of a generator charging secondary batteries); this last condition does not apply, of course, to the case of alternating-current work.

The expression EI for the power in watts is, however, only true when the phase angle φ is zero, as we have seen above.

We have then to find an expression for the product of these two periodic functions E and I for any value of φ. The expression for this product may be written

$$E_{max.} \sin \theta \times I_{max.} \sin (\theta \pm \varphi).$$

The simplest method of finding the average value of this product will be to do just what we did on p. 12 to find the quadratic mean of a sine function; that is, to find the area enclosed by the resultant curve and to divide this area by the length. This will give us the average height of the curve, or, in other words, the mean value of the product.

We will take this average over a complete period, *i.e.*, between the values of $\theta = 0$ and $\theta = 2\pi$.

The mean value will then be

$$\frac{1}{2\pi}\int_0^{2\pi} E_{max.} I_{max.} \sin \theta \times \sin (\theta \pm \varphi) d\theta$$

$$= \frac{1}{2\pi} E_{max.} I_{max.} \cos \varphi \times \pi$$

$$= \frac{E_{max.}}{\sqrt{2}} \times \frac{I_{max.}}{\sqrt{2}} \times \cos \varphi.$$

But $$\frac{E_{max.}}{\sqrt{2}} = E \text{ and } \frac{I_{max.}}{\sqrt{2}} = I.$$

Therefore, the average value of the product of these two functions

$$= \mathbf{EI \cos \varphi},$$

if $\varphi = 0$, $\cos \varphi = 1$, and $EI \cos \varphi = EI$,

if $\varphi = 90°$, $\cos \varphi = 0$, and $EI \cos \varphi = 0$.

That is to say, **when the current in a circuit either lags behind or leads the volts by 90° the power delivered to the circuit is zero.**

This result at first sight appears somewhat startling, and will be dealt with in a moment.

Putting this equation,

$$\text{Power in watts} = \text{EI} \cos \varphi,$$

into words, we arrive at this expression :

The power in watts in any circuit, continuous or alternating, is equal to the product of the pressure, and the component of the current which is in phase with the pressure, *or*, if we prefer it, to the product of the current and the component of the pressure which is in phase with the current ; and the power

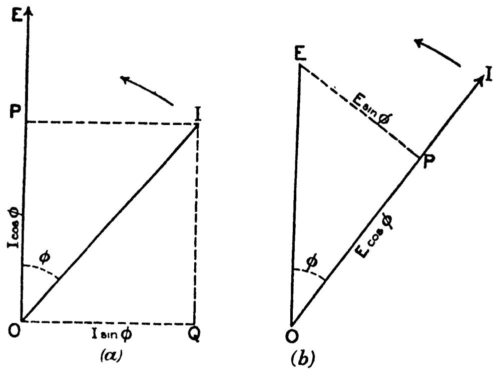

FIG. 39.

absorbed in a given resistance R is equal to the product of the square of the current and the resistance in ohms,

i.e., $$\begin{aligned}\text{Watts} &= \text{E} \times \text{I} \cos \varphi, \\ &= \text{I} \times \text{E} \cos \varphi,\end{aligned}$$

and watts absorbed in resistance R

$$= \text{I}^2\text{R}.$$

The current may then be considered to consist of two components at right angles to one another, and we may construct a " triangle of currents " just as we did a triangle of pressures (p. 26). This is shown in Fig. 39 (*a*), the two components being

I cos φ in phase with E (= OP)

I cos (90° — φ) or I sin φ at right angles to E (= OQ).

We should arrive at precisely the same result by resolving E into its two components, as shown in Fig. 39 (*b*).

E cos φ in phase with I,

E sin φ at right angles to I.

The component I cos φ is frequently known, for the above reasons, as the **watt component** of the current, and I sin φ as the **wattless** or **idle component.**

The latter is also commonly known, in the case of a lagging current—that is, in an inductive circuit—as the **magnetising** current, since it is this component which produces the fluctuating magnetic field on which the E.M.F. of self-induction depends. In a choking coil, for example, which possesses *no* resistance (a purely theoretical case) the whole current is a magnetising one, neglecting losses due to eddy currents and hysteresis, since I^2R, and therefore the power in watts, is zero.

Similarly, in the case of a leading current—that is, when the circuit possesses capacity—this idle component is called the **charging** current.

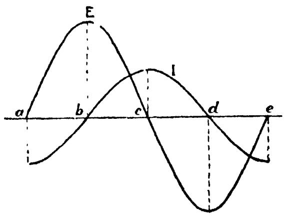

FIG. 40.

To return now to the special case when $\varphi = 90°$, whether lagging or leading. If this phase difference were *exactly* 90°, then, as we have seen, the power in the circuit would be zero. At first sight it would appear impossible that no power would be required to drive an alternator under these conditions (other than that required to drive it light), even though its voltmeter and ammeter read considerable values of pressure and current respectively. Such is, however, the case, and it may help the reader to understand this novel state of affairs by considering the action in the machine when it is supplying a current which is wholly "wattless."

Fig. 40 shows the wave forms of pressure and current in the machine when $\varphi = 90°$. We may divide the cycle into four equal parts, *ab*, *bc*, *cd*, *de*, the points *a*, *c* and *e* being the moments of maximum current value and zero pressure, and the points *b* and *d* being the moments of maximum pressure and zero current.

To understand the significance of this diagram we must draw upon our knowledge of the electrical actions in the case of continuous-current machines. Now we know that in a C.C. generator the current is flowing *in the same sense* as the generated pressure, that is, out from the positive pole of the machine. But in the case of a *motor* the current is flowing *into* the positive pole of the machine, *i.e.*, against, or *in the opposite sense* to the generated pressure, which we are accustomed to call the back E.M.F. of the motor. Precisely the same remarks apply to alternating-current machines; so that, in general, we may say that—

A machine is a *generator* when the current is flowing *with* the generated pressure, and it is a motor when the current is flowing *against* the generated pressure.

Now, during the quarter periods *bc* and *de* in Fig. 40 the current is flowing *with* the pressure, since both E and I are on the same side of the zero line; but during the quarter periods *ab* and *cd* E and I are on opposite sides of the zero line, that is, the current is flowing *against* the pressure.

Our alternator is, then, a *generator* for one-half a complete period—*i.e.*, it is taking power from its engine; but it is a *motor* for the other half period, and is therefore assisting its engine.

The nett result taken over one or more complete periods is that *no power is required to drive it*, its behaviour being equally that of a generator and a motor.

As we have already seen, this value of φ is purely theoretical, since it implies a zero value for R. In certain circuits, however, R may have an extremely low value and this condition may be quite closely approached.

It follows as a corollary of the above remarks that when φ is less than 90° the machine acts on the whole as a generator and *supplies* current; and that when φ is more than 90° the machine acts on the whole as a motor, and *receives* current. This is the case of the reversible machine, and will be dealt with later. At present we need only note that the alternator, like the C.C. generator, is, in fact, reversible, and can be used as a motor.

In order to avoid confusion it is as well to state that φ can never exceed 90° from the point of view of the *supply* pressure, though it may from the point of view of the *back* E.M.F. in a circuit.

Since the power in a circuit is denoted by the expression

$$EI \cos \varphi,$$

it is usual to speak of the term cos φ as the **power factor** of the circuit. It becomes of great importance to keep the value of this factor as near its maximum, or unity, as possible. The reason for this is sometimes hardly grasped by the beginner, but should be quite clear when we realise that in a given resistance, R, the loss of power varies directly as the *square of the current*, and is equal to I^2R. Now, if cos φ is low, it means that the virtual value of the current is considerably greater than its watt-component. In other words, we are sending through our resistance R a current greatly in excess of that which is necessary for the transmission of a given amount of power. If the watt-component is equal to one-half the virtual value of the current—*i.e.*, if cos φ is $\frac{1}{2}$ (which means that φ is 60°)—we are sending a current through the circuit which is twice the value (for a given moment of power) it would have been if cos φ had been 1. Our heat losses then, being equal to I^2R, would be quadrupled.

Since machines are limited in output chiefly by the heating effect of the current on the conductors forming their windings, it is usual to rate alternators so as to be independent of this power factor, and this class of machine is therefore generally rated, not in kilowatts, as are continuous-current generators, but in **kilovolt-amperes,** generally abbreviated to K.V.A. The power in kilowatts which they are supplying will, of course, depend on the power factor, that is

$$\text{K.V.A.} \cos \varphi = \text{KW}.$$

17. Wattmeters.

The product of volts and amperes, that is, the value of EI in any circuit, is known as the **apparent watts** and EI cos φ as the **true watts**.

The apparent watts are arrived at by multiplying the values read on the ammeter and voltmeter. The true watts are read on an instrument designed for the purpose and known as a **wattmeter.** A short description of the principle on which this instrument works is desirable, as we have not met with it in continuous-current work.

The underlying principle of its action is that of the electro-dynamometer, and its essential working parts consist of two coils, which are at right angles to one another when the instrument is at rest. One of these coils is designed to carry the current which is flowing in the circuit whose power is to be measured, and is fixed. The other coil, arranged at right angles to the current coil, and so pivoted as to be free to move on its axis—against a controlling spring—inside the current coil, is composed ot many turns of fine wire, and is therefore

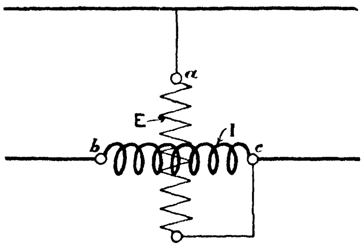

FIG. 41.

of high resistance. This coil is arranged, not in series with the load, but in shunt, or across the leads forming part of the main circuit, just as a voltmeter is connected. This coil, then, is energised by the pressure of the circuit, and the other coil by the current.

The general arrangement and method of connecting up the instrument are shown in Fig. 41. I is the current coil in series with the load, E being the pressure coil, energised by the volts of the circuit. The instrument has three terminals (*a*, *b*, *c*) to which connections are made as shown.

Imagine the two coils to be seen in section as in Fig. 42. When $\varphi = 0$—*i.e.*, when current and volts are in phase—the maximum effect will be produced on the moving coil E. During the half period *pq* the current will be flowing in one sense through the instrument, and we will imagine this sense to be from terminal *c* to *b* through the current coil, and from *c* to *a* through the pressure coil. These currents will produce magnetic fields, whose resultants are shown in Fig. 42 (*a*) and (*b*). Fig. 42 (*a*) represents the direction of the resultant field when *c* is the positive terminal, that is, during the half period *pq*, and Fig. 42 (*b*) represents the same during the half period *qr*. It will be seen that the action of this resultant field is always to produce attraction between the points X and Y, and X′ and Y′. This will mean that the pressure coil, being free to move on a pivot, will rotate counter-clockwise against its controlling spring, and in doing so will carry with it a needle

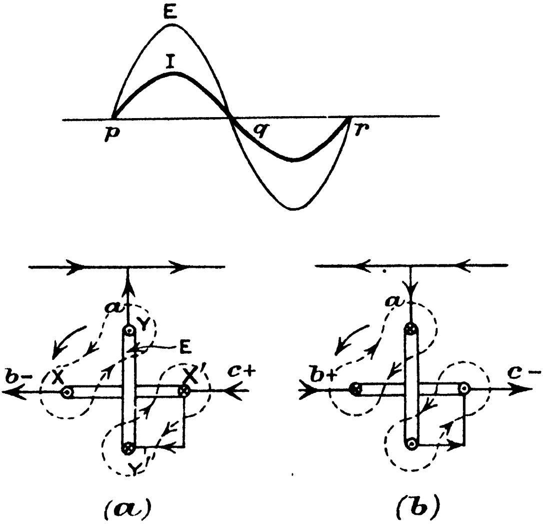

FIG. 42.

which will travel over a scale calibrated in watts. In this case, since current and volts are in phase, cos $\varphi = 1$, and the true and apparent watts are identical. It will be clear that, in order to keep the current in the pressure coil Y strictly in phase with the volts in the circuit, the inductance of this coil, and of any resistance in series with it, must be negligible.

Taking now the other extreme, that is, when the phase-difference between current and pressure is 90°, let us examine the action on the moving coil of the instrument. Proceeding on exactly the same lines as before, we see that during the quarter periods *qr* and *st* there is, as before, attraction between the points XY and X′Y′, with the result that the pressure coil will tend to move counter-clockwise. During the remaining two quarter periods, however, there is *repulsion* between XY and X′Y′, since volts and current are in opposite senses to one another during these quarter periods; with the result that the pressure coil will now tend to move *clockwise*. The result over the whole period will be that the coil (and therefore the needle) will not move at all, owing to its inertia, and the instrument will read zero; that is, when $\varphi = 90°$, $\cos \varphi$, and therefore $EI \cos \varphi$ are zero.

It naturally follows from this that for any values of φ between the extremes 0° and 90°, the movement of the coil

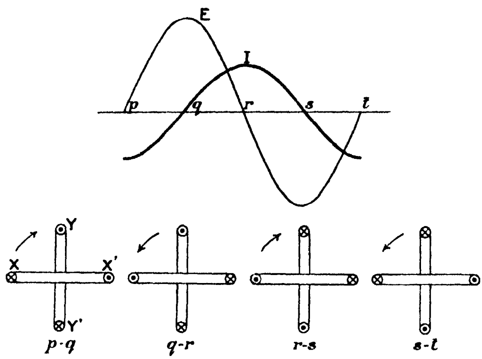

FIG. 43.

will be proportional to $\cos \varphi$, and so will read the true watts of the circuit.

We may then arrive at $\cos \varphi$ (and hence at the value of φ) by dividing the reading of our wattmeter by the product of the ammeter and voltmeter readings; that is

$$\cos \varphi = \frac{EI \cos \varphi}{EI} = \frac{\text{true watts}}{\text{apparent watts}}.$$

Power factor is sometimes expressed as a percentage—*i.e.*, if $\cos \varphi = 0{\cdot}707$, the power factor is said to be 70·7 per cent.

It is obvious that an instrument of this class can be calibrated to read $\cos \varphi$ direct, and is useful in central stations as showing at once the machine power factor. Such an instrument is known as a **power factor indicator,** or sometimes as a **phasemeter.**

CHAPTER VI.

POLYPHASE SYSTEMS.

18. Polyphase Working.—19. Two-phase Systems.—20. Three-phase Systems.—21. Star-connected Generators.—22. Mesh-connected Generators.—23. Measurement of Power in Three-phase Circuits.

18. Polyphase Working.

For some twenty or thirty years past systems have gradually grown up which depend upon the use of two or three alternating currents simultaneously.

The employment of such systems is known as **polyphase** working, those using two currents as **two-phase** and those using three currents as **three-phase** systems respectively.

Their development is due to the fact that alternating-current motors, as a general rule, operate more satisfactorily when they are supplied with two or three currents simultaneously than when they are supplied with only one. In the former case they are known as two, three or polyphase, and in the latter as single-phase motors. This question will be dealt with when we are considering the operation of alternating-current motors generally.

Owing to the rapid and almost daily improvement of the single-phase motor the use of three-phase systems is being more and more confined at the present day to the transmission in bulk and at high pressure of power which is subsequently converted to the continuous-current system by means of rotating machines or **converters,** whose operation is much more satisfactory as three-phase than as single-phase machines.

These systems are also advantageous from a transmission point of view. Just as power can be transmitted more economically on the three-wire continuous-current system than on the corresponding two-wire system, so the cable costs in the case of three-phase systems are considerably lower than those of single-phase at a similar phase pressure.

19. Two-phase Systems.

These depend upon the simultaneous use of two alternating currents, which are of equal amplitude and frequency, but differ in phase by 90°, that is to say, one is at its maximum value when the other is zero, as shown in Fig. 44. It is important for the beginner to realise that this phase difference between the two currents has nothing whatever to do with phase difference between pressure and current. The angle between phase A current, for instance, and the volts responsible for it, may have any value according to the nature of the load, and a similar remark will apply to the angle between phase B current and its volts. To all intents and purposes they are entirely separate currents, but are sent out to line

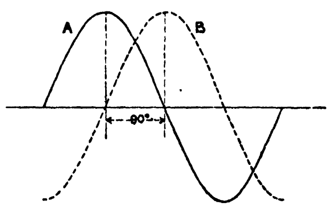

Fig. 44.

with one quarter period between their maximum values, for a purpose which will be made clear when we deal with the subject of induction motors.

Two-phase currents are generated by one alternator, the armature windings for each current being entirely separate. The phase displacement of 90° between them is obtained by spacing these windings half a pole pitch, or 90° (electrical) from one another. That is, when any given winding of phase A is in the working axis of the magnetic flux, and the value of its E.M.F. is therefore a maximum, the corresponding winding of B phase will be in the neutral axis of the flux, and the value of its E.M.F. will be zero.

Now, there is absolutely no electrical reason why each phase should not have its own separate machine, provided that each machine gave exactly the same pressure, and that they both worked and continued working at exactly the same frequency.

It is, however, as one can readily understand, far easier to fulfil these necessary conditions by arranging both sets of windings on a single machine, which is known as a two-phase alternator, than to run separate generators.

As a general rule, four wires are used for transmitting two-phase currents, that is, two for each phase. It is possible to combine the two return leads into one of larger section, and such an arrangement is satisfactory provided that the load on each phase is the same, that is, provided that the virtual values of the two currents are equal.

The current carried by this common return lead is, of course, the resultant of the currents in the two phases, which is readily obtained by means of the vector diagram shown in Fig. 45.

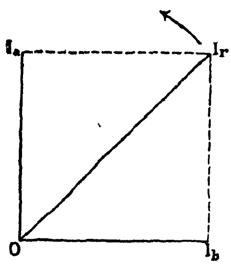

FIG. 45.

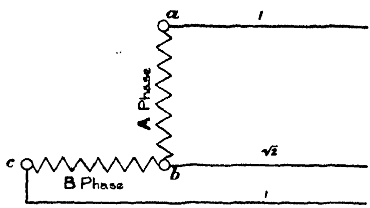

FIG. 46.

If OI_a represents the virtual value of the current in A phase, and OI_b that of the current in B phase (lagging 90° behind A phase current), then OI_r represents the value of the resultant current carried by the common return lead. If $OI_a = OI_b$ then $OI_r = \dfrac{OI_a \text{ (or } OI_b)}{\sin 45°} = \sqrt{2}OI_a$.

The cross-sectional area of the common return lead must then be $\sqrt{2}$ times the area of the other two leads, as shown in Fig. 46.

In such a system we have two distinct pressures available—namely, the pressure between the points *ab* or *bc* and that between the points *ac*. Since the two phase windings are identical, and are wound on the same machine, the pressures between the points *ab* and *bc* are identical, and this is known as the **phase pressure.**

The pressure between the points a and c is found in exactly the same way as was the current in the common return lead, being the resultant of the two-phase voltages at right angles to one another, that is to say, it is equal to $\sqrt{2} \times$ phase volts.

20. Three-phase Systems.

Two-phase systems are hardly used in practice, since at best they show very little advantage over single-phase systems, and their advantages are in no way comparable to those of three-phase working.

Just as two-phase systems depend upon the simultaneous use of two alternating currents, so do three-phase systems make use of three such currents. In this case the full cycle

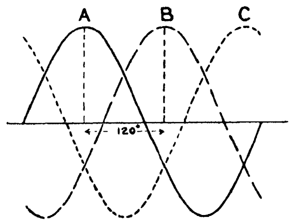

FIG. 47.

of 360° is divided into three equal parts, and the three currents differ in phase by 120°, as shown in Fig. 47.

Now it might be inferred that, just as two-phase systems usually employ four leads—that is, two for each phase—so should six leads be employed for three-phase transmission. Such an arrangement would be practicable, and each phase be used to supply wholly separate circuits. There would, however, be little point in such an arrangement, and it is very much more usual to connect the three windings in the alternator, just as the two sets of windings are connected in a two-phase machine which uses a common return lead. Before considering how this may be done let us examine the curves showing the wave forms of the three currents (Fig. 47).

Now, if the loads on each phase are exactly equal, it follows that the amplitudes of the three currents I_a, I_b and I_c must be the same.

The algebraic sum of the three currents at any instant is then

$$i_a + i_b + i_c,$$

$$= I_{a\,\text{max.}} \sin\,\theta + I_{b\,\text{max.}} \sin\,(\theta + 120°) + I_{c\,\text{max.}} \sin\,(\theta + 240°).$$

But $$I_{a\,\text{max.}} = I_{b\,\text{max.}} = I_{c\,\text{max.}}$$

The sum then

$$= I_{a\,\text{max.}} \{\sin\theta + \sin\,(\theta + 120°) + \sin\,(\theta + 240°)\}$$

$$= I_{a\,\text{max.}}\,(\sin\theta - \sin\theta\cos 60° + \cos\theta\sin 60° - \sin\theta\cos 60° - \cos\theta\,\sin\,60°),$$

$$= I_{a\,\text{max.}}\,(\sin\,\theta - 2\,\sin\,\theta\cos 60°),$$

$$= I_{a\,\text{max.}}\,(\sin\,\theta - 2\,\sin\,\theta \times \tfrac{1}{2}),$$

$$= 0.$$

With equal* loads on the phases, then, the **algebraic sum of the three currents at any instant is zero,** and the phase angle between them is 120°.

21. Star-connected Generators.

Let us now return to the possible ways of connecting our three phases together in the machine. We may represent our

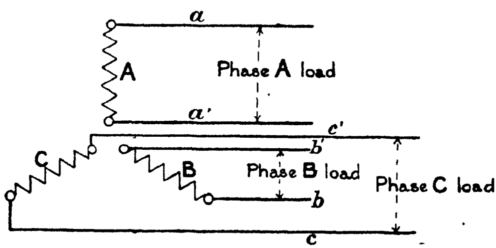

FIG. 48.

three sets of windings, arranged 120° apart from one another by the diagram shown in Fig. 48. If we employed six leads these would be connected to the ends of each phase, that is, the A phase leads to the points *aa'*, the B phase to *bb'* and the C phase to *cc'*.

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* For the case of unequal phase loading, *see* p. 80.

Now, by connecting together in our machine the points $a'b'c'$, we may combine these corresponding leads, and therefore reduce our total number from six to four, as shown in Fig. 49.

This arrangement is known from the form of the diagram as the Y or **star** connection.

Now the lead $a'b'c'$ carries the resultant of the currents in

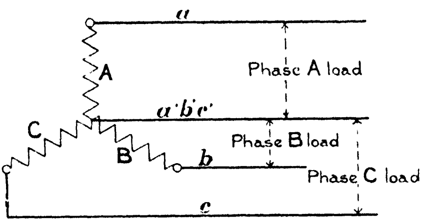

FIG. 49.

the three phases. But when the loads on the phases are equal this resultant current is, as we have already seen, zero. We can, therefore, under these circumstances, entirely dispense with the lead $a'b'c'$ and transmit all three currents by means of *three leads only*, as shown in Fig. 50.

This fact seems at first sight somewhat strange, but will be

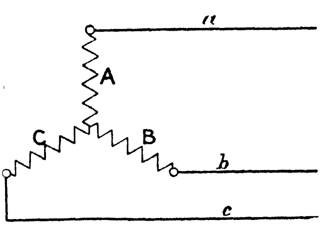

FIG. 50.

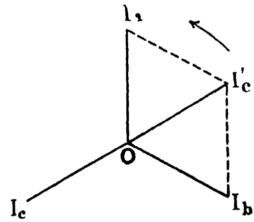

FIG. 51.

readily understood if we draw the vectors of our three currents and find the resultant. Fig. 51 shows these three vectors—namely, OI_a, OI_b and OI_c.

The resultant of OI_a and OI_b is OI'_c, which is equal in magnitude to OI_a, OI_b or OI_c, and is exactly opposite in sense to the last named. The resultant of the three is, therefore, obviously zero.

It has already been mentioned on p. 70 that the advantages of these polyphase systems apply mainly to a motor load, and it is to this class of load, that is, when the generator supplies only motors which make use of the three currents simultaneously, that the condition of equal load on all three phases applies. It is for this reason that three-phase power transmission by three wires only is by far the most common condition in practice.

In the cases already mentioned on p. 70, where a three-phase system is used for the transmission of power in bulk the whole of the load on the generators consists, as a rule, of three-phase motors of one kind or another, which are used to convert the current from an alternating to a continuous supply.

As in the case of two-phase systems, we are concerned with

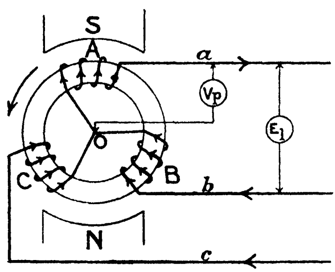

FIG. 52.

two distinct pressures; firstly that between the points *a* and *a′*, Fig. 48 (or the corresponding points at the ends of the other two phases), known, as before, as the **phase pressure,** and that between the points *a* and *b*, *b* and *c*, or *a* and *c*, which is known in star-connected machines as the **line pressure.**

In order to understand thoroughly the relation between phase and line pressure in the case of a star-connected machine, and also to find out what current will flow in our three leads, it will be necessary to take a three-phase armature winding in its most elementary form, and from this deduce the values of pressure and current under consideration.

Let us take a ring armature, as shown in Fig. 52, rotating in a two-pole field, the three phase windings being shown at A, B and C, and the ends of these windings being connected together at O, which is spoken of as the star or neutral point. The other ends of the windings are connected to the three mains which transmit the power from the machine.

This arrangement will then give us a star-connected armature, and our immediate problem is to find the relation between the

Line current (I_l) and phase current;

Line pressure (E_l) and phase pressure.

As regards the first, that is, the current I_l, it will be obvious

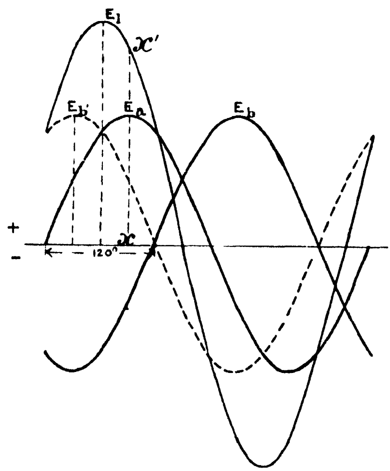

FIG. 53.

from the figure, and also from the current vector diagram shown on p. 75, that the current in any of the three external leads is equal at any moment to the current in the phase to which the lead is connected, since each of these leads is in plain series with the phase winding.

That is, line current = phase current. The line pressure, however, is obviously not the same as the phase pressure. The latter would be shown on a voltmeter V_p connected across line *a* and the neutral point O.

Now, starting from line *b*, and taking the direction of the current (and therefore of the E.M.F.), we reach, first, the *end* of phase B winding, pass through this winding to the neutral point O, from O to the *beginning* of phase A winding, and so on to line *a*; or, in other words, at the instant shown in the figure phase B is in series with phase A.

It will be noticed that at this moment phase A is under the influence of a south pole, and phase B under the influence of a north pole. The direction of the induction in A phase winding is therefore exactly opposite to that in either B or C windings. We may conveniently regard these directions as positive and negative, and say that the induction in coil A is at this moment positive in sense and that in coil B negative. This will be clear if we draw the wave forms of our two pressures A and B 120° apart, as shown in Fig. 53. The point x in this figure corresponds to the moment shown in Fig. 52 when the E.M.F. in coil A is a maximum, that is, when coil A has rotated 90° from its position of zero E.M.F.

At this instant, then, the value of the E.M.F. induced in phase A winding is

$$E_{a\,\text{max.}} \sin 90°, \text{ positive.}$$

Similarly, the value of the E.M.F. in phase B is

$$E_{b\,\text{max.}} \sin (90° - 120°), \text{ negative.}$$

The resultant is then

$$e_l = E_{a\,\text{max.}} \sin 90° - E_{b\,\text{max.}} \sin (90° - 120°),$$

or generally

$$e_l = E_{a\,\text{max.}} \sin \theta - E_{b\,\text{max.}} \sin (\theta - 120°).$$

Now, at the instant x in Fig. 53, the line pressure between a and b

$$\begin{aligned} e_l &= E_{a\,\text{max}} \sin 90° - E_{b\,\text{max.}} \sin (90° - 120°) \\ &= E_{a\,\text{max.}} - (-E_{b\,\text{max.}} \sin 30°) \\ &= 1{\cdot}5\ E_{a\,\text{max.}} \end{aligned}$$

since $\quad E_{a\,\text{max.}} = E_{b\,\text{max.}}$

That is to say, by subtracting a negative quantity ($-E_{b\,\text{max.}} \sin 30°$) from $E_{a\,\text{max.}}$ we have added the voltage values arithmetically, which is a result we should expect from Fig. 52, since at this instant the two-phase pressures are in series.

This process could be repeated for any number of instantaneous values, and the result plotted as a curve, which will be that of the line pressure.

Turning again to Fig. 53, we see that at the moment x the value of the line pressure between a and b is equal to the algebraic difference (in this case the arithmetical sum) of the two phase pressures in A and B. This resultant value is shown at the point x', and the curve of line pressure is plotted as the resultant of the two curves E_a and $-E_b$, the latter being shown as E_b', being always equal in value but opposite in sign to E_b.

The vector diagram of pressures is shown in Fig. 54, where

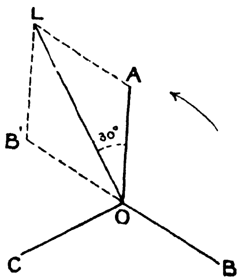

FIG. 54.

OA and OB represent the vectors of the maximum (or virtual) pressures in A and B phases respectively—*i.e.*, curves E_a and E_b in Fig. 53. OB′ represents the vector of curve E_b' (that is, $-E_b$) in this figure, and OL, therefore, represents the vector of the maximum (or virtual) resultant pressure across two mains.

The value of OL is readily seen from Fig. 54 to be $\sqrt{3}$ OA.

That is to say, the **line pressure is equal to $\sqrt{3}$ × phase pressure.** It will also be noticed that, if the phase load is an inductionless one, that is, if the current is in phase with the phase pressure, the line current will lag 30° behind the line pressure.

If a three-phase generator is used to supply other than a pure three-phase motor load, a fourth lead must be added in case the phases are unequally loaded. Incandescent lamps may then be arranged for the phase pressure, and connected as shown in Fig. 55. The size of the fourth or neutral wire will depend upon the loading of the phases, since it is called upon to carry the resultant of the three currents.

The effect of loading the phases unequally, without providing a fourth wire, can be deduced from Kirchhoff's first law,

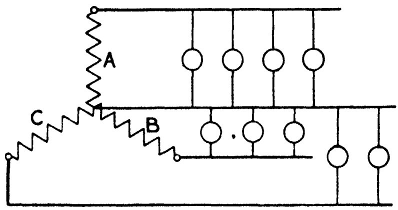

FIG. 55.

which says that if any number of currents meet at a point their resultant is zero. In other words, the currents will automatically take up a position of electrical equilibrium. If then there are three which are unequal in value this will mean that the phase angle between them is no longer 120°.

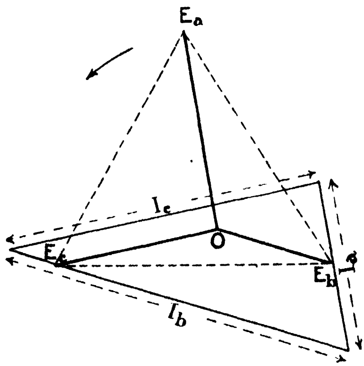

FIG. 56.

Taking the case of a non-inductive unbalanced load, the currents may be plotted as a triangle, since they must be in equilibrium, as shown in Fig. 56.

The vector diagram of pressures is drawn so that the phase pressures are parallel to their respective current vectors,

OE_c being the pressure in phase C, OE_a that in phase A, and OE_b that in phase B. E_cE_b, E_bE_a and E_cE_a represent the three line pressures which are approximately equal to one another, the difference being due to the different IR drops in the circuits. It will be seen from this diagram that the angle between the phase pressures (and therefore also between the currents) is considerably distorted from 120°, and, furthermore, that the pressure on B phase—namely, OE_b—is very low, this being the most heavily loaded phase, as can be seen from the triangle of currents.

The result, therefore, will be very bad **pressure regulation,** the heaviest load being much *undervolted* and the lightest load *overvolted*.

22. Mesh-connected Generators.

The phase windings of the generator armature may be connected in another way, as an alternative to the star system

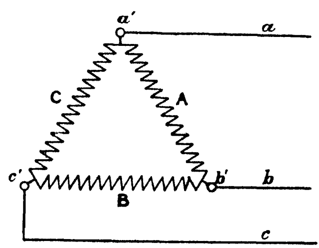

FIG. 57.

of connection which we have already discussed. That is, instead of connecting the beginning of each phase winding to a common point, to connect the end of one phase to the beginning of the next, and so on. This is known as Δ **or mesh** connection, and is shown diagrammatically in Fig. 57, which shows the end of A phase connected to the beginning of B phase, and so on throughout the armature. The method of winding an elementary two-pole ring armature is shown in Fig. 58.

Since there is no neutral point, such a method of connection necessarily implies three leads only for the transmission of power, these leads being connected to the junctions of the phase windings, that is, to the points a', b' and c' in Fig. 57.

Now the pressure between any two of our three leads, a, b and c, must be equal to the phase pressure, since each pair of them is connected across the ends of a phase. We may, for example, imagine the pressure between a and b to be *either* that due to A phase, *or* the resultant of the pressures in B and C phases. Drawing our pressure vector diagram (Fig. 59), we see that these two pressures are always the same, OA′, the

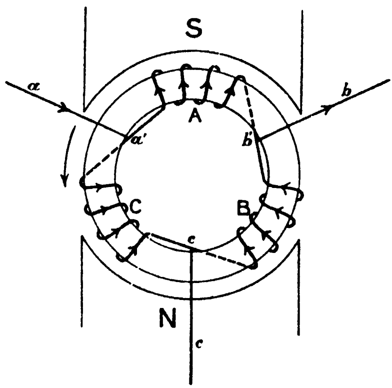

Fig. 58.

resultant of OB and OC, being, of course, equal to OA. This should be compared to the *current* vector diagram in the case of the star-connected machine.

To find the *current* in the mains we proceed in exactly the same way as we did in finding the line *pressure* in the star-connected system.

At any instant the current in the line a is equal to the algebraic difference between the currents in phases A and C, the general equation being

$$i_l = I_{max.} \sin \theta - I_{max.} \sin \ (\theta - 120°),$$

I being the phase current.

At the moment shown in Fig. 58 the current is flowing only in lines *a* and *b*, and the instantaneous value of this current

$$i_l = I_{max.} \sin 90° - I_{max.} \sin (90° - 120°),$$
$$= I_{max.} + I_{max} \sin 30°,$$
$$= 1{\cdot}5\ I_{max}$$

The virtual value of the line *current* in the case of a mesh-connected machine is, therefore, obtained from a similar vector diagram to the star-connected *pressure* vector diagram. Referring to Fig. 60, OA and OC represent the maximum (or virtual) values of the currents in A and C phases respectively, OL being the resultant of OA and — OC, and representing, therefore, the maximum (or virtual) value of the line current.

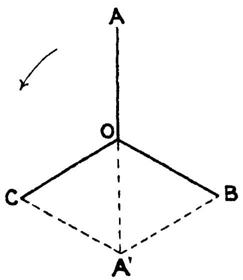

FIG. 59.

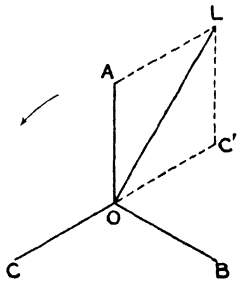

FIG. 60.

That is, the **line current** in the mesh-connected generator is equal to $\sqrt{3} \times$ **phase current.**

Since there is no fourth or neutral wire with a mesh-connected machine, it does not lend itself to unequal phase loading, and is not, in practice, so much used as the star system.

To sum up the points considered in the last two sections, our conclusions are as follows :—

(1) In **star**-connected machine

line current = **phase current,**

line pressure = $\sqrt{3}$ **phase pressure.**

and

(2) In a **mesh**-connected machine

line pressure = **phase pressure,**

line current = $\sqrt{3}$ **phase current.**

23. Measurement of Power in Three-phase Circuits.

If E = phase volts and I = phase current, the watts in any one phase = EI cos φ, where φ is the phase angle between E and I. Obviously, then, the total power of the three phases will be equal to 3 EI cos φ with equal phase loading.

Now for star-connected machines

$$E_l = \sqrt{3}E,$$

$$I_l = I.$$

$$\therefore \quad \text{Power} = \frac{3E_lI_l}{\sqrt{3}} \cos \varphi,$$

$$= \sqrt{3}E_lI_l \cos \varphi.$$

For mesh-connected machines

$$E_l = E,$$

$$I_l = \sqrt{3}I,$$

$$\therefore \text{Power} = \frac{3E_lI_l}{\sqrt{3}} \cos \varphi, \text{ as before,}$$

$$= \sqrt{3}E_lI_l \cos \varphi.$$

Or, in words :—

The power in three-phase circuits, whether star or mesh-connected, with equal phase loading, is equal to $\sqrt{3}$ × line pressure × line current × cos φ.

The actual measurement of power is carried out, as usual, by the use of one or more wattmeters, since without the use of a wattmeter cos φ is an unknown quantity.

There are two distinct methods of measuring actual power, and we will consider these in turn, taking the case of a star-connected system.

1. *When the Neutral Point is Accessible.*—This method involves, with equal phase loading, the use of one wattmeter only, connected as shown in Fig. 61. The current coil is connected in series with one of the mains, the pressure coil being connected between this main and the neutral point of the machine. The wattmeter then reads EI cos φ, and the total power in the circuit is obviously three times the wattmeter reading.

If the phases are unequally loaded three wattmeters must be used simultaneously and their readings added together. If three instruments are not available, then the single one must be connected to each phase in turn

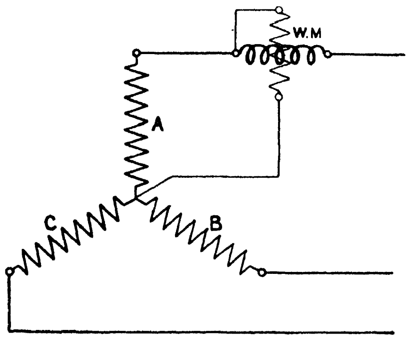

FIG. 61.

2. *When the Neutral Point is not Accessible.*—The more general case, as we have already seen, is that in which each phase is always equally loaded, and transmission is, therefore,

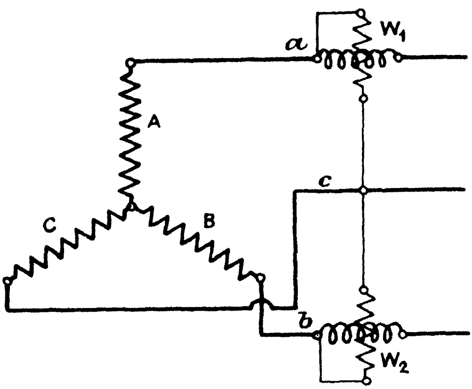

FIG. 62.

by three leads only. It will be necessary in this case to measure the power without having access to the neutral point. This may, in practice, be done by the simultaneous use of two wattmeters, connected as shown in Fig. 62.

It is quite immaterial which two phases we use for our wattmeter connections; we will assume that we have used lines a and b.

As before, let E_l be the line pressure, I_l the line current, the corresponding phase values being E and I.

Now the total power in the circuit at any instant

$$= e_a i_a + e_b i_b + e_c i_c,$$

e and i being instantaneous values of pressure and current.

But since, by Kirchhoff's law,

$$i_a + i_b + i_c = 0 \text{ at any instant,}$$
$$\therefore \quad i_c = -(i_a + i_b).$$

$\therefore$ The power in the circuit

$$= e_a i_a + e_b i_b + e_c(-\{i_a + i_b\})$$
$$= i_a(e_a - e_c) + i_b(e_b - e_c)$$

Now, repeating our pressure vector diagram, we see that the *line* pressure between a and c is equal to the resultant between A phase pressure and C phase pressure *reversed* (*cf.* Fig. 54 on p. 79)—*i.e.*, = resultant of e_a and $-e_c$.

We may then regard the line pressure between a and c as being equal to $e_a - e_c$ in the above equation, and call this e_l. This is the pressure energising the volt coil in W_1.

Similarly, the pressure energising the volt coil in W_2 is $OL' = e_b - e_c = e'_l$.

The currents in the current coils of the wattmeters are respectively i_a and i_b.

The power measured by the wattmeters at any given instant is then

$$W_1 = e_l i_a,$$
$$W_2 = e'_l i_b.$$

The sum

$$= e_l i_a + e'_l i_b,$$
$$= i_a(e_a - e_c) + i_b(e_b - e_c),$$

which is, as we have seen, the total power in the circuit at that moment.

That is, the total power of all three phases is equal to the sum of the two wattmeter readings; and this will always be so whether the machine is star or mesh connected, and whether the circuit is inductive or non-inductive, for balanced or unbalanced loads.

It is worth noting that for any value of φ other than 0° the readings of the two instruments will not be the same.

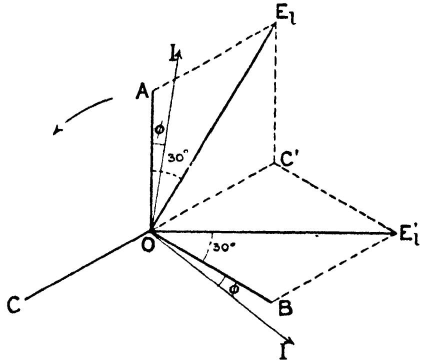

Fig. 63.

If, for example, φ is an angle of lag as shown in Fig. 63, being the same for all three phases, then the deflection on W_1 is proportional to E_l cos $(30° - \varphi)$, and that on W_2 to E_l' cos $(30° + \varphi)$. If φ exceeds 30° the true watts in the circuit will be equal to the *difference* between the two wattmeter readings.

The power factor of the circuit is given by the expression

$$\cos \varphi = \frac{W_1 + W_2}{\sqrt{3}E_l I_l}.$$

CHAPTER VII.

TRANSFORMERS.

24. Transformers.

We shall now investigate the general principle on which depends the action of the most commonly used alternating-current machinery. No attempt will be made to go into details of construction, or even fully into the theory of the various electrical and magnetic actions which are manifested when the machines are at work. Some general idea of these actions are all we can hope to obtain in the short space at our disposal, and there are many excellent text-books which can be read with profit when the general principles are once grasped.

As has already been explained, power can readily be generated and transmitted at very high pressures by adopting the alternating-current system. The economy of this arrangement is obvious, and very large amounts of power can, at these high pressures, be transmitted by comparatively small cables; a very great advantage in practice, since the cost of the power mains is no inconsiderable part of the total cost of any installation.

It will very seldom happen, however, that the power can be *utilised* at this very high pressure, partly owing to the difficulty in designing lamps or motors which would be suitable, but chiefly owing to the very great danger to life which these high pressures would necessarily imply.

For this reason it will be necessary to convert the high-pressure current as sent out from the generating station into a corresponding one at lower pressure. This can be done in the case of an alternating-current system by means of a very simple piece of apparatus known as a **transformer** which contains no moving parts whatever and requires no attention. It is, in fact, the ease with which this transformation can be effected which constitutes one of the chief advantages of alternating-current working.

The underlying principle of a transformer is the same as that of the induction coil, with which we are already familiar; that is, it consists essentially of two coils, a **primary** and a **secondary,** wound in close proximity to one another on an iron core. An E.M.F. will be induced in the secondary winding by any alteration in the value of the primary current, since the magnetic flux produced by the current in the primary will also vary in value, and in doing so will be cut by the coils forming the secondary winding. We get, in fact, **mutual induction** between primary and secondary.

The practical resemblance between a transformer and an induction coil is not, however, very close. In the latter case the necessary current fluctuations are brought about by alternately making and breaking the primary circuit; very small amounts of *power* are dealt with; and, finally, their purpose is always to produce a very high E.M.F. in the secondary, and considerations of efficiency are sacrificed to secure this effect.

(Transformers, on the other hand, are required to deal with very large amounts of power, and efficiency, therefore, becomes of primary importance. So carefully is this considered in practice that efficiencies of 97 per cent. are not uncommon in the case of well-designed machines, that is, if 100 kw. are supplied to the primary 97 kw. can be obtained from the secondary. The variations of the magnetic flux, and consequently of the primary current, necessary to cause mutual induction are produced without any such make and break arrangement as is used in an induction coil supplied with a continuous current; so that the transformer contains no moving parts, and is, as we shall see, entirely automatic in its action. It is also reversible, and can be used either to increase or to reduce the pressure of supply.) As a general rule, in this country its function is to receive power at high pressure and to give out a corresponding amount, depending on its efficiency, at a lower pressure suitable for lamps or other consuming devices.

A full investigation of the action of this apparatus would take very much more space than can be devoted to the subject here, nor is such investigation necessary from the point of view of the user of the apparatus. We shall try and obtain a clear idea of the general principle of its action, and also of the various causes which tend to reduce its efficiency.

Our first step will be to consider the action of the magnetic flux produced by the passage of an alternating current in the primary winding. As we have already seen, this flux rises and falls in strength as the primary current varies, and in doing so is cut by the turns of the primary winding itself, which results in the E.M.F. of self-induction which we have already investigated. It is also cut by the turns of the *secondary* winding, and produces in this an E.M.F. which is known as the **secondary E.M.F.** This latter must of necessity be always **in phase with the E.M.F. of self-induction,** since the same varying magnetic flux is responsible for both. If the secondary circuit be completed through a resistance, or any form of load, this E.M.F. will send a current through that circuit.

The value of the secondary E.M.F. will depend, just as in the case of the induction coil, upon the ratio of the number of turns in the secondary to those in the primary windings. That is, if E_2 is the secondary pressure and E_1 the primary, S_2 and S_1 being the number of turns in the respective windings,

$$E_2 = \frac{E_1 S_2}{S_1}.$$

As has been already mentioned, transformers are generally used as step-down machines, in which case the secondary E.M.F. will be the lower of the two.

It will greatly simplify our future calculations and diagrams if we consider the special case when the ratio of transformation is *unity,* that is, when the number of turns in the primary and secondary windings is the same. It is hardly necessary to say that such a case is not a practical one, but the principle is precisely similar for any ratio.

The losses in any transformer fall under the following heads :—

(1) Ohmic resistance of primary and secondary windings.
(2) Hysteresis.
(3) Eddy currents in the iron core.
(4) Magnetic leakage.

The effect of the first three is to reduce the efficiency of the apparatus by the absorption of some power in the machine itself. The first presents no difficulty. We may quite readily regard the resistance of both as one single quantity, and it may then be compared to the internal resistance of a generator, its effect being precisely similar—namely, to cause the potential difference at the terminals of the machine, when a current is flowing, to be less than the induced E.M.F.

Hysteresis losses present rather more difficulty. Their cause has already been dealt with under the heading of choking coils, and their effect will be considered later.

Eddy currents in the core have also been mentioned. Their effect is simply to put a small extra load on the primary, since the iron core acts as a **secondary on short-circuit,** that is to say, in addition to the usefully employed secondary load, we are also supplying a useless current which, flowing in the core itself, is dissipated in heat. The reactive effect of eddy currents on the primary circuit is precisely similar to that of the *useful* secondary current, which will be discussed when we are dealing with the loaded transformer.

Magnetic leakage is a considerably more difficult subject to understand. It may be here stated, in explanation of the term, that, although every endeavour is made to confine the lines of magnetic force to the iron path of the core, yet some of them will, as it were, escape into the air. In consequence, the whole of the flux which is set up by the primary current will not be cut by the secondary windings, since some of it will escape, or leak, by the way.

The result will be that the secondary E.M.F. will be diminished, and will not have quite the value it should have according to the ratio of secondary turns to primary. In other words, with a unity ratio of transformation, the coefficient of *mutual induction* between primary and secondary will be less than the coefficient of primary *self-induction*.

For reasons which are beyond the scope of this book the effect of magnetic leakage increases with the load on the secondary, and although nearly negligible when the transformer is unloaded, is by no means so when any considerable current is flowing through the coils.

Its effect is not to cause any appreciable lessening of the *efficiency* of the apparatus, but to affect its voltage regulation adversely, as will be explained later.

25. A Transformer on Open Circuit.

When the secondary is not supplying current the primary of a transformer acts simply as a **choking coil.** Under these circumstances, as we have already seen, its effect is to choke back the applied pressure by virtue of the E.M.F. of self-induction, so that only a very small **magnetising** current flows through the primary winding.

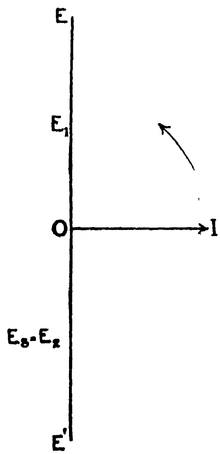

FIG. 64.

As we already know, eddy currents in the core prevent any transformer or choking coil from being theoretically unloaded, since the core acts itself as a secondary circuit. We have, however, already agreed to consider these eddy currents simply as a useless addition to the secondary circuit, and we will therefore omit them from our calculations in this section. We must remember that our no-load diagram will not be really complete if we neglect these eddy currents, but we are not yet in a position to know the effect of a secondary load on the primary circuit. Magnetic leakage can also be omitted as being negligible when the transformer is unloaded.

There remain, then, ohmic resistance and hysteresis to be considered. Before doing so, we will construct the simplest possible vector of pressures and currents, taking the case of a transformer unloaded and without any losses whatever. This is shown in Fig. 64, where I represents the current vector, lagging 90° behind the applied pressure OE, which is equal to the back E.M.F. of self-induction (OE′), or to the secondary induced pressure E_2, since we are considering a transformer with a unity ratio of transformation.

The applied volts are in this case exactly equal and opposite to the E.M.F. of self-induction, since there is no IR component (*see* Fig. 14), and the angle of current lag is, therefore, 90° ; that is, there is no component of the current in phase with the volts, so that the power is zero.

The current flowing is purely a magnetising one, and the magnetic flux is rising and falling exactly in phase with this current, the E.M.F. of self-induction being produced under circumstances with which we are already familiar.

We will now investigate the effect of **hysteresis** on the no-load diagram. It has already been mentioned that this is accountable for an actual loss of *power*, and therefore that the primary current cannot be wholly wattless, even though the secondary is unloaded. As a matter of fact, it is very unusual to find that the power factor of an unloaded transformer is less than 0·5, or, in other words, that φ, the angle of lag between current and pressure, is more than 60°.

Ohmic resistance and eddy currents are partly responsible for this fact, but it is mainly owing to hysteresis, and it will be necessary for us to understand the reason clearly.

In our early investigations of the question of self-induction the assumption was made that "the magnetic flux is varying most rapidly when the current producing it is varying most rapidly," and from this assumption we deduced

$$E_s \propto \frac{di}{dt},$$

i being the current flowing through the coils. Now this is perfectly true so long as the magnetic circuit does not exhibit hysteresis,* as is the case if the solenoid has no iron core, or if the residual magnetic properties of the iron core are absolutely nil. But this latter case is purely theoretical. However

* There would be no hysteresis losses if the permeability of the magnetic circuit were constant throughout the cycle (*see* p. 18), that is to say, if the B/H curve throughout the cycle were a straight line no hysteresis loop would result as the intensity of magnetism rose and fell.

soft and pure the iron may be *some* residual magnetism is unavoidable, and, in the case of an alternating current of normal frequency, when the magnetism must be reversed about one hundred times a second, even a small residual magnetic tendency produces very marked effects. What will these effects be? If the coil contains no iron core the magnetic flux will rise and fall *exactly* in phase with the current which produces it, since air, or indeed any non-magnetic substance, cannot exhibit residual magnetism. In such a case the E.M.F. of self-induction, which depends upon the rate of change of the flux, and therefore, in such cases, of the current, will lag exactly 90° behind the latter.

But if an iron core be introduced the flux will *not* rise and fall exactly with the current. At the instant when the current is reversing, as shown at *a* in Fig. 65, the effect of the residual

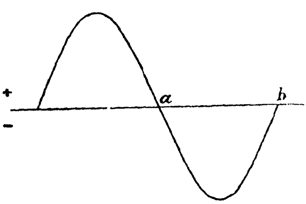

FIG. 65.

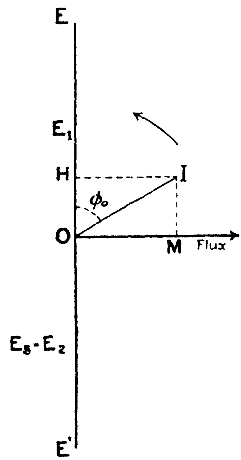

FIG. 66.

magnetism of the core will be to keep the flux in a positive sense, and the current will have to *rise to a certain negative value* before this tendency is neutralised. As soon as this happens the flux will start in the negative sense under the influence of the negative current. At the moment *b* the current again reverses; but, as before, will now have to rise to a certain positive value before the flux is reduced to zero, owing to the lagging effect of residual magnetism. In short, as its name implies, the effect of **hysteresis** is to cause the **magnetic flux to lag behind the current** which produces it. The result on the no-load vector diagram is shown in Fig. 66. The direction of the flux is shown horizontal, lagging behind the current vector OI. The angle of lag φ_0 between the current and the applied volts OE is now no longer 90°, owing to the effect of hysteresis. Obviously, then, the current has now a component OH, *in phase* with the volts OE, with the result that the power supplied to the primary is no longer zero but is equal to OE × OH, that is, to EI cos φ_0.* The E.M.F. of self-induction then lags ∠E′OI behind the current, since it must always, as we have already seen, lag 90° behind the flux which produces it.

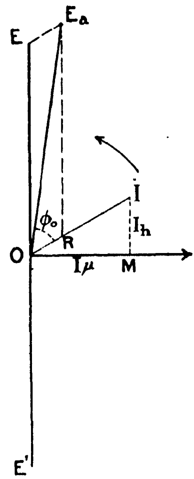

Fig. 67.

The effect of ohmic resistance is, of course, to bring current and volts more into phase with one another. Though this is not marked when the transformer is unloaded it will be as well to consider it now, so that our vector diagram for the case of a loaded secondary will be the more readily understood. The applied pressure will in this case not only have to balance the E.M.F. of self-induction but must also contain an IR component in phase with the current (*see* Fig. 14 on p. 24). This IR component is drawn to the same scale as OE along the current vector, and is shown at OR in Fig. 67. The applied volts will therefore be the resultant of OR and OE, that is, OE_a in Fig. 67.

The final angle of lag φ_0 is therefore seen to be considerably less than 90°, partly because of hysteresis and partly because of the ohmic resistance of the primary. Fig. 67 then shows the complete no-load vector diagram, neglecting the question of eddy currents, which we have agreed to consider as a *load*, albeit a useless one. Making OI equal to the primary current value to any convenient scale, OM will be equal to the magnetising component (I_μ) and IM to the " hysteresis component " (I_h).

* In all these diagrams we shall assume that the wave form of the magnetising current follows the simple sine law, though the effect of hysteresis is to cause, in practice, considerable distortion, the third harmonic being present with a comparatively large amplitude.

26. A Transformer on a Non-inductive Load.

Let us now consider the effect of loading the secondary of the transformer, the load being non-inductive, as, for instance, incandescent lamps, as shown in Fig. 68.

The effect of loading the secondary will be, as was mentioned on p. 21, when we first considered the question of eddy currents, to **increase the current in the primary.** We realised that this *must* be so, since a secondary load implies power, and therefore an extra current load on the generator, which is the ultimate source. Now, although we are quite certain that this

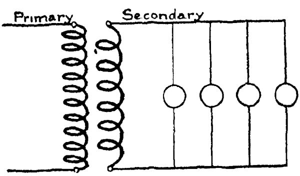

FIG. 68.

must be the case, we do not yet know what actually causes an increase in primary current when a load is put on the secondary.

First, let us again consider what is the actual cause of a secondary E.M.F. being induced. This we know to be the varying magnetic flux. This magnetic flux is responsible for the E.M.F. of self-induction, or the primary induced E.M.F., and also for the secondary induced E.M.F.,

Now the primary induced E.M.F.

$$E_s \propto N \times S_1 \times f,$$

where N is the *flux*, S_1 the number of turns in the primary winding, and f the frequency of supply. Now E_a, the applied E.M.F., is very nearly equal to E_s, since the ohmic resistance of the primary is extremely small. It has purposely been exaggerated for the sake of clearness in Fig. 67. We may then write

$$E_a \propto NS_1 \times f,$$

or

$$N \propto \frac{E_a}{S_1 \times f}.$$

But *whatever the conditions of loading*, E_a, the applied pressure, f, the frequency, and S_1, the number of turns, are *constant*. Therefore N must be constant. That is to say, for all conditions of loading the value of the **magnetic flux must be constant.**

Let us now construct our vector diagram (Fig. 69) for a non-inductive secondary load, on exactly the same lines as Fig. 67. Draw first the vector OM representing the magnetising component of the current, and also the flux value. This will be equal to OM in Fig. 67, since we have just seen that the flux is constant in strength. OI will equal the no-load current as before. OE′ is drawn to show E_s to scale and therefore also E_2 (taking as before a unity ratio of transformation). Since the secondary load is non-inductive, I_2 is in phase with E_2, OI_2 representing this current in magnitude and direction to the same scale as OI. The value of the primary current is found by completing the parallelogram I_2OI_1I, since the effect of the no-load current is the same, as regards the production of the magnetic flux, as the combined effects of primary and secondary currents—that is, it may be regarded as the resultant of these two currents. The vector OE_a of the applied volts is obtained as before by taking OR to represent to scale the resistance component of the applied volts measured along the primary current vector. The effect of a non-inductive load is therefore—

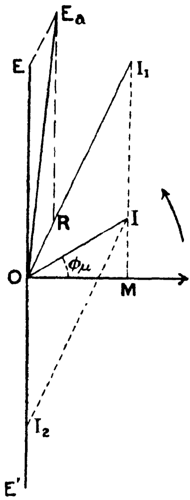

FIG. 69.

(1) **To increase the primary current.**

(2) **To bring the primary current more into phase with the applied pressure.**

Now, as a matter of fact, this diagram has been drawn with the value of the no-load current and also the IR drop in the primary greatly exaggerated for the sake of clearness. Having thereby learnt the details of its construction we will now repeat it in Fig. 70, more in its actual form, and make our observations from this corrected diagram. Even this must be distorted in order to make it readable, but it is much nearer the true state of affairs than Fig. 69.

We can now observe the following facts :—

1. The no-load current is very small compared with the full-load current.

2. The angle φ_μ—that is, the lag of the magnetic flux behind the no-load current—is comparatively large, owing to the effect of hysteresis. It is this angle which is mainly responsible for the comparatively high no-load power factor.

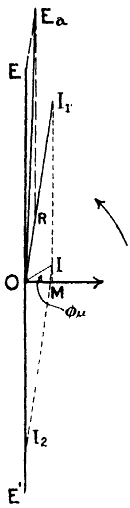

FIG. 70.

3. The effect of a non-inductive secondary load is to bring primary current and volts practically into phase—that is, a non-inductive load in the secondary is equivalent to a **non-inductive load in the primary.**

4. Primary and secondary currents are almost exactly in opposition to one another ; that is, while the **primary** current is trying to **magnetise** the core the **secondary** current is trying to **demagnetise** it. Since the magnetic flux remains constant, it therefore follows that an increase in secondary current necessarily means a corresponding increase in primary current : that is to say, the momentary effect of a rush of current in the secondary is to demagnetise the core, and there is therefore an immediate response in the primary circuit, owing to the applied E. M. F., to counteract the effect of the secondary current and to bring the flux up to its previous strength.

5. The effect of eddy currents on the no-load diagram will be to increase slightly the no-load current, whose power component will then consist of two parts I_h, as before, and I_e.

27. A Transformer on an Inductive Load.

If we have clearly grasped the facts dealt with in the previous sections no difficulty should be experienced in constructing our vector diagram in the case of an inductive load on the secondary. We shall again exaggerate no-load values in order to make the diagram clear.

Drawing first our " no-load triangle " OIM (Fig. 71), OE, and OE′ represent the pressures E_s', and E_s (or E_2) as before. OI_2 represents the secondary current, φ_2 being the angle of lag in the secondary circuit. OE′, or secondary volts, is made up of two components at right angles, namely OR_2

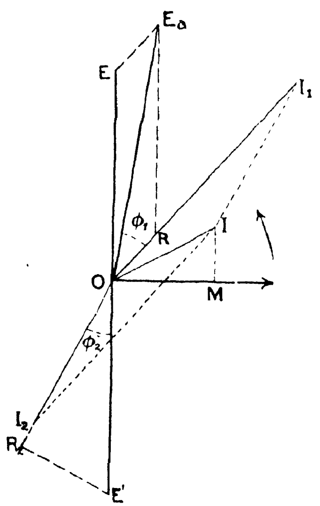

Fig. 71.

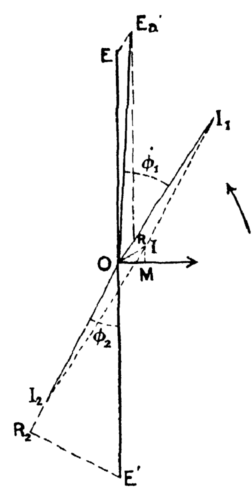

Fig. 72.

which is equal to the ohmic drop in the circuit (*i.e.*, $I_2 \times R_2$) and $E'R_2$, equal to the reactance drop in the secondary. As before, OI_1 is found by completing the parallelogram I_2OI_1I, E_a is found as before (OR being the IR drop in the primary winding of the coil) and φ_1 is the angle of lag in the primary circuit. We should find, by redrawing the diagram and giving no-load and full-load currents their proper values, that primary and secondary currents are again in opposition to one another, so that the phase angle in the primary automatically becomes practically the same as that in the secondary (Fig. 72).

28. The Effect of Magnetic Leakage.

The simplest point of view from which to regard this is to say that, as the whole flux set up by the primary current is not cut by the secondary winding, part of this latter is inoperative as regards useful induction. We must also remember that the converse is also true since the field set up by the secondary current is cut by the primary windings. The practical result of this is that both windings are acting in part as choking coils, and the heavier is the current in the machine the more marked is this choking effect. Now, as we already know, choking coils imply no absorption of power beyond their almost negligible I^2R losses, so that magnetic leakage does not imply any appreciable loss of power, and from this point of view merely means that the machine has to be slightly bigger for a given duty.

Apart from the question of power, however, it has two undesirable effects :—

(*a*) It upsets the voltage regulation of the secondary. that is, between zero and full load the secondary potential difference varies very much more than it would do if we had only the IR drop to consider. This is owing to the fact, as before explained, that the secondary is acting in part as a choking coil.

(*b*) It makes both primary and secondary circuits unnecessarily inductive, and therefore adversely affects the power factor of both.

29. Practical Machines.

The result of our observations has been to show the great practical utility of the transformer. Its efficiency is high and it is absolutely automatic in action, and since it contains no moving parts it can in practice be left wholly without attention. There is as a fact nothing new to us about a machine which automatically regulates its input to suit its output ; as we know, every motor takes, of itself, more or less current according to the mechanical load on the machine, and we may in a sense regard a transformer as a kind of motor in which the necessary relative motion is produced between magnetic field and conductors without any motion of matter.

The practical details of transformer construction all tend to a high efficiency. The use of a closed magnetic circuit, that is a complete iron path for the flux, is therefore essential. Iron of high permeability and possessing as little residual magnetism as possible is essential for the cores, which must of course be completely laminated.

To reduce magnetic leakage, primary and secondary coils must be in the closest proximity, and they are frequently wound in alternate sections on the core.

In the case of very small machines, primary and secondary windings are sometimes combined as shown in Fig. 73 (*a*). The advantage of this is to provide a very compact apparatus,

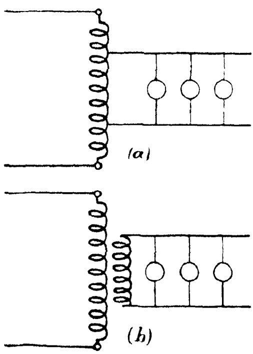

FIG. 73.

but the design is not suitable for any but quite small outputs. Such machines are known as **auto-transformers,** and, as may be seen from Fig. 73 (*b*), are electrically equivalent to the ordinary double-coil type. In principle part of the E.M.F. of self-induction or the induced E.M.F. in the primary is used as a secondary E.M.F., the ratio of transformation depending upon the position of the points from which the secondary circuit is taken. The coil of the machine is uniform, so that if the secondary circuit includes half the coil, the ratio of transformation will be two to one, as shown in Fig. 73 (*a*). Such machines are reversible on exactly the same principle as the double-wound type, and are frequently provided with a number of tappings, so that the ratio of transformation may be varied at will.

Two chief classes of the double winding transformer are found in practice.

The **shell** type is arranged with the core enclosing the coils, instead of the more usual arrangement, known as the **core** type, of winding the coils outside the core. All transformers must be thoroughly ventilated in order that the considerable heat due to the passage of the current through the windings and to core losses may be rapidly dissipated. No such aid to this end, as is supplied by the rapid rotation of moving machinery, is available for transformers, and such artificial means as air-blasts are sometimes employed. A very common method is to immerse the apparatus in oil, which is continually in circulation.

Finally, not only must the working efficiency be high, but the no-load current must be reduced to an absolute minimum, since it commonly happens that the primary of the transformer is kept permanently in circuit with the alternator supplying it. This will be the case, for example, when a building is supplied with current at high pressure, the lamps being lit from the secondary or low-pressure side of a transformer installed for the purpose.

CHAPTER VIII.

GENERATORS.

30. Generators.

The general principle of the alternating-current generator is precisely the same as that of the continuous-current machine with which we are familiar. We are already aware that the elementary generator produces an alternating E.M.F., and in order that this may produce a unidirectional current it is necessary to include a commutator in the machine.

It would appear, then, that the alternating-current generator is in this respect simpler than the continuous-current dynamo, for the reason that we may dispense with the commutator, and connect the two ends of the armature winding to a pair of slip rings on the shaft of the machine, from which an alternating current is collected by rubbing contacts or brushes. This arrangement is, however, seldom used in practice, except in the smallest machines, for the following reason :—

As explained on p. 3, the main reason for the transmission of power in bulk in the form of an alternating rather than a continuous current, is due to the fact that the former can very much more easily and safely be generated **at high pressure,** and that this pressure can subsequently be altered by means of static transformers. There is a distinct practical limit to the pressure at which a continuous-current machine can be worked, and this limit is fixed by the maximum pressure permissible between commutator segments. This limit does not, however, apply to an alternator, and they are in practice worked at pressures of 10,000 volts or even more, which pressure is considerably higher than the practical limit for continuous-current machines. Now, if we adopted the revolving armature type of machine, even though a commutator is unnecessary, nevertheless, we should have this enormous pressure between the two slip rings ; and even if current could be satisfactorily collected by rubbing contacts at this pressure, the danger of such a system would be considerable.

For this reason it is usual to arrange that the field-magnet system shall rotate and that the armature shall be stationary. The former then revolves inside the latter, and the ends of the armature windings are taken from the stationary portion of the machine direct to the high-tension switchboard. There is therefore no danger to the attendant, as there would be if the full pressure of the machine was across the slip rings and brushes necessary with a rotating armature.

The rotating field-magnet system must, of course, be supplied with current in order to energise the magnets, and this current must be a unidirectional one, since the polarity of the magnets must remain constant. In the case of a continuous-current machine either the whole, or a portion of the current generated by the machine is used for this duty. With an alternator, however, this obviously cannot be done, since the current supplied by the machine is not unidirectional. We have therefore to provide the current for the field-magnets, or the **exciting current,** from some independent source. This source may be a battery, or a small continuous-current machine known as an **exciter** specially provided for the purpose. These exciters are frequently small machines mounted on the alternator shaft, but it is quite usual, especially in large stations, to provide an entirely separate plant, whose duty it is to supply exciting current to all the alternators in the station. This exciting current is supplied at low pressure, generally about 100 volts, to the rotating field-magnets by way of brushes and slip rings, and the only live parts of the machine which are exposed are therefore at too low a pressure to cause any danger.

Apart altogether from the question of pressure, the fact that there are no rubbing contacts in the main circuit means that very heavy current outputs can be dealt with. Alternators are in fact nowadays built up to 20,000 K.V.A., which even at 10,000 volts means a full load current of 2,000 amperes. Such a heavy current would necessitate a very large brush contact area if collected from slip rings. In short, as already mentioned on p. 3, the main advantages of alternating-current transmission are due to (*a*) high voltage, (*b*) large units, and these can only be satisfactorily secured by adopting the rotating field type of machine.

31. The Operation of Alternators in Parallel.

In dealing with the parallel working of continuous-current machines there is only one essential point to be considered before a fresh machine is switched on to the bus bars to assist those already running; that is to see that its pressure is the same as or slightly above that of the machines already running.

We are by this time so used to the idea that we must expect extra complications with alternating currents, that it will be no surprise to learn that we have more to consider in the parallel working of alternators than the mere question of pressure values.

Briefly, before we can connect two machines in parallel, we have three conditions to fulfil:—

(1) The virtual values of their volts must be the same.

(2) The two machines must be in phase, that is, the instantaneous values of their pressures must also be the same and in the same sense.

(3) They must be running at exactly the same frequency. If they are not, even though condition (2) may be fulfilled at one instant, it will not be so at the next. Two machines running at the same frequency are said to be in **synchronism.**

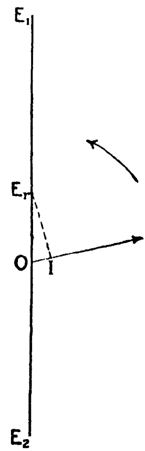

FIG. 74.

We will now briefly investigate the electrical action in the two machines which will follow the neglect of each of these necessary conditions.

(1) *Unequal Pressure Values.*—This will imply that one of the machines is under-excited with respect to the other, and the condition is an unstable one; that is to say, if the two machines are now connected they will at once tend to equalise their pressure values. The vector diagram is shown in Fig. 74

when OE_1 represents the virtual value of the pressure of the over-excited machine and OE_2 that of the under-excited machine. The two are imagined to be in exact synchronism, so that their pressure vectors will be exactly opposed to one another; that is, the terminals which are momentarily positive are connected together, so that the generated pressures oppose one another, this being the ordinary condition of the parallel connection of generators (Fig. 75).

The resultant OE_r of these two pressures is made up of two components, OI and IE_r, the former being the IR component and therefore in phase with the current, IE_r being the reactance component. The current which circulates between the machines will be almost wholly wattless, since the resistance

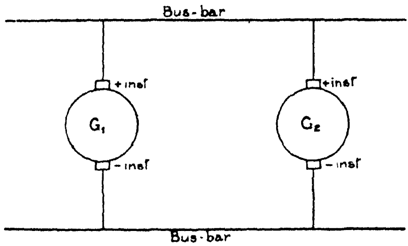

FIG. 75.

of the circuit is inappreciable compared with its inductance. This current will, as seen in Fig. 74, *lag* in the case of the over-excited machine and *lead* in the case of the under-excited machine.

Its effect will be to weaken the field of the former and to strengthen that of the latter and so tend to equalise the two pressures.

(2) *The Two Machines are Out of Phase.*—The result is shown in Fig. 76. The phase difference between the pressures of the machines is shown by the angle θ. The resultant of the two pressures OE_1 and OE_2 is shown at OE_r, which, as before, consists of two components at right angles, OI in phase with the current and IE_r the reactance component. As will be seen, the resulting current which flows between the machines is by no means wattless, having a power component in phase with E_1 in

the same sense, and a component in phase with E_2 but in the *opposite* sense to the latter; that is, a current will flow *with* the pressure in No. 1 machine and against the pressure in No. 2 machine. As we already know, this is therefore a generated current in the former case and a motoring current in the latter, and its effect will be to bring the two machines into phase by assisting one of the engines at the expense of the other and so helping the lagging machine G_2 to increase in speed and come into step with G_1. We shall meet the diagram shown

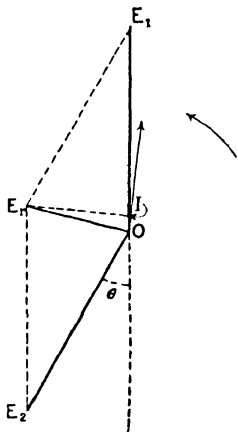

FIG. 76.

in Fig. 76 again when we are considering the subject of synchronous motors.

(3) *Machines not Running at the Same Frequency.*—This will cause "interference" currents between the two machines, of varying values and frequencies. The pressures will be alternately in opposition and in series: the latter condition would cause the current to rise to a dangerous value, and the circuit-breakers would at once open and break the circuit.

32. Synchronising Devices.

In order that the switchboard attendant may know that the above three conditions have been fulfilled before switching in a new machine it becomes necessary to provide a **synchronising** device such as is shown in Fig. 77. G_1 is connected to the bus bars, G_2 being about to be brought in to assist G_1, by closing the double-pole switch S. Two lamps are connected as shown, L_1 being connected to b bus bar and to p terminal of G_2, L_2 being connected to the other bus bar a and the other terminal q of G_2.

The machines are assumed to have been excited to give the same pressure and to have been run up as nearly as possible to the same speed. Suppose that at the instant shown in the figure they are in the proper condition to be connected in parallel, that is, that the two terminals connected to the same bus bar a are both showing a maximum positive polarity, and

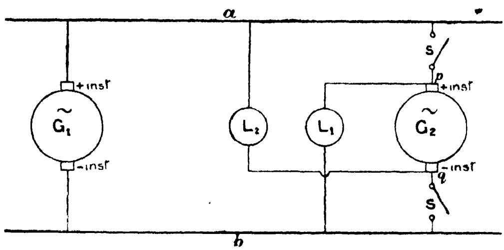

FIG. 77.

the two connected to the other bus bar b a maximum negative polarity.

The pressure across the two lamps will be the same, that across L_1 being the potential difference between b bus bar and p terminal of G_2, the pressure across L_2 being the potential difference between a bus bar and q terminal of G_2. These pressures are obviously at this instant equal to the maximum pressure of either generator, and so long as the machines remain in synchronism the pressure across the lamps will be the virtual value of the pressure of either machine.

Suppose, now, the machines to be exactly 180° out of phase, so that p terminal of G_2 is now negative and q positive, the instantaneous polarity of G_1 remaining the same as before. The potential difference between bus bar b and terminal p of G_2 is now zero, since the machines are of exactly opposite polarities.

Similarly, there is no pressure between bus bar *a* and terminal *q* of G_2. Both lamps will now, therefore, be dark.

The nett result will be that the lamps will alternately glow brightly (when the machines are exactly in synchronism) and be dark when they are directly out of step. These alternations from bright to dark will be slower and slower as G_2 comes up to its proper speed. All, therefore, that the attendant is required to do is to watch the lamps, and as soon as the alternations have become very slow to close his switch when the lamps are glowing most brightly. Once connected in parallel the two

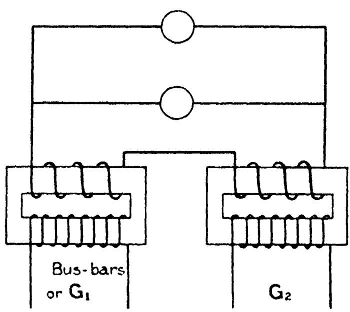

FIG. 78.

machines tend to keep each other in step, as explained in the previous section.

This device is all that is required, provided that the pressure of the machines is low, in which case the synchronising lamps are designed for the generator voltage. In the case of high-tension alternators, however, it will become impossible to connect lamps directly to the bus bars or machines, and some other arrangement must therefore be made. This is shown in its elementary form in Fig. 78. Two small transformers are used, the primary windings being connected to the bus bars (or G_1) and G_2 respectively, the secondary windings being connected in series and to the synchronising lamp as shown. It is usual to connect a second lamp in parallel with the first, the second lamp being alongside the generator, and therefore in view of the engine driver. The ratio of transformation will depend upon the generator pressure and the voltage for which the synchronising lamps are designed.

The action of the apparatus is as follows. When the two machines are in synchronism the two secondary windings are assisting one another, their E.M.F.s being in series, and the lamps will therefore, as before, glow brightly. When, however, the machines are 180° out of phase the pressures of the secondary windings are opposed to one another, and since their values are equal the lamps will be dark. The effect is therefore exactly similar to the case of lamps directly connected.

It is more usual to combine the two transformers, as shown in Fig. 79, into one machine having two primary windings

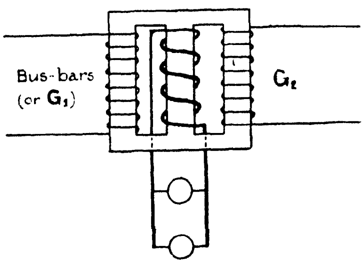

FIG. 79.

connected as before, and one secondary connected to the synchronising lamps.

When the machines are 180° out of phase the two fluxes set up by the currents in the primary windings neutralise one another so that no E.M.F. is induced in the secondary. When they are in synchronism the secondary E.M.F. is a maximum, and the lamps therefore receive their maximum pressure.

There is an obvious drawback to the use of synchronising lamps, and that is that there is nothing to tell the attendant whether the incoming machine is running too fast or too slow. As a matter of fact, he can very soon find out by trial. If the periods of light and darkness in the synchronising lamps become shorter and the flicker therefore more rapid by increasing the speed of the incoming machine, it is obvious that it was running too fast before, and *vice versa*.

Instruments, however, are made, known as **synchroscopes,** in which a pointer is deflected by any want of synchronism between the machines, the deflection being apparently in one constant direction if the incoming machine is too slow, and in the other if it is too fast. The form of the dial is shown in Fig. 80.

The synchronising of **three-phase machines** is carried out as

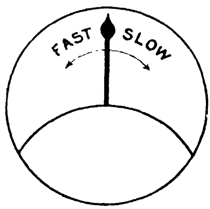

FIG. 80.

a rule in exactly the same way as that of single-phase machines, only one phase being used. It is necessary to ensure that, when the machines are first put to work, the three phases of each machine are operating in the same order. If this is so, all we need do in future is to synchronise one phase, A, for example, in No. 1 generator with A phase in No. 2 generator. B and C phases must then also be in synchronism if they are operating in the same order in each machine.

CHAPTER IX.

SYNCHRONOUS MOTORS.

33. Synchronous Motors.

A brief study of the principle of alternating-current **motors** now remains. These include three main classes, known as **synchronous, induction** and **commutator** respectively. Their characteristics and principles of working are quite distinct, unlike the case of the continuous-current machines which differ only in the matter of field windings, but there are certain fundamental facts which are common to all electric motors. Briefly, these facts may be summed up as follows :—

(*a*) The torque of any motor depends upon the reactions between the stationary and the moving magnetic fields.

(*b*) In every motor there exists a generated or **back E.M.F.,** the value of the current flowing being dependent upon the resultant of this back E.M.F. and the pressure which is applied to the motor terminals from some source of electrical energy.

Synchronous motors are in practice most usually wound for a three-phase supply, but single-phase machines will work just as well in theory, and will be considered in this section for the sake of simplicity. Just as a continuous-current motor is electrically identical with the corresponding generator, so is the synchronous alternating-current motor. In short, any **alternating-current generator will run as a synchronous motor,** but, as we shall see, there are certain limitations to their useful employment, which make such machines by no means of universal application. In fact, their use is practically confined to one special case which will be explained later.

Let us turn to the diagram of the elementary machine shown in Fig. 81. Imagine the poles NS to be excited from some continuous supply, so that their polarity remains constant. Suppose, for the sake of simplicity, that there is only one coil on the armature, which is shown in section at *ab*. If we connect the coil *ab* to an alternator by way of slip rings and brushes, the current in the coil will continually be altering in value and direction under the influence of the alternating supply pressure. At the instant shown in Fig. 81, we will suppose this current to be flowing away from us at the side *a* of the coil, and therefore towards us at the side *b*. Such a state of affairs will, we know, produce a torque on the armature in a clockwise direction. Under the influence of this torque

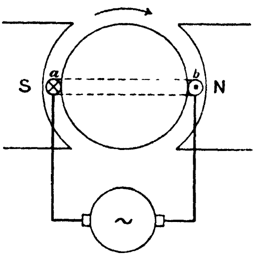

FIG. 81.

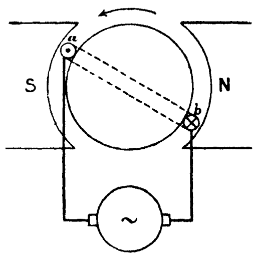

FIG. 82.

the motor will attempt to start, and, although it will not in practice move at all, owing to its inertia, we will imagine a slight movement to take place so that the coil now has the position shown in Fig. 82. Suppose this movement to have taken place in $\frac{1}{100}$ of a second, the supply frequency being 50; by this time the direction of the current in the armature coil has been completely reversed, before the end *a* has left the influence of the south pole. The result will now be a *counter-clockwise* torque, and the armature will therefore merely faintly vibrate instead of rotating.

Now, suppose that the armature is rotated by some external source of power at such a speed that the end *a* of the coil is under the influence of a *north* pole exactly $\frac{1}{100}$ of a second after it was under the influence of a *south* pole; this will mean that when the supply current through the side *a* of the coil is in one direction that side is under the influence of a *south* pole, and when this current is in the opposite direction the side *a* is under the influence of a *north* pole, the coil making one complete electrical revolution in $\frac{1}{50}$ of a second, that is, at a speed which is **synchronous** with that of the generator. Under these circumstances the torque will be in a constant direction, as shown in Fig. 83, and the machine will continue to rotate **synchronously** with the generator supplying it.

Not only will the machine run light under the influence of this alternating pressure, but it can be considerably loaded and will still continue to run, but under all circumstances it can only run **at synchronous speed.**

We can then at once deduce two most important characteristics of the synchronous motor.

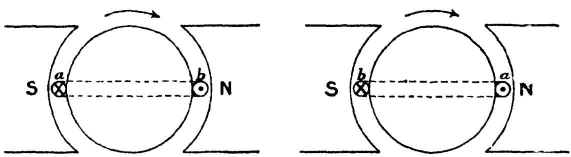

FIG. 83.

(*a*) **It has no starting torque.** In fact, it must, as we have already seen, by one means or another be run up to synchronous speed before it will even run light as a motor.

(*b*) **Its speed is constant** for all loads within its powers. This follows as a natural corollary of (*a*), and if it is so heavily loaded that its speed falls below that of synchronism it will at once stop, and must be restarted as explained above.

In these preliminary investigations the term "electrical revolution" has been purposely employed. The actual number of mechanical revolutions per minute will of course depend upon the respective number of poles in the supply generator and the motor. A 12-pole generator, for example, working at a frequency of 50 $\sim$ will be rotating at $\frac{2}{12} \times 50 \times 60$ or 500 revolutions per minute. A 4-pole motor running in synchronism with this generator will therefore be rotating at 1,500 revolutions per minute.

34. The Back E.M.F.

Now, as we have already seen, in the case of every motor there must be a back E.M.F. which opposes the applied pressure. This will be in the case under consideration a generated E.M.F. due to the cutting of the lines of force of the field by the moving armature conductors; or rather, since in practice either field or armature may rotate, due to the relative motion between armature and field. The current which flows will be that which is due to the resultant of these two pressures, namely the applied E.M.F. (E_a) and the back E.M.F. (E_b). In the case of continuous-current machines this resultant is the arithmetical difference between the two pressures; in the case of alternating-current machines it is the **vectorial** difference, as will be explained later.

There is a second distinction between the two cases of the continuous and alternating-current machine which is of very great practical importance, and it is this. The back or generated E.M.F. of a motor depends, at a given speed, on the strength of the magnetic field of that motor. Now, in the case of the continuous-current machine that field strength depends upon the *applied pressure*. This is *not* the case in a synchronous motor. Quite separate excitation is and must be supplied to the field, and the strength of the latter has therefore nothing to do, directly, with the applied pressure. It will be our purpose to investigate the interesting and unexpected effect of varying the value of our exciting current. Obviously, such variation cannot in any way affect the *speed* of the machine which is absolutely fixed at the outset. It must, however, affect the value of the back E.M.F., and it is this effect which will be studied.

35. Equal Excitation of Generator and Motor.

We will take first the case of the supply generator and motor equally excited, so that the virtual values of their generated E.M.F.s are the same.

(1) When the motor is running *light* no current will flow (in theory) since there is no work to be done. The machines will now behave exactly like two alternators in parallel and their pressures will be equal and opposite, as shown in Fig. 84.

There is no resultant of these pressures, and there will therefore be no current circulating between the machines. The difference between this diagram and that of the theoretically unloaded transformer (Fig. 64) is due to the fact that in the case of the synchronous motor the necessary *magnetising* current is provided from an independent source, the machine being separately excited.

(2) Let us now *load* the motor, but without altering its excitation. At first sight it would appear that the machine must stop, since we have two equal pressures opposing one another;

FIG. 84.

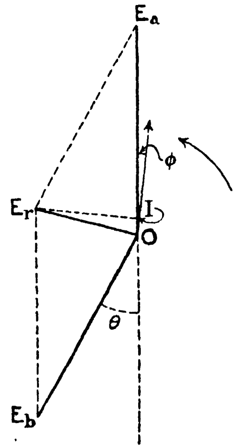

FIG. 85.

and it is not at once obvious how there can be a resultant pressure which will send the necessary current through the motor armature against its generated pressure. The answer is, however, really quite simple. The two pressures come **out of phase,** that is, they no longer *directly* oppose one another. The first effect of putting a load on the motor is to cause the machine to slow up for an instant so that its generated pressure lags behind that of the generator. This action will cease as soon as the resultant of these two pressures is sufficiently great to send the necessary current through the motor, as explained below. The result is clearly seen from Fig. 85, where the back E.M.F. E_b lags the angle θ behind the applied volts E_a. This angle θ will vary according to the load and the relative excitation of the two machines. The resultant of E_a and E_b is shown at OE_r, and is the pressure which sends a current through the motor armature. This pressure consists of two components, since the circuit is an inductive one, namely, OI, which overcomes the ohmic resistance of the circuit, and IE_r, the reactance component at right angles to OI, the current through the motor being, of course, in phase with OI. It will readily be seen that the greater is the load the greater must be the value of OE_r and therefore the more will E_b lag behind E_a.

The power given by the generator will be

$$P_g = E_a \times I \cos \varphi$$

and the power converted to mechanical energy in the motor

$$P_m = E_b \times I \cos (\theta \pm \varphi).$$

It will be seen that the angle of lag between the current and the *resultant* volts is considerable. This is because the ohmic resistance of the windings is very small compared with the apparent resistance due to their inductance—that is, OI is small compared to IE_r.

This diagram should be compared with Fig. 76, which shows the condition of two generators of equal pressures out of step. The cases are precisely similar. The leading machine merely tends to drive the lagging machine as a synchronous motor, and therefore the two, when *both are driven* by some external prime mover, are brought into step with one another.

36. The Effect of Under and Over Exciting the Motor Field.

We have already noted that the back E.M.F. of a synchronous motor is under direct control as regards its magnitude, and that it can be varied at will by altering the value of the exciting current. For example, it can be made considerably to exceed the applied E.M.F. in magnitude; a state of affairs which at first sight appears very strange, but which will be more easily understood when we remember that the resultant pressure is *not* the arithmetical difference between the applied and generated volts, as it is in continuous-current machines.

The effect of varying the excitation of the motor is very important in practice, and we shall proceed to arrive at this effect by taking two extreme cases, firstly when the field is very weak and secondly when the field is very strong.

Turning to Fig. 86, which shows the pressure vector diagram in the former case, E_b is seen to be much smaller than E_a. As before, OE_r represents the resultant pressure which is forcing the current through the motor armature, the current I being in phase with the component OI of this pressure. This current is *lagging* by a considerable angle, φ, behind the applied pres-

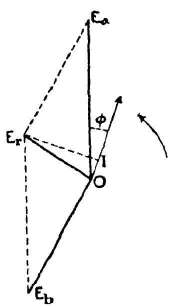

FIG. 86.

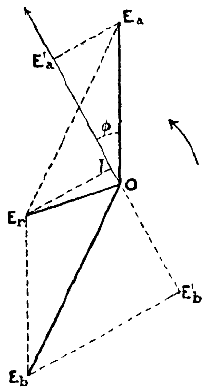

FIG. 87.

sure, so that the power factor of the circuit is low, and the value of I, therefore, considerably in excess of its power component (in phase with E_a).

Let us now increase the value of E_b by strengthening the motor field, and construct a pressure vector diagram under these new conditions, as shown in Fig. 87, E_a, of course, remaining constant. The current I now *leads* the applied pressure by a considerable angle, φ. This result is interesting and somewhat strange, since we are accustomed to regard a leading current as being always caused by capacity in the circuit. In this case, however, the cause is simply the over-excitation of the motor, and the consequent rise in the value of E_b.

Before leaving this diagram let us reconcile it with our previous continuous-current experience. This can readily be done by finding the components of E_a and E_b **in phase with the current,** as shown by OE_a' and OE_b' in Fig. 87. We shall then see that, although the actual value of E_b may be anything according to the excitation of the machine, yet the *component of E_a in phase with the current is always greater than the component of E_b in phase with the current,* which is the general statement giving the relation between applied and back E.M.F. for both continuous and alternating-current motors.

A glance at the two vector diagrams given in Figs. 86 and 87 suggests that it should be possible to find some intermediate value for E_b which should **bring current and applied volts into phase,** that is, should produce a unity power factor, and consequently a minimum current value, for a given power. This is so in practice ; and we can, by altering the excitation of a synchronous motor, *vary the power factor* at will. Naturally, the condition required is that this factor shall be unity, and under these circumstances the motor circuit will behave as though it had no inductance. As we have already seen, we can pass this point and make the current lead. It is often useful to do this in cases where the generator is supplying an inductive load in addition to the synchronous machines. The leading current of the latter will counteract the lagging effect of the former, and so produce a unity power factor in the main generator circuit.

This effect of varying the power factor, and therefore the current required for a given load (remembering that the power supplied is $E_aI \cos \varphi$), is best shown by a curve known as the **V** curve, an example of which is given in Fig. 88. *At a given load* readings of current supplied to the motor are taken for various values of motor field current, and the relations plotted as a curve. The lowest current value represents a unity power factor, values to the left of this point indicating an under-excited field, and consequently a lagging current, values to the right being those of a leading current caused by over-excitation. A power factor curve can also be plotted, its highest point representing unity, this point corresponding, naturally, to the value of the field current which gives the smallest current for any given load.

The value of the motor field current necessary for a unity power factor varies only slightly between no load and full load, and since, in practice, the proper excitation can readily be found by adjusting the motor field rheostat, we need not investigate the somewhat complicated law which governs its variation.

The fact that the power factor of the circuit is within control, coupled with the other characteristics of the synchronous motor of running at a perfectly uniform speed at all loads,

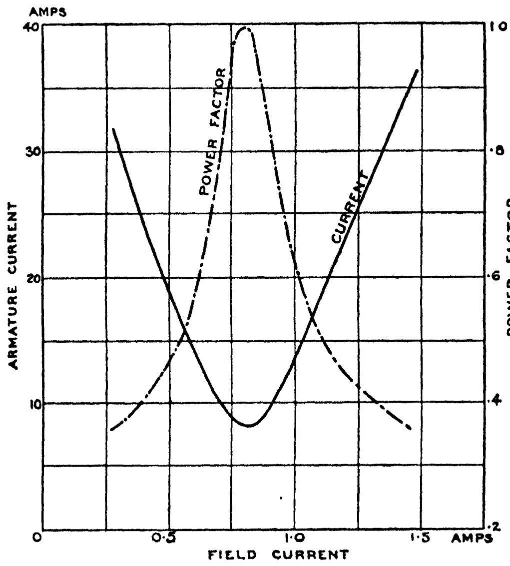

FIG. 88.

renders this type of motor particularly valuable as the driving portion of a rotating machine for converting alternating to continuous current. Such a machine, as a rule, either consists of two distinct units coupled together, one of which is a synchronous alternating-current motor, the other being a continuous-current generator, or it consists of the two combined into one. The former is known as a synchronous **motor generator** and the latter as a **rotary converter.**

Both the above characteristics are most valuable in machines for this duty, since a heavy load on the continuous side of the machine does not reduce the speed of the motor, and so cause a drop in the value of the continuous E.M.F. Furthermore, when very large powers are handled by such machines, as is usual at the present day, the question of power factor is extremely important; the fact that this is under control is, therefore, a valuable property of the synchronous machine.

The use of this type of motor is almost entirely confined to the above class of duty. It is by no means suitable for certain purposes, and has always the following drawbacks:—

(*a*) *It has no starting torque.* This means that some external source of power must be used to run it up to synchronous speed. In the case of converters or motor generators, this may be done by running the continuous part of the machine as a motor from the batteries installed to supply exciting current to the field. A very usual device at the present day, however, is to avoid this somewhat troublesome arrangement by mounting on the shaft an auxiliary starting motor of the *induction* type, whose principle will be explained shortly.

(*b*) *There is no possibility of varying its speed at will.* This objection applies in part to most alternating-current motors other than the commutator type, but it is most pronounced in the case of the synchronous machine, and may be a fatal objection.

(*c*) *It requires some source of continuous-current supply for the excitation of its field.* This is quite enough in itself to rule it out for all ordinary purposes in workshops, etc.

The result, then, is that the synchronous motor has a distinctly limited field of application, being, however, very valuable within these limits. It is nearly always found as a three-phase machine, and is, in construction, very similar to the alternator which drives it.

CHAPTER X.

INDUCTION MOTORS.

37. Induction Motors.—38. The Production of a Rotating Magnetic Field.—39. The Rotor.—40. The Rotor Current.—41. The Frequency of the Rotor Current.—42. The Relation between Current and Induced E.M.F. in the Rotor.—43. The Stator Current.—44. The Starting Torque.—45. Starting an Induction Motor.—46. Measurement of Slip.—47. Speed Regulation of Induction Motors.—48. Reversing.—49. Single-phase Induction Motors.—50. Starting Single-phase Motors.

37. Induction Motors.

This is the type of machine which is most usually employed on alternating-current circuits for such purposes as driving machine tools, and to a certain extent for traction, though it

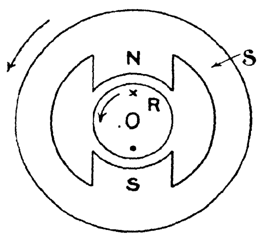

Fig. 89.

is only employed for this latter duty abroad and not at all in this country.

Before considering the actual motor we must try and get an elementary idea of the principle on which it depends for its action. Suppose a circular magnet, S, with two salient poles, NS (Fig. 89), to be rotated in a counter-clockwise direction about a centre, O; a solid cylinder, R, of copper or other electrical conductor being also free to rotate about the same axis. As S rotates R will be situated in a moving magnetic field, and an E.M.F. will, in consequence, be **induced** in it. By Lenz's law this E.M.F. will be in such a direction as will tend to *oppose the motion which produced it.* It cannot stop the rotation of the magnet, but it can prevent *relative* motion between R and S by causing R to rotate in *the same direction* and at the same speed as S. If a mechanical brake be applied to R the effect will be to cause its speed to be less than that of S, so that a certain relative motion between the two will result, and the induced E.M.F. will cause eddy currents to flow in R, as shown in Fig. 89. The greater the load applied to R the greater will be the relative motion between it and S and the heavier will these eddy currents be. The mechanical torque which results is due to the reactions between the magnetic field set up by these currents and the field NS.

Now such an arrangement is not a motor; it is merely a kind of electrical clutch, or coupling, between R and S. If, however, we can arrange that the magnetic field of S shall rotate *without any motion of matter* the machine becomes an **induction motor.**

This can be done by making S an electromagnet, and by supplying its windings with two or three currents, differing in phase, simultaneously. The machine is then called a **two-phase** or a **three-phase induction motor.**

This is the application of polyphase working to motors, and it is the production of this rotating field without the motion of matter which forms the essential feature of such machines.

Since three-phase motors are, in fact, very much more common than two-phase and since the principle of working is precisely the same in either case, we shall consider the former only. Our first step will be to find how this rotating magnetic field is produced, and we shall then consider the electrical actions between this field and the mechanically rotating portion of the machine.

38. The Production of a Rotating Magnetic Field.

Let us first reproduce the wave forms of the three currents supplied to the machine, which, as we remember, differ in phase by 120° (Fig. 90). The magnet system S we will imagine a two-pole one, for the sake of simplicity, and we will take one with three coils only, A, B and C—*i.e.*, one coil per phase. These are shown in section in Fig. 91, their span being 180° (mechani-

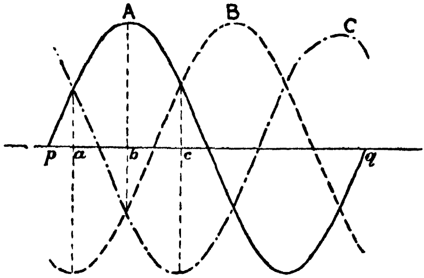

FIG. 90.

cal) since the machine has two poles, the angle between the coils being 120°. They are shown in this figure lying directly across between the points AA′, etc., but they are in practice curved round as shown by the dotted lines (in the case of A coil),

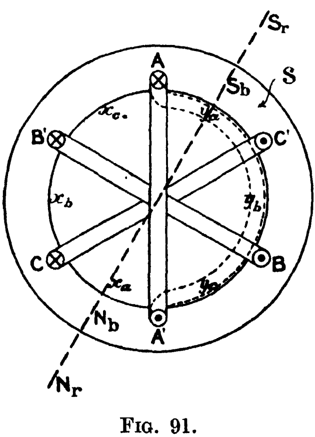

FIG. 91.

in order to keep them clear of the rotating portion of the machine.

We will now consider the direction of the currents in these coils at different instants and their magnetic effect. At the moment *a* (Fig. 90) the current in coil B is a maximum in the negative sense, that in coils A and C being half this maximum value in the positive sense. This state of affairs is shown in Fig. 91, and the coils are so arranged that the currents are flowing *away* from the observer at the points A, B′, and C, and *towards* him at the points A′, B, C′. Now whatever magnetic effect is produced by the three currents, that due to coil B will, at this instant, be the greatest, since the current in this coil has twice the value of the currents in the other two. Coil B is a solenoid, and the direction of the magnetic flux set up by the passage of the current through it is, as we know, at right angles to the line BB′, the North pole being to the left and below the centre of the coil, the South pole to the right and above the centre. The direction of this flux is shown at N_bS_b in Fig. 92 (*a*). On exactly the same principle, the currents in A and C coils are momentarily producing fluxes whose direction is shown by the lines N_aS_a and N_cS_c, these flux values being less than that shown at N_bS_b. The fluxes N_aS_a and N_cS_c have components along N_bS_b, so that their effect is to strengthen this latter flux. The net result will be, at this instant *a*, a powerful magnetic flux N_rS_r (Fig. 91), the point x_a of the magnet system being a North pole and the point y_a a South pole.

An instant later, that is, at the moment *b* in Fig. 90, the disposition of these fluxes has been altered, for the following reasons :—

(1) The current in coil A has *increased* from half maximum positive value to maximum positive value.

(2) The current in coil B has *decreased* from maximum negative value to half maximum negative value.

(3) The current in coil C has *reversed* from half maximum positive value to half maximum negative value.

The new disposition of the magnetic fluxes is shown in Fig. 92 (*b*). N_aS_a has now the greatest value, since the current in this coil is a maximum. N_bS_b remains in the same sense as before, but is reduced in value. N_cS_c has been reversed in sense since the current in the coil has reversed. The net result will be, as before, that the weaker fluxes will assist the "master" flux, the result being a powerful field along the axis $x_b\, y_b$ in Fig. 91.

A few moments' study of Fig. 92 (*c*) will show that N_cS_c is now the master flux, this being the direction of the resultant magnetic field at the moment (*c*) in Fig. 90, its axis lying along $x_c\ y_c$ in Fig. 91.

That is to say, during the time *a*—*c* the North pole has moved from x_a to x_c in a clockwise direction, and the South pole has moved in the same direction from y_a to y_c. We have, therefore, produced a **magnetic field which rotates** in a clockwise direction, and this rotation has been produced without any motion of matter, but by providing the stationary magnet system S with three sets of windings and supplying these windings with three currents simultaneously, 120° out of phase with one another. One complete revolution will be made by the magnetic field in the complete period *p*—*q*, that is to say, it will revolve in synchronism with the generator. S, then, is the magnet system, or, as it is usually called in contradistinction to

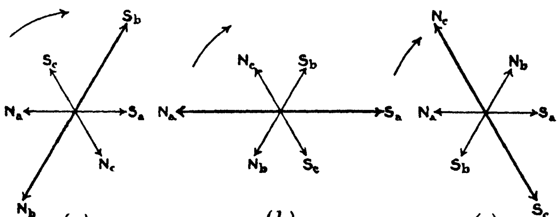

FIG. 92.

the moving part of the machine, the **stator** of a three-phase motor.

A two-pole field has been taken in this elementary investigation, but the usual arrangement is a four-pole machine; in which case the span of the coils AA′, etc., is one quarter of the stator circumference, or 90° (mechanical), instead of half the circumference as shown in Fig. 91.

39. The Rotor.

Turning back to Section 37 and referring to Fig. 89 we see that if any cylindrical conductor free to rotate about its axis be arranged concentrically with the stator, this cylinder will rotate in the same direction as the rotating field, owing to the effort made by the induced E.M.F. to oppose relative motion between the two parts of the machine, that is, owing to the reactions between the rotating field and the field due to the currents which flow under the influence of this induced E.M.F. In practice such a primitive arrangement as a solid cylinder is not used. In the first place it is necessary to reduce the air gap in the path of the flux N_rS_r to an absolute minimum, just as in the case of continuous-current machines. It is, however, even more important in the case of induction motors owing to the detrimental effect which an air gap has on the power factor; this will be discussed later. The core of the rotating portion of the machine or **rotor** must then necessarily be made of iron. This core is laminated in order to confine the induced currents to the copper conductors which are wound in slots in the rotor

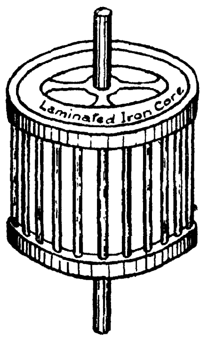

FIG. 93.

as usual. Since these conductors have no connection with the external circuit they may be all joined together at their ends, as shown in Fig. 93. This arrangement is very commonly used in practice, particularly in the case of small machines, and is known from its appearance as a **squirrel-cage rotor.** It consists then simply of a number of short-circuited coils wound on an iron core. Other forms of rotor windings will be alluded to when we deal with the question of starting induction motors.

40. The Rotor Current.

Suppose the stator windings to be supplied with their three-phase currents, and the rotor to be mounted on a shaft so that it is free to rotate in bearings inside the stator. The conductors wound on the rotor are at once situated in a magnetic field which is rotating at full speed in, we will assume, a clockwise direction. The immediate result will be the induction of an E.M.F. in these windings which, as we have already realised, will cause currents to flow which will produce a magnetic field reacting on the stator field. That is, the rotor will revolve clockwise (with the rotating field), and if it is *entirely unloaded* it will rotate synchronously with this field (and, therefore, with the generator supplying it), since, by doing so, it will escape being cut by any lines of force, there being under such circumstances *no relative motion* between field and conductors. It need hardly be said that such a case is purely theoretical, since friction, air resistance, and so on, supply a certain load, however small. But let us be quite clear at the outset, that the effect of induced E.M.F. is at least to try and entirely destroy any relative motion. When the rotor is running absolutely in synchronism with the field, there is, of course, no E.M.F. induced at all. We must then imagine a slight "hunting" effect to be continually produced, that is, the rotor lags for an instant very slightly behind the field; an E.M.F. is therefore induced in it whose action is to speed it up again, and so on. We may express this by saying that in the limit, when the load is infinitely small, the rotor runs synchronously with the rotating field.

Let us then consider the electrical actions in the rotor and stator under two conditions, firstly at no load and secondly when the motor is loaded, that is, when it is doing mechanical work

1. *No Load.*—Under these circumstances, as we have already seen, there is no E.M.F. induced in the rotor and the latter is running at *synchronous speed.* No current will flow in its windings, and the stator will behave simply as a choking coil. The behaviour of the machine will be exactly similar to that of a **transformer** with its **secondary on open circuit.** An induction motor is indeed simply a special case of a transformer, and it will save us a great deal of trouble and needless repetition of diagrams if this is thoroughly understood at the outset. We shall see that the effect on the stator or primary winding of loading the motor is exactly the same as the effect of loading the secondary of a transformer. Hysteresis, eddy currents in the iron of the stator, and magnetic leakage will have exactly

similar effects, though in different degrees. Let it be quite clearly stated again that the two machines are electrically identical in principle, and we should experience no difficulty in pursuing our future investigations.

The no-load stator current is arrived at by just the same methods as we have already used in considering any other unloaded transformer. There is, however, an important point of difference between the two cases—that is, the fact that there must of necessity be an **air gap** in the magnetic circuit of a motor, to allow mechanical clearance between the stator and rotor. This will considerably increase the reluctance of the circuit, and therefore necessitate a proportionately heavier magnetising current. The no-load power factor is therefore extremely low, and the effect of this heavy magnetising current is felt, however much the machine is loaded. Low power factor is indeed one of the most serious drawbacks to the induction motor, and a value of 0·9 is seldom reached even under the best conditions of load with the largest machines. It will be instructive to draw together the no-load vectors of a well-designed transformer and an induction motor, the value of the magnetic flux to be the same in both cases (Fig. 94). OI_t represents the no-load transformer current, OT_μ being the magnetising component, and φ_t the angle of current lag behind the applied volts E_a. The corresponding stator current in the case of the motor is shown at OI_m, OM_μ being the magnetising component necessary to produce the same flux, and φ_m the angle of lag. The effect of the air gap is then to increase considerably the no-load current and to lower the power factor correspondingly. It follows from this that the hysteresis component of the no-load current is small compared with the magnetising component.

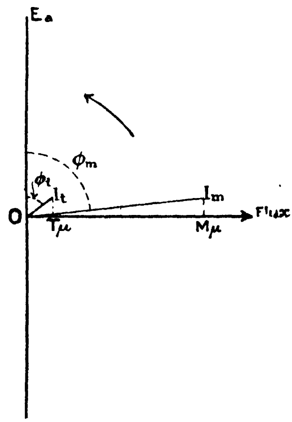

FIG. 94.

2. *When the Rotor is Loaded.*—Let us now give the motor work to do. It is hardly necessary to repeat that this must imply more current through the stator windings. This will mean that a current must flow in the rotor under the influence of the induced E.M.F. The rotor will, in fact, slow up under the load, and relative motion will at once result between the rotating field and the rotor. Now the resistance of the windings of the latter is very small indeed, so that heavy currents will flow in them even when the value of the induced E.M.F. is quite small—that is, when the rotor is running only a very little below synchronous speed. The amount by which the speed of the rotor falls short of that of the rotating field is called **slip**, and is expressed as a percentage of the synchronous speed. For example, supposing that the field is rotating at 1,500 revs. per min., which means, in a two-pole machine, a frequency of supply of 25 $\sim$, and the rotor when fully loaded runs at 1,400 revs. per min. The full-load slip of the machine is then expressed as $\frac{1{,}500 - 1{,}400}{1{,}500}$ or 6·7 per cent.

It will be obvious that there *must* always be some slip, in order that an E.M.F. may be induced in the rotor and a current, therefore, may flow in the latter. So that whereas the principle of a *synchronous* motor depends upon the fact that it runs *at* synchronous speed, that of an *induction* motor depends upon the fact that it must run *below* synchronous speed. Owing to the low resistance of the rotor windings this slip even at full load is quite small, and in large machines does not exceed 2 per cent.

The effect on the stator current of loading the rotor is exactly the same as that of a secondary load on the primary current of a static transformer. That is, the heavier the load, and, in consequence, the greater is the current flowing in the rotor windings, the more current will flow in the stator. These two currents are almost in opposition, so that the field produced by the rotor current is opposing that produced by the stator current, and the latter must, therefore, increase in order to supply the necessary flux. This is a mere repetition of the facts which we have already considered in detail in the case of the static transformer.

41. The Frequency of the Rotor Current.

This will entirely depend upon the relative motion between the rotor and the revolving stator field. Let us take two extreme cases, firstly, at the moment of starting, and, secondly, when the rotor speed is a maximum—that is, when the machine is running light. Suppose the supply frequency to be 25 ∼.

1. *At Start.*—Just at this moment the slip will be 100 per cent., and the frequency of the rotor currents will, therefore, be the same as that of the supply. Calling this f_s we have

$$f_s = \frac{100}{100} \times f$$
$$= 25 \sim.$$

2. *At Full Speed.*—The rotor will now be running very nearly in synchronism with the supply, and the relative motion between it and the revolving field will in consequence be small. The slip will probably not exceed 0·5 per cent., so that calling the frequency of the no-load rotor current f_0 we have

$$f_0 = \frac{0{\cdot}5}{100} f$$
$$= 0.125 \sim.$$

Or generally, if f is the supply frequency, f_r the rotor frequency, and S the percentage slip,

$$f_r = \frac{\mathrm{S}}{100} \times f.$$

In practice, the slip hardly exceeds 5 per cent. at full load, so that the rotor frequency is extremely low ; so low, in fact, that, under running conditions, there will be no appreciable losses from hysteresis in the rotor core. This varying frequency of the rotor currents from start to full speed produces very important effects, which will be discussed in the following section.

42. The Relation between Current and Induced E.M.F. in the Rotor.

The E.M.F. induced in the rotor windings by the action of the magnetic flux set up by the stator currents will cause a current to flow through these windings. The value of this current will depend, for a given induced E.M.F., upon

1. The ohmic resistance of the rotor.
2. The reactance due to inductance.

Of these (1) is constant, whereas (2) depends upon the **frequency of the rotor** E.M.F., which, as we have already seen, depends upon the slip. That is,

$$I_r = \frac{E_r}{\sqrt{R_r^2 + (2\pi f_r L_r)^2}}.$$

Now, under ordinary conditions of running, f_r is very low, so that the apparent resistance due to inductance is inconsiderable compared with the ohmic resistance, and therefore the **angle of lag** between the rotor E.M.F. and the rotor current is small.

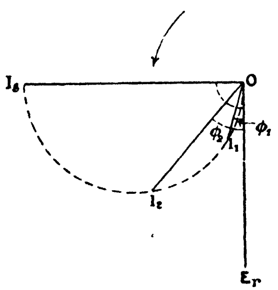

FIG. 95.

At **starting,** on the other hand, f_r has the value of f (the stator frequency) and the angle of lag becomes very considerable. This is best shown by means of a diagram (Fig. 95). OE_r represents the E.M.F. induced in the rotor OI_1, OI_2* the rotor currents when the machine is running light and fully loaded respectively. φ_1 and φ_2 will be the angles of lag under these two conditions of loading. When the rotor is stationary, as at starting, f_r becomes equal to the stator frequency, with the result that the component $2\pi f_r L_r$ of the rotor impedance swamps the component R_r and the angle of lag between OI_s which is the "standstill" current, and OE_r becomes practically 90°.

In short, the *greater is the slip the more does the rotor current lag behind the rotor induced E.M.F.*

This effect is most important in practice, since it has a direct bearing upon the question of the **torque** of the motor. We must remember that this torque is not caused by the *current* in

* The locus of the point I can be proved to be a semicircle.

the rotor conductors, but by the inter-actions of the *magnetic field* set up by this current with the main field.

In the case of induction motors, the field set up by the stator current is this main field. We should then naturally expect that the reactions between stator and rotor fields would be a maximum when the currents in the two were a maximum, that is, at starting. This is, however, not so, owing to **magnetic leakage.** Before discussing this question, however, we will pass on to the consideration of the stator current, and its relation to that in the rotor.

43. The Stator Current.

This can be arrived at, for any condition of loading, in just the same way as the primary current in the case of a static transformer. Let OE_s and OE_r (Fig. 96) represent the stator and rotor voltages respectively, that is, E_s will be equal to the applied volts, neglecting the very small copper drop in the stator windings. OI_0 will represent the no-load stator current, lagging very nearly 90° behind E_s. This diagram should be compared with the vector of an inductively loaded transformer shown in Fig. 71. As shown in Fig. 94, p. 129, the no-load current of an induction motor lags considerably more than that of a static transformer owing to the presence of an air gap in the magnetic circuit in the former case. Turning again to Fig. 96, suppose OI_r to be the rotor current at a certain load lagging behind OE_r by the angle φ_r (as explained in Section 42). Then assuming, for the sake of simplicity, that the number of turns per phase in the rotor is equal to the number of turns per phase in the stator, OI_s represents the stator current at this load, since OI_0, as in the case of the static transformer, is always the resultant of OI_s and OI_r. The stator

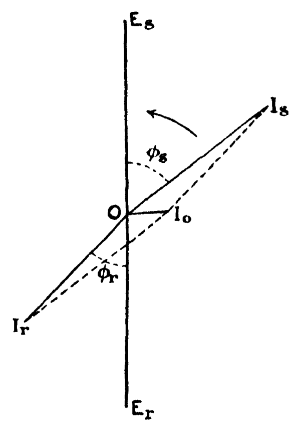

FIG. 96.

current will, therefore, lag φ_s behind the applied volts E_s, the motor power factor at this load being cos φ_s.

The value of OI_s and φ_s for varying conditions of load can be readily found by constructing a number of vector diagrams on this principle in combination with the diagram of rotor currents given in Fig. 95. This construction is shown for one of the phases in Fig. 97, and is known as **Heyland's circle diagram.**

Three conditions of loading are shown—

(1) I_0 .... Motor light.

(2) I_{s1} }
(3) I_{s2} } .... Motor loaded.

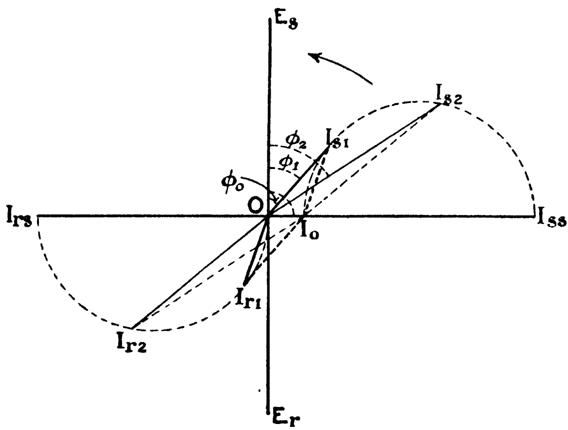

FIG. 97.

(1) When the motor is running theoretically light no currents will flow in the rotor, and the stator current OI_0 will lag φ_0 behind the applied volts, this angle being nearly 90°. The conditions are, in fact, much the same as if the windings on the rotor were not there at all.

(2) When the rotor current is OI_{r1}, corresponding to a certain load, and a certain slip, the stator current will be OI_{s1}, OI_0, being, as before, the resultant of OI_{r1} and OI_{s1} ; φ_1 is the angle of stator current lag. Since the locus of the point I_r is a semi-circle it will readily be seen that that of I_s is also a semi-circle, since at each point the line I_sI_0 is equal to OI_r.

(3) Similarly, when the load and slip have increased so that the rotor current is represented by OI_{r2} the corresponding stator current is OI_{s2} and the angle of stator current lag is φ_2.

Without going into the complete theory of the machine we can deduce from this diagram several points of primary importance.

(*a*) *At no-load the stator current is almost wholly wattless.* The no-load magnetising current is comparatively large, owing to the air gap in the magnetic circuit, the hysteresis component of this current being relatively small for the same reason.

(*b*) As the load rises *the angle of lag φ in the stator circuit decreases up to a certain point and then starts increasing again.* This is at once obvious from Fig. 97, from which it appears that the angle φ decreases between the loads OI_0 and OI_{s1}, being a minimum at this point—*i.e.*, when OI_s *is a tangent to the semi-circle.* This will mean that at this particular load the power factor of the motor will be a maximum. Any load less than, or in excess of this load will imply a reduction in this power factor. As a general rule this will be the *full-load* rated current of the motor, positions of I_s to the left of I_{s1} showing light load currents and to the right of I_{s1} overload currents. Now, the actual value in degrees of φ when this angle is a minimum depends upon the value of the no-load current OI_0. The greater this is in comparison to the value of OI_{s1} the worse will be the full-load power factor. The actual value of the no-load current depends very largely upon the air gap. In practice, as we have already noted, cos φ hardly reaches 0·9 even with the largest machines. In small ones, where the air gap must be proportionately larger, the best value of cos φ is frequently no better than 0·7. We may then state :

(*c*) *The average power factor of an induction motor throughout its range of load is poor.* This is one of the chief drawbacks to this class of machine.

(*d*) *At the moment of starting the stator current is again almost wattless.* A glance at Fig. 97 should make this quite clear. We have already noted on p. 132 that *if the rotor resistance is low* its standstill or starting current is practically wattless, as shown at OI_{rs} in Fig. 97. Since OI_0 is always the resultant of the stator and rotor currents, and since this no-load current is itself practically wattless, it follows that the standstill stator current must also be practically wattless, as shown at OI_{ss} in Fig. 97.

(*e*) *If the rotor resistance is negligible the starting torque of the motor will be practically zero.* This is of the utmost importance in practice, and it is essential that we should clearly understand the reason, and we will devote the following section to its investigation.

44. The Starting Torque.

Let us first reproduce that part of the circle diagram shown in Fig. 97 which deals with the standstill or starting currents.

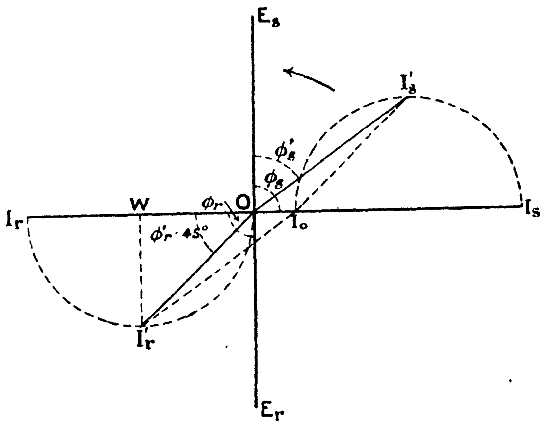

FIG. 98.

Assuming first of all, as we have hitherto done, that, when the rotor frequency is the same as that of supply, as is the case *at the instant of starting*, the rotor resistance is negligible compared with its inductance. The stator and rotor currents then will be as shown in Fig. 98, OI_s being the former and OI_r the latter. It will be observed that, since φ_r is practically 90°, so also will φ_s be.

Now E_s is the vector of the applied pressure, neglecting, as before, the small copper drop in the stator windings. That is to say, the **current supplied from the generator to the stator is**

wattless. This statement naturally implies that, under these circumstances, **no power** is being sent out from the generator and therefore **no torque can be exerted** by the motor, even though both stator and rotor currents are a maximum. Now torque, as we know, is produced by the interlinking of stator and rotor magnetic fields. It follows then that when stator and rotor currents are wattless **their magnetic fields do not interlink,** in other words they **leak.** We are thus brought to the consideration of **magnetic leakage,** and we should realise its immense importance in the case of the induction motor. The question is very much too involved to be considered fully in the space at our disposal, and the reader is referred to the many text-books in which this investigation is completely carried out. It will suffice for our purposes to realise that the greater is the phase angle between rotor and stator volts and currents the greater will be the proportion of the total flux set up by these two currents which leaks—*i.e.*, which does not interlink with the corresponding flux.

How, then, are we to improve this starting torque? Clearly, by arranging that, at starting, the *rotor resistance shall not be swamped by its inductance,* so that φ_r shall be less than 90°, and in consequence φ_s shall also be less than 90°; the current from the generator will then have a considerable watt component, and a torque can therefore be exerted. This we can accomplish by abandoning the squirrel-cage form of rotor, whose resistance will be very low in a machine of any size, and by adopting a **wound rotor** so arranged that **ohmic resistance can be inserted in the rotor circuit on starting the motor.** The result is shown in Fig. 98, where OI_r' and OI_s' represent the standstill or starting currents in rotor and stator respectively when resistance has been added to the rotor circuit. The result has been that, even though the rotor frequency is that of the supply, yet enough resistance has been added to prevent the inductance of the rotor swamping its resistance. The angle φ_r has been altered from 90° to the new value φ_r', and therefore the angle φ_s has been also altered from 90° to the new value φ_s'.

Now the starting torque will depend also upon the *value* of the stator and rotor currents, so that we must obviously not add too much resistance to the rotor circuit. For example, if the rotor resistance was infinite no torque could result, since no current would flow in the rotor windings. There is then a practical limit to the amount of resistance we can add, and it can readily be proved that this limit is reached when φ_r is approximately 45°, in other words, when the **watt component** of OI_r is a maximum. This position has been shown in Fig. 98, the angle φ_r' being 45°. Under these circumstances the watt component of the rotor current—*i.e.*, $I_r'W$—is clearly at its maximum value.

After starting we must cut out this rotor resistance, since it will no longer be necessary. This point has already been discussed in Section 42. When the motor is running the rotor frequency is so low that the inductance of the rotor circuit causes very little reactance, so that φ_1 becomes relatively small without having to add any resistance in the rotor circuit.

45. Starting an Induction Motor.

We have seen above that to start a motor of any size ohmic resistance must be added to the rotor circuit, this resistance being cut out when the machine has started and the rotor windings short-circuited on themselves. The rotor windings in this case are arranged in very much the same way as the stator windings—that is, they are arranged in slots, the span of each coil being arranged according to the number of poles in the stator, *i.e.*, 180° if the stator has two poles, 90° if it has four poles, and so on. The actual number of circuits, or phases, in the rotor varies according to the design. They need not necessarily be the same as those in the stator, but generally are so. A three-phase motor will then have a three-phase rotor winding.

Obviously, the number of rotor phases will not affect the principle of the machine, since, when the rotor has run up to speed, all its windings are short-circuited, and it then behaves much as a squirrel-cage rotor.

We will take the case of a three-phase star-connected rotor, as shown at R_w in Fig. 99. The ends of these phases are connected to three slip rings (SSS) on the rotor shaft, these slip rings being connected to the starting resistance R by the brushes BBB. At the moment of starting, the resistances R are in series with the rotor windings R_w, and are adjustable, being gradually cut out in a series of steps, until when the machine has run up to speed the windings R_w are short-circuited. In the arrangement shown this will be done by connecting the three leads *abc* from the brushes when the resistance R has all been cut out, so that the current will always flow through the brushes. A common arrangement, however, is to short-circuit the ends of the phases in the machine itself, after which the brushes have no further work to do.

Large motors whose rotor resistance is very low are nearly always started in this way, and must then be constructed with wound rotors. In small machines (and in some specially

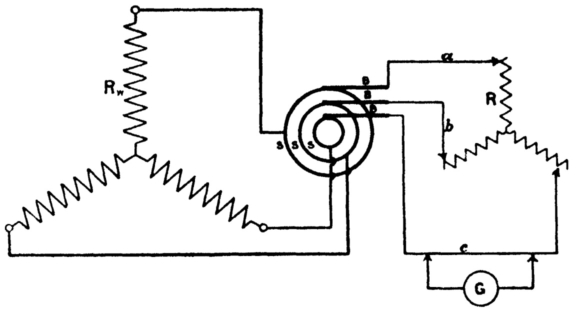

FIG. 99.

designed large ones) squirrel-cage rotors may be used, since their resistance is sufficient to provide a starting torque which is high enough for ordinary requirements, particularly if the motor is not started under load.

46. Measurement of Slip in an Induction Motor.

The rotor slip can be readily observed in any induction motor by observing its revolutions per minute, and deducting these from the number it would have made had it been running at synchronous speed; this latter being a simple calculation depending on the frequency of the supply and the relative number of poles in the motor field and the supply generator. The frequency of supply, however, is not always known with absolute accuracy, and cannot be measured without a special instrument known as a frequency meter. We can, however, in the case of machines having *wound* rotors, whose windings are short-circuited in the starter, arrive at the synchronous speed and so at the slip in a very simple way, by connecting a sensitive galvanometer across any two points in one of the leads between a brush and the starting resistance, as shown at G in Fig. 99. This galvanometer will indicate the passage of a current in this lead, and at the low rotor frequencies met with in practice its needle will be deflected alternately to the left and right as the current in the lead changes from positive to negative. If, then, we count the number of complete throws of the needle from one point back to the same point we have a direct measure of the rotor frequency and hence the slip. In a two-pole machine, for example, one complete double throw will indicate a slip of one revolution behind the revolving field. In a four-pole machine it will indicate a slip of half a revolution, and so on.

Example.—A two-pole machine is running at 1,470 revolutions per minute, and the galvanometer makes 30 double throws or complete swings per minute. What is the slip?

A full period is completed in $\frac{1}{30}$ of a minute, that is, the number of periods per minute in the rotor is 30 or 0·5 per second. Since there are only two poles,

$$\text{Synchronous speed} = 1{,}470 + 30,$$

and

$$f = \frac{1{,}500}{60} = 25.$$

The per cent. slip then

$$= \frac{f_r}{f} \times 100$$

$$= \frac{0{\cdot}5}{25} \times 100$$

$$= 2$$

Generally, then, calling S_s the synchronous speed and n the number of double throws on the needle per minute, s the speed of the rotor in revolutions per minute, p the number of *pairs of poles*,

$$S_s = s + \frac{n}{p}.$$

47. Speed Regulation of Induction Motors.

Although the induction motor is naturally a constant speed machine, in that its speed varies only quite a small percentage from that of synchronism, yet a certain speed regulation can be effected at will. As regards single motors two chief methods are in use.

(*a*) *Inserting Resistance in the Rotor Circuit.*—It will be obvious that this method can only be employed in the case of machines with wound rotors and slip rings. As we have already seen, the effect of added rotor resistance at starting is to improve the starting torque. We may then say that its effect generally is to produce *a given torque at a lower speed*—that is, with a greater percentage slip and a higher rotor frequency. Under these conditions the rotor current will be brought more into phase with the volts and a given torque can be maintained over a considerable range of speed. The power given to the stator will be constant for any required torque of the motor, so that when it is desired to run the machine more slowly at a given load part of the power given to the stator is absorbed as I^2R losses in the resistance which has been added to the rotor. If enough resistance is added the machine will come to rest under the load and will act exactly like a static transformer.

This method then corresponds to the case of the continuous-current motor, whose speed is varied by inserting resistance in series with the armature, and is therefore necessarily wasteful. It has, however, the merit of being simple and within limits effective, but it cannot be employed with machines having squirrel-cage rotors. Unfortunately, it is with just these small machines that it would be most useful, since the I^2R losses involved would not be very serious.

(*b*) *Changing the Number of Poles.*—Since the synchronous speed of an induction motor varies inversely as the number of poles in the stator, it will be obvious that if this number can be varied the machine can be made to run at different speeds at a given torque. For example, a machine with 8 poles, the frequency of supply being 50 ~, will run at a (synchronous) speed of 750 revs. per min. If by means of a change-over switch in the stator circuit four of these poles are rendered inactive synchronous speed will now be 1,500 revs. per min. This method is economical, but necessarily somewhat elaborate,

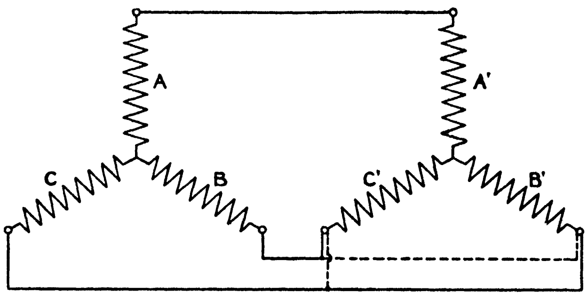

Fig. 100.

particularly when more than a two-speed variation is required. In the case of a wound rotor which is arranged, as we have seen, with the same number of poles as its stator, special winding is necessary to enable it to run when this number is varied.

48. Reversing Induction Motors.

The direction of the rotation will be that of the revolving field. This can readily be reversed by changing two of the

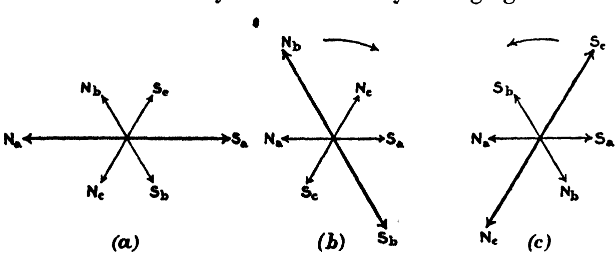

Fig. 101.

stator connections about, as shown in Fig. 100, leaving the third as it was before. The effect will be that the order of the phases will be altered from, for example, ABC to ACB, and the direction of rotation of the master flux will be altered at the same time. If, for example, the flux distribution at any instant is as shown in Fig. 101 (*a*), the direction will be clockwise if phase B is the next to reach its maximum value, as in Fig. 101 (*b*), and counter-clockwise if phase C is the next to reach this value, as in Fig. 101 (*c*). A few moments' study will make it clear that it is immaterial which two connections are changed. The result must be that the order of the phases, and, therefore, the direction of rotation of the field, is reversed.

49. Single-phase Induction Motors.

Although for all forms of alternating-current motors, whether synchronous or induction, polyphase (and preferably three-phase) currents give more satisfactory results than single-phase, yet there are many instances in which only the latter are available, and it therefore becomes essential to find a motor which will run satisfactorily on a single-phase system. The synchronous motor has already been shown to have only a limited field of application, so that our attention is naturally turned to the induction type of machine, and the question at once suggests itself: Is a single-phase induction motor a possibility? The answer is in the affirmative; but let it be at once understood that this type is never so economical or so satisfactory as the corresponding three-phase machine. The theory of its action is very much more complex, but it is thought that some description is essential in view of the fact that single-phase alternating-current systems are very common at the present time, particularly in the Colonies, where they are in fact the normal case.

If, while a polyphase induction motor is running, connections to the stator are altered so that *only one phase is working*, the motor will continue to run and will exert considerable torque; in fact, it will behave much as though it were still a polyphase motor. The reason, briefly stated, is that the interaction between stator and rotor currents actually does produce a very similar rotating field, although the primary flux due to the stator current is only a pulsating one. The full investigation of this fact is beyond the scope of this book, and would indeed be out of place in any elementary treatment of the subject, but a very short description of the lines on which such investigations proceed may be of some assistance.

The general principle, which is due to Ferraris, is to consider the single pulsating stator flux as the resultant of two fluxes which *rotate in opposite directions*. Referring to Fig. 102, it will be seen that the resultant of the two equal fluxes f_1 and f_2, rotating clockwise and counter-clockwise respectively, always lies along the axis FF′. This resultant field will be a maximum and equal to $2 \times f_1$ (or f_2) when the two fluxes lie along the axis FF′, and will be zero when these two fluxes are at right angles to FF′, since the two components neutralise one another under these conditions. We can see then from the figure that such a single alternating flux as is produced by the stator current of a single-phase motor, rising to a maximum positive value along OF and a maximum negative value along OF′, may always be represented by the two equal fluxes f_1 and f_2 rotating at exactly the same speed in opposite directions, and further that the effect of the flux FF′ on the rotor will be exactly the same as the combined effect of the fluxes f_1 and f_2 at any instant.

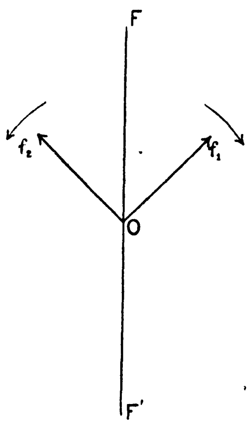

Fig. 102.

Now the objection might be raised that, since f_1 and f_2 are exactly similar fluxes rotating at exactly the same speed in opposite directions, whatever effect f_1 had on the rotor would always be exactly opposed to the effect of f_2; in other words, that the result of the two combined would be nil. This is perfectly true **at the moment of starting,** since the rotor currents produced by the action of f_1 and f_2 are then of equal frequencies. In other words, a simple single-phase motor is **not self-starting.** But when once the motor has been started *in either direction* it will continue to run and will exert a torque in the direction in which it was started. The reason is that the effects of the two fluxes f_1 and f_2 on the rotor are no longer similar. Imagine the simplest case, when the motor is running at synchronous speed in a clockwise direction. The effect of f_1 on the rotor is now nil, since the latter is running in the same direction as this flux and at the same speed. The effect of f_2 is however considerable. The slip of the rotor with respect to f_2 is now 200 per cent., since it is revolving at a speed of n revolutions per second *clockwise*, and f_2 is rotating at a speed of n revolutions per second *counter-clockwise*. The relative motion between f_2 and the rotor is therefore $2n$ revolutions per second.

The practical construction of the vector diagram at any particular load is then proceeded with as though these two rotating fluxes were produced by the currents in two sets of stator windings and their results are combined. The general lines which this diagram follows are indicated in Fig. 103, where the lines marked 1 represent the vectors of the currents producing the clockwise field, and the lines marked 2 those producing the counter-clockwise field.

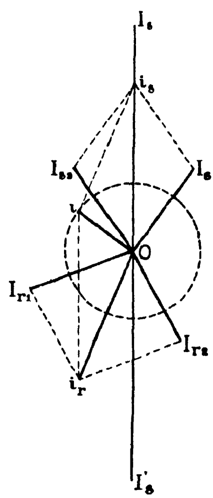

FIG. 103.

OI_s and OI'_s represent the positive and negative (maximum) values of the simple alternating stator current. The rotating components of OI_s are shown at OI_{s1} and OI_{s2} at the instant when the stator current is Oi_s (positive). We will imagine the rotor to be running with a 2 per cent. slip clockwise, that is, its slip with respect to the clockwise field will be 2 per cent., and with respect to the counter-clockwise field 198 per cent. The rotor currents OI_{r1} and OI_{r2} corresponding to these stator currents are found by exactly the same methods which we employed in the case of the polyphase motor in Fig. 96. It will be noticed that since the slip of the rotor with respect to OI_{s1} is very much less than it is with respect to OI_{s2}, I_{r1} will be less in direct opposition to I_{s1} than is I_{r2} to I_{s2} (*cf.* Fig. 97). The resultant i_r of I_{r1} and I_{r2} will therefore give us the actual rotor current at this instant. The resultant of Oi_s and Oi_r is shown at Oi. Now by taking other instantaneous values of OI_s at this load, and proceeding to find

O*i* on exactly similar lines, it will be found that the locus of the point *i* is very nearly a circle. In other words, the interaction between stator and rotor currents produces **a rotating field.**

It need hardly be said that the diagram given in Fig. 103 is by no means complete, but all that has been attempted is to show that a rotating field actually does result, and the *direction* of this rotating field will always be *that in which the motor was started.* A few moments' consideration will show that this is so ; the only difference between the two directions being that the relative slips between the rotor and the two rotating fluxes are reversed.

As compared with polyphase motors, the following characteristics should be noted :—

(*a*) Special starting arrangements must be made. This will be dealt with in the following section.

(*b*) The power factor is not so good.

(*c*) I^2R losses are about twice as great.

(*d*) The no-load current is about twice as great.

(*e*) The maximum torque is considerably less.

On the other hand :—

(*a*) It requires only two leads instead of three.

(*b*) It will run in either direction without altering any connections.

(*c*) It can, by the addition of a **commutator,** be made to give a considerable starting torque.

50. Starting the Single-phase Induction Motor.

Since no rotating field results until the rotor is moving, special arrangements must be made if this type of motor is to be self-starting. The usual method is as follows :—

The stator is supplied with two sets of windings P_r and P_s which are displaced 90° from each other, as though the machine was a two-phase motor. One of these sets of windings is used at starting only, and is energised by a branch circuit tapped from the two mains as shown in Fig. 104. In series with P_s is a choking coil L, which has the effect of considerably increasing the angle of lag between the current in the starting phase and the applied volts. In effect, imagining the inductance of P_r

to be zero, and the resistance of P_s to be zero, the angle between the currents in the two phases will be 90°, and the action of the stator will be exactly the same as in the case of a two-phase motor. In practice, of course, these ideal conditions are not realised, with the result that the phase angle between the two currents is considerably less than 90°, and the rotating field is flattened from a circle to an ellipse. That is, the starting torque

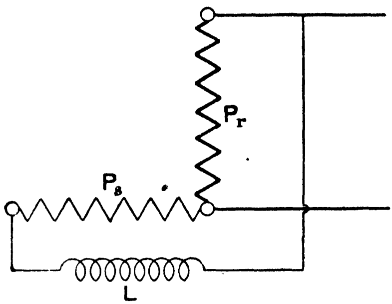

FIG. 104.

is poor, and these motors are therefore not suited to start up under any considerable load.

Sometimes the auxiliary phase has an added capacity instead of inductance, but in a motor of any size the necessary condenser would be somewhat bulky if its charging current were to have a reasonable value.

This arrangement is known as **phase splitting,** and the starting phase is cut out as soon as the motor has run up to speed. Owing to the fact that this phase is only required for a few seconds, its windings are worked at very high-current densities, so that they may take up a minimum amount of space on the stator.

CHAPTER XI.

COMMUTATOR MOTORS.

51. Single-phase Traction Systems.

Induction motors, whether for single or three-phase circuits, suffer from three defects: a comparatively poor starting torque, a small range of speed regulation, and a restricted overload capacity. For driving machine tools and for similar purposes, these defects are as a rule unimportant, but for **traction** work, lifts and so on, they constitute a serious drawback. Three-phase motors are, as we have seen, much superior to single-phase, but necessitate three conductors instead of two. This is a serious objection for traction work, since it necessarily implies two insulated conductors (the third being the running rails) from which the current must be collected by means of rubbing contacts. Many three-phase traction systems are in use in different parts of the world, but cannot be regarded as wholly satisfactory, both for the reasons given above, and also because of the heavy losses during acceleration, due to the low power factor of this type of motor when starting under load and running up to speed. The single-phase induction motor is worse in every respect than the three-phase machine, so that a more satisfactory type must be looked for if the single-phase alternating-current motor, with its single insulated conductor, is to be a serious rival to the continuous-current series machine which is so well suited to this class of work. The problem has been solved during recent years by the introduction of the alternating-current **commutator** motor, which is in its simplest form very similar both in principle and construction to the corresponding continuous-current machine; and while embodying a large number of the good features of the latter, possesses the great advantage common to all alternating-current machinery, namely, high-pressure distribution from the generating station without the necessity for rotating machinery at the sub-stations. Expert opinion is, at the present time, divided between the relative advantages of the two systems. In both cases, for an undertaking of any considerable size, the central station generates alternating current at very high pressure. This may be fed direct to the insulated conductor—generally a bare overhead wire—in the case of railways equipped with alternating-current motors, but must be converted in the case of a continuous-current motor system by rotating machinery in various sub-stations situated at different points along the railway.

Commutator motors are of two types: (*a*) the **Series** and (*b*) the **Repulsion.** In either case the rotor is wound in very much the same way as is the armature of a continuous-current machine, except for one or two practical points of difference which will be dealt with later. Before dealing with the above two types in detail, we will take the general case of a closed armature winding rotating in an alternating flux. We will consider a ring armature, since its windings are more easily followed, and the general principles apply equally well to the universally employed drum winding.

52. The Transformer E.M.F.

Fig. 105 shows an ordinary ring armature rotating between two poles which are energised by an alternating current, so that their polarity is continually reversing. That is, there is flowing between the poles an **alternating flux,** whose axis is horizontal in Fig. 105. This ring armature then is in the position of the **secondary winding of a transformer,** in that its windings are continually being threaded by this flux as it changes in magnitude and sense. There will, in consequence, be induced in the windings an E.M.F. under the ordinary laws of induction, and we shall speak of this as the **transformer E.M.F.** Now at first sight, since the windings form a closed circuit, it might appear that this transformer E.M.F. would result in a heavy current throughout the armature winding; in fact, that the latter would behave like a secondary on short circuit. But a few moments' consideration will show that this is not the case. The coils above the axis TT′ have an E.M.F. induced in them, this E.M.F. being a maximum in coil *a* and zero in coils *bb*′. This should be clear from Fig. 105, the plane of coil *a* being at right angles to the flux axis, and that of coils

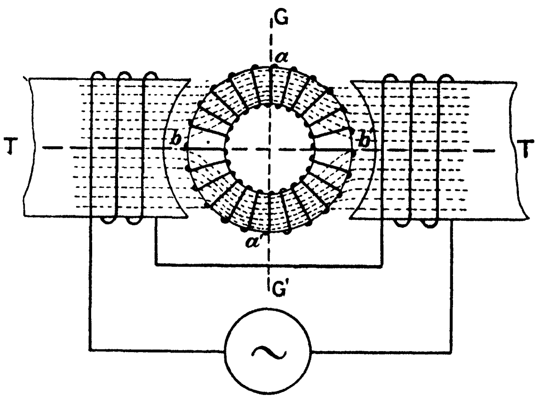

FIG. 105.

bb′ parallel to the flux axis, so that the former is linked by a maximum and the latter by a minimum number of lines of force; it being remembered that it is the *horizontal* motion of the lines of force which is responsible for the induction of the transformer E.M.F. Similarly, the coils *below* the axis TT′ have an E.M.F. induced in them in exactly *the same sense* as that in the upper coils (being a maximum in coil *a*′ and zero in coils *bb*′), since it is the same flux which is inducing both; that is to say, when the E.M.F. in coil *a* is a maximum in one sense, that in coil *a*′ is also a maximum in the same sense, and so on with the remaining coils. As regards the whole armature circuit, then, these two E.M.F.s oppose one another, and no current will flow. We are familiar with exactly the same principle of opposing E.M.F.s in the case of a continuous-current generator on open circuit, where the generated E.M.F. in one half of the armature winding opposes that in the other half. This distribution of transformer E.M.F.s is conveniently represented in Fig. 106, the E.M.F. being a maximum in the coils *aa′* and zero in the coils *bb′*, as explained above. It will be clear from this figure that the points *a* and *a′* are at equal potentials as regards this transformer E.M.F., and consequently that the axis of maximum transformer potential *of the whole armature* is TT′. The value of this transformer E.M.F. is not in any way affected by the *rotation* of the armature, but depends only upon the strength of the main flux, and the rate of change of this

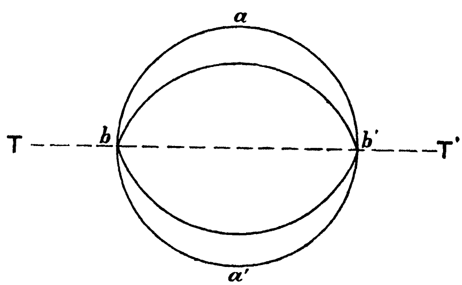

FIG. 106.

flux, that is, upon the **frequency of supply.** This point is an important one, and will be referred to again later.

53. The Generated E.M.F.

If the armature is made to *rotate*, an E.M.F. will be **generated** by the cutting of the lines of force of the main field by the armature windings, alternately in an upward and downward direction, that is, by relative *vertical* motion between the conductors and the lines of force of the field. This implies that coils *bb′* (Fig. 105) are the seat of maximum *generated* E.M.F., as indeed we are aware from the elementary principles of the dynamo. Similarly the generated E.M.F. is zero in coils *aa′*. The value of this E.M.F. depends upon the field strength, and the speed at which the armature is rotating; it is not affected by the supply frequency, as is the transformer E.M.F., which in its turn is unaffected by the speed of rotation.

The distribution of generated E.M.F.s in the coils is shown in Fig. 107, from which it will be clear that the axis of maximum generated E.M.F. of the whole armature is GG′, this being, as we know, the normal brush position in the case of the continuous-current generator or motor.

In an alternating-current motor of this type we are then concerned with *two* induced pressures, which we call the transformer and generated E.M.F.s respectively, and we have just seen that the axes of these two are at right angles to one another. We shall see also in due course that these two

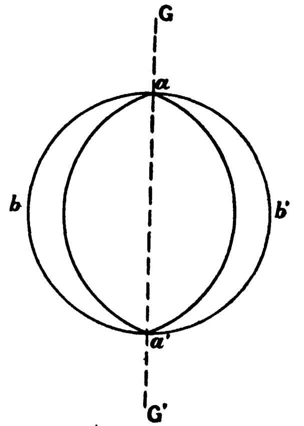

FIG. 107.

E.M.F.s are also 90° *out of phase* with one another—that is, that at the instant when the transformer E.M.F. is a maximum, the generated E.M.F. is zero and *vice versa.* Both these facts, namely, that the axes of the generated and transformer E.M.F.s are at right angles to one another in *space,* and also that these two pressures are 90° out of phase with one another, that is, that they are at right angles *in time,* have very important effects. We shall now proceed to investigate the action of the series motor, and see how this transformer E.M.F., which does not exist in the corresponding continuous-current machine, necessitates considerable modification in the design of the alternating-current series motor.

54. The Series Motor.

Broadly speaking, this is very similar to the continuous-current series motor. It will be remembered that the direction of rotation of the latter is unaffected by reversing the terminal connections; that is, it is independent of the direction of the supply pressure. The reason is, of course, that reversing the terminal connections results in the reversal of the current in the field coils and the armature **simultaneously**, the torque therefore remaining in the same sense as before. If then we supply such a motor with a single-phase alternating-current, it will run in a constant direction, just as though it was being

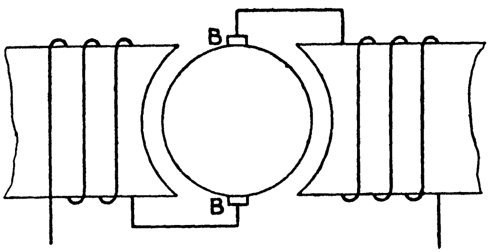

FIG. 108.

supplied with a continuous current. Certain modifications in design are, however, necessary—

(1) In the construction of the *field magnets*, remembering that they are energised by an alternating current,

(2) To obtain *sparkless commutation*,

(3) To obtain a reasonably good *power factor*.

These points will be dealt with in order. Fig. 108 shows the simple series motor, the field being bi-polar with its axis horizontal. The brushes BB are set in their normal position, that is, at right angles to the field axis. The armature windings are omitted, but may be imagined to be the normal drum winding for a two-pole field.

(1) As in the case of all other alternating-current electro-magnets, the core, that is, the iron of the poles and magnet frame, must be completely **laminated**. The reason for this has been fully explained elsewhere and need not be considered further.

(2) The effect of the transformer E.M.F. is to render sparkless **commutation** difficult. Consider the electrical actions in the coil *a* (Fig. 109) when it is short-circuited by a brush in the act of commutation. We shall again take the case of a ring armature in order to make the matter clear.

We know that the main impediment to sparkless commutation in the case of a continuous-current machine is the self-induced or **reactance** voltage, which tends to prolong the flow of the current in coil *a* after it has been short-circuited. This E.M.F. we are accustomed to neutralise either by increasing the brush lead or by using commutating poles, that is, by impressing on the short-circuited coil an E.M.F. which is more or less directly opposed to the reactance voltage. High resistance carbon brushes, such as are universally employed at the

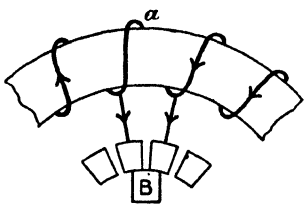

Fig. 109.

present time, are also very helpful to sparkless commutation in that they tend to limit the value of the "sparking" current which flows in the short-circuit path under the influence of the reactance voltage. Further, in all reversible traction motors a fixed central brush position is essential. This can be secured in the case of continuous-current machines not only by the use of commutating poles, but also, as is invariably done, by using field magnets whose ampere-turns are so much in excess of those of the armature as to prevent any appreciable flux distortion (due to armature reaction) under all conditions of load. The necessity for this fixed brush position is just as vital in the case of the alternating-current traction motor, but the problem is rendered more difficult since we are debarred from using field magnets with too many ampere-turns, owing to considerations of power factor which will be dealt with in the next section.

The effect of the transformer E.M.F. on commutation is as follows :—

Coil *a* at the moment when it is short-circuited by a brush is the seat of the **maximum transformer E.M.F.,** as has already been shown in the previous section (Figs. 105 and 106). This transformer E.M.F. will give rise to a heavy short-circuit current which will flow between commutator segment and brush, and will therefore result in destructive sparking between the two. Here, then, we have a very serious difficulty to contend with, and the solution of the problem is by no means simple. Let us now follow, in as few words as possible, the steps which can be taken to minimise the difficulty.

(*a*) Introduce a high non-inductive resistance R between the

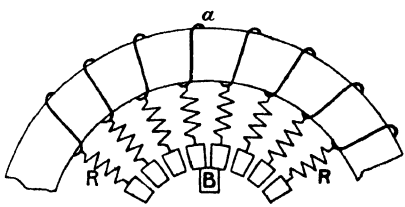

FIG. 110.

armature coils and commutator segments, as shown in Fig 110. This is almost invariably done in practice, and has the effect of reducing very considerably the value of the short-circuit current caused by the transformer E.M.F. At the same time the IR drop in the machine is increased, and its efficiency thereby reduced to some extent.

(*b*) Arrange for the armature winding to be subdivided as completely as possible, so that as few coils as may be are short-circuited at any one time by the brushes. This subdivision is carried to the limit in practice, so that each armature slot contains only one coil ; which implies a large number of very narrow commutator segments, and therefore proportionately narrower brushes. This accounts for the very large diameter of the commutator which is characteristic of this class of machine.

(*c*) Use a low supply frequency, since, as we have seen, the value of the transformer E.M.F. depends directly upon this. This is usually done, ordinary traction frequencies being about 25$\sim$, and sometimes as low as 15$\sim$. In fact, except in certain cases the alternating-current commutator motor will not operate satisfactorily at the ordinary supply frequency of 50$\sim$.

(*d*) Use commutating poles, whose function is to neutralise the E.M.F.s which are responsible for sparking. We must remember that there are two of these to be considered, namely, the reactance and the transformer voltages, and further, that they are 90° out of phase with one another for the following reasons :—

The reactance voltage must necessarily be in phase with the current, since its value at any instant depends upon the value of the current in the short-circuited coil at that instant. For example, at regular intervals, which depend upon the relation between the speed of rotation and the frequency of supply, a coil will be short-circuited when the current value in the armature is zero (the frequency of commutation being so high compared with that of supply that the current may be considered practically constant during the process), and it will be obvious that under these circumstances the act of commutation cannot give rise to any self-induced E.M.F. That is, the reactance voltage will also be zero. Similarly, at corresponding intervals of time, a coil will be short-circuited when the current in the armature has its maximum value ; under these circumstances the act of short-circuiting the coil will give rise to a maximum self-induced E.M.F. In short, the reactance voltage is always in phase with the armature current. Now the transformer E.M.F., on the other hand, is always 90° out of phase with the flux which gives rise to it, since it depends upon the *rate of change* of that flux. This fact has already been explained in detail in Chapter II., and need not be elaborated here. This flux is produced by the main field windings, and will be practically in phase with the current in these windings, since the effect of the air gap in the magnetic circuit is to render the hysteresis component of this current inappreciable compared with the magnetising component, exactly as in the case of the induction motor (§ 40). Since the armature and field windings are in series with one another, the transformer E.M.F. is therefore practically 90° out of phase with the armature current, and so also 90° out of phase with the reactance voltage.

The field produced by the commutating poles must therefore, for perfect commutation, be so arranged that the E.M.F. generated by the short-circuited coil in this commutating field is exactly equal and opposite to the *resultant* of the transformer and reactance voltages, as shown in Fig. 111. OE_t represents in phase and magnitude the transformer voltage, OE_s the reactance voltage, and OE_g the voltage generated in the short-circuited coil as it moves in the com-

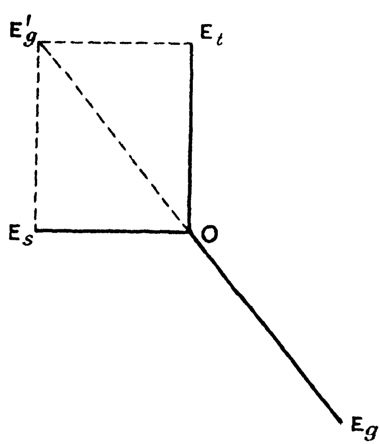

FIG. 111.

mutating field. If this is exactly equal and opposite to OE'_g (the resultant of OE_t and OE_s) commutation will be perfect. It will be obvious that these conditions can only be fulfilled at one definite speed, since the value of OE_g depends upon the strength of the commutating field, and both OE_g and OE_s depend upon the speed of the machine. At starting, for example, the transformer E.M.F., which, unlike the reactance voltage, is wholly independent of the speed, cannot be compensated for at all, since OE_g does not exist until the armature begins to rotate.

One arrangement which is used to secure the correct phase for OE_g is to provide the commutating poles with two sets of windings, one in series with the armature (as in the case of the continuous-current machine) and the other in parallel with the armature through an inductance, so that the current through this shunt winding is considerably out of phase with that in the series commutating winding. This arrangement is shown in Fig. 112. FF are the main field windings, S_c being the series and Sh_c the shunt commutating windings respectively, L being the inductance in the shunt circuit.

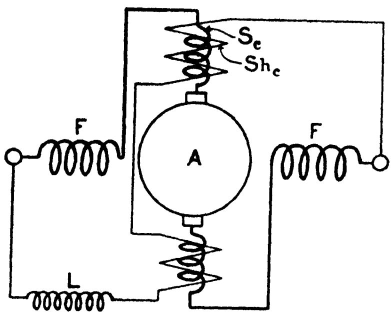

FIG. 112.

55. The Power Factor of the Series Motor.

Commutator motors resemble induction motors in that they are self-exciting; that is, part of the duty of the current from the generator is the supplying of the main field, and it must therefore include a magnetising component. As a result, the supply current will, as a rule, lag to a greater or less extent behind the applied volts, and the power factor of the machine will be less than unity. As we shall see later on, there are types of machines other than the pure series motor in which the power factor can be made practically unity under certain conditions of speed, and they may even be made to take a leading current.

As regards the plain series motor, the factors which affect the question are as follows :—

(1) The self-inductance of the field and armature windings,
(2) The air gap in the magnetic circuit,
(3) The back, or generated E.M.F.,
(4) The ohmic resistance of the motor as a whole.

Taking these points in order, we will proceed to investigate the effect of each in turn.

(1) Both field and armature are inductive circuits, and are in series with one another. There is therefore no transformer effect to be considered other than that in the coils which are short-circuited by the brushes, which effect has already been dealt with. These coils form, in fact, with the brushes and short-circuited commutator segments, a small local circuit which is equivalent to a short-circuited transformer secondary, and therefore exercise a small demagnetising effect on the main field.

The field and armature windings as a whole, however, act

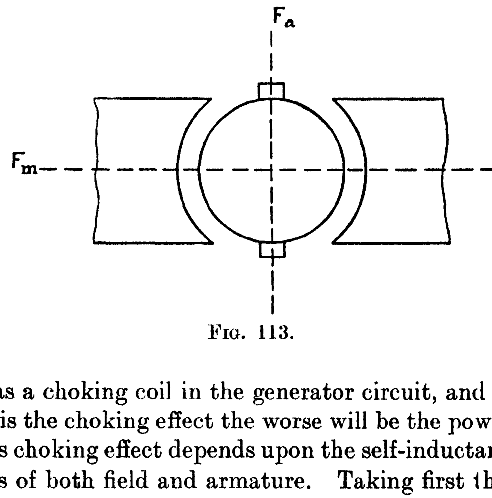

FIG. 113.

simply as a choking coil in the generator circuit, and the more marked is the choking effect the worse will be the power factor. Now this choking effect depends upon the self-inductance of the windings of both field and armature. Taking first the case of the field windings, this self-inductive effect is unavoidable, since by suppressing it we should destroy the main motor field. It is possible, however, to reduce the self-inductance to a minimum by working with a comparatively weak field, that is, by reducing the number of ampere-turns forming the winding. This is invariably done in practice, and has already been referred to on p. 154.

The self-inductance of the armature winding is, however, wholly harmful. The armature ampere-turns produce a flux at right angles to the main field, as shown in Fig. 113, F_m being the main flux, and F_a the cross flux produced by the current in the armature. This cross flux is responsible for the flux distortion which is more or less apparent in all continuous-current machines, and, in their case, has no harmful effect apart from the question of commutation. It is not, however, active in producing torque, and in alternating-current motors it adversely affects the power factor, in that the armature circuit is unnecessarily inductive. This can be remedied by arranging on the stator of the machine a winding whose ampere-turns directly oppose those of the armature, so that the inductance, and therefore the choking effect of the motor circuit as a whole, is distinctly reduced. This is shown diagrammatically in

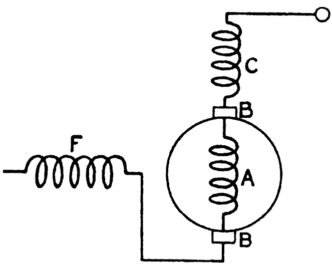

FIG. 114.

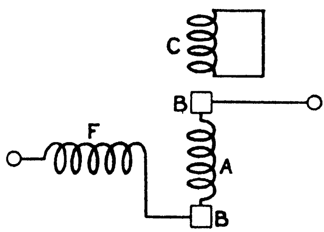

FIG. 115.

Fig. 114, these additional stator ampere-turns being known as a " compensating " winding, and the machine being called, in consequence, a **compensated** series motor. The main field windings are shown at F, the armature ampere-turns at A, the brushes at BB, and the compensating winding at C. The flux produced by A is always at right angles to that produced by F, and in direct opposition to that produced by C. The net result is that the portion of the circuit A—C is practically non-inductive, and the power factor of the machine is therefore improved.

The commutating windings referred to in § 54 have been omitted in Fig. 114 for the sake of clearness, but it will be noticed, by comparing Figs. 112 and 114, that the fluxes produced by the series commutating winding and the compensating winding are similar in sense and direction. Their functions, in fact, overlap, in that the compensating winding also assists commutation, and the series commutating winding also tends to improve the machine power factor.

The compensating winding need not necessarily be in series with the armature, as was shown in Fig. 114 ; it is sometimes arranged as shown in Fig. 115, short-circuited on itself. It then acts as the short-circuited secondary of a transformer, of which A is the primary winding, and therefore exerts a demagnetising effect on the armature under ordinary transformer laws.

(2) The effect of the *air gap*, which must necessarily exist between stator and rotor, will be to affect the power factor adversely, in that a proportionately heavier magnetising current will be required to produce a given flux. This point has already been dealt with in § 40, in the case of the induction motor.

(3) The *generated E.M.F.* exercises a considerable influence on the machine power factor, and its effect is best studied by constructing the vector diagram shown in Fig. 116.

The line OE_a represents, in magnitude and phase, the applied pressure, which is made up of three components, when the machine is running. That which is necessary to balance

(*a*) The generated E.M.F.,
(*b*) The IR drop in the motor,
(*c*) The E.M.F. of self-induction in both field and armature windings.

As regards (*a*), the generated E.M.F. (OE_g) at any instant depends upon the strength of the main field at that instant. It must be remembered that the main flux is an alternating one—that is, it is continually varying in sense and magnitude. At the instant when this flux is at its maximum value, in either sense, the generated E.M.F. will be a maximum in the corresponding sense, because, at a given speed of rotation, the rate of cutting of the lines of force by the armature conductors is then most rapid. Similarly, at the instant when the field strength is zero there will be no generated E.M.F., since there are for that instant no lines of force being cut by the conductors. In short, the **generated E.M.F. is in phase with the main flux,** which is practically in phase with the current producing it, as we have seen on p. 156. We may therefore say that the generated E.M.F. is in phase with the motor current. It will naturally be in direct opposition in *sense,* since, in any motor, the effect of the generated E.M.F. is always to oppose the flow of the current; this fact directly follows from Lenz's law. The component of the applied volts necessary to balance OE_g will then be in the same sense as and in phase with the current. This is shown at OE'_g in Fig. 116.

(*b*) The IR drop in the motor will also be in phase with the current.

(*c*) The component necessary to balance the self-induced E.M.F. of the motor circuit will be at right angles to the current, just as is the transformer or secondary induced E.M.F. in the short-circuited coil (already referred to on p. 156). They both, in fact, depend upon the *rate of change* of the main field, and are therefore 90° out of phase with it, as we are aware from elementary principles.

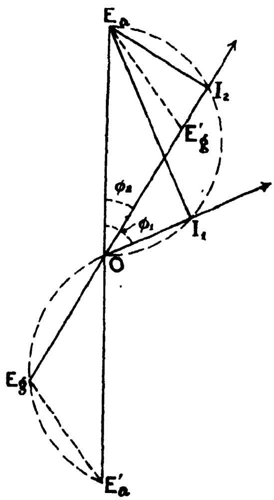

FIG. 116.

We may now proceed with the construction of the vector diagram shown in Fig. 116. Since OE_a, the applied pressure, always consists of two components at right angles to one another, if we describe a semicircle on OE_a as diameter, this semicircle will be the locus of the point I.

Taking first the conditions at **starting**—*i.e.*, when the generated E.M.F. is zero—the components of the applied volts are now OI_1, the IR drop in the machine, and I_1E_a (at right angles to OI_1), the reactance drop in the machine, equal to ωLI. OI_1 is therefore the current vector at starting, lagging by the considerable angle φ_1 behind the applied volts.

Secondly, let us take a certain condition of load and speed in which the value of the generated E.M.F. is OE_g. The applied volts now consist of three components: OE'_g, in phase with the current, as explained above, equal and opposite to OE_g;

$I_2E'_g$, representing the IR drop in the machine, also, of course, in phase with the current; and finally I_2E_a, the reactance component, at right angles to the current vector OI_2. The angle of lag (φ_2) is now considerably less than it was at starting, *owing to the effect of the generated E.M.F.*

Lastly, in the limit, when the machine is *entirely* unloaded, the generated E.M.F. becomes equal and opposite to the applied volts, as shown at OE'_a. No current will now flow since there is no load, the vector of the motor current coming more and more into phase with the E.M.F. vector OE_a as this condition is approached.

The following observations may be made from this diagram:

(1) Since the ohmic resistance and the inductance * of the motor circuit are both constant, the triangle $E_aIE'_g$ is similar for all conditions of loading—that is, the value of $\frac{E_aI}{IE'_g}$ or $\frac{\omega LI}{IR}$ is constant. The power factor obviously depends, *inter alia*, on this expression, so that, by reducing the value of L, *i.e.*, the motor self-inductance, the value of E_aI is reduced, and the current therefore comes more into phase with the volts. This is the function of the compensating winding, as already explained.

(2) The generated E.M.F. has the effect of increasing the component OI of the volts, and therefore of improving the power factor. High rotation speeds, and a consequently high value for OE_g, are therefore advantageous from this point of view, as they are also from the point of view of commutation.

(3) Since the expression $\frac{E_aI}{IE'_g}$ is constant, and the angle E_aIO is always a right angle, it follows that the angle $E_aE'_gO$, and therefore the angle E'_aE_gO, is constant—that is, the locus of the point E_g is the arc of a circle.

56. Characteristics of the Series Motor.

The mechanical characteristics of the alternating and continuous-current motors are very similar, though the efficiency of the former is never so good as that of the latter, largely owing

* Assuming, as is nearly true provided that the iron is not saturated, that the permeability of the magnetic circuit is constant.

to the I^2R losses in the short-circuited coils and the high resistance commutator connections. The power factor is much better in the compensated motor than in the induction motor, and approaches unity at high speeds. The starting torque is a maximum, the speed torque curve being, in fact, very similar to that of the continuous-current machine. It is, therefore, well suited for traction work, for which duty it is widely used, particularly in the modified form which will be described later. The frequency of supply should be low, as can be readily arranged when the generators are dealing wholly with a traction load.

57. The Repulsion Motor.

One of the drawbacks to the series motor is the fact that it cannot be used on high-pressure systems without the introduction of a transformer. The E.M.F. which can be applied to the terminals is, in fact, limited by the maximum pressure permissible between adjacent commutator segments, and is somewhere in the region of 300 volts. This difficulty is overcome in the **repulsion** type of motor, in which current is supplied from the mains to the stator windings only, the necessary torque-producing current being induced in the rotor windings by transformer action. Commutation is also somewhat better in the repulsion type, as will be explained in due course.

Referring back to Fig. 115, which shows a compensated series motor with the compensating winding short-circuited on itself, it will be seen that a very similar state of affairs will result if the compensating winding is arranged in series with the main field winding, and the *armature* windings are short-circuited through the brushes, which are connected by a conductor of negligible resistance. The two cases are shown together in Fig. 117 for the sake of comparison.

The main field winding is shown at F, C being the compensating winding, A the armature winding and BB the brushes, connected in Fig. 117 (*b*) by the conductor as shown. Electrically, the only difference between the two figures is that, whereas in the series type A is the primary and C the secondary transformer winding, in the repulsion type C becomes the primary and A the secondary. The winding F produces the main flux as before, the winding C being responsible for the transformer flux, and therefore for the induction in A of an E.M.F. which will send a current through the circuit formed by the armature windings, brushes and short-circuiting brush

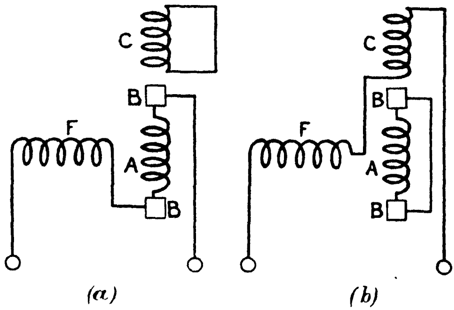

FIG. 117.

connector. In the actual machine, the currents in each half of the armature winding will unite at a brush, flow together along the connector, and divide again at the opposite brush. This current, induced in the armature by transformer action, will set up a flux, which, by interaction with the main flux set up by the current in coil F, will produce the torque in the machine.

Since F and C in Fig. 117 (*b*) produce fluxes at right angles to one another in space, it is obvious that they may be replaced by a single winding, whose axis will be that of the *resultant* of the two fluxes produced by F and C. This is shown diagrammatically in Fig. 118, the axis of the combined coil FC making an angle, α, with the brush axis. This machine is the simple repulsion motor, and the field or stator windings have a double duty to perform :—

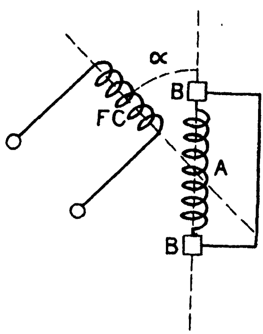

FIG. 118.

(*a*) To set up the main or torque-producing flux in which the rotor revolves.

(*b*) To set up the transformer flux which will induce the necessary current in the rotor windings.

It will be obvious that the angle α must have some value between 0° and 90° in order that a torque may result—that is, that the flux produced by FC must have a component *along* the brush axis and also *at right angles* to the brush axis. The arrangement of the motor is shown in Fig. 119. The value of

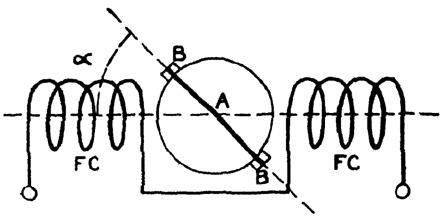

FIG. 119.

the angle α may obviously be altered at will by shifting the brush axis.

Taking first the case of a *zero* value for this angle, the field axis will lie along the brush axis, and no torque will result, provided that the machine is exactly symmetrical with respect to this axis. The reason is that, although the field windings are, with this brush position, producing the maximum transformer effect on the armature (the brushes being set in the axis of

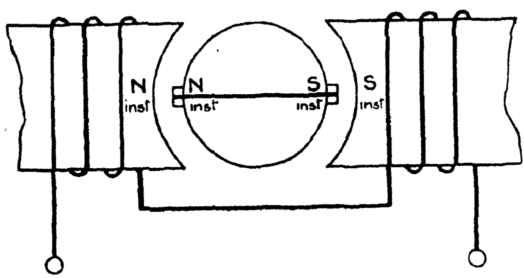

FIG. 120.

maximum transformer E.M.F., as explained on p. 151), the necessary torque-producing component at right angles to the brush axis is lacking. The condition is one of unstable equilibrium, as will be seen from Fig. 120; the armature current sets up a flux which *directly* opposes the main flux at each instant, and the rotor has therefore an equal tendency to revolve in either direction. The whole machine will, in fact, behave as a static transformer on short circuit, the effect of the current in the upper half of the armature being exactly balanced by that in the lower half.

Taking now the other extreme, when the brush axis is at right angles to the flux—that is, when a is 90°, as shown in Fig. 121—it will be clear that no torque can result, since the brushes are now set in the axis of *zero* transformer E.M.F. and no current will flow in the armature windings (other than that in the coils actually short-circuited by the brushes, the effect of which is balanced), the armature now acting as the secondary of a transformer on *open* circuit. The conditions are, in fact, the same as though the brushes were not there, neglecting again

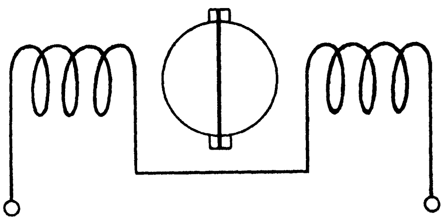

FIG. 121.

the short-circuited coils. In short, the main field has now no component *along* the brush axis, and its transformer effect is nil.

Lastly, if we take some intermediate value for the angle a, as shown in Fig. 122, it will be at once evident that a clockwise torque is exerted on the rotor owing to the mutual repulsion of the two like poles NN and SS. The main flux has now a component at right angles to the brush axis, in addition to that along the brush axis. The torque will be in a constant direction and continuous, since the armature flux has always a component which directly opposes the main flux at each instant, and as soon as one coil passes from under the brush another takes its place. The current which flows in the armature windings will not be as heavy as in the case first considered (Fig. 120), since the brushes are no longer set in the axis of maximum transformer E.M.F. The actual value of a to produce a maximum torque is found in practice to be about 20°.

It follows as a natural corollary of our investigations that the direction of rotation will depend upon the direction in which the brush axis is set. For example, with the brushes set as shown in Fig. 123, the direction of rotation will be counter-clockwise.

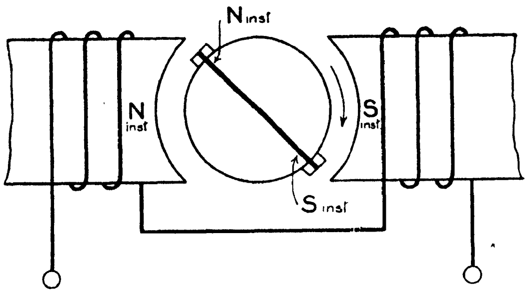

FIG. 122.

58. The Stator Current of a Repulsion Motor.

The action of the motor is best studied by considering the field or stator windings as consisting of two sets of coils at right angles to one another in space, as shown in Fig. 117 and

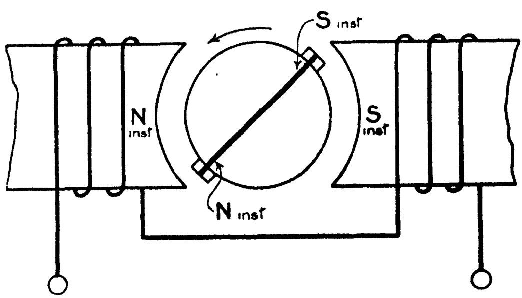

FIG. 123.

repeated here in Fig. 124. This arrangement is obviously identical, from an electrical point of view, with that shown in Fig. 119. Now we have already seen that, of these windings, coil C is responsible for the induction of the armature or rotor current, by transformer action; we shall, therefore, speak of this as the **primary transformer winding.** Coil F provides the working magnetic field, which, by interlinking with the armature field, produces the necessary torque in the machine; this coil we shall speak of as the **field winding.** Each of these coils gives rise to an alternating magnetic flux, and these fluxes are always at right angles *in space*, but differ *in phase* by a varying angle which depends upon the position of the brushes and the load on the machine. It must not be forgotten that, in actual fact, these two coils are combined into one; the effect of varying the brush position is, therefore, to strengthen the transformer flux and to weaken the main flux, or *vice versa.*

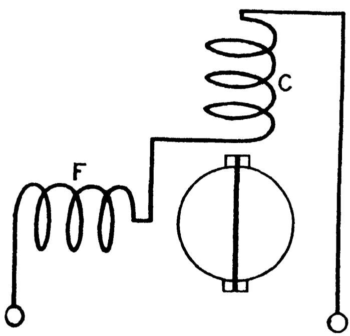

FIG. 124.

The vector diagram of a repulsion motor will be, in effect, a modified transformer diagram, the modifications being caused mainly by two facts :—

(*a*) The armature or secondary winding *rotates* in the main field set up by the current in coil F, and there is in consequence an E.M.F. *generated* by the armature windings as they cut the lines of force of this field.

(*b*) The primary transformer winding C is in series with a highly inductive winding, F, which acts as a choking coil and therefore lowers the power factor of the circuit. This fact is very important, and we shall follow later the evolution of a type of machine which is designed to overcome this difficulty.

Before proceeding to the case of the machine running under load, we will construct the simplest possible vector diagram for a special case—that is, when the brushes are lifted from the commutator, so that the armature will not rotate. Under these circumstances the armature will behave like the secondary of a transformer on open circuit and no current will flow in its windings. We will take as our vector of reference the transformer flux produced by coil C, as we have been accustomed to do in the case of the static transformer and the induction motor. This is shown at F_t in Fig. 125. Flux F_t is responsible for the induction of the "transformer" E.M.F. in the rotor; this E.M.F. will lag 90° behind the flux, as shown at OE_t. It will simplify these diagrams if we consider (as we did in the case of the transformer and the induction motor) that the rotor, or secondary, has the same number of turns as the primary or stator *transformer* winding —that is, as coil C. OE'_t will be the component of the applied volts necessary to balance OE_t, and will be equal and opposite to the latter. The primary, or stator, current will be practically in phase with the flux F_t, as usual; its vector is shown at OI_0. So far, the diagram has been a mere repetition of that of any unloaded transformer, with an air gap in the magnetic circuit. We have now to consider the effect of coil F, a highly inductive winding in series with the primary transformer winding C. This causes the E.M.F. of self-induction in the primary to exceed the secondary induced E.M.F. by a considerable amount, and we must, therefore, add to the applied volts a component to balance this self-induced E.M.F. This component will be 90° out of phase with the flux which gives rise to it—that is, the flux F_m set up by the current in coil F. This flux will obviously be *in phase* with F_t, since neither flux is being in any way reacted on, and the coils are in series with one another. The component to balance the self-induced E.M.F. in coil F, and also any unbalanced self-induced E.M.F. in coil C, will therefore be as shown at $E'_tE'_s$.

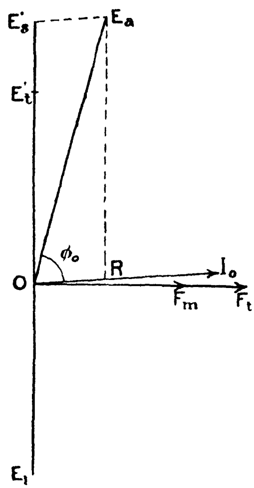

FIG. 125.

The applied volts will then be the resultant of OE'_s and OR, as shown at OE_a. The machine is, in fact, behaving like a very leaky transformer, since a considerable part of the stator flux—that, in fact, set up by the current in the coil F—is not cutting the secondary windings. The effect is to cause the no-load power factor to be very low. Not only is the no-load power factor affected, but, as we shall see later from the load vector diagram, the effect of this coil F on the power factor is apparent at all loads. It is desirable to emphasise again the distinction between the relation of the two fluxes F_t and F_m in *space* and *in phase* or *time*.

59. The Repulsion Motor Running under Load.

We will now proceed to the more difficult case of the loaded repulsion motor—that is, when the brushes are down and the machine is running. Before constructing this vector diagram it will be desirable to note in order the various conditions which must be borne in mind.

(*a*) A current is now flowing in the armature or rotor windings by way of the commutator, brushes and short-circuiting connection. This current is produced by the transformer action of the flux F_t, and therefore reacts on this flux, as in the case of any other transformer.

(*b*) The flux F_m, being at right angles in space with the flux F_t and with the brush axis, has no transformer effect on the rotor; it is not, therefore, in any way reacted upon by the rotor current, and so remains practically in phase with the primary or stator current which produces it. We shall ignore the slight lag of a flux behind the current which gives rise to it—that is, the iron losses in the cores—in order to simplify the diagrams, remembering that this lag is caused by the eddy currents in and the hysteresis of the iron.

(*c*) The armature is now rotating, and will, therefore, *generate* an E.M.F. by the motion of the conductors in the flux F_m. It must be remembered that the axis of maximum generated E.M.F. in all motors is at right angles to the brush axis with an ordinary ring armature. This generated E.M.F. will be, as we have already seen in the case of the series motor, in phase with the flux responsible for it—that is, the flux F_m

(*d*) Owing to the high (leakage) self-inductance of coil F, a component of the applied volts will be necessary which will be at right angles *in phase* with the flux F_m, as we have already noted in the previous section.

(*e*) The resultant of the generated E.M.F. and the transformer E.M.F. will cause a current to flow in the rotor windings and the short-circuited brush connection. The phase angle between this rotor current and the *resultant* rotor E.M.F. which causes it will be determined by the relation between the apparent resistance due to the inductance of the rotor windings and their ohmic resistance.

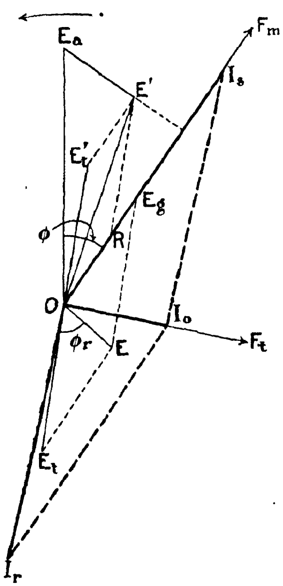

FIG. 126.

The most straightforward method to adopt in constructing the vector diagram, for any one condition of load and speed, will be to proceed as though we were plotting the results of actual readings. In order to do this, the following data must be obtained :—

1. The value of the applied E.M.F. as read on a voltmeter.

2. The value of the stator current, as read on an ammeter.

3. The phase angle between applied volts and stator current, deduced from 1 and 2 above, and from a wattmeter reading.

4. The resistance and the apparent resistance due to the inductance of both stator and rotor windings. These must be ascertained before the machine is started.

5. The no-load stator current, found by taking a reading with the brushes off the commutator, as explained in Section 58.

The complete diagram (Fig. 126) may now be constructed as follows :—

Draw OE_a equal to the applied pressure to any scale (of volts) from 1 above. Draw OI_s equal to the stator current to any scale (of currents) from 2 above, at the correct angle φ with OE_a from 3 above. The direction of OI_s will be that of the flux F_m (*v.* (*b*) above), and it will also be that of the generated E.M.F. vector (*v.* (*c*) above).

Now the applied pressure OE_a is made up of three components :—

(i.) The reactance drop in the stator windings—*i.e.*, a component to balance the E.M.F. of self-induction in coil F at right angles to the flux F_m. This is shown at E_aE', and will be considerable owing to the choking effect of this coil (*cf.*, $E'_tE'_s$ in Fig. 125).

(ii.) The IR drop in the stator winding, in phase with the stator current. This is shown at $E'E'_t$ (or OR). Both these are known from 2 and 4 above.

(iii.) A component to balance the transformer E.M.F. in the rotor. This will be OE'_t to the scale of volts.

The transformer E.M.F. can now be drawn at OE_t, equal and opposite to OE'_t, assuming, as before, an equal number of turns in the rotor and coil C.

The vector of the transformer flux (OF_t) will be 90° in advance of OE_t (as in Fig. 125). This will also be practically the direction of the no-load current OI_0, which can be drawn to the scale of currents from 5 above. The rotor current OI_r is found in phase and magnitude by completing the parallelogram $OI_rI_0I_s$.

Finally, we have to determine the *magnitude* of the generated E.M.F. whose *phase* we know to be that of the flux F_m. This is done by plotting the *resultant* rotor E.M.F., whose phase can be deduced from 4 above, and whose magnitude depends upon the value of the rotor current OI_r. This resultant E.M.F. is shown at OE. The value of the generated E.M.F. is found by completing the parallelogram OE_tEE_g.

From this completed diagram the observations dealt with in the following sections may now be made.

60. The Power Factor of the Repulsion Motor.

The power factor of the machine depends (1) upon the value of $E'E_a$ in Fig. 126—that is, upon the self-inductance of the stator winding. This is bound to be comparatively high owing to the presence of coil F, as explained above. (2) Upon the value of the generated E.M.F. OE_g—that is, upon the product of the speed at which the machine is rotating and the strength of the flux F_m. Obviously, from Fig. 126, if the armature is stationary OE_g disappears, the resultant rotor voltage being OE_t, and the angle between OI_s and OE_a being therefore correspondingly increased. This should be clear from Fig. 127, in which a portion of the full vector diagram is reproduced for the two cases, firstly, in Fig. 127 (*a*), when the rotor is revolving fast

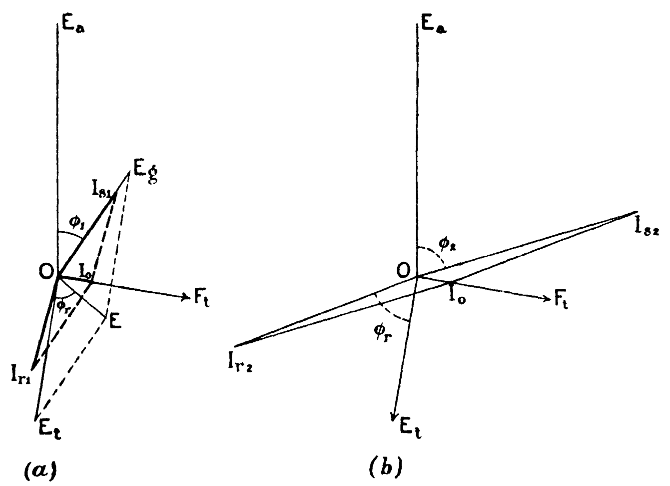

FIG. 127.

enough for OE_g to reach a considerable value, and, secondly, in Fig. 127 (*b*), when the rotor is just about to start, and OE_g is therefore non-existent.

In the first case (Fig. 127 (*a*)) the rotor pressure—the resultant of OE_t and OE_g—is shown at OE; the angle φ_r between this resultant pressure and the rotor current OI_r is constant (*v.* (*e*) § 59). The angle of lag between stator current and applied volts is shown at φ_1.

In the second case (Fig. 127 (*b*)), since there is now no generated E.M.F., the resultant rotor pressure is OE_t, φ_r remaining the same as before. The stator phase angle φ_2 is considerably greater under these conditions, the power factor being now a minimum. The effect of speed on the power factor is therefore the same as in the case of the series motoı, and for a similar reason.

61. Commutation in the Repulsion Motor.

It will be seen from Figs. 126 and 127 that the *generated and transformer E.M.F.s are nearly in phase opposition,* instead of being in quadrature as they are in the case of the series motor. This has a very important bearing on the question of commutation. Since these two E.M.F.s almost directly oppose one another, the opposition being more and more complete as the speed rises, the question of sparkless commutation is not so difficult as it is in the case of the series motor. The ordinary precautions against excessive sparking, such as high-resistance commutator connections and subdivided armature windings, must be taken, since at the actual moment ot starting the conditions are no better than in the case of the series motor, and the transformer E.M.F. in the short-circuited coils cannot be balanced. But as the machine speeds up and the generated E.M.F. rises, commutation becomes practically sparkless, without the necessity for the special commutating windings employed in the series type of machine.

62. The Speed Limit of the Repulsion Motor.

The alternating-current series motor, like the corresponding continuous-current machine, will race if the load is entirely thrown off, and for precisely the same reason. The speed of the repulsion motor, however, cannot exceed a certain value, which depends, in the actual machine, upon the brush position —that is, upon the relative values of the fluxes produced by the currents in coils F and C (Fig. 124). As the speed of the motor rises, the vectors of the transformer and generated E.M.F.s (Fig. 127) come more and more directly into opposition until, at one definite speed—depending upon the brush position—they are equal and opposite. No current will then flow in the rotor and the torque will fall to zero. Speed regulation *can* be, though seldom in practice is, obtained by shifting the brush position. From the point of view of efficiency, power factor and commutation the machine is best suited to a more or less constant speed in the neighbourhood of synchronism.

It will be apparent, from the vector diagrams, that this class of machine resembles closely the single-phase induction motor, the difference being that the rotor windings are not short-circuited *individually*, as they are in the induction type, but *as a whole*, by way of the commutator and brushes. The practical result of the latter arrangement is to produce a considerable starting torque and a better starting power factor, since the reactance of the rotor does not swamp its ohmic resistance, a state of affairs which is particularly noticeable in all induction motors with squirrel-cage rotors of low resistance (§ 44). As a matter of fact, the plain repulsion motor is seldom used as such, though single-phase motors are frequently *started* as repulsion motors, by providing them with wound rotors and a commutator, as was mentioned in § 49. After starting, the commutator segments are short-circuited, either by hand or by some centrifugal device, and the machine then runs as an induction motor

63. The Compensated Repulsion Motor.

As has been mentioned in § 58, one of the objections to the simple repulsion motor is its comparatively low power factor, which is due to the high leakage self-inductance of the stator winding. This we found to be unavoidable, owing to the fact that the stator winding has two duties to perform, namely, the inducing of the armature current by transformer action, and also the production of the working flux of the motor. We considered this double function by imagining the stator winding to be split up into two components, F and C, at right angles to one another in space (*v.* Fig. 124).

In the **compensated** repulsion motor, component F disappears, and the working flux of the machine is provided by the current in the *rotor* windings. This arrangement is shown diagrammatically in Fig. 128 (*b*), the simple repulsion motor being shown, for the sake of comparison, in Fig. 128 (*a*).

In the compensated motor shown in Fig. 128 (*b*) coil C is the only one which is wound on the stator, and it induces the necessary rotor current along the short-circuited brush axis by producing the transformer flux F_t. The working or motor flux F_m is produced, not by a coil, F, on the stator as in Fig. 128 (*a*), but by providing the rotor with a *second* winding at right angles to F_t, this winding being in series with coil C by way of brushes and commutator. As a matter of fact, these two rotor windings are combined into one, and the rotor is wound as usual, in the same way as a continuous-current motor armature, and both sets of brushes rub on the same commutator.

At first sight it would appear that we have gained nothing by this arrangement. We have, in fact, substituted for coil F on the stator a similar winding, F_r, on the rotor, which is also

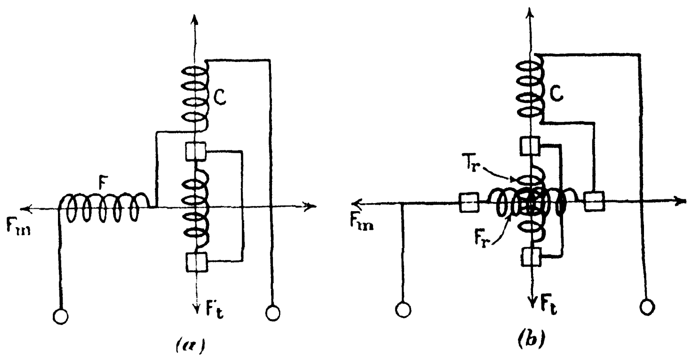

FIG. 128.

highly inductive and in series with coil C. The conditions are, however, really quite different. Although F_r is highly inductive, and therefore tends to exercise a considerable choking effect on the stator circuit (being in series with the stator windings), this choking effect is more or less completely neutralised, owing to the fact that the winding F_r *rotates*, and is therefore the seat of a *generated E.M.F.*, in addition to its E.M.F. of self-induction. This point we will now proceed to investigate, and it will be convenient to regard the rotor windings as split up into two parts, F_r and T_r as in Fig. 128 (*b*), at right angles to one another, just as we did the stator windings in the case of the simple repulsion motor.

As regards the influence of coil C on the power factor, nothing need be added to what has already been said in § 60. It forms the primary winding of a transformer of which T_r is the secondary, and the E.M.F. generated by T_r in the motor flux F_m (produced in this case by the series rotor winding F_r) tends to produce a unity power factor in the stator circuit, as shown in Fig. 127.

We will therefore pass on to the consideration of coil F_r and its effect on the machine power factor. This coil is the seat of a considerable E.M.F. of self-induction, which is at right angles *in phase* to the flux F_m, and hence practically at right angles, in phase, to the stator current. When the machine is running, as we have already seen, this stator current is considerably in

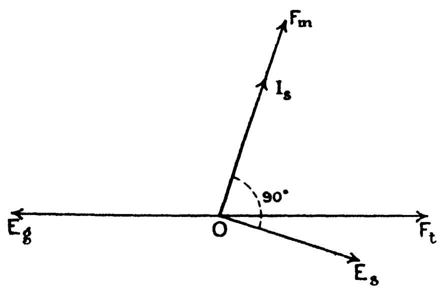

FIG. 129.

advance, in phase, of the transformer flux F_t (Fig. 126). Were this self-induced E.M.F. the only one to be considered in coil F_r, the power factor would be low, as we have already noted. But, in addition to this E.M.F. of self-induction, an E.M.F. is generated in this winding by its **rotation in the transformer flux F_t**, since this flux lies at right angles in space to the axis of F_r (*v.* p. 172). This generated E.M.F. will be, as we have already seen, in phase with the flux F_t responsible for it. But F_t is considerably out of phase with F_m, this phase angle approaching 90° at high speeds. The result is that **the E.M.F. of self-induction and the rotation E.M.F. in coil F_r are nearly in phase opposition,** the former being at right angles to F_m and the latter being in phase with F_t. This is shown in Fig. 129, OI_s being the vector of the stator current and of F_m, OE_s being that of the

E.M.F. of self-induction and $OE_{//}$ that of the generated E.M.F. in the rotor winding F_r at a certain definite speed.

The result is that the E.M.F. of self-induction in this coil is more or less **neutralised by the generated E.M.F.**, so that F_r behaves as though it were a non-inductive winding. The power factor of this type of machine is therefore very good, being practically unity at a speed approaching synchronism,

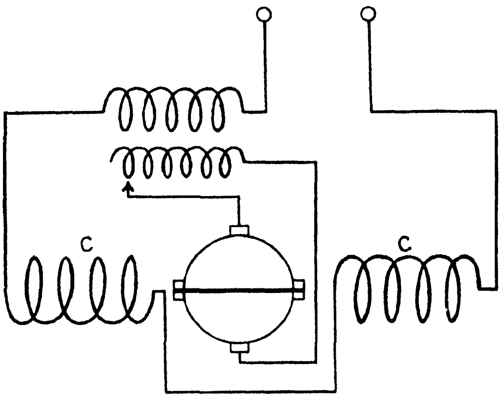

FIG. 130

owing to the compensating effect of the generated E.M.F.s in the coils F_r and T_r as explained above.

To this class of machine belongs the **Latour-Winter-Eichberg** motor (Fig. 130), which is practically the same as the type shown in Fig. 128 (*b*), except that the current is supplied to the series rotor windings by way of an adjustable transformer, instead of direct. This is partly in order that the machine may be used on high-pressure circuits and partly to effect **speed regulation** by varying the pressure across the series rotor windings F_r, and so the value of the motor flux F_m.

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