A Brief History of NUMBERED SYSTEMS

Douglas Clarkson believes it has taken quite some time to credit ancient cultures with sophisticated numeric skills, here he discusses the proof.

More and more of our technical landscape mirrors the embrace of a digital culture, reflecting the increasing use of numbers to store and convey knowledge and information. This also seems to be stirring an increase in the science of numbers, especially where in systems these appear to have elusive or curious properties.

Early Beginnings

From whatever period of recorded history a basis of numeric representation of that era can be identified. We in the West are familiar with Babylonian; Ancient Egyptian, Greek and Roman number systems though somewhat less familiar with the cultures of South America and even less familiar generally with Chinese/Japanese systems. The records of many cultures, however, have vanished without trace.The culture that built the ancient megaliths of Britain - Stonehenge, Avebury and Callanish have left only the stones themselves as witness of their numeracy.

From the widespread use of counting in units of ten and the similarity in structures used in Indo-European languages, it is conjectured that the basic Indo-European mother tongue had the same method of counting.

Counting can also be undertaken in pairs of two as in one, two, two one, two two, two two one, two two two etc. This can usually be used for numbers up to ten. Some cultures in South America have used 4-12 to feature in counting systems while the Aztecs used a 5-20 count. These systems are indicated in Table 1. There is a trace of 20 count in some languages of Western Europe such as English, French and Danish.

Value	Representation 4/12 count	Representation 5/20 count
1	1	1
2	2	2
3	3	3
4	4	4
5	4 + 1	5
6	4 + 2	5 + 1
7	4 + 3	5 + 2
8	4 x 2	5 + 3
9	4 x 2 + 1	5 + 4
10	4 x 2 + 2	10
11	4 x 2 + 3	10 + 1
12	12	10 + 2
13	12 + 1	10 + 3
14	12 + 2	10 + 4

Table 1: Representation of 4/12 count and 5/20 count

Since many cultures have disappeared without trace, the greater part of the historical development of number systems and number understanding has been lost. Often the key to appreciating the number systems of ancient cultures has been through the discovery and decipherment of a single artefact - such as the Rhind papyrus of Ancient Egypt.

The Numbers of Babylon

Some of the oldest records of numbers are in the first half of the third millennium BC among the Sumerians whose culture was supplanted by the Babylonians. From this

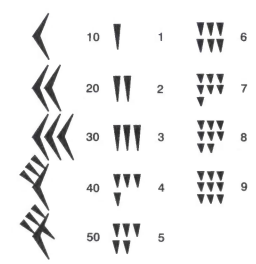


Figure 1a: Examples of representation of Babylonian numbers as used in Plimpton 322 tablet: Symbols are used to establish digit values between 1 and 59.

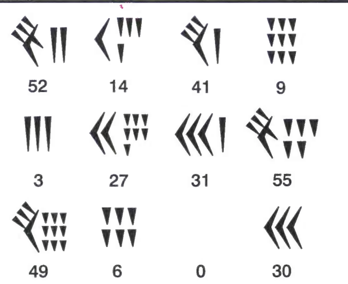


Figure 1b: Representation of numbers in Babylonian cuneiform script using dialect of Plimpton 322 tablet - 52:14:41:09 = 11,284,869; 03:27:31:55 = 747,115; 49:06:00:30 = 10,605,630

culture, around the time of Hammurabi in 1750 BC, surviving records provide an insight into numeric skill and competencies.

The method of writing numbers in this period is described as cuneifrom (Latin cuneus - wedge) and utilises the base number 60. What looks like a exclamation mark represents 1 and a sharp left arrow 10. Some examples of numbers are indicated in Figure 1a. The significant feature of this number system was the use of place notation. We very much take notation for granted in our Arabic number system. Figure 1b gives examples of this use of place notation. In some records, there is some ambiguity as to whether such numbers record integers or fractions as recorded in the Plimpton tablet 322 and which subsequently will be described in some considerable detail.

Babylonian Multiplication and Division

In Babylonian mathematics, multiplication was undertaken by means of the expression:-

$ab = ((a + b)^2 - a^2 - b^2)/2$

Thus as an example 35 x 22 is given by :-

$(57x57-35x35 - 22x22)/2 = 770$

One clay tablet found at Senkerah on the Euphrates in 1854 is thought to date from 2000 BC and held details of squares of numbers up to 59 and cubes of numbers up to 32.

The Babylonians undertook long division of integers by recognising that 7/5 could be expressed as the product of 7 times (1/5). The Babylonians established tables of reciprocals as indicated in Table 2.

	Base 60	fraction	component
Denominator	1/60	1/3600	1/216000
2	30	0	0
3	20	0	0
4	15	0	0
5	12	0	0
6	10	0	0
8	7	30	0
9	6	40	0
10	6	0	0
12	5	0	0
15	4	0	0
16	3	45	0
18	3	20	0
20	3	0	0

Table 2: Representation of reciprocals which have terminating base 60 fractions.

Thus the fraction 1/12 can be expressed as 5/60 in base 60 representation. As with multiplication and division in decimal, an indication needs to be maintained of place significance of columns being manipulated. Numbers such as 7,9,11,13,14,17,19 etc. do not produce terminating base fractions but would be written as approximations. By comparison with denominators between 2 and 20, there are only seven terminating base ten fractions.

Babylonian Square Roots

There is clear evidence that the Babylonians had a method of calculating square roots. This prompts a puzzle as to whether this method represented a trial and error approach that was found to work or reflects a deeper insight into number theory.

In an example, we start with an integer A and with N the largest integer such that its square is less than A. In the example of A=7, we can select N=2. A sequence of integers s(j) of increasing value is determined where

s(j) = 2 N s(j-1) + (A-N2) s(j-2)

In this example this becomes

s(j) = 4 s(j-1) + 3 s(j-2)

Where s(0) is set to 0 and s(1) to 1, the sequence becomes

0,1,4,19,88,409,8827,41008 etc.

The actual expression giving the approximation of the square root is:-

sqrt(A) = N + (A-N2)(s(j)/s(j+1))

where j has a value which gives a reasonable approximation to the square root value.

Table 3 indicates the results obtained in determining the square root of 7.

Derived expression

r	r^2
2 + 3 (1/4)	7.5625
2 + 3(4/19)	6.9252
2 + 3(19/88)	7.0104....
2 + 3(88/409)	6.99854...
2 + 3(409/1900)	7.000201..
2 + 3(1900/8827)	6.9999719..

Table 3: Results of solution of derivation of Babylonian square roots for value 7.

Jumping ahead many centuries, Archimedes (287-212 BC) is reported to have stated that the square root of 3 is given by the relationship:-

(265/153) < sqrt(3) < (1351/780)

This is from use of the ratio terms (896/2448) and (18272/49920) in the expansion of terms in the series. Thus either Aristotle had used the Babylonian square root generator system or had invented a comparable system. Leastways it gives the students of the history of mathematics something to debate - especially if you are a Greek mathematician seeking to demean the achievements of the Babylonians.

Various of the Babylonian tablets read like school geometry books. One tablet number 7289 in the Yale Babylonian collection demonstrates a value of sqrt(2) as 1 24' 51'' 10''' which is accurate to 6 parts in 10^{-7}. It comes complete with diagram. The short QuickBasic programme EXAMPLE1 indicates the nature of the terms developed in the process of square root calculation for values of N less than 34.

```
10 REM Babylonian square roots (N<34)
15 DIM s(20) AS DOUBLE
20 PRINT "input value of number for square root:A"
30 INPUT a: IF a = 0 THEN STOP
40 PRINT "input value of N ( N*N<A)"
50 INPUT N
55 IF N * N > a THEN GOTO 20
60 s(0) = 0: s(1) = 1
70 FOR jj = 2 TO 8
80 s(jj) = 2 * N * s(jj - 1) + (a - N * N) * s(jj - 2)
90 NEXT jj
100 PRINT " s(jj) s(jj+1) r r*r"
110 FOR jj = 1 TO 7
115 r# = N + (a - N * N) * (s(jj) / s(jj + 1))
120 PRINT USING "####################"; s(jj); s(jj + 1);
130 PRINT USING "####.#############"; r#;
140 PRINT USING "####.#############"; r# * r#
150 NEXT jj
160 PRINT : PRINT
170 GOTO 20
```

In line 115, r# signifies double precision real number representation.

Plimpton 322

Photo 1: The Famous Plimpton 322 tablet which displays four columns of cuneiform numbers and has been interpreted as representing geometrical relationships involving Pythagoras theorem.

All students of the history of mathematics are familiar with Plimpton 322. This is a clay tablet in the G. A. Plimpton Collection at Columbia University and is shown in Photo 1. Estimates of its age range between 1900 BC to 1600 BC - so that it could be nearly 4000 years old. It was initially thought to record commercial transactions before O. Neugebauer interpreted the numeric content in an altogether different way.

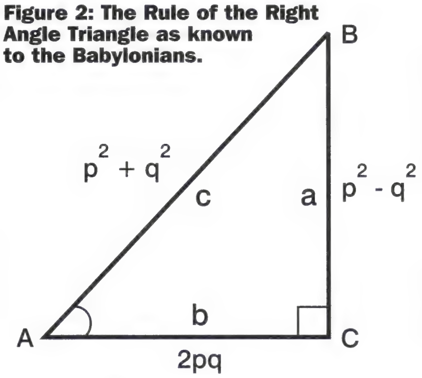
With a small element of understanding of elementary geometry and arithmetic it is entirely possible to grasp the mathematical structure of the table which is now widely accepted - although this is still a topic of some debate. The table with corrected numerical interpretation is indicated in table 4 in the base 60 notation and consists of fifteen lines of four columns of numbers. Numbering the columns from left to right, column 1 is more likely to represent a fraction, while columns 2, 3 and 4 integers. This set of values contains four corrections which are widely agreed and terms are indicated with * in Table 4. Column four is acting just like an index of the lines of the tablet.

Column 1	Column 2	Column 3	Column 4
1:59:00:15	1:59	2:49	1
1:56:56:58:14:50:06:15	56:07	1:20:25*	2
1:55:07:41:15:33:45	1:16:41	1:50:49	3
1:53:10:29:32:52:16	3:31:49	5:09:01	4
1:48:54:01:40	1:05	1:37	5
1:47:06:41:40	5:19	8:01	6
1:43:11:56:28:26:40	38:11	59:01	7
1:41:33:45:14:03:45	13:19	20:49	8
1:38:33:36:36	8:01*	12:49	9
1:35:10:02:28:27:24:26	1:22:41	2:16:01	10
1:33:45	45	1:15	11
1:29:21:54:02:15	27:59	48:49	12
1:27:00:03:45	2:41*	4:49	13
1:25:48:51:35:06:40	29:31	53:49	14
1:23:13:46:40	56	1:46*	15

Table 4: Base 60 notation values of corrected Plimpton 322 tablet. The 1: of the first column is added to all term as indicated.

The Babylonians are thought to have been aware of the 'rule of the right angle triangle' as indicated in Figure 2. This is where for integer values of p and q, the sides of a right angle triangle are given by 2pq, p^2-q^2 and p^2+q^2 and relate to sides b, a and c. This would be the basis of what was later to become known as Pythagoras Theorem. The values in the second and third columns can be interpreted as p^2-q^2 and p^2+q^2 for relevant values of p and q. This holds except for line 11. So far so good. What about the first column?

Figure 2: The Rule of the Right Angle Triangle as known to the Babylonians.



This corresponds to the ratio $(c/b)^2$ if we use the number in column one as a fraction and add value one as indicated in the table. The ratio (c/b) corresponds to the trigonometric function of secant or (1/COS) of the angle at vertex A on the triangle with reference to Figure 2. The table would appear to be that of the square of secants for angles between 45 and 30 degrees. The steps in angles are provided by the set of triangles for which the 'rule of the right angle triangle' holds. There is a missing line in this sequence between lines 11 and 12. It can never be proved, however, that this was the intended use of the table.

The values of angles and of term $(c/b)^2$ are often reported in books, papers and on the Internet. It is obvious, however, that care is not being taken in ensuring sufficient resolution in undertaking the calculations to interpret the table.

The real surprise of the Plimpton 322 tablet is that the values of the fraction in column one are expressed as fully resolved to exact base sixty fractions. There is no point in calculating the value of a given fraction to see if it equals the value derived from the computer from calculating the value $(c/b)^2$. The 4000 year old fraction is exactly correct on paper - since the values are base 60 terminating fractions. What we do see, however, for six lines in the table is that double precision arithmetic, at 15 digits numeric resolution, demonstrates a finite difference between the value of the fraction and the calculated value $(c/b)^2$. This is, however, a limitation of the numeric representation of our modern digital computers and not of Babylonian mathematics. For line three we see, for example, that the Babylonians were able to express the ratio (343768681/182250000) as an exact series of base sixty fractions.

(p,q)	Difference p^2-q^2	Sum p^2+q^2	Index
12,5	119	169	1
64,27	3367	4825	2
75,32	4601	6649	3
125,54	12709	18541	4
9,4	65	97	5
20,9	319	481	6
54,25	2291	3541	7
32,15	799	1249	8
25,12	481	769	9
81,40	4961	8161	10
,	45	75	11
48,25	1679	2929	12
15,8	161	289	13
50,27	1771	3229	14
9,5	56	106	15

Table 5: Integer values of sum and difference for Plimpton 322 for specific values of p and q.

If we consider this issue further, we see that the denominator of the expression $(c/b)^2$ has a value which is written as $4p^2q^2$ can be expressed as a sequence of base sixty terminating fractions. Thus for index line seven, for example, the denominator can be expressed as 2x2x3x3x3x2x3x3x3x2x5x5x5x5. The Babylonians apparently did not like irrational numbers.

For entry seven for example, a calculated value of the term $(c/b)^2$ is exactly equal to the corresponding base 60 fraction. If we change a least significant digit in the fractional expression (i.e. 01:43:11:56:28:26:41) then the value of the difference is expressed as $2.2\ e^{-16}$. If we used single precision real numbers we would not be able to show the real accuracy of the Babylonian fractional values. Your typical pocket calculator would be of limited value.

The short programme EXAMPLE2, written in QuickBASIC, indicates how inputs of values in column 3 (sum) and column 2 (difference) yields parameters of the triangle and a value of $(c/b)^2$ expressed in base sixty notation.

```
10 REM Babylonian Fractions EXAMPLE2
20 REM look at tablet Plimpton 322
30 PRINT "Input SUM of squares term (0 exits)"
40 INPUT sum: IF sum = 0 THEN STOP
50 PRINT "Input DIFFERENCE of squares term"
60 INPUT dif: IF dif > sum THEN GOTO 30
70 p# = ((sum + dif) / 2) ^ .5
80 q# = ((sum - dif) / 2) ^ .5
90 PRINT "p = "; p#
100 PRINT "q = "; q#
110 PRINT ""
120 a# = p# * p# - q# * q#
130 PRINT " a = "; a#
140 b# = 2 * p# * q#
150 PRINT " b(2pq) = "; b#
160 c# = p# * p# + q# * q#
170 PRINT " c = "; c#
180 x# = (c# * c#) / (b# * b#)
190 PRINT "(c/b)^2 = ";
200 PRINT USING "#.##################^^^^"; x#
210 a = ATN(a# / b#)
220 PRINT " angle A = ", a * 57.2958
230 PRINT "base 60 fraction:"
240 num# = sum * sum
250 den# = b# * b#
260 cyc = 10
270 FOR jj = 1 TO cyc
280 dig(jj) = INT(num# / den#)
290 PRINT dig(jj);
300 PRINT ":";
310 num# = num# - den# * dig(jj)
320 num# = num# * 60
330 NEXT jj
340 PRINT : PRINT
350 GOTO 20
```

The discovery of the tablet indicates how a single artefact can drastically revise the appreciation of the mathematical skills of ancient cultures.

Numeric Scale and Precision

When it comes to scale and precision of numbers, we are basically determining a numeric representation with calculators or computers.

Base 60 Notation	Fraction	Value	
0:1	(1/60)	1.666666666666667	E-2
0:0:1	(1/602)	2.777777777777778	E-4
0:0:0:1	(1/603)	4.629629629629630	E-6
0:0:0:0:1	(1/604)	7.716049382716049	E-8
0:0:0:0:0:1	(1/605)	1.286008230452675	E-9
0:0:0:0:0:0:1	(1/606)	2.143347050754458	E-11
0:0:0:0:0:0:0:1	(1/607)	3.572245084590763	E-13
0:0:0:0:0:0:0:0:1	(1/608)	5.953741807651272	E-15
0:0:0:0:0:0:0:0:0:1	(1/609)	9.922903012752122	E-17

Table 6: Value of base 60 fractions expressed in decimal format to double precision - 15 digits.

The two aspects involved are digit precision and range of exponent. The range of exponent is not really the issue rather it is the precision of digits in which decimal numbers are expressed. Single precision calculations (typically 7 digit decimal resolution) are seen as especially weak when set against the demonstrated Babylonian resolution of (1/607) used in the Plimpton 322 tablet and where Babylonians fractions are more typically expressed as exact derivations due to the greater number of factors of base 60.

Great care is therefore required in trying to analyse 'old base 60' numbers since the assumptions of unlimited accuracy in base 10 arithmetic algorithms in digital computers do not always hold. It is a little disconcerting that the ancient Babylonians could write down fractional values of numbers with a precision that many of our modern calculating devices cannot match. Is this progress or were the Babylonians just fussy about numbers?

One thing to be careful of in computer calculations, when using standard real number representation (short) the significance is represented to only about 7 digits though numbers can be expressed with 15 digits of resolution.

Egyptian Representations

The earliest Egyptian process of writing numerals used essentially repetition of symbols for one, ten, hundred, thousand, ten thousand, hundred thousand and million in a hieroglyphic system. The representation of such numbers is indicated in Figure 3. The hieroglyphs were written from right to left, and in this way it didn't matter which way the symbols were written, there was no place significance as used today in Arabic numbers or in the Sumerian/Babylonian system. There must have been a temptation, however, to write a single character for 1000 as an approximation rather than write a true value of 999 which would have required 27 characters.

In time this system of repetition of basic units of numbers developed to one of encipherment - the process of assignment of an individual symbol to a more complex number. Such hieratic forms were developed for numbers up to 1000.

The Rhind papyrus was purchased by the Scottish Egyptologist A. Henry Rhind in Luxor in 1858 and is shown in Photo 2. The papyrus is about 6m long and 30cm wide and is thought to be written by the scribe Ahmes as a copy of a document some 2000 years older. The papyrus itself is something like a 'how to' guide to basic arithmetic.

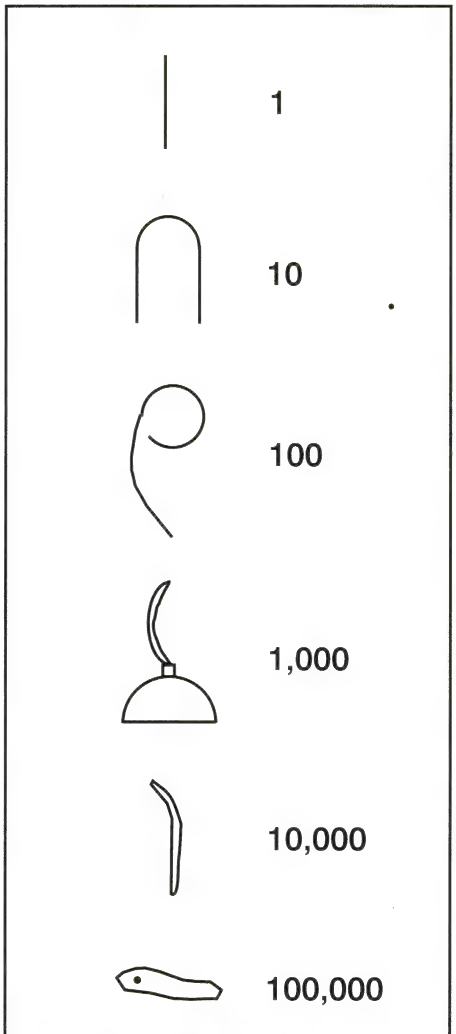


Figure 3: Representation of Egyptian Numeric notation (hieroglyphics)

Another famous papyrus, the Moscow papyrus is shown in Photo 3.Taking a stroll down history, it is instructive to see how the Ancient Egyptians would multiply 43 and 67.

The series of terms would be written of equivalent value:-

1	67	*
2	134	*
4	268	
8	536	*
16	1072	
32	2144	*

= 2881

where the indicated terms (1, 2, 8 and 32) which add up to 43 are totalled. This looks very much binary arithmetic - multiplication by two. Its equivalent in binary representation is moving a binary number one place to the left. This concept was extended to dividing numbers - say 67 by 8.

A column of values would be written to represent:-

/8	1	*
/4	2	*
/2	4	
1	8	
2	16	
4	32	
8	64	*

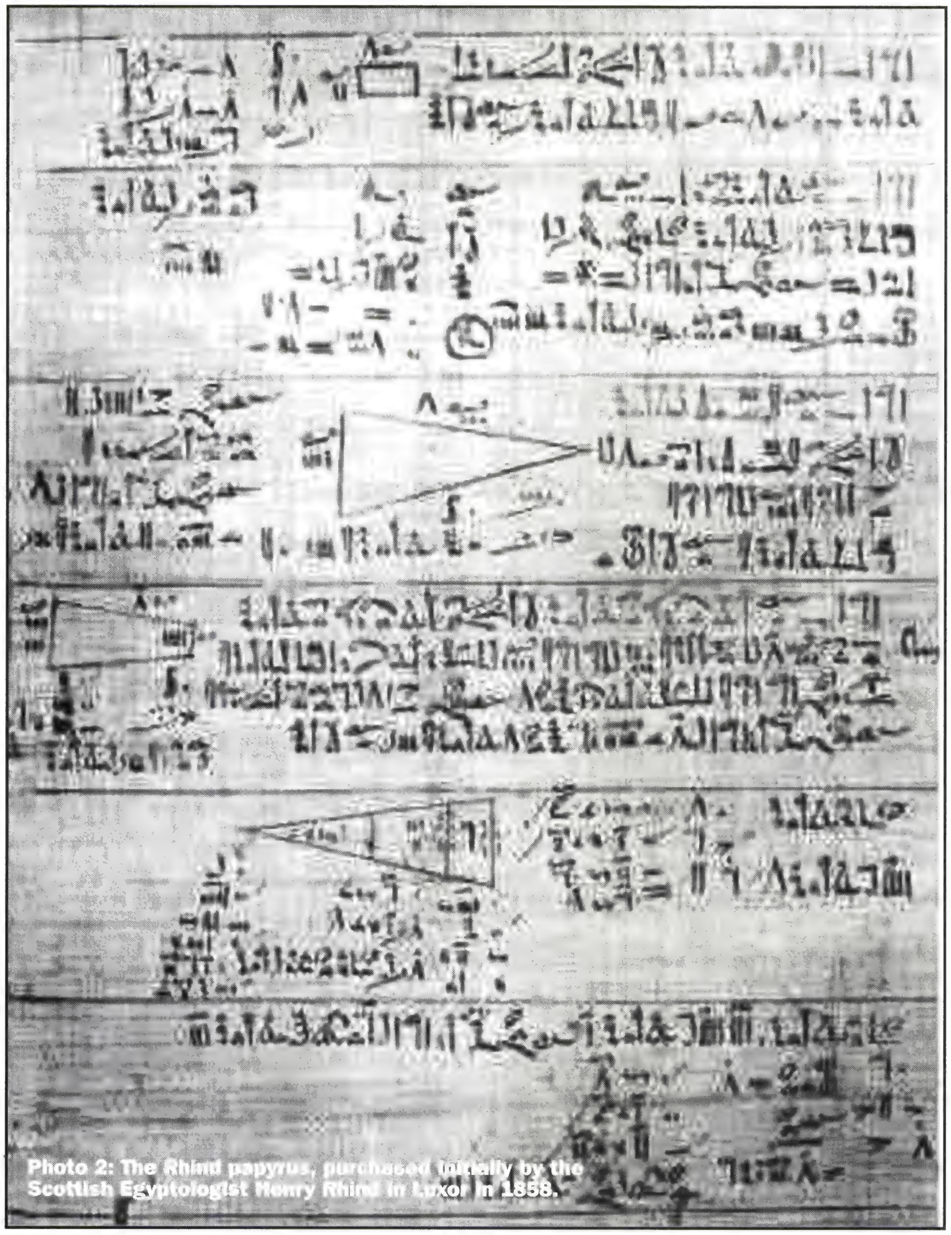


Photo 2: The Rhind papyrus, purchased initially by the Scottish Egyptologist Henry Rhind in Luxor in 1858.

where /8 represents the fraction 1/8.

The numerator value of 67 is given by adding 64, 2 and 1 in the right column which is equivalent to 8 + 1/4 + 1/8.

If we want to divide 171 by 12, this method provides a solution but indicates a problem:-

/8		
/4	3	*
/2	6	
1	12	
2	24	*
4	48	*
8	96	*

By luck the division yields a value 14 +1/4. If the number had been 174 we would have to have added additional fraction values that would reduce to unit values as indicated:-

/12	1	*
/6	2	*
/4	3	*
/2	6	
1	12	
2	24	*
4	48	*
8	96	*

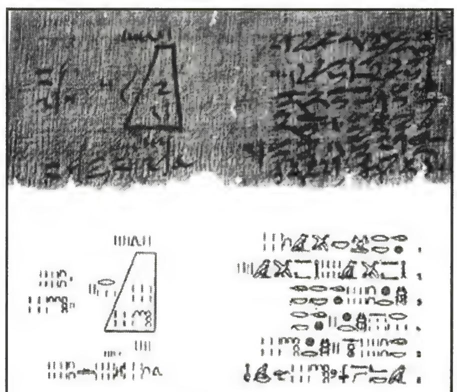


Photo 3: The Moscow papyrus.

This gives a value of 14 + 1/4 + 1/6 + 1/12.

Using these basic techniques the Ancient Egyptians could correctly calculate expressions as complex as:-
2/3 * (16 + 1/56 + 1/679 + 1/776) x (1 + 1/2 + 1/7)

Egyptian Fractions

The concept of division provides a good introduction to the world of Egyptian Fractions. We are perhaps familiar with the expression :-

$1/R = 1/R1 + 1/R2$
as describing Resistors R1 and R2 in parallel.

The Egyptians would express a given fraction as the sum of a series of unique fractions with unit numerator. Thus the fraction 2/7 would be written as 1/4 + 1/28. As a niche in the study of ancient number systems, there are groups who still study the significance of the use of Egyptian fractions in solving problems in modern number theory.

Ancient Greece and Rome

The Ancient Greek system essentially used 27 characters to describe numbers up to 9999. The initial sequence of characters, based on the 24 letters of the Greek alphabet with some exotic additions described 1,2,....9, 10, 20, ...90, 100, 200 to 900. The comma was used to indicate a character describing 1000, 2000 etc. A dot could be used to indicate a scale factor of 10,000. This method of recording data was, however, of not much use in undertaking serious arithmetic.

The Greeks, however, tended to focus more on abstract mathematical thought than the very practical arithmetic of the Egyptians. The greater part, however, of Western mathematics has grown out of the legacy of mathematics left to use by the Greeks.

The Roman system had been derived from the Etruscans, with letters I, V, X, L, C, D and M representing 1, 5, 10, 50, 100, 500, 1000. Symbols had also been used to describe 5000, 10000, 50000 and 100000. The significance of such a system is that it was probably used as a means of recording numbers rather than undertaking the calculations. Most of the hard mental effort of calculating was probably done using counting boards where counters would signify numeric values.

Hindu Arabic Numerals

The most significant development, in number representation, was that of Hindu-Arabic numerals.

The core language of Sanskrit in use around 2500 BC had incorporated within it a basic decimal system with terms for numbers 1 to 9, 10, 100 and higher powers of ten. By about the 3rd century BC the set of Brahmi numerals as indicated in table 7 were in common usage.

UNITS	1	2	3	4	5	6	7	8	9
TENS	10	20	30	40	50	60	70	80	90
HUNDREDS	100	2x100	3x100	4x100	5x100	6x100	7x100	8x100	9x100
THOUSAND	1000	2x1000....							

Table 7: Description of the Brahmi number system in use around 300 BC.

Numeric representation here used a 'composite' approach. Individual digits were defined and in addition individual symbols for the sequence 10, 20, 30,...90. For counting above 100, two symbols, one for 100 and one for 1000 were introduced and used with place significance so that in this approach a number could be written (2) x (1000) + (30) + 6 for 2036 where if no term for hundreds was included a zero is implied. Thus numbers up to 9999 could be represented from a selection of only 20 different symbols. This system further developed and around 570 AD the 'Arabic' system of numbers was in use within India.

The migration of this system out of India probably took place around 662 AD and with it first being assimilated by the Arabs who had no number system of their own. It is known that in 773 AD an Indian brought some writings in Astronomy by Brahmagupta to the Court of Caliph Al-Mansur in Bahgdad. After having studied this work it was Al-Khwarizmi the mathematician who first described the use of the Hindu system, and subsequently Europeans began to access this information through Latin translations. The oldest mathematical European manuscript is the Codex Vigilanus from about 976 AD which contains the digits 1 through 9 but no zero.

Spain with its Moorish occupation at this period, was an interface between cultures which allowed the passage of the Hindu system into Western culture. The most important books in Latin describing the Arabic-Hindu system appeared in the 12th century.

We take our number system very much for granted with its implicit use of place notation. It was a very difficult leap to take in those times and in fact there are many indications that there was a period when Roman and Arabic were all mixed up within the representation of a single number.

The close trading links between the states of Italy and the Arabs around this period was a further factor which increased the acceptance of the Arabic number system. Leonardo of Pisa (also known as Finonacci) around 1200 AD was to study the Arabic system extensively and conclude that it was a superior system. It took some 500 years, however, from the first awareness of the system within Europe to its firm acceptance. The rest, as it were, is history.

Mayan Numbers

The Mayan culture, unlike that of the Greek, Egyptian and Babylonian cultures was a living culture when it was first encountered by the Conquistadors around 1524 AD. The toppling of its power structure and wholesale and deliberate destruction of its culture caused a great deal of value to be lost. Also, while archaeological efforts have been primarily directed towards Indo-European cultures, serious study of Mayan culture and its system of counting and mathematics is fairly recent.

The number system of the Mayas was essentially vigesimal - to base twenty. Figure 4 depicts the basic representation of numbers between 0 and 19. This was achieved by using only three symbols - for zero, one (a dot) and five (a horizontal bar). Numbers were typically written vertically and with place notation. Zero was specifically designated with a unique symbol. Two separate 'type' of numbers,

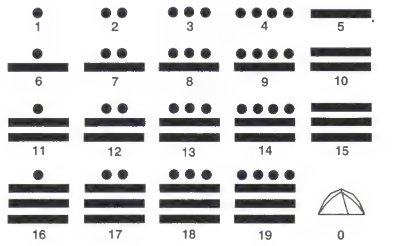


Figure 4: Representation of the Mayan numbers 0 through 19.

however, can be identified - the mathematical count and the calendric or 'long count as indicated in Table 8.

Digit Position	Mathematical Count	Calendric Count
2	20 (20)	20 (20)
3	400 (20x20)	360 (18x20)
4	8000 (20x20x20)	7200 (18x20x20)
5	160,000 (20x20x20x20)	144,000 (18x20x20x20)

Table 8: Place significance values of Mayan count systems

We can represent numbers the decimal numbers 64404,146849 and 122074 in the Mayan mathematical count representation as indicated in Figure 5.

The Mayan system of numbers provided a convenient means for recording numbers for everyday practical reckoning. Adding numbers was especially simple and straightforward. A series of glyphs were also associated with the numbers 0 to 19.

The most important use of such numbers systems, in particular the 'long count' system was in calendrical/astronomical observations. As studies of archaeological records progress, deeper insight is being provided into the processes of such astronomical observations. Observation of the planet Venus was of particular importance, and exploits of war and sacrifice were apparently linked to Venus's path in the sky. Such observations identified a Venus/Earth cycle of 584 days and with a longer cycle of 2922 days involving the position of the earth, Venus and the stars.

It has been suggested that the Mayan 'long count' 360,7200,14400,2880000 etc. could be related to the Platonic Year, the period of 25920 years for a complete precession of the equinoxes. Some researchers are trying to link 'important numbers' of the Maya culture, Ancient Egypt and Babylon to indicate a shared or common basis for systems of numeric representation.

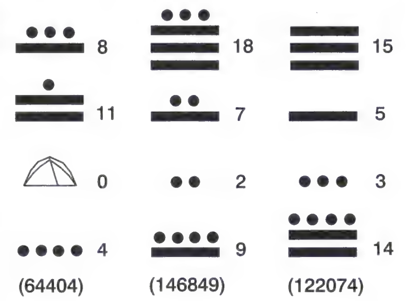


Figure 5: Representation of numbers in Mayan notation (mathematical count).

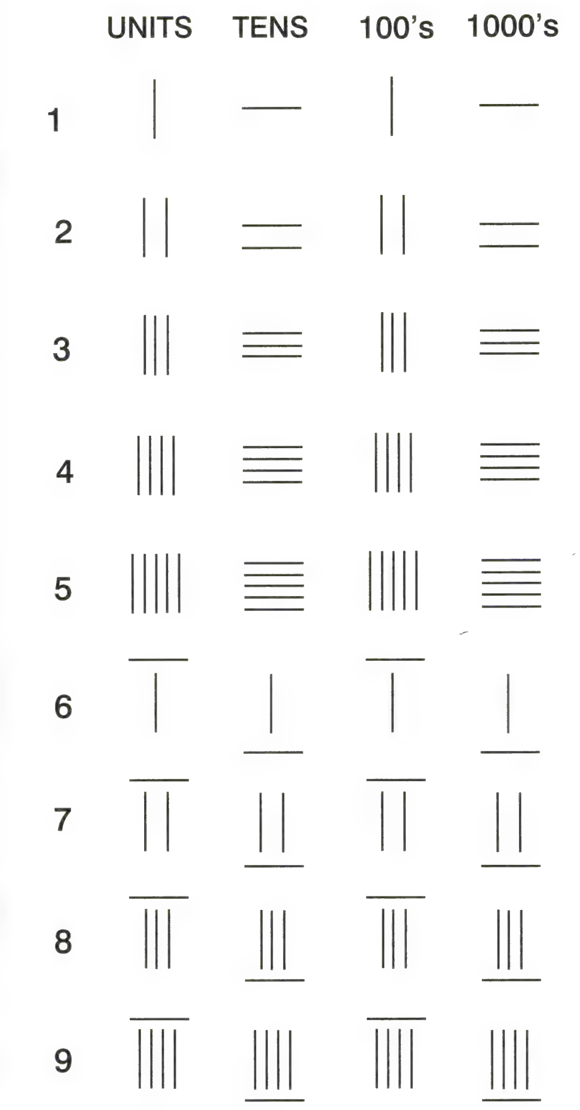


Figure 6: System of Chinese reckoning blocks.

Chinese Number Systems

There is generally less information available on Chinese number systems. The basic division of knowledge is between the reckoning devices such as count boards and the written text of numeric representation. It is likely that these two activities were kept relatively separate between the marketplace and the intellectual/literate class of China and this division is also characteristic of many parallel cultures. One approach used in counting was to use so called reckoning blocks, as indicated in Figure 6, where these can be considered numeric tokens used to represent numbers. The Chinese numerical notation was derived from these reckoning blocks but did not facilitate calculation. Its role would have been possibly comparable with Roman letters used to record an arithmetic result. The basic arithmetic system was based on factors of ten. In China the modern abacus was developed from about 900 AD. In the orient, the modern abacus is still used to great effect in undertaking highly complex calculations.

Proficiency tests in Japan during the 1960's, for example, required the adding of ten sets of 15 numbers, each up to 10 digits long, in 10 minutes. In the use of the abacus, properties of numbers such as symmetry and complimentary values are harnessed to speed up number processing. This aspect, however, is almost completely lacking when we use computers and calculators to do the reckoning.

The Role of the Abacus in Counting Systems

It is possible to distinguish two main divisions of number use and development. One level related to commercial, market place transactions while the other related to more abstract mathematical thought and astronomical reckoning. We are familiar with the abacus as beads on columns, but counting by means of counters such as pebbles can be traced to the very dawn of history. The word abacus in fact is derived from 'abax' meaning flat surface. Even the word calculate is derived from the Latin 'calculus' meaning pebble.

Pythagoras is credited with introducing the abacus into Greece which led to its subsequent uptake throughout the Roman empire. But it is not readily appreciated just how strong an influence the abacus/counting board would have on western culture. A typical Roman counting board is indicated in Figure 7 and displays the number 2862.

Counters cast between M and C (1000 and 100) would imply a value 500 and with 50 and 5 being also implied in mid lines. Numbers to be added would be represented side-by-side and the counters amalgamated from the units upwards. There were rules for subtracting multiplying and dividing. When a value was determined, it was recorded as the number of counters on the lines of the counting board. The real arithmetic was not done by manipulating the written number.

Western Europe used this same method of counting board arithmetic widely until the 16th century though it was the French Revolution that would ultimately rule it out of favour. The term Chancellor of the Exchequer relates to the use of a large calculating table - the exchequer - which probably went out of use around 1783. The counters used to signify value in such a system were known as jettons. It is only relatively recently, therefore, that 'counting house' methods have been replaced by book work using Arabic numerals.

Summary

It has taken quite some time to credit ancient cultures with sophisticated numeric skills. The example referenced in the Plimpton 322 tablet, when interpreted with double precision arithmetic shows that the Babylonians could compute with this accuracy or better around 4000 years ago. It is likely, however, that just as the last 150 years has witnessed a development in understanding such systems of records, even more rapid progress can be made using modern computers to decipher such records with caution and an ample measure of respect.

There is great curiosity, however, in revisiting ancient concepts of numbers since mathematics is itself a form of universal language which can pass from one culture to another. So in our highly numerate age, it is surely useful to obtain a sense of perspective in the way in which we use numbers now and how also previous cultures used them. This could even add a level of interest in acquiring numeracy skills.

Further Reading

The Book of Numbers. John H. Conway and Richard K. Guy, Springer - Verlag, 1996.
The History of the Abacus, J.M. Pullan, Hutchinson & Co., 1968.
Advanced Abacus: Japanese Theory and Practice, Takashi Kojima, Tut Books, 1980.

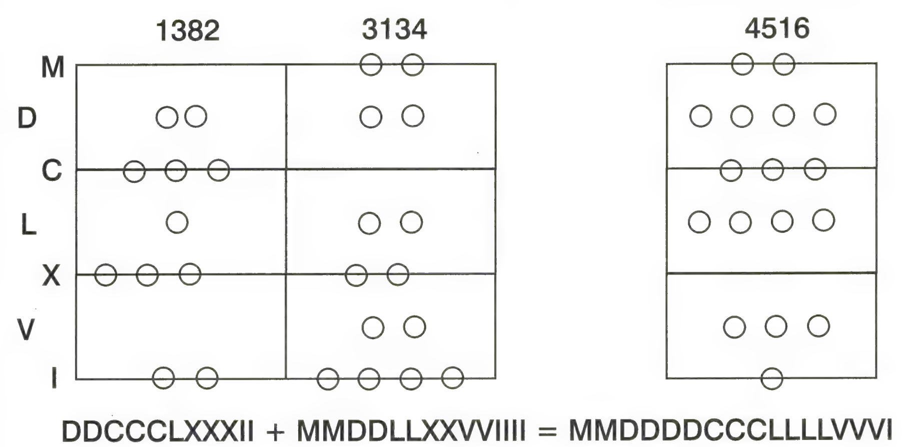


Figure 7: Roman style abacus with example of adding 1382 and 3134.