Structural properties of bolted joints

Engineers need to understand the behavior of bolts, flanges, seals, and gaskets to design reliable connections.

Sound designs of bolted joints rely on a good grasp of their structural behavior, and structural properties of preloaded bolted joints — including the bolt load-sharing ratio, separation load ratio, and joint stiffness — depend only on the stiffnesses of the bolt and clamped members. Nonetheless, bolted joints are rather complicated. Here's how engineers can approximate bolt and clamped-member stiffnesses and derive structural parameters useful for design.

Bolts and clamped members can be modeled as bars connected in series for stiffness calculations. The stiffness of a bar with uniform cross-sectional area A and length L is related to its modulus of elasticity $E = \sigma/\varepsilon$, the ratio of stress and strain. If a force F elongates a bar by δ, then the bar stiffness is:

$$k = \frac{F}{\delta} = \frac{A(F/A)}{L(\delta/L)} = \frac{A\sigma}{L\varepsilon} = \frac{AE}{L}.$$

Bolt stiffness

Bolts consist of shank and threaded sections, as shown. Stiffness of the shank is:

$$k_s = \frac{A_s E}{L_s}$$

where $A_s = \pi d^2/4$.

The threaded-section stiffness is:

$$k_t = \frac{A_t E}{L_t}$$

where A_t = tensile-stress area of the screw.

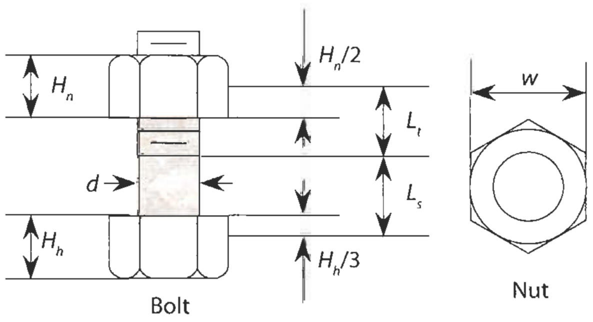
The tensile-stress area of the screw can be found in handbooks and textbooks. Or calculate it based on the equation from ASME Standard B1.1-1989 (revision of ANSI B1.1-1982) for unified inch screw threads:

$$A_t = \pi\left[\frac{d_{2b}}{2} - \frac{3H}{16}\right]^2 = \frac{\pi}{4}\left[D - \frac{9\sqrt{3}}{16}\frac{1}{1/P}\right]^2 = \frac{\pi}{4}\left[D - \frac{0.9743}{1/P}\right]^2$$

where D = basic major diameter. The basic pitch diameter $d_{2b} = D - 0.75\,H$; H = height of a sharp V-thread or fundamental triangle, $H = \sqrt{3}(P/2)$; and P = thread pitch. $1/P$ is the number of threads per inch.

Or determine the stress area of the screw based on the area of a circle with the average diameter, d_a, calculated

Typical bolt dimensions



Account for the head, threaded, and shank sections in bolt-stiffness calculations.

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Key points:

- Structural properties of bolted joints depend on bolt and clamped-member stiffnesses.
- Clamped members can be treated as a sleeve or hollow cylinder.

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from the basic pitch diameter, d_{2b}, and minimum minor diameter $d_{1m} = D - 1.5\,H$.

$$d_a = \frac{d_{2b} + d_{1m}}{2} = D - \frac{9}{8}H$$

$$A_t = \frac{\pi}{4}\left[d_a\right]^2 = \frac{\pi}{4}\left[D - \frac{9}{8}H\right]^2 = \frac{\pi}{4}\left[D - \frac{9\sqrt{3}}{16}P\right]^2$$

The effective length, L_t, of the threaded section is from the interior end to the midpoint of the nut height. This assumes that bolt tensile force decreases linearly from the seating surface to the free surface of the nut. The effective length of the shank, L_s, includes one-third of the bolt-head height. This assumes bolt force diminishes linearly along the height of the bolt head starting at the mean area of the seating surface. Because the widths across flats of the bolt head are typically $w = 1.5d$, the mean diameter of stress area in bolt head is $d_m = (d+w)/2$.

Equivalent stiffness of the head in terms of the shank is

$$\frac{(\pi/4)d^2E}{L_e} = \frac{(\pi/4)d_m^2E}{H_h/2}$$

or

$$\frac{L_e}{H_h} = \frac{d^2}{2d_m^2} = \frac{d^2}{2\left[(1+1.5)d/2\right]^2} = 0.32 \approx \frac{1}{3}$$

For studs with or without threaded inserts, include one-half of the thread-engagement length in the threaded-section length for stiffness calculations. These approximations are judgment calls. Other methods use $d/4$ in lieu of both $H_n/2$ and $H_h/3$. The series spring-connection method may be extended to bolts with multiple diameters.

Calculate total bolt stiffness, k_b, from series connection of the section stiffnesses:

$$\frac{1}{k_b} = \frac{1}{k_s} + \frac{1}{k_t}$$

or

$$k_b = \frac{1}{1/k_s + 1/k_t}.$$

Clamped members

Clamped members can be treated as a sleeve. The stress area of the hollow cylinder is:

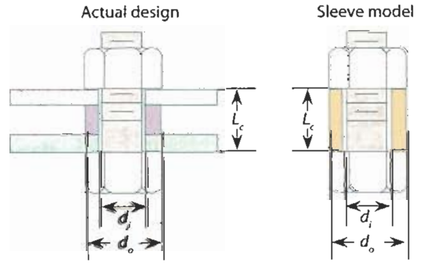
$$A_c = \pi(d_o^{\,2} - d_i^{\,2})/4$$

where d_o and d_i = outside and inside diameters, respectively. The clamped-member stiffness is:

$$k_c = A_cE_c/L_c.$$

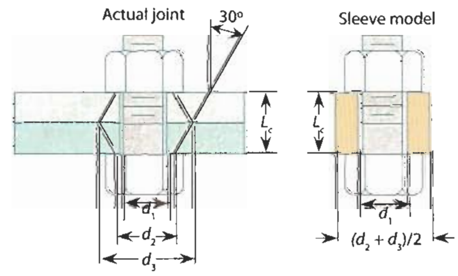
Calculate the clamped member stiffness of a solid structural connection using a double-cone load-path model, as

Clamped members with a spacer



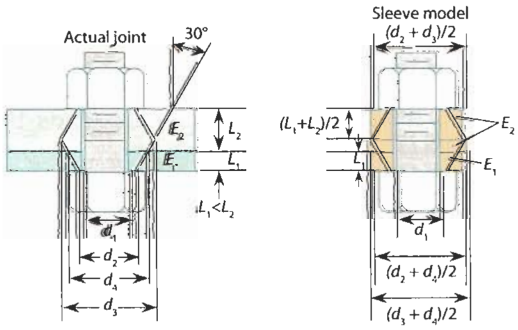
Clamped members can be modeled as a hollow sleeve.

Effective area of clamped members



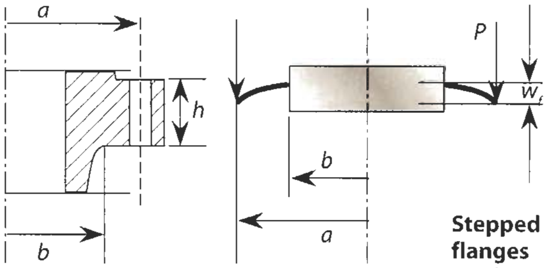
This model depicts clamped members of with the same thickness and modulus of elasticity.

Two materials and two thicknesses



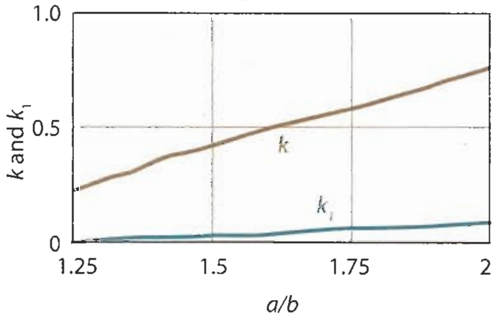
The double-cone models is modified to handle clamped members made of different materials and thicknesses.

Flange stiffness model



Stepped flanges can be modeled as a plate.

Bending constants



Use these constants for modeling flange-plate bending behavior.

Nomenclature

A_s = Shank cross-sectional area
A_t = Tensile stress area of the screw
D = Basic major diameter
d = Bolt diameter
d_m = Mean diameter of bolt-head stress area
d_{1m} = Minimum minor diameter
d_{2b} = Basic pitch diameter
E = Modulus of elasticity
H = Height of a sharp V-thread
H_n = Nut height
H_h = Bolt-head height
k = Stiffness
k_b = Total bolt stiffness
k_c = Clamped member stiffness
k_s = Bolt shank stiffness
k_t = Bolt threaded-section stiffness
L_c = Clamped member stiffness (thickness along bolt axis)
L_e = Equivalent length of head in terms of the shank
L_s = Shank length
L_t = Bolt threaded length
P = Thread pitch
w = Width across bolt-head flats
ε = Strain
σ = Stress

shown in the graphic, "Effective area of clamped members." A 30° cone angle is customarily used in industry. If clamped members have the same modulus of elasticity, the double cone shown is equivalent to the sleeve model.

The double-cone model can be modified to accommodate clamped members with different moduli of elasticity and thickness as shown in "Two materials and two thicknesses." The stiffness of clamped members is calculated from series connections of three hollow cylinders shown in the sleeve model, assuming $L_1 < L_2$.

Gaskets and seals

Some O-rings rely on internal pressure to entrap rubber in the groove for sealing. These seals may not be in the structural load path and can be ignored for stiffness calculations. In other cases an O-ring may dominate the compliance (inverse of stiffness) of clamped members. For O-rings with solid, hollow, or other cross sections, stiffness data should be translated to the clamped member stiffness for each bolt. For example, if it takes linear force f (force/length) to deform the seal by δ, and the joint has N bolts equally spaced on the bolt center circle with diameter D_c, then each bolt's share of the spring rate of the seal is: $k_s = \pi D_c f/N\delta$.

Flange stiffness

A pair of flat flanges with a gasket can be modeled as three hollow cylinders in a series connection. Stepped flanges with a gasket diameter substantially smaller than the bolt-center circle diameter are subject to bending. Calculate the stiffness of those flanges using the model shown in the accompanying graphic. The stress and deflection equations of this plate model and coefficients k and k_1 are based on data in Timoshenko (*Strength of Materials, Part II, Advanced Theory and Problems*, 3rd ed., Van Nostrand Reinhold Co, New York, 1958). For stress:

$$\sigma_m = k\frac{P}{h^2}$$

and deflection:

$$w_f = k_1\frac{Pa^2}{Eh^3}$$

Stiffness of the entire flange as a plate is:

$$K_f \equiv \frac{P}{w_f} = \frac{Eh^3}{k_1 a^2}.$$

The combined clamped member compliance is the sum of compliance of two flanges and one gasket:

$$\frac{1}{K_c} = \frac{1}{K_f} + \frac{1}{K_g} + \frac{1}{K_f}.$$

Stiffness of clamped members for one of the N bolts on the flanges is $k_c = K_c/N$.

In future installments of this series, we'll look at bolted-joint behavior under external load, loading-plane factors, the effects of preload, finite-element models, and reactions to internal loads. **MD**