Transformer Design for "Zero" Impedance Amplifiers

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A rigorous discussion of a practical method that can reduce weight and volume of an output transformer for zero-impedance output stages without increasing distortion.

TRANSFORMERS have been irksome to the designer of high-efficiency program amplifiers. He knows that higher efficiency yields a smaller power transformer. But the output transformer, as used in class AB and B operation, ordinarily presents him with rather severe problems, especially when high power, low distortion, and wide bandwidth are desired simultaneously.

Progress toward minimizing problems associated with the output transformer seems to take three major directions:

1. Eliminate the output transformer entirely. Apparently this procedure substitutes new problems for old.
2. Use special circuitry to overcome the "limitations" of the output transformer. MacIntosh[1] and Peterson,[2] using high-quality transformers of special design have achieved notable success.
3. Devise circuitry which overcomes transformer "limitations," using small and comparatively inexpensive output transformers. This article is concerned with a development in this category.

The "Zero-Impedance Transformer"

This writer's experience indicates that a comparatively small output transformer specifically designed for the "zero" impedance output stage[3,4] provides performance characteristics at least comparable with the "large" transformer designed for the conventional high-quality feedback amplifier.

The advantages this type of transformer affords when fed by a zero-impedance stage are as follows:

1. Small size. Typical weight reduction is approximately 40 per cent (See TABLE I). It appears feasible to design a high-quality output transformer only 1½ to 2½ times the size of a 60-cycle power transformer of the same power rating. The smallest size is obtained by using a B supply with perfect regulation and fixed bias.
2. Wider bandwidth.
3. Lower inter-primary leakage reactance—reduces "switching" transients.
4. Lower effective primary capacitance. High transformer input capacitance limits the amount of low-distortion high-frequency power from those class AB, and B_1 amplifiers that utilize large transformers with bifilar windings.
5. The foregoing advantages contribute to the feasibility of an economical high-performance wide-band Class B_1 amplifier employing a regulated B supply.

TABLE I

TYPICAL WEIGHT REDUCTION DATA

AUDIO POWER	STANDARD			ZERO IMPEDANCE			W_s/W_z	WEIGHT REDUCTION %
	LAMINATION	STACK	W_s (LBS)	LAMINATION	STACK	W_z (LBS)		
15	EI112	1¼	3.28	EI12	⅞	2.12	1.55	35%
35	EI125	1¾	5.69	EI125	1	3.31	1.72	42%
70	EI13	1¾	8.66	EI125	1½	5.4	1.6	38%

The limitations are:

1. This type of transformer appears to be limited to use in the "zero" impedance output stage.
2. The continued need for the type of elaborate winding schedule often used in high-quality large transformers.
3. The continued employment of the same high-quality core material as used in the traditionally large transformer.
4. While the cost is substantially less than that of the large transformer, its cost is higher than the small P.A.-type transformer which it may superficially resemble because of its smaller size.

Distortion

Low distortion has a decisive effect on the size of the output transformer. The equations pertaining to distortion, based on the equivalent low-frequency circuit (*Fig.* 1), follow:

Exciting current

$$I_e = I_C - jI_M \qquad (1)$$

where I_C = in-phase core loss current

and I_M = quadrature magnetizing current

total primary current

$$I_1 = I_e + I_2 \qquad (2)$$

$$= (I_C + I_2) - jI_M \qquad (3)$$

Primary EMF

$$E_p = \frac{I_M}{X_L} \qquad (4)$$

The voltage drop across $(R_G + R_1)$ is

$$I_1(R_G + R_1) = [(I_C + I_2) - jI_M](R_G + R_1) \qquad (5)$$

$$= [(I_C + I_2)(R_G + R_1)] - jI_M(R_G + R_1) \qquad (6)$$

Since I_M is the quadrature magnetizing current containing the distortion harmonics, it is clear that the distortion producing voltage drop $I_M(R_G + R_1)$ subtracted from the input voltage E_G produces a component of the voltage E_p, having a distortion term across the primary reactance X_L. If we assume I_M is *all* harmonics, then the maximum fractional distortion D_T appearing across the effective primary reactance is

$$D_T = \frac{I_M(R_G + R_1)}{E_G - I_1(R_G + R_1)} \qquad (7)$$

$$= \frac{I_M(R_G + R_1)}{E_p} = \frac{E_p}{X_L}\frac{(R_G + R_1)}{E_p}, \qquad (8)$$

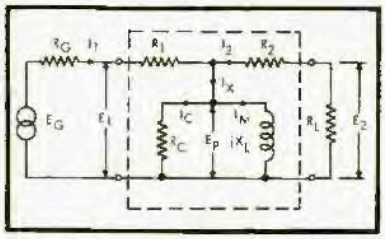

Fig. 1. Low-frequency equivalent circuit.

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[1] MacIntosh & Gow, "Description and analysis of a new 50-watt amplifier circuit." AUDIO ENGINEERING, Dec. 1949.

[2] A. P. Peterson, "A new push-pull amplifier circuit." *General Radio Experimenter*, Oct. 1951.

[3] J. Miller, "Combining positive and negative feedback," *Electronics*, March, 1950.

[4] C. A. Wilkins, Pat. Pending: "*Controlled Positive Feedback.*"

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or[5] $$D_T = (R_G + R_1)/X_L, \qquad (9)$$

and the distortion factor due to the transformer alone is

$$D_s = R_1/X_L \qquad (10)$$

The EMF, E_p, across the primary shunt reactance (with the distortion component), in turn causes the load current I_2 to flow through the secondary d.c. resistance and load resistance.

Since we expect to handle the distortion parameters in some detail, it is now desirable to establish the *exact* transfer ratio between input and output. If we solve each loop of the network (*Fig.* 1) using Kirchoff's Law we can write the exact equation:

$$\frac{E_1}{E_p} = 1 + \frac{R_1}{R_G} + \frac{R_1}{R} - j\frac{R_1}{X_L} \qquad (11)$$

where $R = R_2 + R_L$, and

$$\frac{E_p}{E_2} = 1 + \frac{R_2}{R_L} \qquad (12)$$

Multiplying these two equations and separating the real and quadrature terms,

$$\frac{E_1}{E_2} = \left(1 + \frac{R_2}{R_L}\right)\left(1 + \frac{R_1}{R_G} + \frac{R_1}{R} - j\frac{R_1}{X_L}\right) \qquad (13)$$

$$E_1/E_2 = (1 + R_2/R_L)(1 + R_1/R_G + R_1/R)\left(1 - j\frac{R_1}{X_L}\frac{1}{1 + \frac{R_1}{R_G} + \frac{R_1}{R}}\right) \qquad (14)$$

so that the transformer distortion factor (again assuming that the magnetizing current I_M is all harmonics) is the j term in Eq. (*14*),

$$D_s = \frac{R_1}{X_L}\left(\frac{1}{j + R_1/R_G + R_1/R}\right) \qquad (15)$$

or $$D_s = R_1/X_L \qquad (16)$$

What is especially significant is that if $R_G \ll R_1$ or better still if $R_G = 0$, then the *total* circuit distortion is determined by the transformer *alone*.

Size of the Conventional Transformer

Before attempting to reduce the size of the conventional output transformer let us see why it is larger than a power transformer of the same rating and at the same time derive an expression for its size in terms of copper losses and distortion.

Assuming $R_1 = R_2$, which is the usual case in a transformer designed along traditional lines, its loss factor $1/Q_s$ will be

$$1/Q_s = R_1/X_L + R_1/X_L = 2R_1/X_L. \qquad (18)$$

X_L is established after a number of considerations:

(a) The lowest frequency the amplifier is to pass in accordance with relative low-frequency response in

$$db = 20 \log \frac{1}{1 - jR_p/X_L}, \qquad (19)$$

where

$$1/R_p = [1/(R_G + R_1)] + 1/R; \qquad (20)$$

(b) The maximum permissible distortion. This is a function, at high power levels, of the flux density B_M in the primary winding. Although the maximum distortion factor is

$$D_T = (R_G + R_1)/X_L, \qquad (9)$$

the inductance, being a non-linear function depends on what the effective core permeability μ_e (see *Fig.* 2), is for a given B_M, which is, in turn, a function of the maximum voltage across the primary, in accordance with

$$E_p = 1.14\, NAB_M f\, 10^{-8} \qquad (22)$$

where N = primary turns,
A = effective core area,
and f = lowest frequency of operation.

Then to be certain that at low-frequency high power levels we do not exceed D_T, we must select a reasonable value for B_M to prevent X_L from falling below the minimum established by Eq. (*9*).

(c) In addition to the foregoing, the load line at high power also must be considered. At power levels near maximum, the effective load on the output tubes becomes reactive, the load line becomes "elliptical," and R_G increases due to phase shift in the feedback loop. The impedance Z of X_L in parallel with R_L reduces the effective plate load impedance to a value lower than the optimum (established at mid-frequency) required for maximum power transfer. The lower impedance increases[6] the voltage drop across R_G (power wasted), thereby decreasing the available low-distortion power at the lowest operating frequency.

It is the last two considerations which then traditionally have the most bearing on the size of X_L. Therefore X_L is established by Eq. (*9*) and the following relationship

$$X_L = d_s R_L, \qquad (23)$$

where d_s is usually an integer in the range of 3 to 6 (empirically determined) in order to keep the "ellipse as narrow as practicable" in the high-quality conventional output transformer.

The weight W_s of the transformer (which we will also refer to as the "standard") after combining Eqs. (*18*) and (*23*) may be expressed[7] by

$$W_s \propto Q_s^{3/2} \propto (d_s R_L/2R_1)^{3/2} \qquad (24)$$

Size Reduction Methods

Equation (*24*) suggests several ways of reducing the transformer size.

(a) Increase the copper losses by K; that is, to $2R_1K/R_L$. This however would produce more distortion than the conventional transformer in a zero-impedance circuit.

(b) Reduce the primary reactance by K; that is, make new reactance X_L/K. This would also produce more distortion from the small transformer than the large one used in a zero-impedance stage.

(c) Make the distortion of the small transformer the *same* as the large one in the zero-impedance stage. A special procedure for accomplishing this will be described.

A Special Design Procedure

Equation (*15*) tells us we would obtain zero distortion if R_1 could be reduced to zero. Although this is not phys-

[5] The writer is indebted to D. Wildfeuer of Arma Corporation for this simple formulation of maximum distortion.

[6] R. Lee, "*Electronic Transformers and Circuits.*" Wiley: 2d ed. 1955 pp. 158, 167.

[7] M.I.T. Staff, "*Magnetic Circuits and Transformers.*" Wiley: 1943, p. 228, wherein it is demonstrated that $Q \propto K_d^6$, where K_d is a factor by which the *linear* dimension of the transformer is changed. Q is therefore proportional to volume$^{2/3}$ (and weight$^{2/3}$).

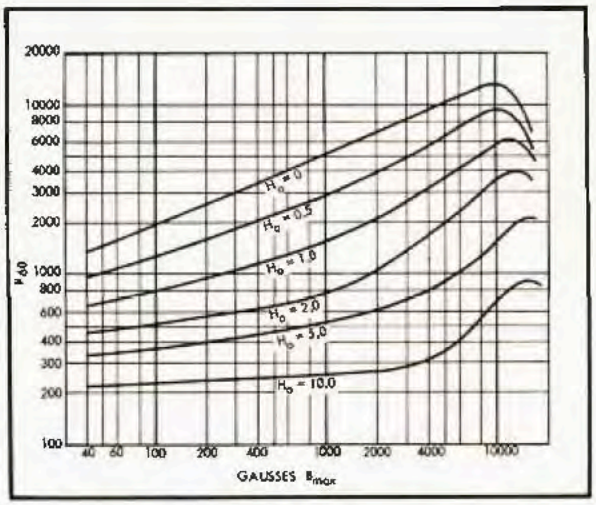

Fig. 2. Flux density vs. Mu. (Courtesy Magnetic Metals Co., Camden, N. J.)

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ically possible, we can make R_1 very low by decreasing it from its customary value $R_1 = R_2$, and increasing R_2. This is tantamount to putting most of the copper losses into the secondary. In transformer vernacular—run the secondary "hot" and the primary "cold." The transformer designer can now proceed as follows:

Keep the new transformer copper efficiency the same as that of the standard, but make new transformer primary d.c. resistance

$$R_A = R_1/K, \qquad (25)$$

where K is the factor representing the degree to which we expect to reduce the size of the standard, and new transformer secondary resistance R_B

$$R_B = R_1(2 - 1/K), \qquad (26)$$

so that:

$$R_A + R_B = R_1 + R_2 \qquad (27)$$

New transformer primary Q_A is the same as large

$$Q_A = Q_1 = X_A/R_A \qquad (28)$$

where

$$X_A = X_L/K \qquad (29)$$

Transformer secondary

$$Q_B = X_A/R_B, \qquad (30)$$

$$Q_B = (R_A Q_1)/[R_1(2 - 1/K)]. \qquad (31)$$

Now total new-transformer dissipation $1/Q_X$ is

$$1/Q_X = (1/Q_1) + K(2 - 1/K)/Q_1 \qquad (32)$$

$$= 2K/Q_1. \qquad (33)$$

$$1/Q_X = 2KR_1/(d_a R_L) \qquad (34)$$

And multiplying (34) by (24),

$$Q_s/Q_x = K. \qquad (35)$$

The weight ratio of large and small transformer is then

$$W_s/W_x = K^{3/2} \qquad (36)$$

Examine the consequences of this procedure. The total distortion with our standard transformer is

$$D_T = (R_G + R_1)/X_L \qquad (9)$$

and of our modified transformer is

$$D_o = \frac{R_1/K}{X_L/K} = R_1/X_L \qquad (37)$$

And comparing the distortion factors D_T and D_o by dividing (9) by (37) we have

$$D_T/D_o = 1 + (R_G/R_1) \qquad (38)$$

The intrinsic distortion of the standard transformer is the *same* as that of the smaller transformer. But when the standard is used with a substantial source resistance, it yields more distortion than our new smaller transformer in a zero-impedance output stage.

Here we have a technique that looks very promising. However, having ignored a number of important parameters, we should try to see what practical limits must be assigned to K by studying each neglected parameter:

1. Flux density and non-linear nature of X_L;
2. Power supply regulation at low frequencies;
3. Core loss;
4. Temperature rise of the transformer;
5. Bandwidth; and
6. The Class B_1 high-frequency response and regulation.

1. Flux Density and Distortion

(a) When X_L is reduced by a factor of K, the flux density is generally increased (see Eq. 22), so it is essential to insure that μ_e in the equation

$$L = 3.19N^2 A\mu_e 10^{-8}/l \qquad (39)$$

where l = length of magnetic path does not fall below the value needed to maintain $L_o = L_s/K$.

(b) It is also necessary now to reconsider the previous formulation for distortion which was based on the supposition that *all* of the magnetizing current was harmonic. Because, as Partridge[8] has shown, the percentage of third and fifth harmonics in the magnetizing current is a function of the operating flux density, Table II summarizes such data.

TABLE II

Typical Silicon-steel Magnetizing Current Harmonic Components with Zero-Impedance Source[9]

B_M Gauss	Percentage of 3rd Harmonic	Percentage of 5th Harmonic
100	4	1
500	7	1.5
1,000	9	2.0
3,000	15	2.5
5,000	20	3.0
10,000	30	5.0

On the basis of this data we shall define K_d as the factor by which the distortion increases when operating at B_o (due to size reduction) rather than B_M (the flux density of the standard transformer). Then

$$D_o = \left(\frac{R_1/K}{X_L/K}\right) K_d = K_d R_1/X_L \qquad (40)$$

and

$$\frac{D_T}{D_o} = \frac{(R_G + R_1)X_L}{(R_1/X_L)\ K_d} = \left(1 + \frac{R_G}{R_1}\right)\frac{1}{K_d} \qquad (41)$$

If, for example $B_M = 5000$ gauss and $B_o = 10{,}000$ gauss then $K_d = 0.3/0.2 = 1.5$ (neglecting fifth harmonics). Equation (41) is therefore a more accurate version of Eq. (38).

For help in designing the new transformer, Table III lists likely values of K_d and the corresponding values of R_G/R_1 and $D_T D_o$.

[8] N. Partridge, "Harmonic Distortion in a. f. transformers," *Wireless Engr.* Sept.–Nov., 1942.

[9] R. Lee, *op. cit*, p. 163.

TABLE III

RELATIVE DISTORTION

D_T/D_o	R_G/R_1	K_d
0.75	0.5	2
1.0	0.5	1.5
1.0	1	2
1.33	1	1.5
1.5	2	2
2.0	2	1.5

Values of 1.5 and 2 for K_d are deliberately chosen to represent the probable maximum distortion increase if the flux density doubles due to a choice of $K \cong 2$. Table III indicates that the small transformer produces the same or less distortion than the large, provided K (and thereby K_d) is not made too large.

(c) Since there is usually some degree of output tube unbalance, polarizing d.c. in the primary has two important effects:

(1) the effective μ_e is decreased, and is now obtained from the family of curves in *Fig.* 2 after estimating the ampere-turns per inch, H:

$$H = NI/l \qquad (42)$$

where N = primary turns
I = polarizing d.c. current
l = length of magnetic path, inches

(2) the magnetizing current now contains *even* harmonics as well as odd. It is therefore desirable when reducing the size of the standard transformer that H be kept the same (or smaller). This can be easily accomplished in practice.

2. Power Supply Regulation

Power supply regulation is quite important to the *high-level* low-frequency performance.

(a) In *Fig.* 3 neglect R_G (to be studied later), and R_1 (since $R_1 \ll R_L$). First assume perfect power supply regulation so that the output tubes must furnish the following VA:

$$VA = \frac{E_p^2}{Z} \qquad (43)$$

$$VA = \frac{E_p^2}{R_L} - j\frac{E_p^2}{R_L}\frac{1}{d} \qquad (44)$$

$$VA = P_o - jP_o/d \qquad (45)$$

$$VA = P_o(1 - j/d) \qquad (46)$$

where P_o = real watts audio

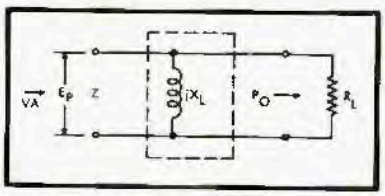

Fig. 3. Low-frequency impedance.

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Equation (45) tells us that at high power levels P_o/d reactive *VARS* will be supplied to the magnetic core. Since the *VA* furnished by the output comes ultimately from the plate supply, it is apparent that distortion is higher at high power levels when using an unregulated power supply.

We can now attempt to establish suitable criteria for $d_o = X_L/(KR_L)$.

The VA ratio between small and large transformers is

$$\frac{VA_x}{VA_s} = \frac{1 - jK/d_x}{1 - j/d_s} = \frac{\sqrt{1 + (K/d_x)^2}}{\sqrt{1 + (1/d_x)^2}} \qquad (47)$$

If we assume certain values of d, we can tabulate (Table IV) the *VA* increase

TABLE IV
VA INCREASE

d_x	d_o	$M = VA_x/VA_s$	10 log M, db
5	2.5	1.055	0.232
3	1.5	1.11	0.45
2	1.0	1.27	1.04
1	0.5	1.59	2.02

demanded of the power supply at high-level low frequencies.

If, for example $d_s = 3$ at lowest useful frequency and is reduced by $K = 2$ (a weight saving of 65 per cent) only 11 per cent more *VA* is demanded from the power supply at that frequency. Under these conditions the measured distortion in the zero impedance circuit at the lowest frequency is found to be unusually low, and *easily comparable* to the distortion figure for the large transformer in a conventional feedback amplifier.

(b) When $R_G = 0$, but the power supply is *not* perfectly regulated, experience indicates that a quite sizeable but less dramatic size-reduction is feasible. Examination of Eq. (41) has suggested to the writer a more modest value of K in this case. Experience has shown that with a choice of $K = 1.4$, a weight saving of about 40 per cent is obtainable with a distortion figure *equal to or lower than* that of the large transformer in an unregulated conventional feedback amplifier.

3. Core Loss

Analysis of the effect of increasing core loss (decreasing R_C) when $K = 2$ indicates a negligibly small change in high-level low-frequency performance, so core loss may safely be ignored except perhaps when $K \gg 2$. This would be the case where we succeeded in reducing the output transformer to the size of a 60-cps power transformer of the same power rating.

4. Temperature Rise

Since our (low-distortion) conventional output transformer is much larger than its equivalent power transformer of the same rating, it runs "cool," that is its temperature rise is in the approximate range of 10°–20° C.

If we use an approximate equation for temperature rise[10]

$$\theta_o = cP_l/w^{2/3} \qquad (48)$$

where θ_o = temperature rise, °C.

c = a constant

P_l = losses

w = weight

The relative temperature rise of the new transformer will be

$$\frac{\theta_x}{\theta_s} = \left(\frac{W_x}{W_s}\right)^{-2/3} = \left(\frac{1}{K^{3/2}}\right)^{-2/3} = K \qquad (49)$$

So that our new transformer rise for $K = 2$ will be in the range of 20° to 40° C. A temperature rise of 40° to 55° C is permitted for military and commercial transformers, respectively, with Class A (105° C final temperature) insulation. On a thermal basis, therefore, the *conventional transformer may be reduced in weight by at least 65 per cent.*

5. Bandwidth

If we neglect the various winding capacitances and shunt core loss R_C, the following bandwidth equation[11] is informative (see *Fig.* 4)

$$B_T = \frac{f_h}{f_l} = \frac{R_s}{R_p}\frac{L_p}{L_s} \qquad (50)$$

where f_h = 3db down high-frequency ($X_s = R_s$)

f_l = 3db down low-frequency ($R_p = X_L$)

$$R_s = R_G + R_1 + R_2 + R_L \qquad (51)$$

$$1/R_p = 1/(R_G + R_1) + 1/(R_2 + R_L) \qquad (20)$$

L_s = total leakage inductance,

X_s = leakage reactance

L_p = primary shunt inductance

The relative high-frequency response is

$$20 \log \frac{1}{1 + jX_s/R_s} \qquad (52)$$

and

$$L_s = 3.2N^2 \frac{M}{b}\left(d + \frac{a}{3}\right) 10^{-8} \qquad (53)$$

where M = coil mean length turn

b = winding length

d = insulation between windings

a = total copper depth of windings

Substituting the equations for L_p and L_s, into (50) we get

$$B_T = \frac{f_h}{f_l} = C_h\mu_e \frac{R_s}{R_p} \qquad (54)$$

where C_h is a complex constant describing the geometry of the core. If $R_G \ll R_l$ we can write

$$B_s = C_h\mu_e \frac{R_l + R_2 + R_L}{R_l} \qquad (55)$$

By substituting R_A and R_B in (55), we get for the bandwidth of the small transformer

$$B_x = C_h\mu_e \frac{R_l/K + R_l\left(2 - \frac{1}{K}\right) + R_L}{R_l/K} \qquad (56)$$

$$B_x = C_h\mu_e \frac{K(2R_l + R_L)}{R_l} \qquad (57)$$

Dividing (57) by (54)

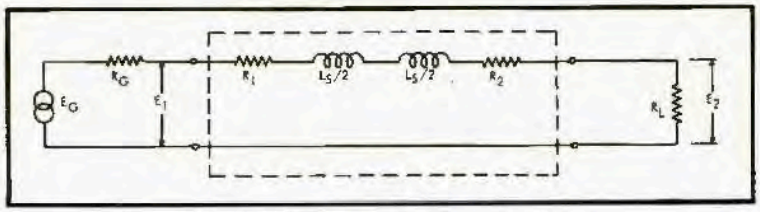

Fig. 4. High-frequency equivalent circuit.

$$B_x/B_T = K\frac{R_p}{R_l}\frac{(R_s - R_G)}{R_s} \qquad (58)$$

or

$$B_x/B_T \cong K\left(1 + \frac{R_G}{R_l}\right)\left(1 - \frac{R_G}{R_s}\right) \qquad (59)$$

In words, our new transformer in a zero-impedance output stage has *at least K times the bandwidth as the large.*

6. Class B_1 High-Frequency Performance

The "*K*-modified" transformer has important beneficial results in a class B_1 amplifier with zero source resistance and a well regulated plate supply.

(a) The interprimary (half-primary to half-primary) leakage reactance is lower. If $K = 2$, the dreaded "notch" due to switching transients[12] is moved up an octave, approximately.

(*Continued on page 68*)

[10] R. Lee, *op cit.* p. 60. This equation, while intended for large transformers, serves our purpose here.

[11] MIT Staff, "*Magnetic Circuits and Transformers.*" Wiley: 1943, p. 484. Also see F. Terman, "*Radio Engineers' Handbook.*" McGraw-Hill: 1943, p. 388, *Fig.* 26.

[12] A. P. Sah, "Quasi-transients in Class B audio-frequency amplifiers." *Proc. IRE*, Nov. 1936.

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(from page 30)

(b) Keeping interprimary inductance very low by using a bifilar winding technique usually increases the effective primary capacitance. Higher capacitance in turn reduces the available power at low distortion.

The performance is then quite analogous to that at low frequencies where

$$VA = P_o - jP_o/d \qquad (45)$$

Analysis indicates, in a similar way, that at high frequencies volt-amperes VA_h must be furnished (at high power levels) in accordance with

$$VA_h = P_o + jP_o/d_h \qquad (60)$$

where $d_h = X_C/R_L$ (61)

and X_C = effective primary capacitive reactance

We can now recall that capacitance, being basically a measure of length, has a proportionality

$$C \propto V^{1/3} : W^{1/3}, \qquad (62)$$

where V is volume and W the weight.

If $K = 2$, then W is reduced by $2^{3/2} = 2.83$ and the capacitance of the smaller transformer is reduced by a factor $2^{1/2} = 1.41$, almost a half octave.

(c) The combination of greater bandwidth, lower interprimary leakage and lower capacitance can now contribute substantially to the design of an economical high performance B_1 amplifier.

Limits of K

For over a decade it has been known that a low generator impedance obtained with negative feedback would improve the performance of a mediocre output transformer. But, unfortunately, feedback amplifiers using such transformers perform poorly at *high* power levels.

The writer has used small values of K (in the range of 1.4–1.6) and maintained adequate high-level primary inductance for low distortion in conjunction with grain-oriented laminations, and employed winding techniques common to the traditional output transformer.

Tests on stable zero-source-resistance (attained by controlled positive feedback) amplifiers with output transformers designed in accordance with the procedures outlined in this article confirm the advantages stated herein.