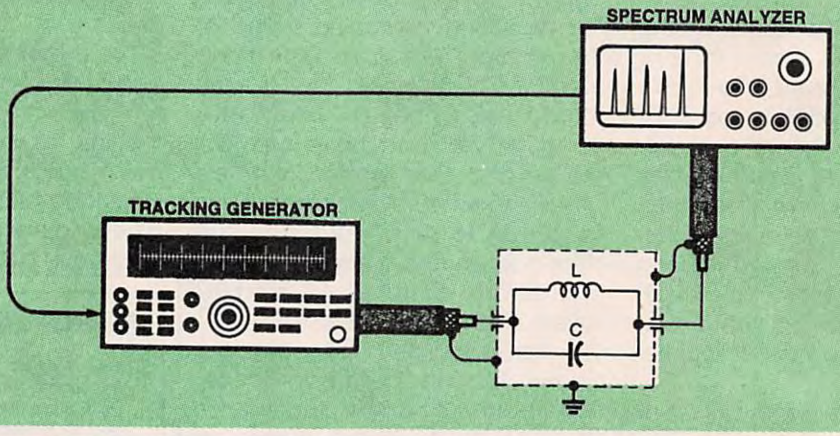
Measuring Inductors and Capacitors at RF Frequencies



At low frequencies, measuring inductance and/or capacitance is no big deal, but get to RF frequencies and things get a lot more interesting.

JOSEPH J. CARR

The techniques needed to measure the values of inductors (L) and capacitors (C) at radio frequencies differs somewhat from what is needed to make the same measurements at low frequencies. In short, although similarities exist, the RF measurement is a bit more complicated. One of the reassons for that is that stray or "distributed" inductance and capacitance values of the test set-up will affect the results. Another reason is that capacitors and inductors are not ideal components, but rather all capacitors have some inductance, and all inductors have capacitance. In this article we will take a look at several effective methods for measuring inductance and capacitance at RF frequencies.

VSWR Method. When a load impedance is connected across an RF source, the maximum power transfer occurs when the load (Z_L) and source (Z_S) impedances are equal ($Z_L = Z_S$). If those impedances are not equal, then the *voltage standing wave ratio* (VSWR) will indicate the degree of mismatch. We can use that phenomenon to measure values of inductance and capacitance using the scheme shown in Fig. 1A. The instrumentation required includes a signal generator or other signal source, and a VSWR meter or VSWR analyzer.

Some VSWR instruments require a transmitter for excitation, but others will accept the lower signal levels that can be produced by a signal generator. An alternative device is an SWR-analyzer type of instrument. It contains the signal generator and VSWR meter, along with a frequency counter to be sure of the actual test frequency. Whatever signal source is used, however, it must have a variable output frequency. Further, the frequency readout must be accurate (the accuracy of the method depends on knowing the actual frequency).

The load impedance inside the shielded enclosure consists of a non-inductive resistor (R) that has a resistance equal to the desired system impedance resistive component (50 ohms in most RF applications, and 75 ohms in television and video). An inductive reactance (X_L) and a capacitive reactance (X_C) are connected in series with the load. The circuit containing a resistor, capacitor, and inductor simulates an antenna-feedpoint impedance. The overall impedance is:

$$Z_L = \sqrt{R^2 + (X_L - X_C)^2} \qquad (1)$$

Note the reactive portion of Equation 1. When the condition $|X_L| = |X_C|$ exists, the series network is at resonance, and VSWR is minimum (see Fig. 1B). This gives us a means for measuring the values of the capacitor or inductor, provided that the other is known. That is, if you want to measure a capacitance, then use an inductor of known value. Alternatively, if you want to know the value of an unknown inductor, use a capacitor of known value.

Using the test set-up in Fig. 1A, adjust the frequency of the signal source to produce minimum VSWR.

1. For finding an inductance from a known capacitance:

$$L_{\mu H} = \frac{10^{12}}{4\pi^2 f^2 C_{PF}} \qquad (2)$$

Where:

$L_{\mu H}$ = inductance in microhenrys (µH)
C_{pF} is the capacitance in picofarads (pF)

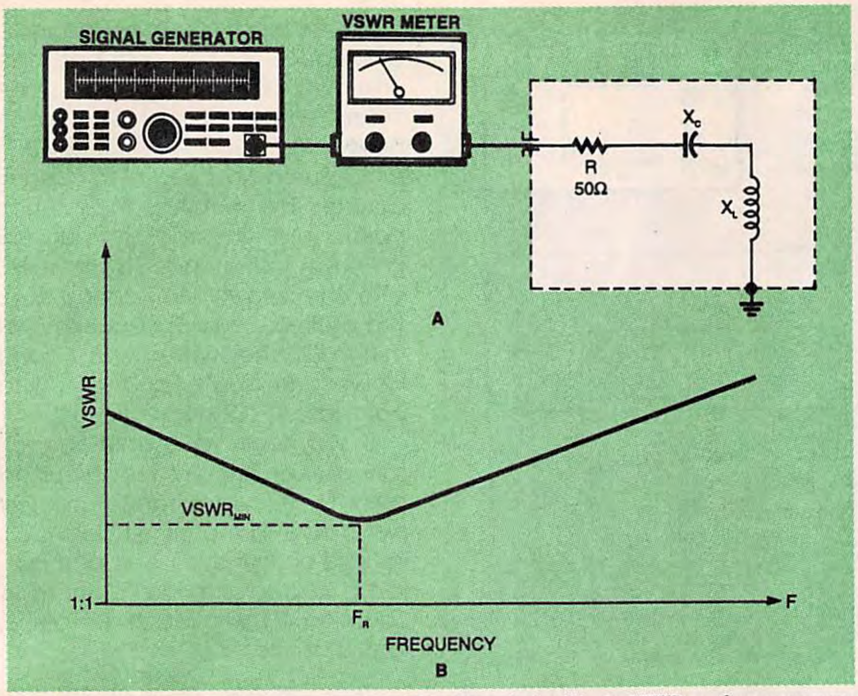


Fig. 1. The VSWR method for measuring L and C is shown in A while the VSWR-vs.-frequency curve is shown in B.

In capacitors the series resistance is typically too small to be of consequence. We can replace L in the model of Fig. 2B with a capacitor, and again measurie voltage E. The value of the capacitor will be:

$$C = \frac{2\pi f \times 10^6}{R \times \sqrt{\left(\frac{V}{E}\right)^2 - 1}} \qquad (6)$$

The value of resistance selected for R1 should be approximately the same order of magnitude as the expected reactance of the capacitor or inductor being measured. For example, if you expect the reactance to be, say, between 1000- and 10,000-ohms at some frequency, then select a resistance for R1 in this same range. That will keep the voltage values manageable.

f is the frequency in hertz (Hz)

2. For finding a capacitance from a known inductance:

$$C_{pF} = \frac{10^{12}}{4\pi^2 f^2 L_{\mu H}} \qquad (3)$$

The accuracy of this approach depends on how accurately the frequency and the reactance are known, and how accurately the minimum VSWR frequency can be found.

Voltage Divider Method. A resistive voltage divider is shown in Fig. 2A. This circuit consists of two resistors (R1 and R2) in series across a voltage source V. The voltage drops across R1 and R2 are V1 and V2, respectively. We know that either voltage drop is found from:

$$V_X = \frac{V R_X}{R1 + R2} \qquad (4)$$

Where: V_X is V1 and R_X is R1 or, V_X is V2 and R_X is R2, depending on which voltage drop is being measured.

We can use the voltage divider concept to find either inductance or capacitance by replacing R2 with the unknown reactance. Consider first the inductive case. In Fig. 2B resistor R2 has been replaced by an inductor (L). Resistor R1 is the inductor series resistance. If we measure the voltage drop across R1 (*i.e.* "E" in Fig. 2B), then we can calculate the inductance from:

$$L = \frac{R}{2\pi f} \times \sqrt{\left(\frac{V}{E}\right)^2 - \left(1 + \frac{R_S}{R1}\right)^2} \qquad (5)$$

As can be noted in Equation 5, if R1 >> R_S, then the quotient R_S/R1 becomes negligible.

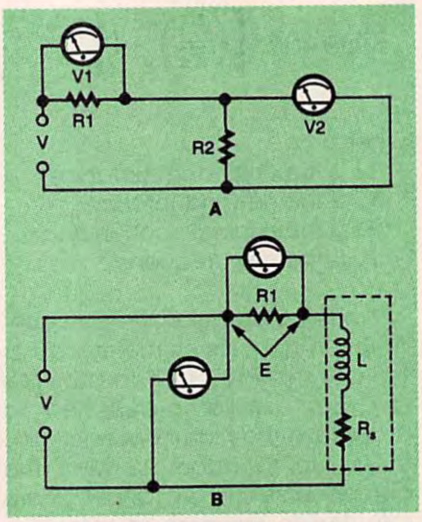


Fig. 2. A simple voltage divider is shown in A and a reactance voltage divider is in B.

Signal Generator Method. If the frequency of a signal generator is accurately known, then we can use a known inductance to find an unknown capacitance, or a known capacitance to find an unknown inductance. Figure 3 shows the test set-up for this option. The known and unknown components (L and C) are connected together inside a shielded enclosure. The parallel-tuned circuit is lightly coupled to the signal source and the display through very low-value capacitors (C1 and C2). The rule is that the reactance of C1 and C2 should be very high compared with the reactances of L and C at resonance.

The signal generator is equipped with a 6-dB resistive attenuator in order to keep its output impedance stable. The output indicators should be any instrument that will read the RF voltage at the frequency of resonance. For example, you could use either an RF voltmeter or an oscilloscope.

The procedure requires tuning the frequency of the signal source to provide a peak output-voltage reading on the voltmeter or scope. If the value of one of the components (L or C) is known, then the value of the other can be calculated using Equation 2 or 3, as appropriate.

Alternate forms of coupling are

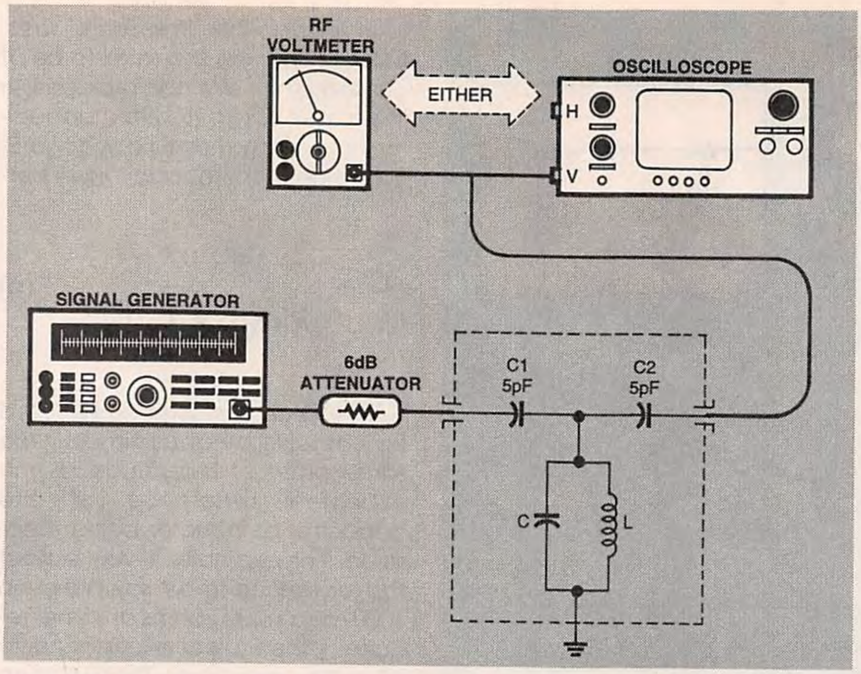


Fig. 3. If the frequency output of a signal generator is accurately known, we can use a known inductance to find an unknown capacitance, or vice versa.

shown in Fig. 4. In either case, the idea is to isolate the instruments from the L and C elements. In Fig. 4A the isolation is provided by a pair of high-value (10,000-ohm to 1 megohm) resistors, R1 and R2. In Fig. 4B the coupling and isolation is provided by a one- or two-turn link winding over the inductor. The links and the main inductor are lightly coupled to each other.

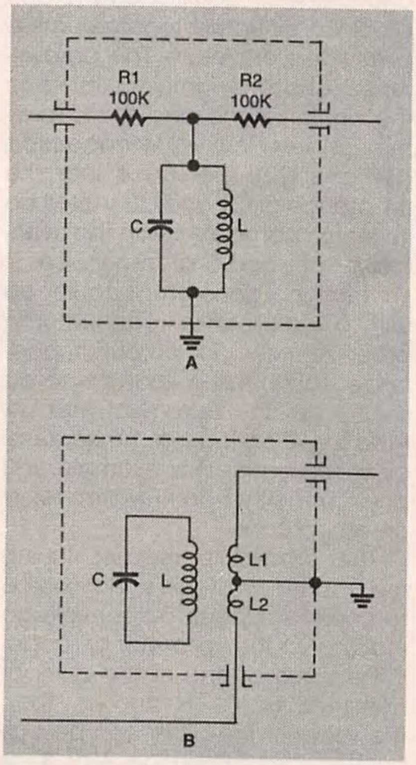


Fig. 4. Different ways to couple L and C elements to the test instruments.

Frequency-Shifted Oscillator Method. The frequency of a variable-frequency oscillator (VFO) is set by the combined action of an inductor and a capacitor. We know that a change in either capacitance or inductance produces a frequency change equal to the square of the component ratio. For example, for an inductance change:

$$L2 = L1 \times \left[\left(\frac{F1}{F2}\right)^2 - 1\right] \quad (7)$$

Where:

- L1 is the original inductance
- L2 is the new inductance
- F1 is the original frequency
- F2 is the new frequency

From this equation we can construct an inductance meter such as the one shown in Fig. 5. This circuit is a Clapp oscillator designed to oscillate in the high-frequency (HF) range up to about 12 MHz. The components L1, C1, and C2 are selected to resonate at some frequency. Inductor L1 should be of the same order of magnitude as L_X. The idea is to connect the unknown inductor across the test fixture terminals. Switch S1 is set to position "b" and the frequency (F1) is measured on a digital frequency counter. The switch is then set to position "a" in order to put the unknown inductance (L_X) in series with the known inductance (L1). The oscillator output frequency will shift to F2. When we know L1, F1 and F2 we can apply Equation (7) to calculate L_X (L2 in Equation 7).

If we need to find a capacitance, then modify the circuit to permit a capacitance to be switched into the circuit across C1 instead of an inductance as shown in Fig. 5. Replace the "L" terms in Equation (7) with the corresponding "C" terms.

Using RF Bridges. Most RF bridges are based on the DC *Wheatstone bridge* circuit (see Fig. 6). In use since 1843, the Wheatstone bridge has formed the basis for many different measurement instruments. The *null condition* of the Wheatstone bridge exists when the voltage drop of R1/R2 is equal to the voltage drop of R3/R4. When the condition R1/R2 = R3/R4 is true, then the voltmeter (M1) will read zero. The basic measurement scheme is to know the values of three of the resistors, and use them to measure the value of the fourth. For example, one common scheme is to connect the unknown resistor in place of R4, make R1 and R3 fixed resistors of known value, and R2 is a calibrated potentiometer marked in ohms. By adjusting R2 for the null condition, and then reading its value, we can use the ratio (R2 x R3)/R1 = R4.

The Wheatstone bridge works well for finding unknown resistances from DC to some relatively low RF frequencies, but to measure L and C values at higher frequencies we need to modify the bridge. Three basic versions are used: Maxwell bridge (Fig. 7), Hay bridge (Fig. 8), and Schering bridge (Fig. 9).

Maxwell Bridge. The Maxwell bridge is shown in Fig. 7. The null condition for this bridge occurs when:

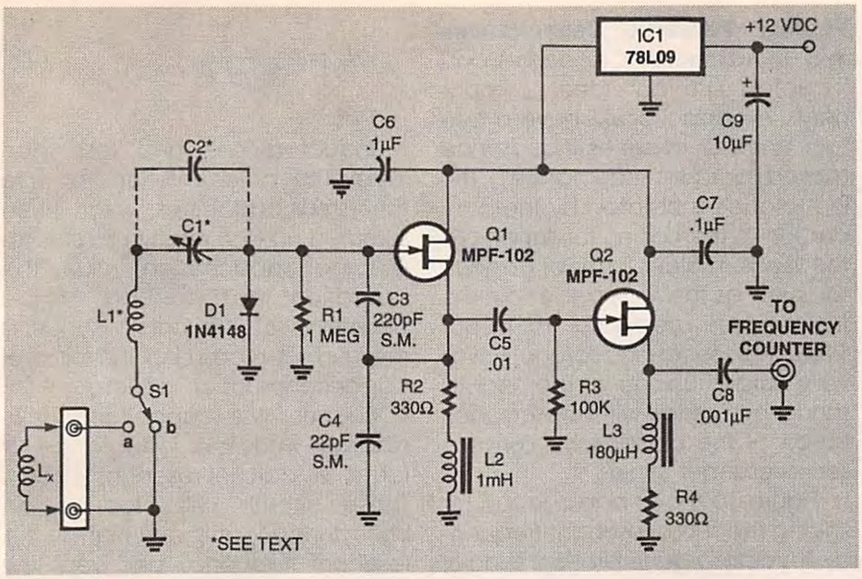


Fig. 5. This circuit uses the frequency-shift method to measure the value of an unknown inductance. It is easily modified to measure an unknown capacitor instead.

$$L1 = R2 \times R3 \times C1 \qquad (8)$$

and

$$R4 = \frac{R2 \times R3}{R1} \qquad (9)$$

The Maxwell bridge is often used to measure unknown values of inductance (*e.g.* L1) because the balance equations are totally independent of frequency. The bridge is also not too sensitive to resistive losses in the inductor (a failing of some other methods). Additionally, it is much easier to obtain calibrated standard capacitors for C1 than it is to obtain standard inductors for L1. As a result, one of the principal uses of this bridge is inductance measurements.

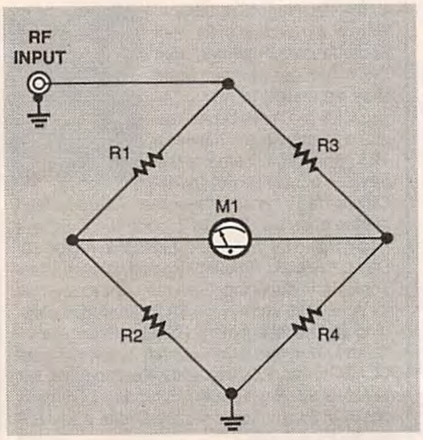


Fig. 6. The Wheatstone bridge has been used since 1843.

Maxwell bridge circuits are often used in measurement instruments called *Q-meters,* which measure the quality factor (Q) of inductors. The equation for Q is, however, frequency sensitive:

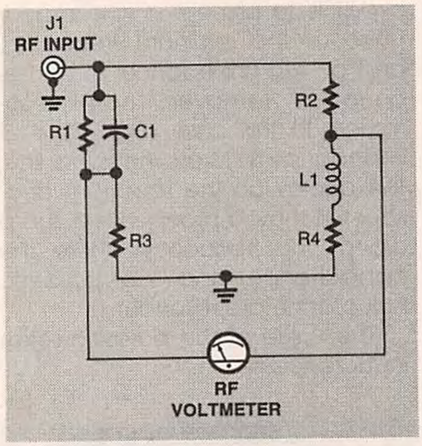


Fig. 7. In the Maxwell bridge, the balance equations are independent of the frequency.

$$Q = 2 \times \pi \times F \times R1 \times C1 \qquad (10)$$

Where: F is in Hertz, R1 in ohms, and C1 in farads.

Hay Bridge. The Hay bridge (see Fig. 8) is physically similar to the Maxwell bridge, except that the R1/C1 combination is connected in series rather than parallel. Unlike the Maxwell bridge, the Hay bridge is frequency sensitive. The balance equations for the null condition are also a little more complex:

$$L1 = \frac{R2 \times R3 \times C1}{1 + \left[\frac{1}{Q}\right]^2} \qquad (11)$$

$$R4 = \left[\frac{R2 \times R3}{R1}\right] \times \left[\frac{1}{Q^2 + 1}\right] \qquad (12)$$

Where:

$$Q = \frac{1}{\omega \times R1 \times C1} \qquad (13)$$

The Hay bridge is used for measuring inductances with high Q figures, while the Maxwell bridge is best with inductors that have a low Q value.

Note: A frequency-independent version of Equation (11) is possible when Q is very large (i.e. >100):

$$L1 = R2 \times R3 \times C1 \qquad (14)$$

Schering Bridge. The Schering bridge circuit is shown in Fig. 9. The balance equations for the null condition are:

$$C3 = \frac{C2 \times R1}{R2} \qquad (15)$$

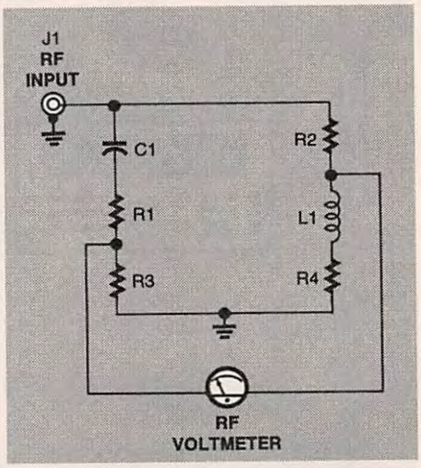


Fig. 8. The Hay bridge is used to measure inductances with high Q values..

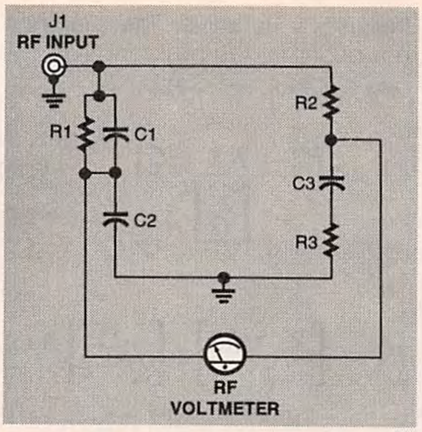


Fig. 9. The Schering bridge is used to find the capacitance and power factor of capacitors.

and

$$R3 = \frac{C2 \times R1}{R2} \qquad (16)$$

The Schering bridge is used primarily for finding the capacitance and the power factor of capacitors. In the latter applications no actual R3 is connected into the circuit, making the series resistance of the capacitor being tested (*e.g.* C3) the only resistance in that arm of the bridge. The capacitor's Q factor is found from:

$$Q_{C3} = \frac{1}{\omega \times R1 \times C1} \qquad (17)$$

Finding Parasitic Capacitances and Inductances. Capacitors and inductors are not ideal components. A capacitor will have a certain amount of series inductance (called "parasitic inductance"). This inductance is created by the conductors in the capacitor, especially the leads. In older forms of capacitor, such as the wax-paper dielectric devices used prior to about 1960, the series inductance was very large. Because the inductance is in series with the capacitance of the capacitor, it forms a series-resonant circuit.

Figure 10 shows a test set-up for finding the series-resonant frequency. A *tracking generator* is a special form of sweep generator that is synchronized to the frequency sweep of a spectrum analyzer. They are used with spectrum analyzers in order to perform stimulus-response measurements such as Fig. 10.

The nature of a series-resonant circuit is to present a low impedance at the resonant frequency, and a high impedance at all frequencies removed from resonance. In this case (Fig. 10), that impedance is across the signal line. The display on the spectrum analyzer will show a pronounced, sharp dip at the frequency where the capacitance and the parasitic inductance are resonant.

The value of the parasitic series inductance is:

$$L = \frac{1}{2^2 \pi^2 f^2 C} \qquad (18)$$

Inductors are also less than ideal. The adjacent turns of wire form small capacitors, which when summed up can result in a relatively large capacitance value. The illustration that appears at the beginning of this article shows a method for measuring the parallel capacitance of an inductor.

Because the capacitance is in parallel with the inductance, it forms a parallel-resonant circuit. Those circuits will produce an impedance that is very high at the resonant frequency, and very low at frequencies removed from resonance. In the lead illustration, the inductor and its parasitic parallel capacitance are in series with the signal line, so will (like the other circuit) produce a pronounced dip in the signal at the resonant frequency. The value of the parasitic inductance is:

$$C = \frac{1}{2^2 \pi^2 f^2 L} \qquad (19)$$

Conclusion. There are other forms of bridges, and other methods, for measuring L and C elements in RF circuits, but those discussed here are the most practical. That's especially true if you do not own or have access to specialized instruments.Ω

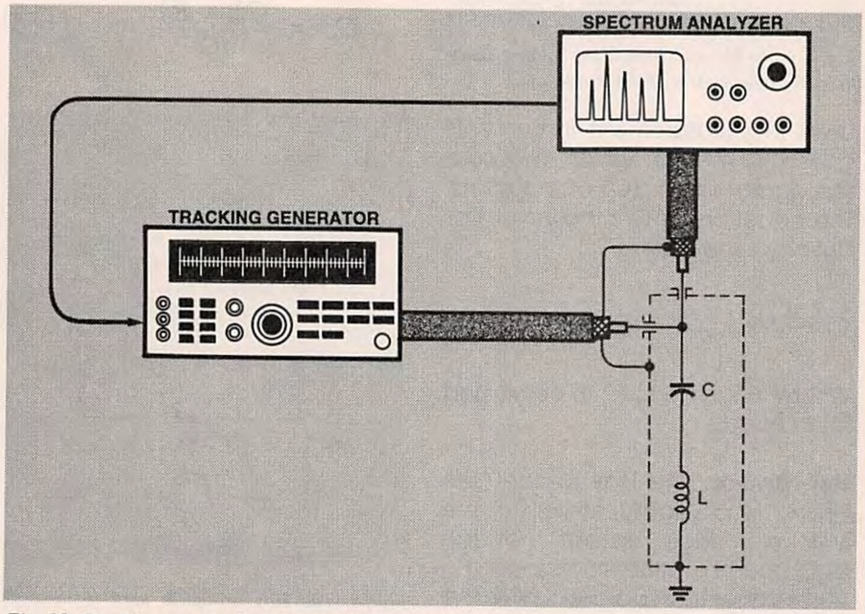


Fig. 10. Use this set up to find the parasitic series inductance of a capacitor.