Vented Speaker

1: Detailed discussion of the principles

This is the first of two articles on vented speaker systems. In this part the emphasis is on understanding vented system operation and on identifying factors which ensure a satisfactory acoustic response of such systems at bass frequencies. The second article will give simple procedures whereby a suitable vented enclosure may be designed for a given bass driver.

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Vented speaker systems, designed to extend the bass response of a given woofer, have been around for half a century. The first patent application was made by Thuras in the US in 1930 but for the first 30 years or so the subject was shrouded in mystery. There was no satisfactory theory which would enable the synthesis of a suitable design, just a number of recipes which were not always in agreement with each other. In some quarters the vented system came to be called the "boom box" and this description was certainly appropriate to a system which I remember listening to in the 1950s.

During the 1950s, a number of papers were written which together constituted the genesis of a theory. The crucial step taken in these papers was to represent the acoustical behaviour in terms of equivalent electrical networks to which conventional circuit analysis could be applied. The big leap forward was achieved by A. N. Thiele in 1961 when he used a simplified model for which an exhaustive mathematical analysis was possible. Having analysed the model, Thiele then went on to identify a wide range of possible combinations of woofer and enclosure which would lead to acceptable results. He also showed how the system parameters could be determined through measurements of the voice-coil impedance, making it relatively easy to implement the theories without recourse to an expensive acoustic laboratory. A number of papers have been written since then, notably by R. H. Small and P. J. Snyder. With the exception of the latter, all of the various authors have used complicated mathematics which is appropriate for only professional engineers. As a result, their conclusions have remained inaccessible to a larger audience. The purpose of these articles is not so much to add to what has already been written in other places, but to explain existing theories to non-professional readers.

The method of this first article is to work from the simple to the complex. For this reason only "ideal" systems are discussed in this part, because they exhibit all of the features which are necessary for an understanding of how vented systems work.

Woofer characteristics

To understand the behaviour of an isolated speaker at low frequencies, it is useful to regard the cone and voice coil assembly as a rigid piston which is suspended from the frame by a spring. The springiness is provided by the spider which centres the voice coil in the magnet and by the surround which attaches the edge of the cone to the frame. The assumption of rigid behaviour is generally good for frequencies below 200Hz or so, which is the interesting area when analysing bass performance.

When a mass is suspended by a spring, it has a natural frequency of vibration. This may be observed by giving the mass an initial displacement and then releasing it. The importance of this resonant frequency in the present context is that it is relatively easy to make the mass vibrate at frequencies near its resonance. For a speaker, the resonant frequency is determined by the balance between the total vibrating mass which includes a contribution from the air moving near the cone, and the stiffness of the suspension. From this, we can proceed to the following conclusions: (i) if the mass is increased the resonant frequency decreases because the system is more sluggish. (ii) if the stiffness is increased, the resonant frequency increases as a converse effect. Thus the resonant frequency is a measure of the ratio of stiffness to mass.

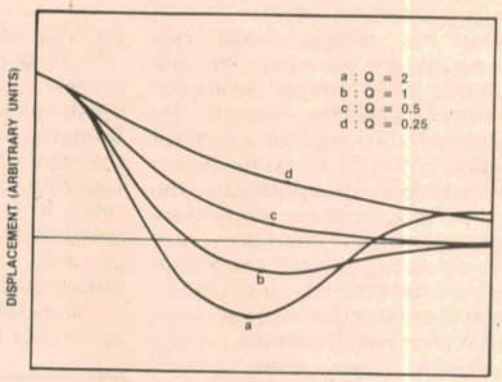
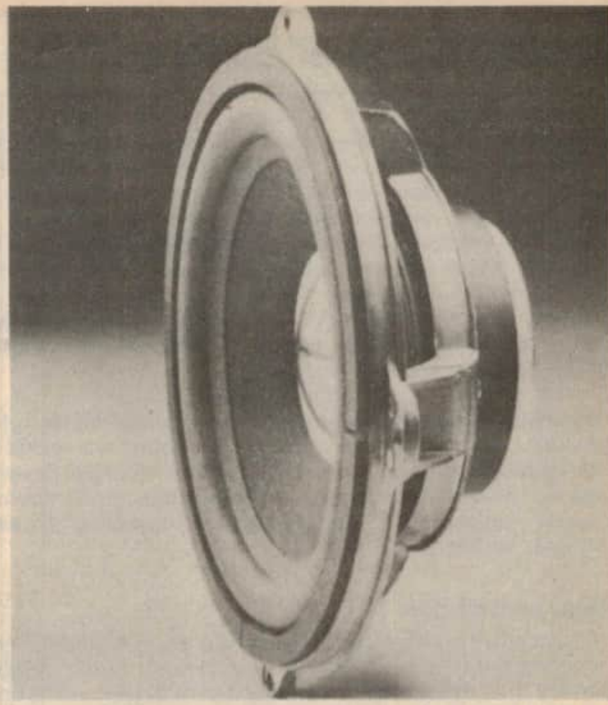


Fig. 1

Suppose that we now connect the woofer to an amplifier, which is turned on but to which no input is connected. If the cone is displaced from its normal position and then released, its subsequent vibrations will be damped, as shown in Fig. 1. The chief cause of the damping is electrical. As the voice coil moves in the magnetic field, it generates a voltage and this causes a current to flow with magnitude limited by the combined resistance of the voice coil, the connecting cables, and the amplifier internal resistance. (In this article, we shall assume that all resistances external to the voice coil are zero.) This current, because it flows in a magnetic field, exerts a force on the voice coil in a manner which damps the motion. The other important factors contributing to the damping are various frictional losses in the suspension. The effectiveness of the damping is specified by a number, Q. Small Q corresponds to heavy damping and large Q to little damping, again as shown in Fig. 1.

Systems of operation



8MV Mk 2 20cm high-power woofer from Magnavox. V_{AS} = 67 litres; f_s = 35Hz; and Q_T = 0.39.

The purpose of the cone is not just to vibrate, but to produce pressure variations in the air, which we regard as sound. Because the movement of the cone simultaneously produces low pressure on one side and high pressure on the other, an unmounted speaker is not able to radiate low frequencies effectively, due to pressure cancellation at the long wavelengths involved. Mounting it in an enclosure brings about a separation of the front and back waves, but it also changes the stiffness of the suspension.

Let us now imagine that the woofer is mounted in an infinite (ie, very large) box, often called an infinite baffle. We can then consider its behaviour when we apply a sine wave through the amplifier, varying the frequency but keeping the driving voltage constant. The displacement amplitude of the cone will be greatest for frequencies around the resonant frequency, falling off on either side at the eventual rate of 6dB per octave. Acoustic output is not equal to this amplitude, but to the product of amplitude with frequency. This corrects the 6dB per octave fall at frequencies above resonance, and turns the rate below resonance into a 12dB per octave slope. Thus the resonant frequency f_r is also a fundamental cut-off frequency. The response near resonance depends critically on the Q value, and this is shown in Fig. 2 where a number of curves are shown for different Q values. Two conventions have been adopted in drawing these curves; (i) the 0dB level is the high frequency limit. (ii) frequencies are not given in Hertz, but as a ratio with the resonance frequency (ie, f/f_r). Also shown in Fig. 2 is a dotted line representing the 12dB per octave bass roll off. For typical high quality woofers Q is in the range 0.25 to 0.4, so it is seen that the infinite baffle arrangement leads to a steady but inexorable roll off from well above the cut-off frequency. This trend is often seen in manufacturer's published response curves, which are for infinite baffle conditions. It is interesting also to reflect that the purpose of transmission line designs is to approximate infinite baffle conditions in a reasonable size of box!

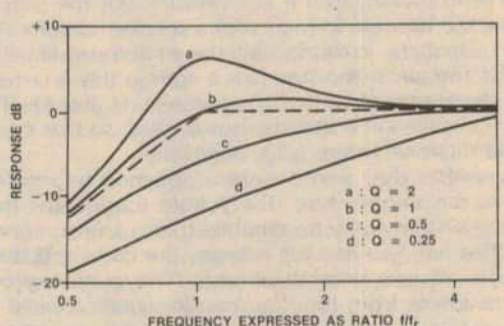


Fig. 2

Ideal sealed box

Suppose that we mount the speaker in a fully sealed box. This will prevent the back wave from escaping but it will also increase the resonant frequency. To understand the latter effect, remember that the vibrations of the cone will successively compress and rarify the air in the box. When the air is compressed its pressure is raised above normal atmospheric pressure, and the difference between the pressures acting on the front and back of the cone acts as an additional restoring force. In general the air in the box does not act as a perfect spring, rather it introduces some additional frictional forces which add to the damping. For the present I shall assume that these effects are negligible, and acknowledge this assumption by calling the box "ideal". Later, I shall drop this assumption. Note also that the box acts as a simple spring only if its dimensions are small compared with the wavelength of the sound: this is no problem as the velocity of sound is in excess of 300m/s, and our attention is confined to low frequencies.

The stiffness of the box depends on the ratio of the area of the moving piston to the volume of the box: that is, it depends both on the woofer and on the box. An important parameter is the volume which the box must have so that its stiffness is the same as the woofer's original stiffness. This quantity is denoted by V_{AS}, and it is essential to know its value when designing speaker enclosures. Since the box stiffness is inversely proportional to the box volume, we may write the following relationship between the box stiffness k_B and the woofer stiffness k_S:

$$k_B = k_S V_{AS}/V_B \qquad (1)$$

For example, if the box has volume to one-third V_{AS}, the box will be three times as stiff as the woofer. The total stiffness is the sum of k_S and k_B, since both restoring forces act in concert. The effect of this on the resonant frequency is given by the formula

$$f_r = 2\pi\,(k/m)^{1/2} \qquad (2)$$

where m is the total vibrating mass. This shows that the resonant frequency increases in proportion to the square root of the stiffness. For a box of volume one-third V_{AS}, the total stiffness is increased by a factor four, and the resonant frequency is doubled. More generally, if we denote the resonant frequency of the woofer by f_S and the resonant frequency when it is mounted in a box by f_o, we have

$$f_o = f_S\,[(V_B+V_{AS})/V_B]^{1/2} \qquad (3)$$

In order to determine the acoustic output of the sealed enclosure we need to know the Q value of the woofer when it is in the box. Now the damping is caused by frictional forces and these are unaffected by the box. I have already stated that the resonance f_S is related to the ratio of restoring force to mass, through equation (2). Similarly, to find the effect of the box on the Q value, we need to know that the quantity f_r/Q measures the ratio of frictional forces to mass, at the responant frequency. This is why small Q corresponds to high damping. What happens when we mount the woofer in an ideal box is that the ratio f_r/Q remains constant while f_r is increased. For this reason we must distinguish between the Q value of the woofer in isolation Q_T, and the Q value when it is in the box, Q_o. The relationship is

$$f_S/Q_T = f_o/Q_o \quad (4)$$

which shows that Q_o is larger than Q_T by the factor f_o/f_S. In practice, when the chosen speaker enclosure is a sealed box the volume is chosen so that Q_o is close to unity. Since this usually means that the box is many times stiffer than the original suspension, this method of mounting is called acoustic suspension.

Ideal vented box

In designing an ideal sealed box for a given woofer, there is only one parameter which is at the disposal of the designer, namely the box volume V. By using a vent or port in the box, a second parameter is introduced. To see why, consider first the vented box with the speaker cut out temporarily blocked in. Such a system is known as a Helmholtz resonator, after the scientist who investigated it last century. For the purpose of designing the bass performance of a speaker system a simple model is adequate. In this model, the air in the vent acts like a mass and the air in the box like a spring: this is a resonant system characterised by two parameters, f_B and Q_B. For the present, I shall assume that the box is ideal, so that Q_B is very large and frictional forces f_B/Q_B negligible.

Now consider the vented system obtained by mounting a woofer in this vented box. The simple models for the two resonating systems may be combined into a new, composite model. This has two moving masses, the cone and the vent, each radiating sound as it vibrates. The cone assembly is subject to forces from four sources: (i) signals applied to the voice coil impress a force on the cone; (ii) the woofer suspension provides a restoring force, acting between the cone and the box itself; (iii) there are frictional forces characterised by Q_T; (iv) the air in the box provides a restoring force, only it no longer acts between the cone and a rigid box, but between the cone and the air in the vent. This latter force is the one and only force which is applied to the vent air mass, but it causes it to vibrate and radiate sound as well as the cone.

The effect of coupling the two resonant systems via the stiffness of the box is quite profound. The frequencies f_o and f_B are no longer resonances; rather there are two new resonant frequencies f_L and f_H. These frequencies may be calculated by solving the equations.

$$f_L f_H = f_S f_B \quad (5)$$

$$f_L^2 + f_H^2 = f_o^2 + f_B^2 \quad (6)$$

Formulas for the solution may be given, but they are not important for understanding the acoustic response. However, the reader may find it helpful to have one concrete example. For one system built by the author, the values were

$f_L = 21Hz$
$f_S = 35Hz$
$f_B = 36Hz$
$f_o = 52Hz$
$f_H = 59Hz$

To understand the role of these frequencies, it is helpful to think of a simpler coupled mechanical system which is easily visualised. Suppose we set up two identical pendulums, hung side by side from a horizontal support. Each is a mass m suspended by a light string of length ℓ. The two masses are connected by a spring, whose mass is negligible compared with m. With the connecting spring omitted it is possible to set the two pendulums oscillating in unison by starting them in identical manners. This must also be one of the two coupled modes of oscillation since it would have no effect on the length of the spring. The other mode has the two pendulums moving in opposition to each other, and since the spring is providing extra restoring forces in this case, its frequency must be the higher of the two. The general principle, which applies to less symmetrical systems such as the vented speaker is that, when two natural resonators are coupled, there are two coupled modes of vibration. One of them, the lower resonant frequency, minimises the effect of the coupling. The other, at the higher resonance, maximises it.

In the present context, the lower resonant frequency f_L minimises the effect of the coupling between cone and vent. In this mode, the cone moves out while the vent takes in air, and vice versa, rather like a pump with a hole in it, and so there is very little net acoustic output from this action. At the higher resonant frequency f_H the effect of the coupling is maximised because the cone and vent air move in and out together, radiating constructively. The actual level of output is governed by the damping at f_H, and here the effect of the coupling has a second profound effect. To see this, recall that before the coupling is introduced, only one of the two natural resonators (the woofer) had any damping. However, both of the coupled modes involve movement of the cone, and the frictional forces represented by f_S/Q_T have to be shared between the two. Thus the Q value of the upper resonance is greater than $f_H Q_T/f_S$, so that it is possible to achieve a damping factor of unity for this mode without making f_H/f_S as large as the corresponding ratio f_o/f_S for the sealed box.

This is not the end of the story. At the box resonant frequency f_B, which is well below f_H, the vent will give a large output without the cone making large excursions. The output level of the vent is completely under the control of the driver damping, even for an ideal box. Thus the vented system works by employing the vent as an auxiliary radiator below the frequency f_H. For this reason f_H is sometimes called the cross over frequency.

Optimally flat response

Since the ideal sealed box is less complicated than the ideal vented box, I will return to it briefly. Reference to Fig. 2 reveals that the response curves for $Q_o = 0.25$ and 0.5 show a steady fall in response with falling frequency, whereas those with $Q_o = 1.0$ and 2.0 have a peak. Regardless of the Q value, the system is second order – it falls off at 12dB per octave – and the main features of the response are obtained by drawing a straight line representing this 12dB rate. The details of the curve near to resonance are controlled by the Q value, and in this respect the value $Q_o = 0.707$ is important. It is the largest value that Q_o may have without causing a peak, and the corresponding response is known as optimally flat. The response function G(f) will be given in the next section. It is not a complicated formula, but there is one frequency where it is particularly simple, namely the resonant frequency f_o. Here the formula is

$$G(f_o) = Q_o \quad (7)$$

This shows that the optimally flat system is down 3dB at the cut-off frequency.

For an ideal vented box the response formula is naturally more complicated. The vented system is fourth order and falls off at 24dB per octave. The crucial question is whether there is an acceptable response curve joining these two straight lines. For the analogous fourth order electrical network, it was known that it is possible to achieve an optimally flat response (no peaks −3dB at cut-off) before Thiele published his

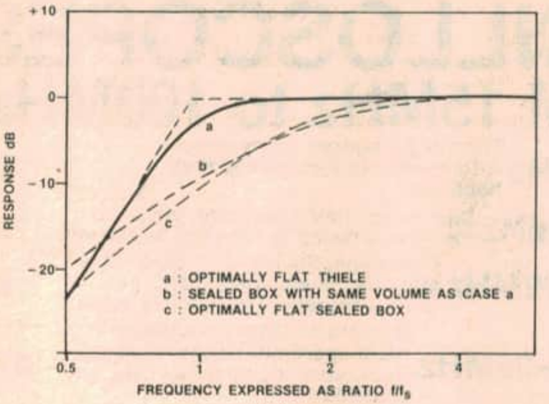


Fig. 3

important paper. Thiele found that there is a unique choice of parameters for the ideal vented speaker system which gives the optimal response. This is the now-famous Thiele optimally flat alignment and its response is shown as the solid curve in Fig. 3. The parameters for this alignment, including the cut-off frequency f_c, are

$$Q_T = 0.383$$
$$f_B = f_S$$
$$V_B = 0.707\ V_{AS}$$
$$f_c = f_S$$

Obviously this alignment is only possible if a woofer is available with the optimum Q_T value. Assuming that this is the case, it is interesting to compare the results of using the same woofer in a sealed box. First, if the volume of the box is given by the Thiele alignment, then the cut-off frequency is $f_o = 1.55\ f_S$ with a Q_o of 0.595; the low Q_o means that the response is −4.5dB at f_o. Choosing an optimally flat sealed system raises the cut-off frequency to 1.85 f_S. The response curves for these two choices are shown as dotted lines in Fig. 3. Note that the frequency scale uses f/f_S for all three curves, so as to enable direct comparison of results.

Response functions for ideal boxes

It is not the purpose of this article to delve into mathematical details. Nevertheless it is very useful to write down the formula for the response function and spend some time considering its interpretation. This will lead to an understanding of how the response depends on the choice of parameters V_B and f_B. In the second article this information will be turned round so that the parameters can be chosen to provide practical designs. The formulas take on their simplest appearance when we use the fundamental frequencies of the system as the basic units. That is, frequencies will not be absolute (in Hertz) but specified as the ratio of the cut-off frequency to other important frequencies such as f_S, f_o, f_B, f_L and f_H.

For a sealed box the cut-off frequency is simply the resonant frequency of the woofer in the box, f_o, for which a formula has already been given in equation (3). The response function is

$$G(f) = 1/[(1 - f_o^2/f^2)^2 + f_o^2/Q_o^2f^2]^{1/2} \qquad (8)$$

The prominent feature of this formula is that it involves the sum of two squares. Since the square of a number can never be negative, the smallest value which either term may have is zero and in fact the first term is indeed zero at the resonant frequency f_o. Thus the importance of this frequency is apparent from a cursory inspection of the response function. Another way of viewing this is that, if the response function is derived in terms of the woofer parameters f_S, Q_T, V_{AS} and the box volume V_B, then we would immediately recover the formula for the resonant frequency f_o by observing that one of the terms becomes zero at this frequency. The other point which follows from equation (8) is that, when f is small, the response function is well approximated by the ratio f^2/f_o^2. This tells us that the system is second order and that f_o is the cut-off frequency.

For an ideal vented system (not necessarily the Thiele optimally flat alignment) the response function is given by the formula

$$R(f) = 1/\,[(1-f_L^2/f^2)^2(1-f_H^2/f^2)^2 + (f_S^2/Q_T^2f^2)\,(1-f_B^2/f^2)^2]^{1/2} \qquad (9)$$

This is more complicated than the sealed box response but the interpretation of its major features is no more difficult.

The first point of interest is the cut-off frequency. This is found by observing that for small values of f the response is well approximated by the ratio $f^4/f_L^2f_H^2 = f^4/f_S^2f_B^2$. [The second form, with f_Lf_H replaced by f_Sf_B follows from equation (5).] This shows that the system is fourth order, and that the cut-off frequency is

$$f_c = (f_Sf_B)^{1/2} \qquad (10)$$

The second point is that there are three frequencies where one of the two squares in the response function becomes zero, namely f_L, f_B and f_H. At these frequencies the response formula is simple in form and also of great interest.

Since the *raison d'etre* of the vent is to maintain response down to the box frequency f_B, even though the woofer itself has ceased to make a useful contribution, it is natural to calculate $R(f_B)$. Some simple algebra involving equations (3), (5) and (6) gives the result

$$G(f_B) = (f_B^2/f_S^2)(V_B/V_{AS}) \qquad (11)$$

In most practical vented designs f_B^2/f_S^2 is between one half and two; moreover we shall see that the choice of this ratio depends almost entirely the Q_T value of the driver. Consequently, this formula shows that once the woofer has been selected and f_B chosen, the response at f_B is determined by the size of the box. As an example, with the Thiele optimally flat alignment $f_B = f_S$ while $V = 0.707\ V_{AS}$, so that the response is 3dB down at the box frequency.

The other important frequency is the cross over, f_H. Again the response is easy to calculate, namely

$$G(f_H) = (Q_Tf_H/f_S)[f_H^2/(f_H^2-f_B^2)] \qquad (12)$$

In the Thiele optimally flat alignment the response at f_H (= 1.76 f_S) is less than 0.1dB down. Generally speaking, vented systems are designed so that $G(f_H) = 1$. Equation (12) shows how the vent assists in achieving this objective. This is because the factor Q_Tf_H/f_S, which is the damping predicted by the application of the principle that Q/f_r is a constant [see equation (4) and the discussion there] is multiplied by $f_H^2/(f_H^2-f_B^2)$ which is always greater than one. For the Thiele optimally flat alignment, the values are

$$Q_Tf_H/f_S = 0.67$$

$$f_H^2/(f_H^2-f_S^2) = 1.48$$

This illustrates one of the profound effects of coupling the cone and vent. In this particular alignment, about two-thirds of the woofer damping is applied to the upper frequency resonance, with the other third applying to f_L. This enables the use of a larger box than the fully sealed design, which keeps down the various natural frequencies to a minimum.

Conclusion

This concludes the first article which was concerned with the principles whereby vented systems achieve their results. A number of questions arise, which must be answered in order to turn the principles into a set of design rules. They are:

(i) What effect does the box damping have on the response functions?

GLOSSARY

Quite a few terms are used in this article and some of these have not been defined. To help you keep track while reading the article, we have compiled this glossary:

f_B: resonant frequency of a vented box with woofer cutout sealed.
f_c: cut-off frequency; the −3dB point on the system frequency response curve beyond which the response usually falls at 12 or 24dB/octave.
f_H: upper resonant frequency of a vented speaker system. Also referred to as the mechanical crossover frequency.
f_L: lower resonant frequency of a vented speaker system.
f_o: resonant frequency of a woofer when mounted in a sealed box.
f_r: generalised term for resonant frequency.
f_s: resonant frequency of a woofer in free air.
k_B: stiffness of air in a sealed box when acted upon by a woofer.
k_s: stiffness of a woofer suspension.
m : total vibrating mass of a system.
Q: figure of merit for a resonant system. A high-Q figure refers to an undamped system while a low-Q refers to a heavily damped system.
Q_B: Q of a vented box resonance with woofer cutout sealed.
Q_o: Q of woofer resonance when mounted in a box.
Q_T: Q of woofer resonance when in free air.
V_{AS}: equivalent volume; the volume of air that offers a compliance to the woofer that is equal to the compliance of the woofer's suspension.
V_B: Volume of air in box.

(ii) How can equations (11) and (12) be turned around so as to tell us the appropriate values of f_B and V_B for a given woofer?
(iii) How can the parameters f_S, Q_T and V_{AS} be measured for a woofer if they are not specified in the manufacturer's literature?
(iv) How do we choose the dimensions of the vent?
(v) Having built a prototype, how can we check that its critical frequencies accord with the theory, and if necessary adjust the vent?
All of these questions will be addressed, and answered, in the second article.

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