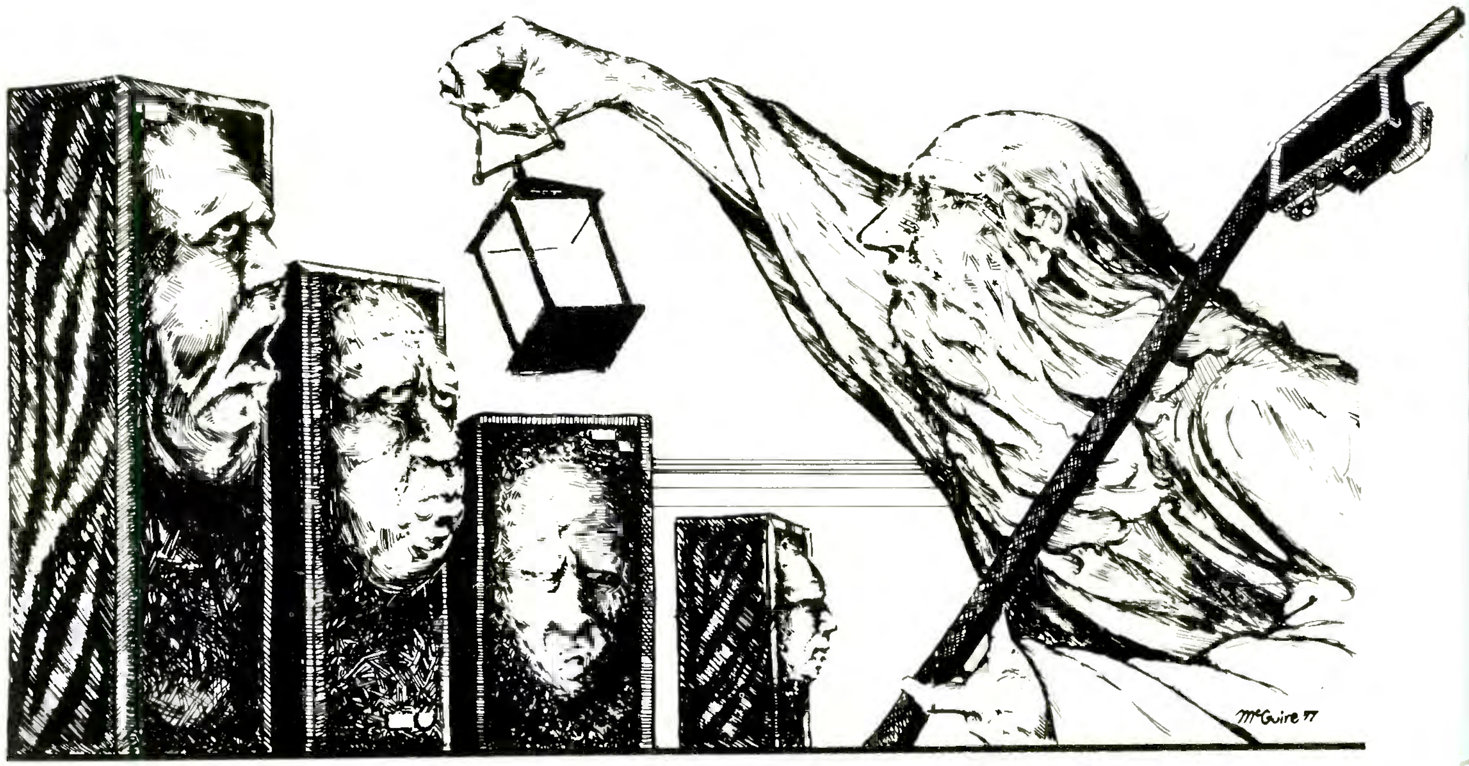
The Search for an Optimum Transmission Line Speaker

W. J. J. Hoge*



The first description of what is normally called a transmission line loudspeaker was published in 1936 by Benjamin Olney (1). The system was the "acoustical labyrinth" which he patented in 1934 (2) and represented an attempt to overcome the poor performance of the open-back cabinets of console radio sets. Olney's employer, Stromberg Carlson, produced the system for a few years during the '50s until they left the component high fidelity market. Transmission line systems did not really begin to catch on until after 1965. In that year A. R. Bailey published a transmission line system construction article (3). Since then, several manufacturers have placed such systems on the market.

In the past few years the performance of direct-radiator loudspeakers has been well analyzed and methods for synthesizing optimum design specifications have been developed (4, 5, 6). These techniques have been successful in many applications (7, 8), however, until very recently, the theory of transmission line loudspeakers has not been very well understood (9).

Direct-radiator loudspeakers are divided into three types, closed-box, vented-box, and passive radiator systems. Similarly, there are three types of transmission line systems. For the sake of brevity, let us call them Type A, Type B, and Type C. In Type A systems, the back side of the driver radiates into a sealed enclosure, while the front is coupled to a transmission line. The system output is solely from the output end of the line.

Type B and C systems allow the front side of the driver to radiate into the listening area, while the rear of the driver is connected to the transmission line, usually via a coupling volume. For Type B systems, the far end of the line from the driver is blocked. Type C systems have an aperture at the far end of the line so that the signal in the room is the sum of the outputs of the driver and the transmission line. What goes on in these systems, and is one better than the other? To answer these questions, we must analyze the systems.

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Using Signal Flow Graphs

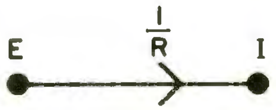
There are several common techniques for system analysis. The most popular is the dynamic analogy method which allows an equivalent electrical circuit of the loudspeaker to be drawn. However, another method, state-variable analysis, is this author's favorite. This method uses signal flow graphs instead of equivalent circuits (10).

That's nice. What's a signal flow graph?

Well, a signal flow graph is a way of writing a set of equations for a system and then interconnecting them so that the system can be analyzed. Consider the system in Fig. 2. (Kindly Editor's Note: This is the newly designed symbol for the U.S. Patent Office.) A voltage is applied to the lamp by the battery, and a resulting current flows. If the battery potential is E volts and the resistance in the current is R ohms, then the current I is given by Ohm's Law:

$$I = E/R \qquad \text{(Eq. 1)}$$

A signal flow graph of the equation would look like this:

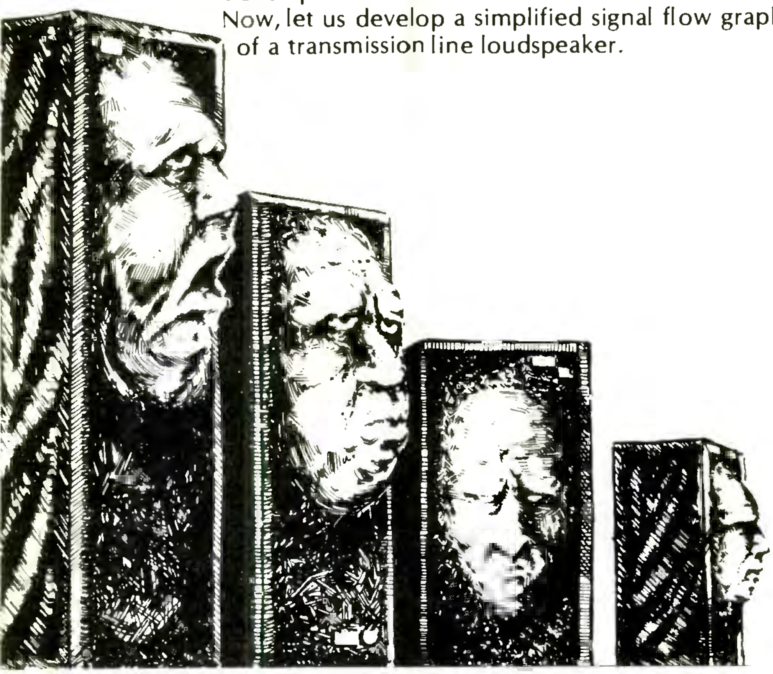


(Sfg. 1)

The dot

is called a node. The line with the arrow

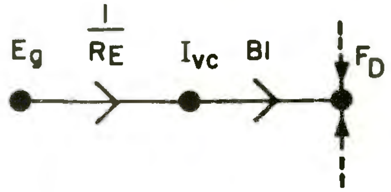
is called a branch. A node represents some physical quantity in which we are interested, while a branch shows the relationship between the two nodes which it connects. Now, let us develop a simplified signal flow graph of a transmission line loudspeaker.



We start with the electrical input from the generator E_g. It causes a current in the voice coil I_{VC}. If R_E is the voice coil resistance, then we have

(Sfg. 2)

The current in the voice coil interacts with the magnetic field to produce a driving force on the diaphragm F_D. If B is the flux in the gap and l is the length of the wire in the gap, then

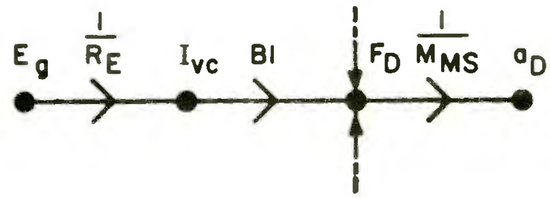


(Sfg. 3)

Note that some additional branches are entered in the F_D node. This is because other parts of the system are "pushing" on the diaphragm and contribute to the total force. We'll crank them in a bit later.

Newton's Second Law of Motion tells us that if we push on something, it will accelerate. The acceleration of the diaphragm a_D with effective mass M_{MS} is given by

(Sfg. 4)



The velocity of the diaphragm u_D is found by the equation

$$u_D = \int a_D \, dt \qquad \text{(Eq. 2)}$$

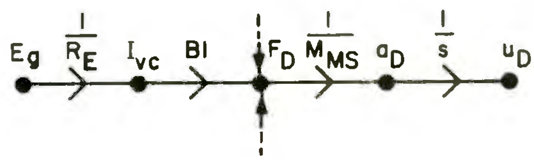
At this point we hear screams of dispair from those Gentle Readers who did not take calculus (and some who did and know that integral calculus is a pain in the neck, or perhaps someplace lower). But have no fear! The author has a trick up his sleeve. Under certain conditions (This Engineering technique is known as "arm-waving" and is usually accompanied with the magic words, "It can be shown that..."), of which this is one, we can turn calculus into simple algebra by saying that

$$s = d/dt \qquad \text{(Eq. 3)}$$

If this is true, then

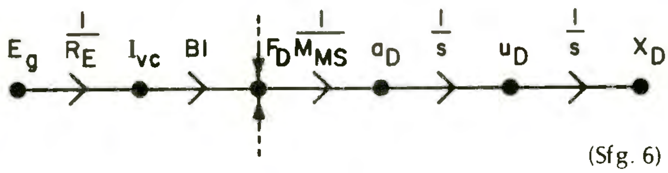
$$1/s = \int dt \qquad \text{(Eq. 4)}$$

Thus, it can be shown that



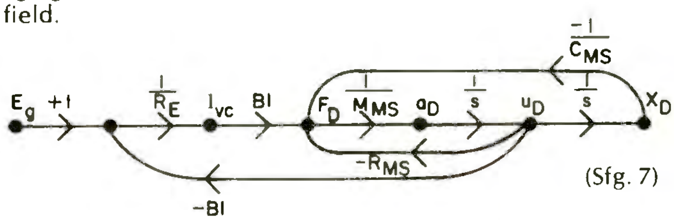
(Sfg. 5)

In a similar manner we integrate u_D to find the displacement of the diaphragm x_D.



(Sfg. 6)

We can complete the signal flow graph for the driver by adding a branch from x_D to F_D to give the restoring force from the compliance of the suspension C_{MS}, a branch from u_D to F_D to give the force opposing motion of the diaphragm caused by mechanical losses R_{MS}, and a branch from u_D back to the voice coil circuit to represent the voltage generated when the coil of wire moves in the magnetic field.



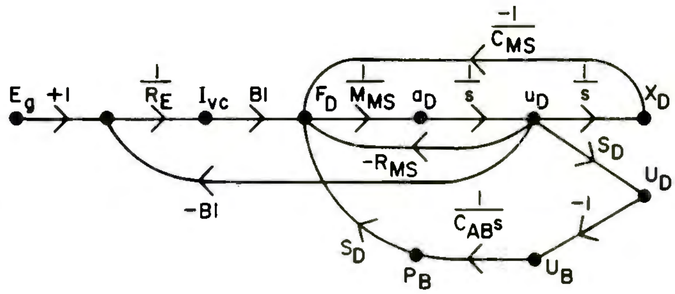
(Sfg. 7)

The minus signs indicate that the restoring force and the losses oppose the driving force and that the voltage generated by coil motion opposes the input signal.

The mechanical motion of the diaphragm causes a current of air (or volume velocity) U_D to flow into the room. Also, a volume velocity U_B flows into the enclosure. Obviously, U_D is 180° out of phase with U_B, but the two are of equal magnitude. U_B causes a pressure variation p_B in the enclosure given by

$$p_B = 1/C_{AB} \int U_B \, dt \qquad \text{(Eq. 5)}$$

where C_{AB} is acoustic compliance of the air in the enclosure (or coupling chamber). If S_D is the area of the diaphragm, then

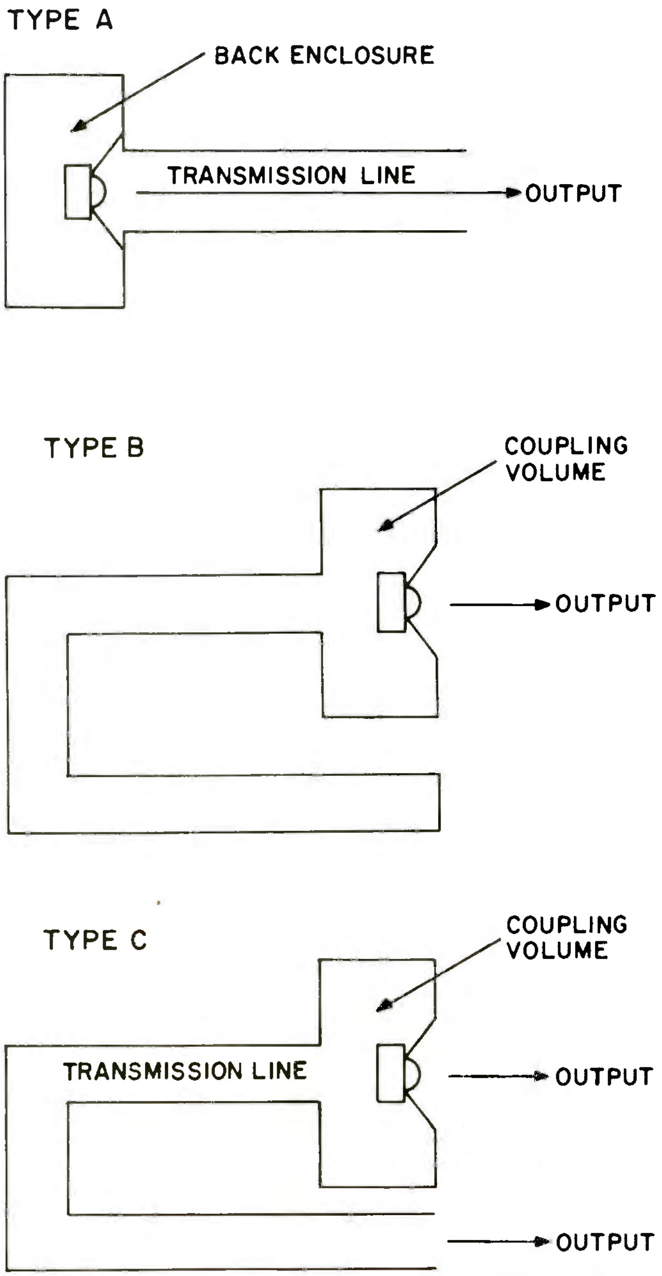


(Sfg. 8)

which is the signal flow graph of a closed-box loudspeaker. To get a Type A system, we need only add the throat impedance z_{AT} of the transmission line to the system. We can then find the throat pressure p_T and the system output power, P_A. Fig. 3 gives the signal flow graph of a Type A system.

For the type B and C systems z_{AT} is connected to the back of the diaphragm. See Figs. 4 and 5.

Fig. 1—Transmission line enclosure types.



Analyzing System Types

Now that we've got an analytical picture of the system, we should be able to figure out how it works. Type B is the simplest case. Note that we have used an italic *z* rather than a plain Z for z_{AT}. This is to signify that z_{AT} varies with frequency. What effect does this have on performance? Consider the case of a constant diameter tube for the line. The behavior of the driver is easily determined at various frequencies. Acoustic waves from the driver travel down the line and reached the blocked end. The sealed end is an infinite acoustic impedance (a situation analogous to an open-circuited electrical line) (24). This termination of the line yields a reflection coefficient of -1. At some frequency, the line length is equal to a quarter-wavelength of the acoustic signal. The reflected wave travels back down the line and strikes the driver diaphragm in such a way as to assist its motion. This produces a peak in the output. At twice that frequency, the line is a half-wave length long. In this case, the reflected wave opposes diaphragm motion and system output drops. Whenever the line length is equal to an odd number of quarter-wavelengths, there is an increase in diaphragm output. When the line is some multiple of a half-wavelength, the diaphragm output drops. Since the diaphragm is the only source of output, the response has many peaks and dips. This is certainly unsuitable for high fidelity reproduction.

The response may be smoothed by filling the line with some sort of damping material (e.g., fiberglass or long-fiber wool). This turns z_{AT} into a resistor. It also makes the system perform like a leaky closed-box system, and low efficiency results. A simple, well-designed closed-box system would clobber a Type B system of equal cabinet volume. So, Type B systems are not the optimum approach.

Type C systems, however, have some possibilities. As in a vented-box direct-radiator system, we have an additional source of output (the line mouth) to assist the driver. Once again, the line throat impedance z_{AT} varies with frequency. The line is terminated with an open aperture which has a relatively low acoustic impedance (analogous to a shorted electrical line). The behavior is opposite of the Type B; the reflection coefficient is essentially +1. When the line is an odd number of quarter-wavelengths long, the line presents a high acoustic impedance to the driver and most of the driver's output is delivered to the line.

When the line is an odd multiple of a half-wavelength long, the reflected wave will assist the driver and the driver

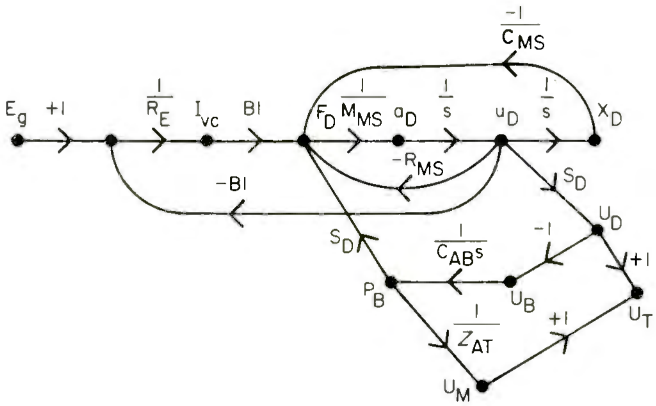


Fig. 5—Signal flow graph of a Type C transmission line loudspeaker.

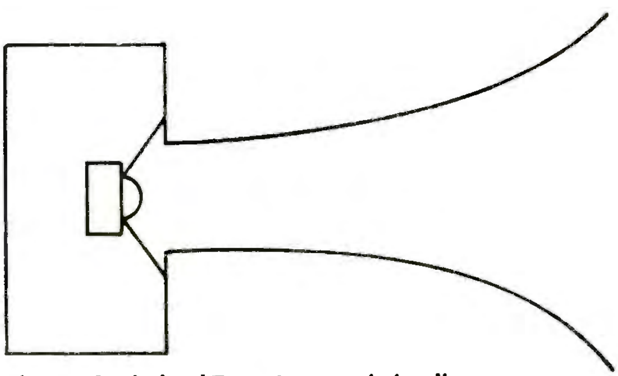


Fig. 6—Optimized Type A transmission line loudspeaker.

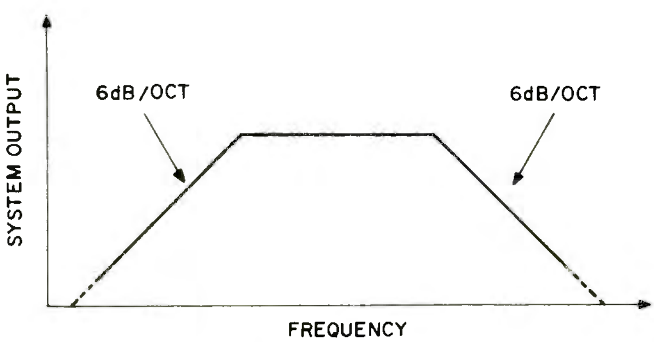


Fig. 7—Frequency response of the simplified model.

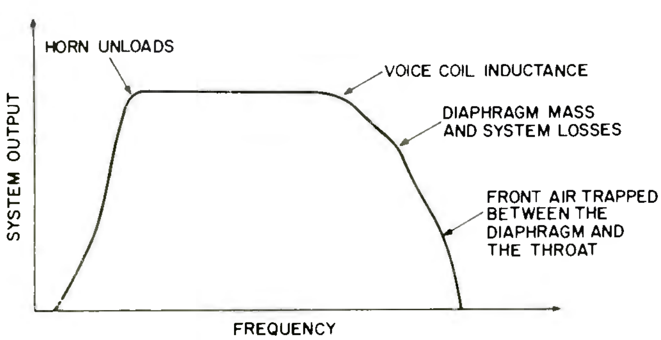


Fig. 8—Frequency response of the more complete model.

distortion are the *only* considerations, a horn-loaded Type A system is optimum. If size and price are part of the picture, then a vented-box direct-radiator (at least for the woofer) may make some sense.

Acknowledgements

The analysis of the Type B and C transmission line systems was based on a method developed by G. S. Letts. The horn analysis was based on a model originally developed by D. B. Keele, Jr., whose comments, along with those of R. H. Small, J. R. Ashley, and W. M. Leach, were most helpful.

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Fudge Factor Omission

Dear Sir:

Mrs. Edsel Murphy has asked me to thank you for the kind mention of her Fudge Shop & Speaker Works. However, she points out that someone has erred in omitting the fudge factor K from Equation 6 in the article "The Search for an Optimum Transmission Line Speaker" (*Audio,* Aug. 1977). Equation 6 should read:

$$T(s) = \frac{K_s}{s^2 + \left(\frac{2\pi f_s}{Q_{TS}} + \frac{R_{AT} S_D}{M_{MS}}\right)s + 4(\alpha + 1)^2 f_S{}^2}$$

Investigation into the cause of the missing K found a wee beastie lurking in Mrs. Murphy's shipping department. This creature had apparently eaten the K off the manuscript before it could be sent to you. Such action tends to decrease the information content of the paper and increase its entropy, we feel certain that is must be the dread Maxwell's Daemon. We trapped it in in one of Mrs. Murphy's cookie jars, and I have enclosed it for your examination. We feel that with your great experience in these matters, you can properly identify the creature.

I have heard many good remarks about the artwork for the article, particularly the "talented illustrator's" rendering of my search. While searching for an honest loudspeaker is often more difficult than searching for an honest man (it's more like searching for an honest politician), modern equipment does help. I didn't use a lantern, I used a five-cell flashlight. Also, I would like to point out that I am only *mumblety-mumble* years old. My whiskers are not quite as grey as pictured, and I still have most of my hair.

W.J.J. Hoge
Boy Wonder