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LINKWITZ-RILEY ACTIVE CROSSOVERS UP TO 8TH-ORDER: AN OVERVIEW

INTRODUCTION

In 1976, Siegfried Linkwitz published his famous paper [1] on active crossovers for non-coincident drivers. In it, he credited Russ Riley (a co-worker and friend) with contributing the idea that cascaded Butterworth filters met all Linkwitz's crossover requirements. Their efforts became known as the Linkwitz-Riley crossover alignment. In 1983, the first commercially available Linkwitz-Riley active crossovers appeared from Sundholm and Rane [2].

Today, the de facto standard for professional audio active crossovers is the 4th-order Linkwitz-Riley (LR-4) design. Offering in-phase outputs and steep 24 dB/octave slopes, the LR-4 alignment gave users the tool necessary to scale the next step toward the elusive goal of perfect sound.

Now a new tool is available: the 8th-order Linkwitz-Riley (LR-8) active crossover [3]. With incredibly steep slopes of 48 dB/octave, the LR-8 stands at the door waiting for its turn at further sound improvements. Using a LR-8 cuts the already narrow LR-4 crossover region in half. Just one octave away from the crossover frequency the response is down 48 dB. The LR-8 represents a major step closer to the proverbial brick wall, with its straight line crossover region.

Before exploring the advantages of LR-8 designs, it is instructional to review just what Linkwitz-Riley alignments are, and how they differ from traditional Butterworth designs of old.

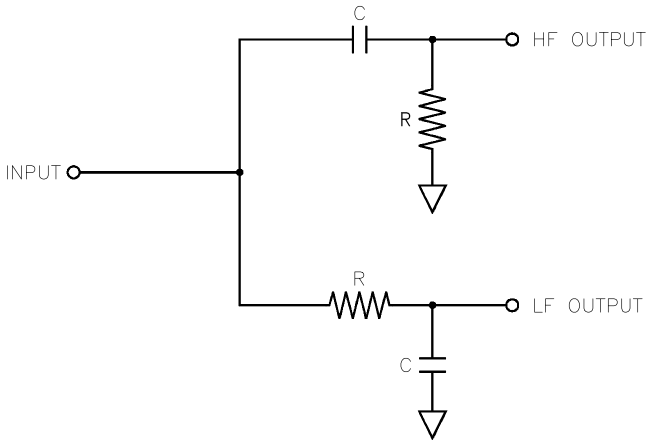

Figure 1. 1st order crossover network

1st-Order Network

It begins with a resistor and a capacitor. It never gets more complicated than that—just resistors and capacitors: lots and lots of resistors and capacitors. Resistors are the great emancipators of electronics; they are free of frequency dependence. They dissipate energy without frequency prejudice. All frequencies treated equally. Capacitors, on the other hand, selectively absorb energy; they store it, to be released at a later time. While resistors react instantly to any voltage changes within a circuit, capacitors take time to charge and discharge.

Capacitors are so frequency dependent, they **only** pass signals with frequency associated with them. Direct-current (what we call zero frequency) will not pass at all; while, at the other end of the spectrum, very high frequencies will not absorb. Capacitors act like a piece of wire to high frequencies; hardly there at all.

We use these facts to create a crossover network. Figure 1 shows such a circuit. By interchanging the positions of the resistor and capacitor, low-pass (low frequencies = LF) and high-pass (high frequencies = HF) filters result. For the low-pass case (LF), the capacitor ignores low frequencies and shunts all high frequencies to ground. For the high-pass case (HF), the opposite occurs. All low frequencies are blocked and only high frequencies are passed.

1st-Order Amplitude Response

Using 1kHz as an example and plotting the amplitude versus frequency response (Figure 2) reveals the expected low-pass and high-pass shapes. Figure 2 shows that the 1st-order circuit exhibits 6 dB/octave slopes. Also, that 6 dB/octave equals 20 dB/decade. Both ways of expressing steepness are useful and should be memorized. The rule is: **each order, or degree, of a filter increases the slopes by 6 dB/octave or 20 dB/decade**. So, for example, a 4th-order (or 4th-degree—interchangeable terms) circuit has 24 dB/octave (4x6 dB/octave) or 80 dB/decade (4x20 dB/decade) slopes.

Using equal valued resistors and capacitors in each of the circuits causes the amplitude responses to 'cross over' at one particular frequency where their respective -3 dB points intersect. This point represents the attenuation effect resulting when the impedance of the capacitor equals the resistance of the resistor.

The equivalent multiplying factor for -3 dB is .707, i.e., a signal attenuated by 3 dB will be .707 times the original in level. Ohms law tells us that if the voltage is multiplied by .707, then the current will also be multiplied by .707. Power is calculated by multiplying voltage times current. Therefore, a voltage multiplied by .707, and a current multiplied by .707, equals 0.5 power. So the -3 dB points represent the one-half power point—a useful reference.

Lastly, Figure 2 shows the flat amplitude response resulting from summing the LF and HF outputs together. This is called **constant voltage**, since the result of adding the two output voltages together equals a constant. The 1st-order case is ideal in that **constant power** also results. Constant-power refers to the summed power response for each loudspeaker driver operating at the crossover frequency. This, too, results in a constant. Since each driver operates at ½ power at the crossover frequency, their sum equals one—or unity, a constant.

1st-Order Phase Response

Much is learned by examining the phase shift behavior (Figure 3) of the 1st-order circuit. The upper curve is the HF output and the lower curve is the LF output. The HF curve starts at +90° phase shift at DC, reduces to +45° at the cross-over frequency and then levels out at 0° for high frequencies.

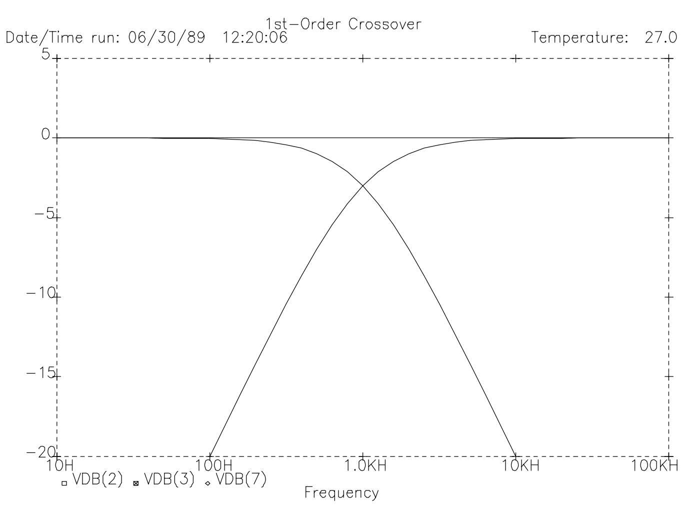

Figure 2. 1st-order amplitude response

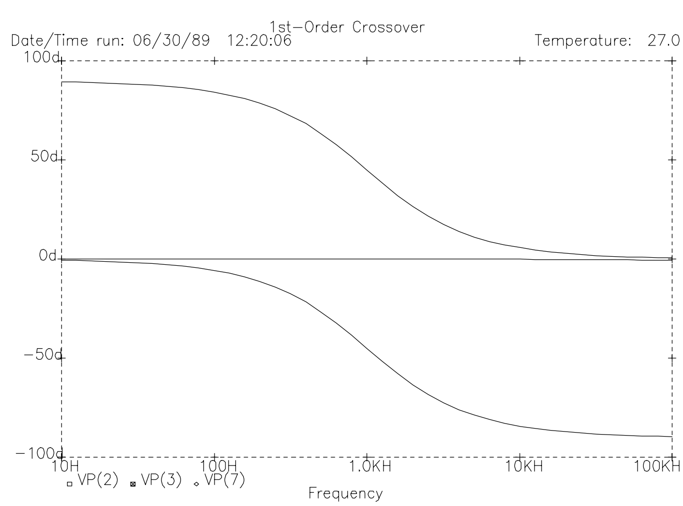

Figure 3. 1st-order response

The LF curve starts with 0° phase shift at DC, has -45° at the crossover frequency and levels out at -90° for high frequencies.

Because of its reactive (energy storing) nature each capacitor in a circuit contributes 90° of phase shift, either positive or negative depending upon its application. Since the HF section places the capacitor directly in the signal path, this circuit starts out with +90° phase shift. This is called **phase lead**. The LF section, which starts out with 0° and eventually becomes -90° is called **phase lag**.

Examination of Figure 3 allows us to formulate a new rule: **each order, or degree, of a crossover network contributes ±45° of phase shift at the crossover frequency** (positive for the HF output and negative for the LF output).

Once again, Figure 3 shows the idealized nature of the 1st-order case. Here the result of summing the outputs together produces 0° phase shift. Which is to say that the summed amplitude and phase shift of a 1st-order crossover equals that of a piece of wire.

1st-Order Group Delay Response

We shall return to our rules shortly, but first the concept of **group delay** needs to be introduced. Group delay is the term given to the ratio of an incremental change in phase shift divided by the associated incremental change in frequency (from calculus, this is the first-derivative). The units for group delay are seconds. If the phase shift is **linear**, i.e., a constant rate of change per frequency step, then the incremental ratio (first-derivative) will be constant. We therefore refer to a circuit with linear phase shift as having **constant group delay**.

Group delay is a useful figure of merit for identifying linear phase circuits. Figure 4 shows the group delay response for the Figure 1 1st-order crossover circuit. Constant group delay extends out to the crossover region where it gradually rolls off (both outputs are identical). The summed response is, again, that of a piece of wire.

The importance of constant group delay is its ability to predict the behavior of the LF output **step response**. A circuit with constant group delay (linear phase shift) shows no overshoot or associated damping time to a sudden change (step) in input level (Figure 5). The circuit reacts smoothly to the sudden change by rising steadily to meet the new level. It does not go beyond the new level and require time to settle back. We also refer to the step response as the **transient** response of the circuit. The transient response of the summed outputs is perfect since their sum is perfectly equal to one.

For clarity purposes normally only the step response of the LF network is shown. Nothing is learned by examining the step response of the HF network. A step response represents a transition from one DC level to another DC level, in this case, from -1 volt to +1 volt. A HF network, by definition, does not pass DC (neither does a loudspeaker), so nothing particularly relevant is learned by examining its step response. To illustrate this, Figure 5 shows the HF step response. It begins and ends with zero output since it cannot pass DC. The sharp edge of the input step, however, contains much high frequency material, which the HF network passes. So, it begins at zero, passes the high frequencies as a pulse, and returns to zero.

The HF and LF outputs are the exact complement of each other. Their sum equals the input step exactly as seen in Figure 5. Still, we learn everything we need to know by examining only the LF step response; looking for overshoot and ringing. From now on, just the LF output will be shown.

Vector Diagrams

A vector is a graphical thing (now we're getting technical) with magnitude and direction. We can use vectors to produce diagrams representing the instantaneous phase shift and amplitude behavior of electrical circuits. In essence, we freeze the circuit for a moment of time to examine complex relationships.

We shall now apply our two rules to produce a vector diagram showing the relative phase shift and amplitude performance for the 1st-order crossover network at the single crossover frequency (Figure 6a). By convention, 0° points right, +90° points up, <R>-90° points down, and ±180° points left. From Figures 2 & 3 we know the HF output amplitude is <R>-3 dB with +45° of phase shift at 1 kHz, and the LF output is -3 dB with -45° phase shift. Figure 6a represents the vectors as being .707 long (relative to a normalized unity vector) and rotated up and down 45°. This shows us the relative phase difference between the two outputs equals 90°.

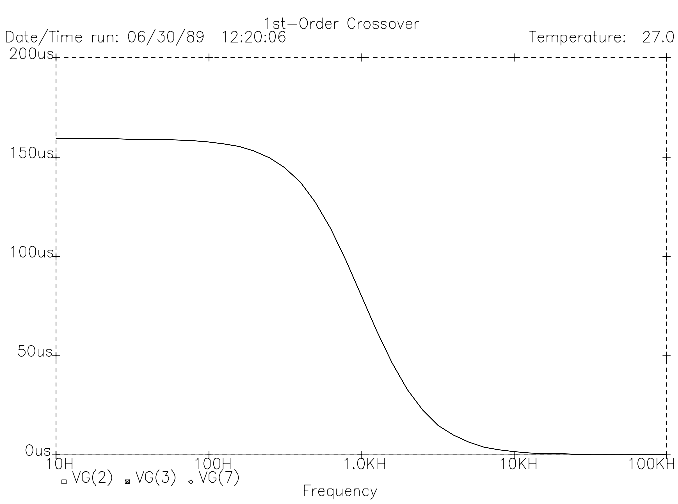

Figure 4. 1st-order group delay response

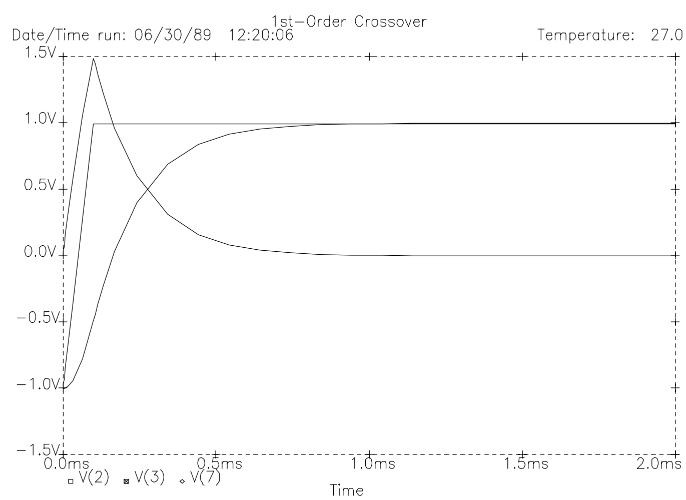

Figure 5. 1st-order transient response

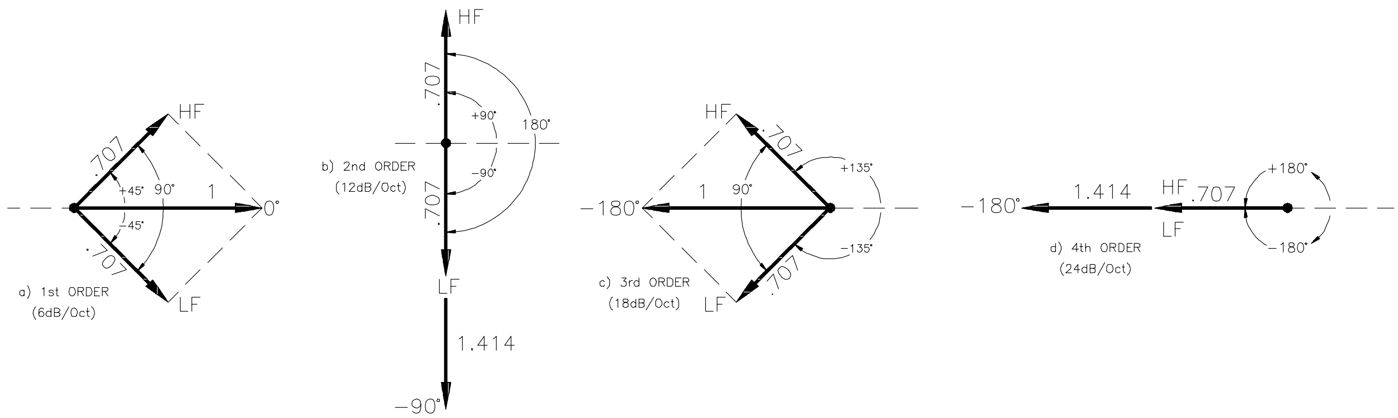

Figure 6. 1st through 4th-order butterworth vector diagrams

Next we do vector addition to show the summed results. Vector addition involves nothing more complex than mentally moving one of the vectors to the end of the other and connecting the center to this new end point (it is like constructing a parallelogram). Doing this, results in a new vector with a length equal to 1 and an angle of 0°. This tells us the recombined outputs of the HF and LF networks produce constant voltage (i.e., a vector equal to 1), and is in phase with the original input of the circuit (i.e., a vector with 0° phase rotation).

The 1st-order case is ideal when summed. It yields a piece of wire. Since the responses are the exact mirror images of each other, they cancel when summed, thus behaving as if neither was there in the first place. Unfortunately, all optimized higher order versions yield flat voltage/power response, group delay or phase shift, **but not all at once**. Hence, the existence of different alignments and resultant compromises.

2nd, 3rd & 4th-Order Butterworth Filters

There are many types of filters (most named after mathematicians). Each displays a unique amplitude characteristic throughout the passband. Of these, only **Butterworth** filters have an absolutely flat amplitude response. For this reason, Butterworth filters are the most popular for crossover use. Butterworth filters obey our two rules, so we can diagram them for the 2nd, 3rd and 4th-order cases (Figures 6b-6d). The 2nd-order case has ±90° phase shift as shown. This results in the outputs being 180° out of phase. Vector addition for this case produces a zero length vector, or complete cancellation. The popular way around this is to reverse the wiring on one of the drivers (or, if available, electronically inverting the phase at the crossover). This produces a resultant vector 90° out of phase with the input and 3 dB (1.414 equals +3 dB) longer. This means there will be a 3 dB amplitude bump at the crossover region for the combined signals.

The 3rd-order Butterworth case (Figure 6c) mimics the 1st-order case at the crossover frequency, except rotated 180°. Hence, we see the HF vector rotated up 135° (3x45°) and the LF vector rotated down the same amount. The phase shift between outputs is still 90°. The resultant is constant voltage (unity) but 180° out-of-phase with the input.

The 4th-order Butterworth diagram (Figure 6d) shows the HF vector rotated up 180° and the LF vector rotated down the same amount. The phase difference between outputs is now zero, but the resultant is +3 dB and 180° out-of-phase with the input. So, the 4th-order and the inverted phase 2nd-order produce 3 dB bumps at the crossover frequency.

Linkwitz-Riley Alignment

Two things characterize a Linkwitz-Riley alignment: 1) In-phase outputs (0° between outputs) at *all* frequencies (not just at the crossover frequency as popularly believed by some) and 2) Constant voltage (the outputs sum to unity at all frequencies).

Linkwitz-Riley in-phase outputs solve one troublesome aspect of crossover design. The acoustic lobe resulting from both loudspeakers reproducing the same frequency (the crossover frequency) is always on-axis (not tilted up or down) and has no peaking. This is called zero lobing error. In order for this to be true, however, both drivers must be in correct time alignment, i.e., their acoustic centers must lie in the same plane (or electrically put into equivalent alignment by adding time delay to one loudspeaker). Failure to time align the loudspeakers defeats this zero lobing error aspect. (The lobe tilts toward the lagging loudspeaker.)

Examination of Figure 6 shows that the 2nd-order (inverted) and 4th-order Butterworth examples satisfy condition 1), but fail condition 2) since they exhibit a 3 dB peak. So, if a way can be found to make the amplitudes at the crossover point -6 dB instead of -3 dB, then the vector lengths would equal 0.5 (-6 dB) instead of .707 (-3 dB) and sum to unity—and we would have a Linkwitz-Riley crossover.

Russ Riley suggested cascading (putting in series) two Butterworth filters to create the desired -6 dB crossover points (since each contributes -3 dB). Voila! Linkwitz-Riley alignments were born.

Taken to its most general extremes, cascading any order Butterworth filter produces 2x that order Linkwitz-Riley. Hence, cascading (2) 1st-order circuits produces a 2nd-order Linkwitz-Riley (LR-2); cascading (2) 2nd-order Butterworth filters creates a LR-4 design; cascading (2) 3rd-order Butterworth filters gives a LR-6, and so on. (Starting with LR-2, every other solution requires inverting one output. That is, LR-2 and LR-6 need inverting, while LR-4 and LR-8 do not.)

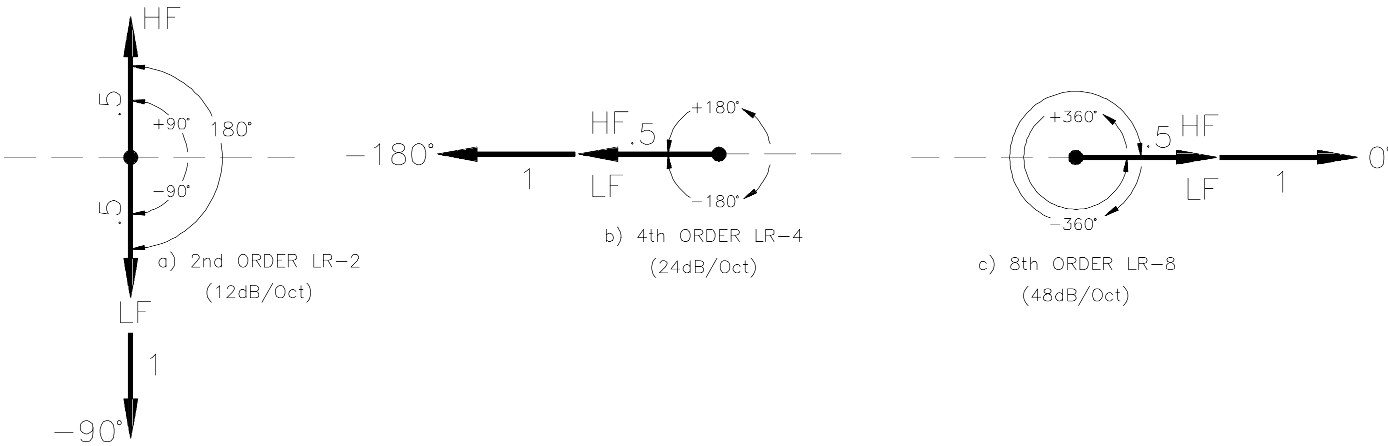

Figure 7. Linkwitz-Riley vector diagrams for 2nd to 8th-order cases.

LR-2, A Transient Perfect 2nd-Order Crossover

As an example of this process, let's examine a LR-2 design. Referring to Figure 1, all that is required is to add a buffer amplifier (to avoid interaction between cascaded filter components) to each of these two outputs and then add another resistor/capacitor network identical to the first. We now have a 2nd-order Linkwitz-Riley crossover.

The new vector diagram looks like Figure 7a. Each vector is .5 long (from the fact that each 1st-order reduces by 0.707, and .707 x .707 = .5) with phase angles of $\pm 90^o$. Since the phase difference equals 180^o, we invert one before adding and wind up with a unity vector 90^o out of phase with the original.

Figure 8 shows the amplitude response. The crossover point is located at -6 dB and the slopes are 12 dB/octave (40 dB/decade). The summed response is perfectly flat. Figure 9 shows the phase response. At the crossover frequency we see the HF output (upper trace) has $+90^o$ phase shift, while the LF output (lower trace) has -90^o phase shift, for a total phase difference of 180^o. So, we invert one before summing and the result is identical to the LF output.

These results differ from the 1st-order case in that the summed results do not yield unity (a piece of wire), but instead create an all-pass network. (An all-pass network is characterized by having a flat amplitude response combined with a smoothly changing phase response.) This illustrates Garde's [4] famous work.

Cascading two linear phase circuits results in linear phase, as shown by the constant group delay plots (all three identical) of Figure 10. And constant group delay gives the transient perfect LF step response shown in Figure 11.

LR-4 and LR-8 Alignments

Looking back to Figure 7b., we see the vector diagrams for 4th and 8th-order Linkwitz-Riley designs. The LR-4 design shows the resultant vector is unity but 180^o out of phase with the input at the crossover frequency.

Cascading (2) 4th-order Butterworth filters results in an 8th-order Linkwitz-Riley design. Figure 7c. shows the vector diagram for the LR-8 case. Here, we see the phase shift for each output undergoes 360^o rotation returning to where it began. The resultant vector is back in phase with the original input signal. So, not only, are the outputs in phase with each other (for all frequencies), they are also in phase with the input (at the crossover frequency).

8th-Order Comparison

A LR-8 design exhibits slopes of 48 dB/octave, or 160 dB/decade. Figure 12 shows this performance characteristic compared with the LR-4, 4th-order case for reference. As expected, the LR-4 is 80 dB down one decade away from the corner frequency, while the LR-8 is twice that, or 160 dB

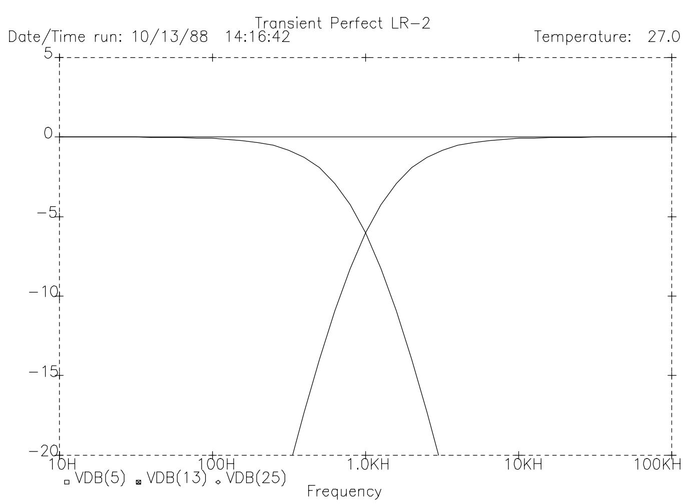

Figure 8. LR-2 amplitude response

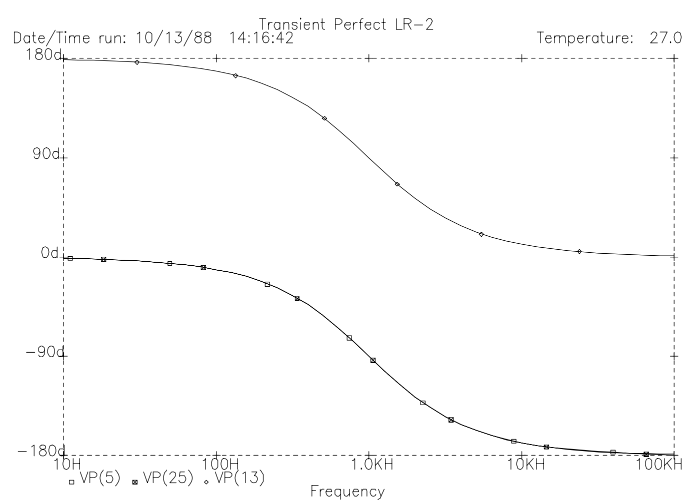

Figure 9. LR-2 phase response

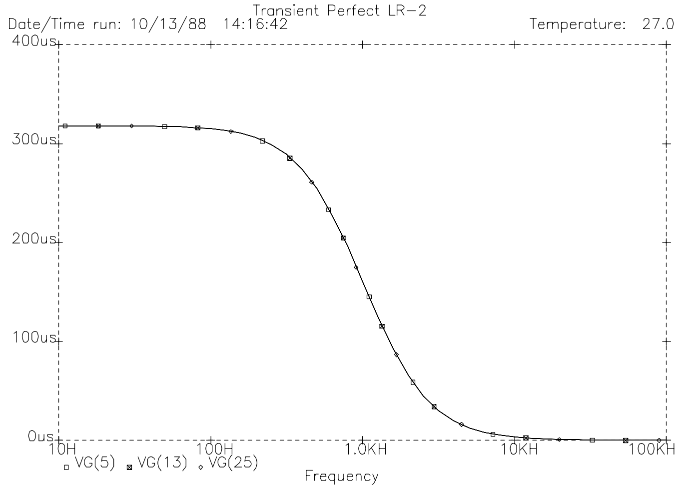

Figure 10. LR-2 group delay

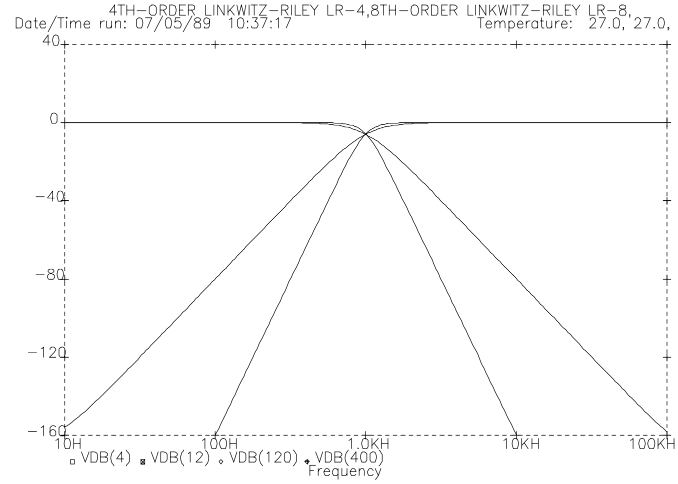

Figure 12. LR-4 and LR-8 slopes

down. Of interest here, are the potential benefits of narrowing the crossover region by using a LR-8 alignment.

Figure 13 magnifies the responses shown in Figure 12 to reveal a clearer picture of the narrower crossover region, as well as showing the flat summed responses. (The slight difference in summed amplitudes at the crossover frequency is due to a slight gain difference between the two circuits.) The critical crossover region for the LR-8 case is one-half of what it is for the LR-4 case. The exact definition of where the crossover region begins and ends is ambiguous, but, by whatever definition, the region has been halved.

As an example of this, a very conservative definition might be where the responses are 1 dB down from their respective passbands. We would then refer to the crossover region as extending from the -1 dB point on the low-pass response to the -1 dB point on the high-pass response. For LR-8, these points are 769 Hz and 1301 Hz respectively, yielding a crossover region only ¾-octave wide. As a comparative reference, the LR-4 case yields -1 dB points at 591 Hz and 1691 Hz, for a 1.5-octave wide region.

For the LR-8 case, it is interesting to note that the -1d B point on the low-pass curve corresponds almost exactly to the -20 dB point on the high-pass curve (the exact points occur at 760 Hz and 1316 Hz). So if you want to define the region as where the response is down 20 dB, you get the same answer. The entire region for the LR-8 case is ¾-octave wide, or it is one-half this number for each driver. That is, the loudspeaker driver (referred to as ‘driver’ from now on) has to be well behaved for only about 0.4 octave beyond the crossover point. This compares with the 4th-order case where the same driver must behave for 0.8 octave.

The above is quite conservative. If other reference points are used, say, the -3 dB points (895 Hz & 1117 Hz), then the LR-8 crossover region is just 1/3-octave wide, and drivers only have to stay linear for 1/6-octave. (1/6-octave away from the crossover frequency the drive signal is attenuated by 12 dB, so the output driver is operating at about 1/16 power.)

The extremely steep slopes offer greater driver protection and linear operation. Beyond the driver’s linear limits all frequencies attenuate so quickly that most nonlinearities and interaction ceases being significant. Because of this, the driver need not be as well behaved outside the crossover frequency. It is not required to reproduce frequencies it was not designed for. For similar reasons, power handling capability can be improved for HF drivers as well. And this narrower crossover region lessens the need for precise driver time

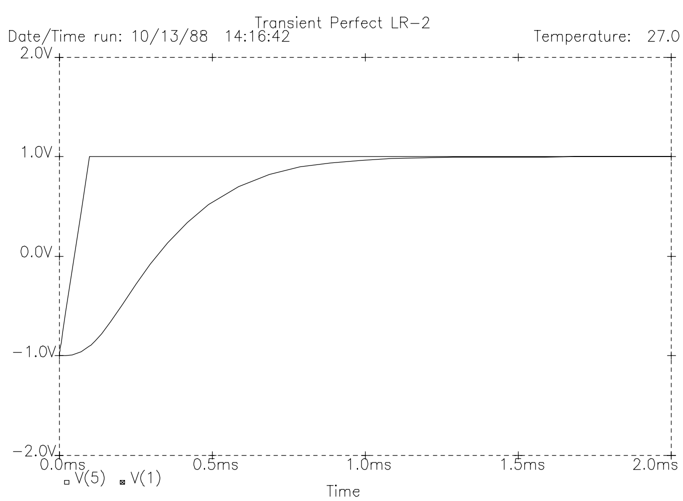

Figure 11. LR-2 transient response

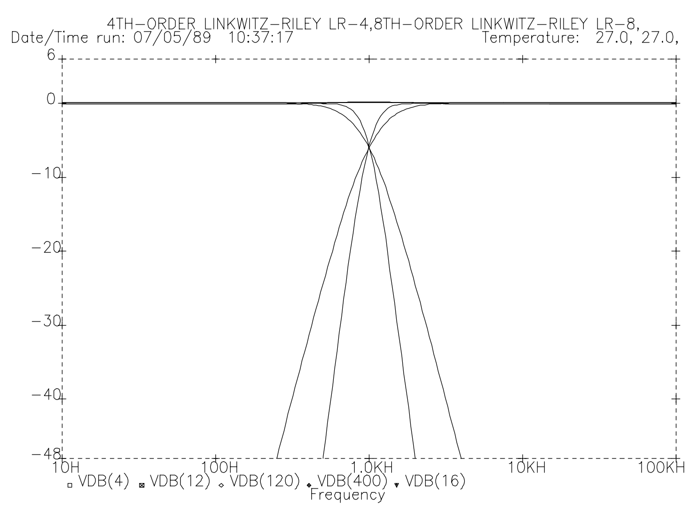

Figure 13. Figure 12 mangnified

alignment since the affected spectrum is so small.

The caveat, though, is an increased difficulty in designing good systems with sharp slopes. The loudspeakers involved have differing transient responses, polar patterns and power responses. This means the system designer must know the driver characteristics thoroughly. Ironically, sometimes loudspeaker overlap helps the system blend better even when on-axis amplitude response is flat.

LR-8 Phase Response

Figure 14 shows the respective phase response for LR-4 (upper trace) and LR-8 (lower trace) designs. As predicted by the vector diagram in Figure 7b, the LR-4 case has 180^{o} ($4x45^{o}$) of phase shift at the crossover frequency. Thus, the output signal is out-of-phase with the input signal at the crossover frequency for the LR-4 case. Both outputs are in-phase with each other, but out-of-phase with the input.

The LR-8 design eliminates this out-of-phase condition by bringing the outputs back in sync with the input signal at the crossover frequency. The lower trace shows the 360^{o} phase shift for the LR-8 alignment.

LR-8 Transient Response

Butterworth functions do not have linear phase shift and consequently do not exhibit constant group delay. (First-order networks are not classified as Butterworth.) Since Linkwitz-Riley designs (higher than LR-2) are cascaded Butterworth, they also do not have constant group delay.

Group delay is just a measure of the non-linearity of phase shift. A direct function of non-linear phase behavior is overshoot and damping time for a step response. The transient behavior of all Linkwitz-Riley designs (greater than 2nd-order) is classic Butterworth in nature. That is, the filters exhibit slight overshoot when responding to a step response, and take time to damp down.

Figure 15 compares LR-8 and LR-4 designs and shows the greater overshoot and damping time for the 8th-order case. The overshoot is 15% for the LR-4 case and twice that, or about 30%, for the LR-8 case. As expected, the LR-8 design takes about twice as long to damp down. The initial rise-time differences are due to the group delay value differences.

Is It Audible?

The conservative answer says it is not audible to the overwhelming majority of audio professionals. Under laboratory conditions, some people hear a difference on non-musical tones (clicks and square waves).

The practical answer says it is not audible to anyone for real sound systems reproducing real audio signals.

Linkwitz-Riley Power Response

Linkwitz-Riley alignments produce constant voltage response (voltage vectors sum to unity) at the crossover frequency, but they do not produce constant power. At the crossover frequency, each voltage output is ½ of normal. This produces ½ the normal current into the loudspeakers. Since power is the product of voltage times current, the power is ¼ of normal. Considering a simple two-way system, the combined total power at the crossover frequency will be ½ of normal (¼ from each driver), producing a dip of 3 dB at the crossover frequency in the overall power response.

For LR-2 designs, this may be a practical problem, but for all higher ordered Linkwitz-Riley designs, it is not. The reason is due to the steep slopes for LR-4 (and higher) designs. The steep slopes reduce the crossover region to such a small spectral area that this power dip is rarely a real problem.

Future of LR-8 Designs

The ultimate success of the LR-8 design is yet to be determined. It is up to the system designers and end-users to evaluate the design and decide whether the positives outweigh the negatives, and the performance factors offset the additional cost.

Meanwhile, you've got a new hammer to swing.

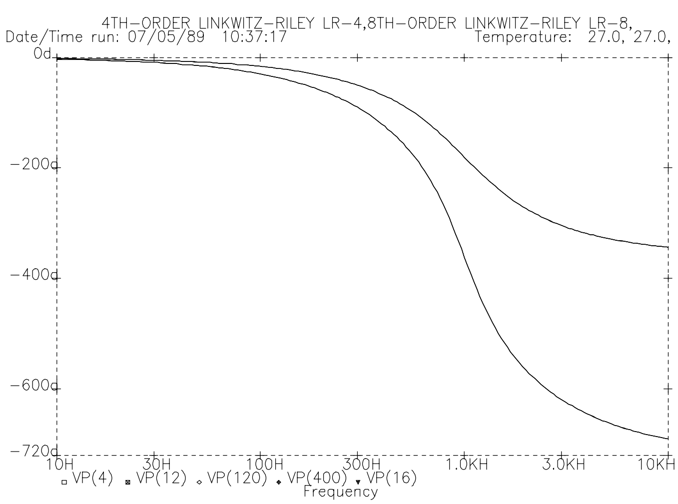

Figure 14. LR-4 and LR-8 phase response

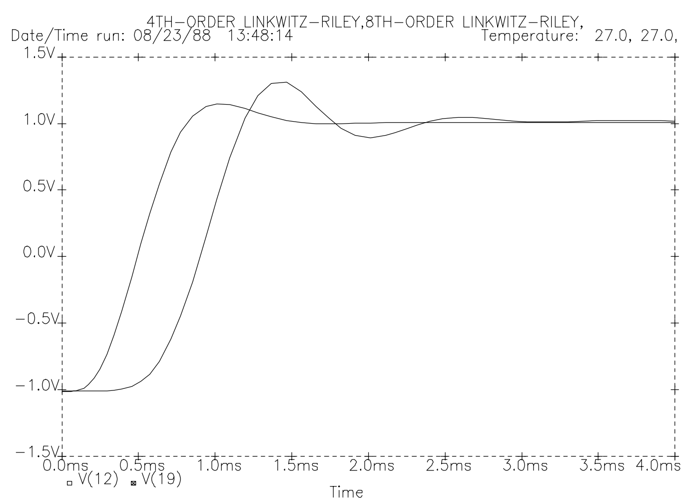

Figure 15. LR-4 and LR-8 transient response

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