NORMAN S. CROWHURST

db Theory & Practice

Use of Analogs: Part I

• Some recent questions I have received have related to comparisons among different kinds of loudspeaker enclosures, such as acoustic suspension, loaded reflexes, and so forth, and some problems in modifying such designs, or in converting from one to the other. In tackling these procedures, it is not long before someone tries to draw an analog diagram to see what he is doing. And that is where the problems in understanding *really* start. If you study one of the classic textbooks, you will find a whole set of analogs rather than just one, which seems to add to our confusion.

An excellent paperback published as a translation in English, by Bent Gehlshøj from the Danish Academy of Technical Sciences, in 1947, has been sitting on my bookshelf (but is well worn) for many years. Here in TABLE I will repeat a table of mechanical analogies from that book:

VELOCITY

The acoustical analogy is similar, but complicated by the fact that *volume* current, rather than just current, substitutes for velocity. The introduction of area through which air flows adds an extra dimension that is not in the mechanical or electrical systems. But that is not the main cause of most peoples' difficulty.

The real problem is more basic than that. In a mechanical system, all units break down to simple integral exponents of basic dimensions. Starting with the fundamental dimensions of d for distance, or length, m for mass, and t for time, velocity is d/t, acceleration is d/t^2 and force is md/t^2.

From that, mechanical impedance is force divided by velocity, which becomes m/t. You can complete the table, if you wish. But now turn to the electrical analogies. Using basic definitions of force and motion, work done, etc., we derive units in terms of mechanical basics, based on electrostatic or electromagnetic phenomena.

TABLE 1

Mechanical system		Analogy I Impedance Diagram		Analogy II Admittance Diagram	
force	f	voltage	v	current	i
velocity	v	current	i	voltage	v
impedance	z	impedance	z	admittance	Y
mass	m	inductance	L	capacitance	C
compliance	c	capacitance	C	inductance	L
elastic energy	$\frac{1}{2}cf^2$		$\frac{1}{2}Cv^2$		$\frac{1}{2}Li^2$
kinetic energy	$\frac{1}{2}mv^2$		$\frac{1}{2}Li^2$		$\frac{1}{2}Cv^2$

Table 1. The table of mechanical analogies.

Whichever you use, you end up with fractional exponents of m and d, and when you correlate the two systems the ratio of their dimensions is always a velocity (d/t), a reciprocal of a velocity, or the square of a velocity, or its reciprocal.

You know by now, of course, that this velocity is the propagation velocity of electromagnetic waves. But the problem arises because we have difficulty thinking about fractional exponents as dimensions. I have met a lot of people who have trouble with the exponent in acceleration, d/t^2. When that is compounded with m and d to various fractional powers, we tend to give up trying to think in those terms at all!

What was a help to me was the theory of interrelation between electric currents and magnetic fields as they appear in inductances and transformers. First take a simple, solid cored, gapped inductor (FIGURE 1).

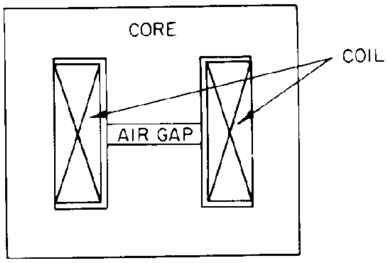


Figure 1. Cross-section of inductance, using a core with an air gap. How does inductance depend on the gap dimensions?

Suppose the air gap is 1 inch square by 0.2 inch between the poles and the inductance of 1000 turns on such a core is 420 millihenries. What will be the inductance if the gap is reduced to 0.1 inch, or if it is increased to 0.4 inch?

You think of the analogy between magnetic field and an electrical circuit. Reducing the length of the magnetic path in air will reduce the magnetic potential needed; increasing the length will increase it. So you might conclude that reducing the gap would reduce inductance—increasing the gap would increase it. But you would be wrong.

Inductance depends on the *permeability* of the magnetic circuit which, in electrical analogy, is equivalent to admittance. So reducing the gap from 0.2 inch to 0.1 inch will increase the overall permeability of the magnetic circuit to about twice because most of it is in the air gap. Reducing the air gap from 0.2 inch to 0.1 inch will increase inductance from 420 millihenries to about 840 millihenries.

Going the other way will reduce inductance. But a gap of 0.4 inch will cause considerable fringing of the field, increasing the effective area considerably above 1 square inch. So the inductance will not fall to half, but to something above that.

LEAKAGE INDUCTANCE

So much for simple inductance using a core, the main reluctance of which—the air gap—can be adjusted or controlled. Now let us turn our attention to something a little more complicated—leakage inductance. First consider it as two windings, as in a transformer, on the same core (FIGURE 2).

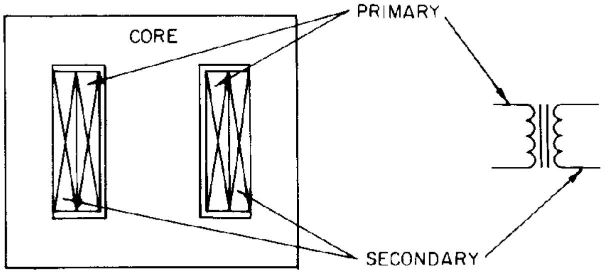


Figure 2. Cross-section of an ordinary transformer, to show relationships relative to transformation.

The main inductance is due to the high-permeability core, which is a complete magnetic circuit. If one winding has 1000 turns and the other has 2000 turns, and the permeability of the core is very high, the inductance of the 1000 turns may be more than 100 henries and that of the 2000 turns will be more than 400 henries.

I say *more than* because the inductance of a high-permeability core carrying a winding is not only high, but variable. Normally other values in the circuit will be such that this inductance is not critical anyway. So, suppose the 2,000 turn winding connects to a load of 10,000 ohms and delivers 4 volts across it. The current into the 10,000 ohms will be 400 microamps.

Now, by transformer action, because the inductances are so high that negligible current flows due to them, the 1000 turns will require 2 volts across it to produce the 4 volts across the 2000 turns. And the 1000 turns will require 800 microamps into that to produce the 400 microamps taken from the 2000 turns. What does this mean?

A resistance that takes 800 microamps at 2 volts is 2,500 ohms, or one fourth the load connected to the 2000-turn winding. Impedance is transformed by the square of the turns ratio.

LEAKAGE BETWEEN THE WINDINGS

But so far we have not considered leakage inductance *between* the windings. If the main inductance is in hundreds of henries, the leakage inductance of a good transformer will be in tens of millihenries. But here is the question: how does leakage inductance depend on the dimensions of the windings and their disposition one to the other?

One simple configuration is shown in FIGURE 3, where dimensions l_t, L_L

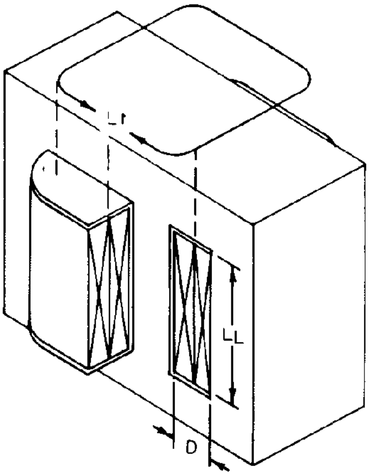


Figure 3. Dimensions of transformer windings, relevant to leakage inductance. How does leakage inductance depend on these dimensions?

and D are identified. How does leakage inductance vary as each of these dimensions is changed, keeping the others constant? The best way to resolve this is to get a clear picture in your mind what leakage inductance really is—an inductance due to imperfection in coupling.

In one sense, it is like the inductance due to the air gap of FIGURE 1. Lengthening the leakage flux path will reduce the inductance while increasing its area so there is a greater amount of leakage flux will increase the leakage inductance. So applying that principle, increasing L_t and D will increase inductance; increasing L_L will reduce leakage inductance.

In transformer design, especially for audio work, the usual problem is to keep leakage inductance as small as possible and primary inductance, due to the core, as large as possible. Pursuing the techniques by which this can be done is a very interesting study, but there is an even more interesting application. Instead of regarding leakage inductance as an enemy, to be minimized, can we actually use it, as an element in filter design. for example?

USED IN FILTERS

Over the years, I have designed some quite successful filters in which leakage inductance has been used as an element. It has several advantages over ordinary inductances that I will come to later. For now, I want to settle something that comes in the realm of theory and practice and that has caused many arguments over the years, some of which have been settled only by demonstrating that it works!

Many transformer books treat leakage inductance as being the amount by which a coupling factor falls short of 100 per cent. In a good audio transformer, the coupling factor is probably 99.99 per cent, or even better. But there is a danger in viewing it that way. The primary inductance, which is 100 per cent, uses a core of some magnetic material with permeability and thus has an essentially variable inductance.

So many people believe that because leakage inductance is the remaining 0.01 per cent, or whatever it is, that it must be variable too. The fact is, it is not, because the path of the leakage flux to which this inductance is due is totally in air, or at least in nonmagnetic material, such as the turns of the windings. But, while it is thus a sort of *air-cored* inductance, it can have a higher Q than ordinary air-cored inductors of the same size, because the use of a core changes the basic dimensions very radically.

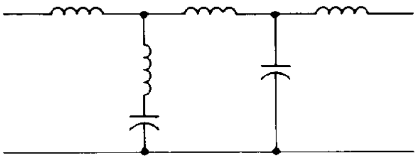


Figure 4. A form of m-derived low-pass filter that can easily be tailored to use leakage inductance for all the inductor elements. How would you go about designing it? Answer next month!

Now, how do you use this inductance to build a filter? FIGURE 4 shows a filter that lends itself very well to the use of leakage inductance. Space has gone for this month, so I suggest that, as an exercise in theory and practice, or if you prefer, an exercise in how new methods of instruction, leaving you with a challenge, can be employed, you spend some odd moments between now and when next month's issue arrives trying to figure out how to build the filter. I'll go into it reasonably thoroughly (after all, this is not an inductance design course) next month. Then, you can check your own findings with mine! ■

NORMAN S. CROWHURST

db Theory & Practice

Use of Analogs: part 2

● Last month I left you with a problem that is a useful half-way introduction to the use of analogs. I finished up showing you the physical dimensions that determine leakage inductance between windings, without considering how you will represent such an inductance in a schematic. This is where it gets into a kind of analog situation.

I did point out something that has been argued from time to time, that leakage inductance is highly linear because it is effectively an "air-cored" inductance even on an iron-cored transformer, unless you go to the trouble of inserting magnetic material in the leakage path—which is unusual but has been done.

Figure 3 of last month's article indicated that most winding spaces are longer than they are deep. Figure 1 repeats that sketch, with an additional one to illustrate the point. If the space occupied by the windings is approximately three times as long as it is deep and if it is totally occupied by two windings, a primary and a secondary, what will be the comparative leakage inductances between the two windings, arranged in the alternative manners shown?

Because the roles of the dimensions identified as L_L and D are interchanged, the leakage inductance, assuming the same number of turns in each arrangement will be approximately nine times the value, in the arrangement on the right, of that on the left. And, like the primary form of inductance, its value is proportional to the square on the number of turns in the winding.

Suppose one winding has 1,000 turns, and the other one has 1,500 turns, what does that mean? All right, suppose that, in the arrangement on the left, the leakage inductance referred to the 1,000-turn winding is 10 millihenries. Then, referred to the 1,500-turn winding, it will be 1.5 squared, or 2.25 times this, which is 22.5 millihenries.

Now, going to the arrangement at the right, referred to the 1,000-turn winding, the leakage inductance will be nine times the value, or 90 millihenries. Referred to the 1500-turn winding, it will be 202.5 millihenries.

CAPACITOR ACROSS THE WINDING

Next, as a step toward designing the filter I introduced at the end of last month's column, what would be the effect of connecting a capacitor across the 1,500-turn winding, say a 0.01 microfarad value? This depends on where you look at the circuit. Let us take the arrangement on the right because that most nearly corresponds with the arrangement we shall use to design the filter.

A capacitor of 0.01 microfarad will tune an inductance of 202.5 millihenries to about 3,500 hertz. Looked at through the transformer, the capacitor will have its reactance divided by 2.25 or its capacitance value multiplied by 2.25 to look like a value of 0.0225 from the 1000-turn side. So from that side, it looks like 90 millihenries with 0.0225 microfarad, which also tunes to about 3,500 hertz.

Figure 2 shows how the circuit will look with external impedance components. On the secondary, or 1,500-turn side, the capacitor is in parallel with any other load impedance connected. But on the primary, or 1,000-turn side, the leakage inductance and capacitance are in series.

This means that if the impedance on the 1,000-turn side is very low in value, and the impedance on the 1,500-turn side is very high except for the 0.01 microfarad capacitor, the equivalent circuit looks like a series-resonant circuit from the 1,000-turn side and a parallel-resonant circuit from the 1,500-turn side. The Q-value of the resonance will increase by making the impedance on the 1,000-turn side lower and by making the impedance on the 1,500-turn side higher.

The fact is that this arrangement can act as a resonant, as well as an ordinary, transformer in the same package. The "operative impedance" of the resonant transformer will be $\sqrt{L/C}$, which figures to 2,000 ohms on the 1,000-turn side, or 4,500 ohms on the 1,500-turn side. What this means is that at the resonant frequency, putting 200 ohms on the primary will look like 45,000 ohms on the secondary, or vice versa.

At that frequency, a 200-ohm resistance on the 1,000-turn side will provide a maximum-energy transfer match for a 45,000-ohm resistance on the 1,500-turn side. Change the resistance on the 1,500-turn side to 22,500 ohms, half the previous value, and the matching resistor on the 1,000-turn side will be double the previous value, or 400 ohms.

BUILDING A FILTER

When building the filter shown at Figure 3, which was mentioned at the end of last month's column, each of the inductors can be a leakage inductance. Figure 4 shows a possible physical configuration in cross-section. The first inductance, L_1 in Figure 3, can be the leakage inductance between section A and section B of Figure 4. The second inductance, L_2 in

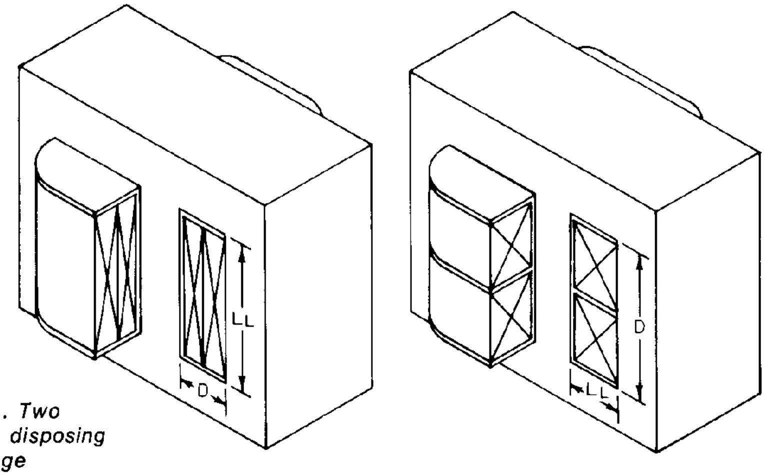


Figure 1. Two ways of disposing of leakage inductance within a transformer winding.

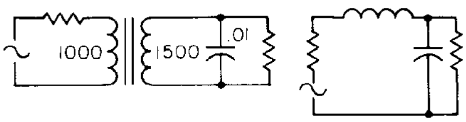


Figure 2. Circuit and equivalent circuit of leakage inductance, tuned with a capacitance.

FIGURE 3, can be the leakage inductance between section B and section C of FIGURE 4.

So across section C you connect capacitor C_1. One advantage of this kind of design is that you make the turns in section C suitable for using some convenient standard value for C_1, just as the 1,000 to 1,500 turns ratio transformed the effective value of the capacitor connected across the second winding. Nothing but the capacitor is connected across section C.

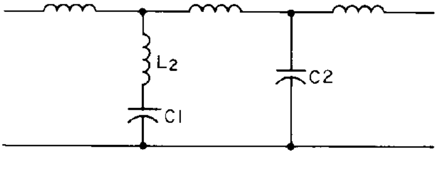
Now, the relative widths of spaces A and C in conjunction with the one common to both, section B, determine the relative values of L_1 and L_2. In practice what you do to design this part of the circuit is to arrange spaces A, B and C, so that the leakage inductances are in the ratio determined by the theoretical design. You then pick the number of turns on section A to suit the designed input impedance and the number of turns on section C to suit the capacitor value you intend to use as standard.

Now we have two inductors and a capacitor of the filter. Winding B provides the output point for connecting to the other elements, for which we use spaces D, E, and F, which have to be equal in their total to the total of A, B, and C, as a matter of physical convenience.

We make the spaces D, E, and F to suit the ratio between the remaining two inductances, D to E, making L_3 and E to F, making L_4. Now what we have to do is make the two sets fit. As sections A, B, and C are on a different core limb from sections D, E, and F, windings B and D may need to have different numbers of turns. The coupling is achieved by connecting the two windings together.

The difference in the number of turns will arrange that the total space A + B + C is equal to the total space D + E + F when the relationship between inductances L_1 and L_2, and L_3

Figure 3. The filter circuit for which leakage inductance is to be used. (See text to find out how values are realized.)



and L_0, is satisfied. The turns on space E are designed to suit the standard capacitor used for C_2 in the same way used for section C.

At this point we have a complete filter which will match any convenient values of capacitor, to the required design values, and will also match different input and output impedances, something that is not possible with any other way of constructing this kind of low pass filter.

REVIEW

If you are not sure how that worked, go over it again carefully. The relationship between the first two inductors is fixed by the relative spaces, A, B, and C. Their effective value is fixed by the turns used on A. The relationship between the second two inductors is fixed by the relative spaces, D, E, and F. Their effective value is fixed by the turns in D, relative to the turns in B. Any sets of turns in the correct ratio, which will not usually be far from equal, will serve here, just changing the voltage at which the transfer is made. For example, if the required ratio proves to be 1.15, you could use 1,000 turns on B and 1,150 turns on D, or 2,000 turns on B with 2,300 turns on D equally well.

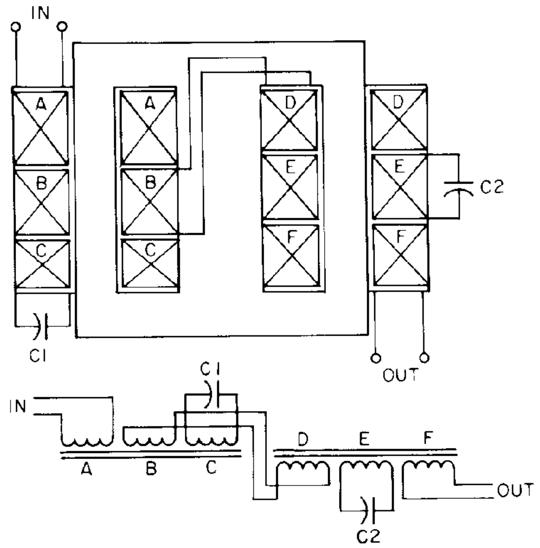


Figure 4. Physical configuration of windings to achieve a filter circuit.

Finally, the relationship between the effective values of the capacitors, and the value actually used can be adjusted by the number of turns wound in spaces C and E while the output impedance can be adjusted by changing the turns of F.

This is a very interesting design procedure, and provides intriguing possibilities as a practical matter for isolation of the various circuits, not possible with any ordinary low pass filter.

TWO TRANSFORMERS

Where is the trick, you may be asking? Well, of course, any transformer also performs a high-pass function as well as a low-pass function. This is really two transformers, using a common core. Because you use two legs of the same core, each leg will have its own primary inductance, and thus you have two inductances that I have not talked about so far.

These inductances have magnetic cores, which the leakage inductances we have been talking about do not. If they should saturate anywhere in the pass band, below the low-pass cut-off frequency, they could introduce distortion. Or if, in conjunction with the circuit impedances, these inductances produce a high-pass action or low-frequency loss within the desired pass band, that could detract from your intended performance.

So you really do not get something for nothing. But the use of leakage inductances, which have physically controlled values based on the dimensions of the structure, are highly stable, and possess considerably better Q values than simple air-cored types, can be quite attractive. By being inside the core, although using air spaces, they are effectively shielded from outside fields, which air-cored inductors are not, unless you put them in a shielding can, which both changes their values and downgrades their Q.

Candidly, I believe the only reason that leakage inductance has not been used more in equipment design is that few engineers have figured out how to put it to use, rather than having to "fight" it as a source of high frequency losses. ■