Analog implementation of equalizers and filters

This document contains an in-depth look into filters and equalisers.

The Low-cut and High-cut filters:

This filters is usually used in the begining of the signal path to filter out any unwanted humming and hiss.

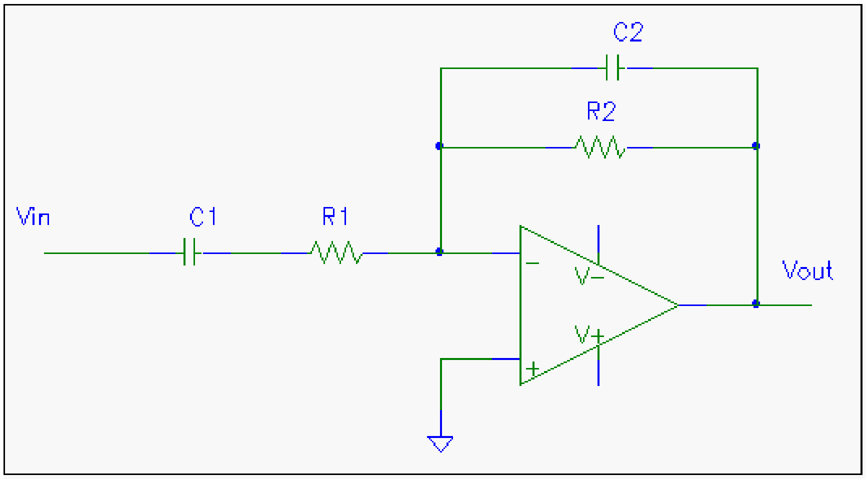

Figure 1 - First order filter

The transfer function of this filter is as follows:

$$A_v = \frac{V_{out}}{V_{in}} = \frac{s\frac{1}{R_1C_2}}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}$$

The time constant of R_1C_1 determines the low-cutoff frequency while the time constant of R_2C_2 determines the high cutoff frequency. The pass-band gain, $A_{vpass}=R_2/R_1$.

Figure 2 shows the standard topology used for an second order filter.

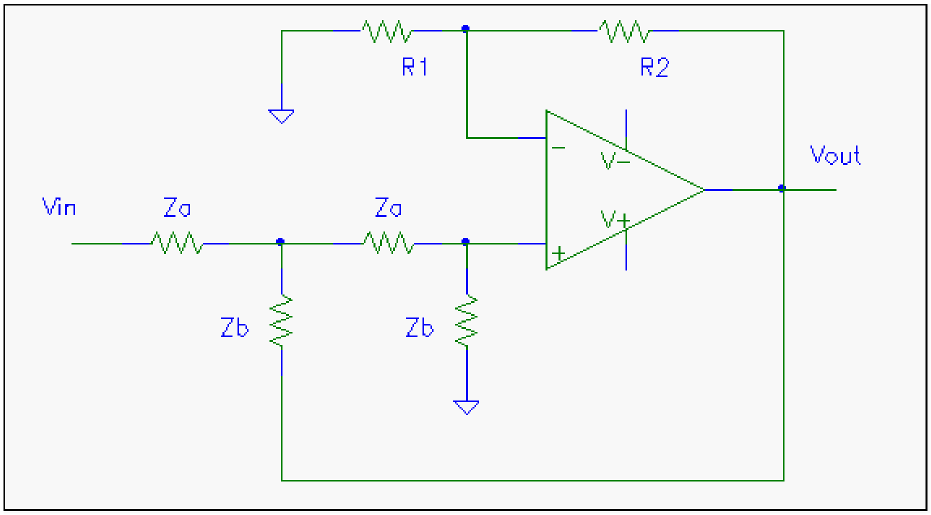

Figure 2 - Second-order lowpass or highpass filter

The transfer function of this circuit is as follows:

$$A_v = \frac{V_{out}}{V_{in}} = \frac{KZ_b^2}{Z_a^2 + (3-K)Z_aZ_b + Z_b^2}$$ with $$K = 1 + \frac{R_2}{R_1}$$

For a low-cut filter, that is a high-pass filter, Z_a must be capacitors and Z_b resistors. By substituting Z_a=1/sC and Z_b=R into above equation, the following equation is obtained:

$$A_{vhp} = \frac{V_{out}}{V_{in}} = \frac{Ks^2}{s^2 + s\frac{3-K}{RC} + \frac{1}{R^2C^2}}$$

For a high-cut filter, that is a low-pass filter, Z_a must be resistors and Z_b capacitors. By substituting Z_a=R and Z_b=1/sC into above equation, the following equation is obtained:

$$A_{vlp} = \frac{V_{out}}{V_{in}} = \frac{\frac{K}{R^2C^2}}{s^2 + s\frac{3-K}{RC} + \frac{1}{R^2C^2}}$$

For both circuits the following equations apply:

$\omega_o = \frac{1}{RC}$, $f_o = \frac{1}{2\pi RC}$ and $Q = \frac{1}{3-K}$

The best choice for the Q-factor is 0.707. This results in the sharpest curve, without any overshoot. If this Q-factor is selected, the value of K must be 1.586. Note that this is also the pass-band gain. The signal should therefore be attenuated if constant signal level is desired. If a slope steeper than 12db/oct is desired, a higher order filter can be used. It is best to use the Butterworth response for audio, because of its flat response.

The shelving EQ circuit:

The simplest type of equaliser is the low and high shelving equalisers. This is usually called Bass and Treble on Hi-fi sets. Figure 3 shows the basic topology for such an equaliser. The potentiometer, R_p, sets the boost cut ratio. The impedance of Z(s) must be high at the frequencies to boost/cut and low at other frequencies, effectively shorting out R_p.

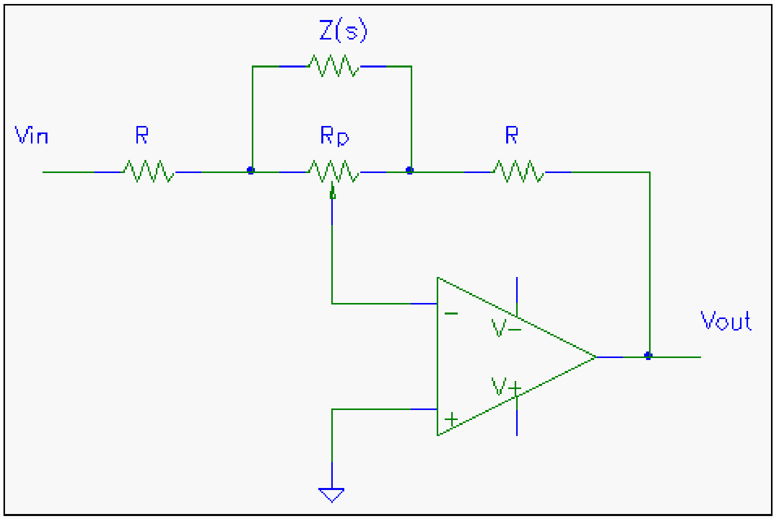

Figure 3 - Basic shelving equaliser circuit

The transfer function of this circuit is as follows:

$$A_v = \frac{V_{out}}{V_{in}} = -\frac{Z(s)((1-\beta)R_p + R) + RR_p}{Z(s)(\beta R_p + R) + RR_p}$$

If a capacitor is substituted for Z(s), a low shelving equaliser is realised, while an inductor is needed for a high shelving equaliser. Figure 4 shows a circuit for a combined Lo and Hi shelving equaliser as used in conventional hi-fi sets and mixing consoles. As can be seen, the inductor in parallel with R_{treble} is omitted, C_2 is instead inserted. At the high frequencies, R_{bass} is shorted out by C_1. The higher the frequency becomes, the lower C_2?s impedance become, giving the gain setting on R_{treble} higher priority over the unity gain network formed by the lo section.

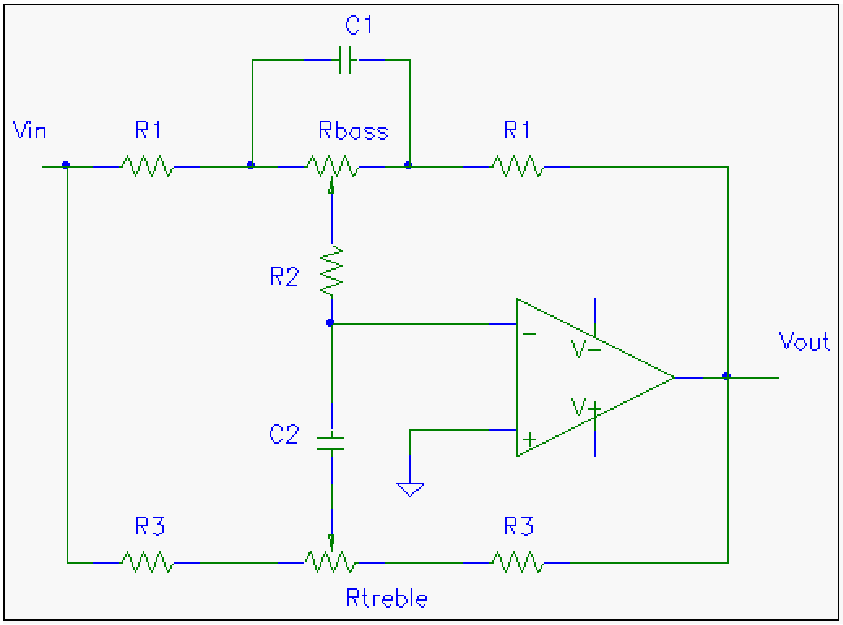

Figure 4 - Lo and Hi shelving equaliser circuit

Basic variable Q graphical EQ circuit:

The circuit in Figure 5 shows the basic circuit for a graphical equaliser. The frequency dependant impedance, Z(s), determines the centre frequency of the equaliser. The impedance of Z(s) must be low at the centre frequency, while high at other frequencies. By replacing Z(s) by a short it can be easily seen that the left part of R_p forms an attenuator feeding the input signal to the non-inverting operational amplifier, which gain is controlled by the right half of R_p. If Z(s) is replaced by an open-circuit, R_p is only connected between the two input terminals of the op-amp, which is at the same potential and R_p does not effect the circuit. Multiple bands can be implemented by placing more than one pot in parallel, each with its wiper connected to a different Z(s) circuit.

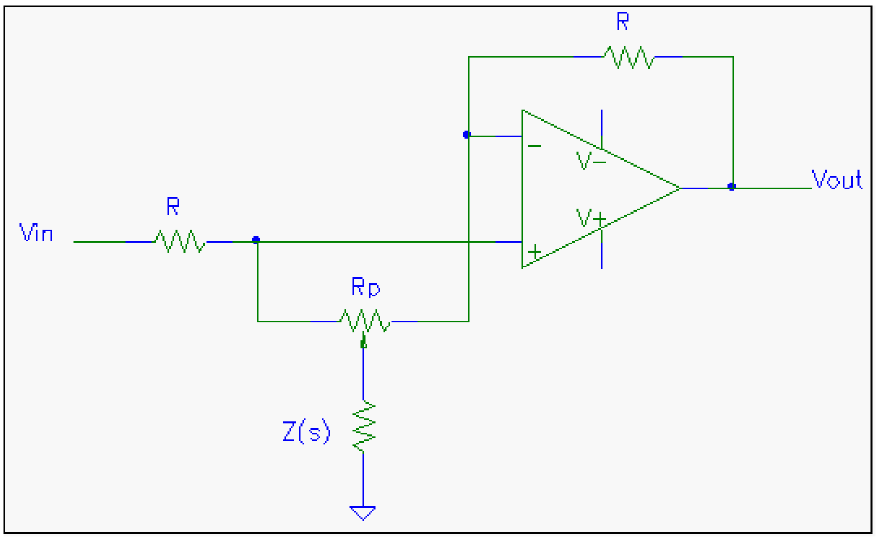

Figure 5 - Basic graphic equaliser circuit

If it is assumed that Z(s) has zero impedance at the centre frequency, the transfer function of the filter will be as follows:

$$A_v = \frac{V_{out}}{V_{in}} = \frac{\left(R + (1-\beta)R_p\right)\beta R_p}{\left(R + \beta R_p\right)(1-\beta)R_p}$$

It can be seen that if b<0.5, that is R_p?s wiper is moved toward the left, the gain is below unity, while if b>0.5, that is R_p?s wiper is moved toward the right, the gain is above unity. The maximum boost/cut ratio is controlled by the impedance of Z(s), which effectively limits the value of b so that a<b<(1-a) where $a=Z_{pass}/R_p$.

Figure 6 shows a practical circuit for Z(s). On the left side is the classic series RLC circuit, while the circuit on the right side uses a gyrator to replace the inductor. To simplify the analysis of the circuit on the right, it is assumed that the impedance of C_1, R_1 is much higher than the impedance of C_2, R_2.

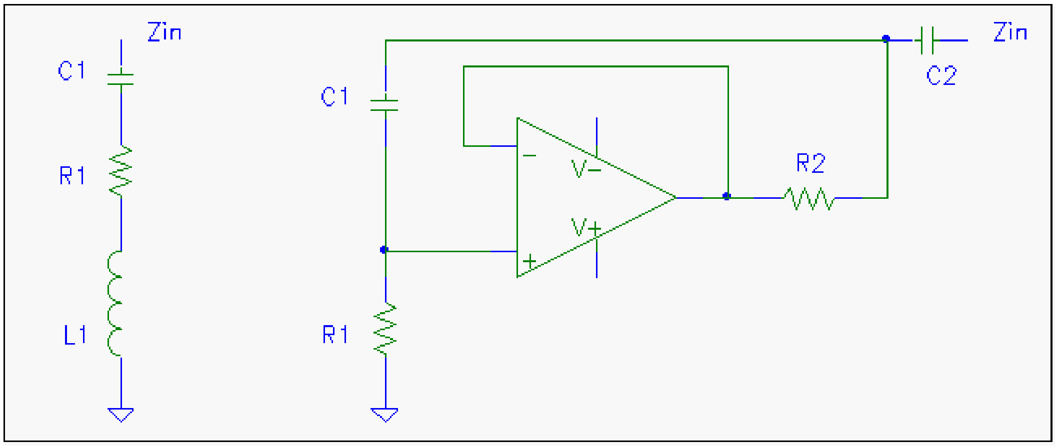

Figure 6 - Frequency dependent impedance circuit

The impedance of the circuit is described by the following equations:

$$Z(s) = \frac{s^2R_1R_2C_1C_2 + sR_2(C_1 + C_2) + 1}{sC_2(sR_2C_1 + 1)} \approx \frac{s^2R_1R_2C_1C_2 + sR_2C_2 + 1}{sC_2}$$

$$\omega_c = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

$$|Z_{pass}| = \frac{R_2(C_1 + C_2)}{C_2\sqrt{\frac{R_2C_1}{R_1C_2} + 1}} \approx R_2$$

$$Q \approx \frac{\sqrt{R_1R_2C_1C_2}}{R_2(C_1 + C_2)}$$

A shelving equaliser can also be realised with this circuit topology. For a Lo shelving equaliser the circuit in Figure 6 can be replaced by a single capacitor in series with an resistor. For a Hi shelving equaliser, the circuit in Figure 6 is used as it is, except that the high frequency roll-off of the impedance is above the audio spectrum. The same scheme can also be used for a Lo shelving equaliser. The advantage of this is that the circuits would not boost any unwanted noise beyond the audio spectrum.

Parametric equaliser circuit:

Figure 7 shows a parametric equaliser circuit which is used in most older mixing consoles. In this case Z(s) is a bandpass filter. The transfer function of this circuit is as follows:

$$A_v = \frac{V_{out}}{V_{in}} = \frac{2Z(s)(1-\beta) - 1}{1 - 2Z(s)\beta}$$

For the equations following, Z(s) must be positive and its bandpass gain must be below 0.5.

$$A_{vcut} = 2Z(s) - 1 \quad A_{vflat} = \frac{Z(s) - 1}{1 - Z(s)} = -1 \quad A_{vboost} = \frac{1}{2Z(s) - 1}$$

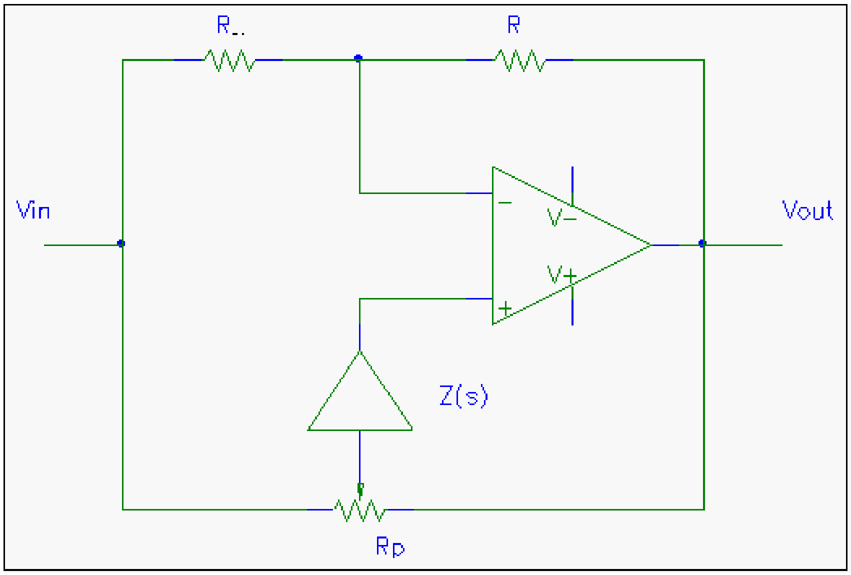

Figure 7 - Basic parametric equaliser circuit

The drawback of this circuit is that it is not a constant Q design. Another drawback is that the filter, Z(s), is always in the circuit, even if the boost/cut control is flat. The following circuit takes care of these two problems:

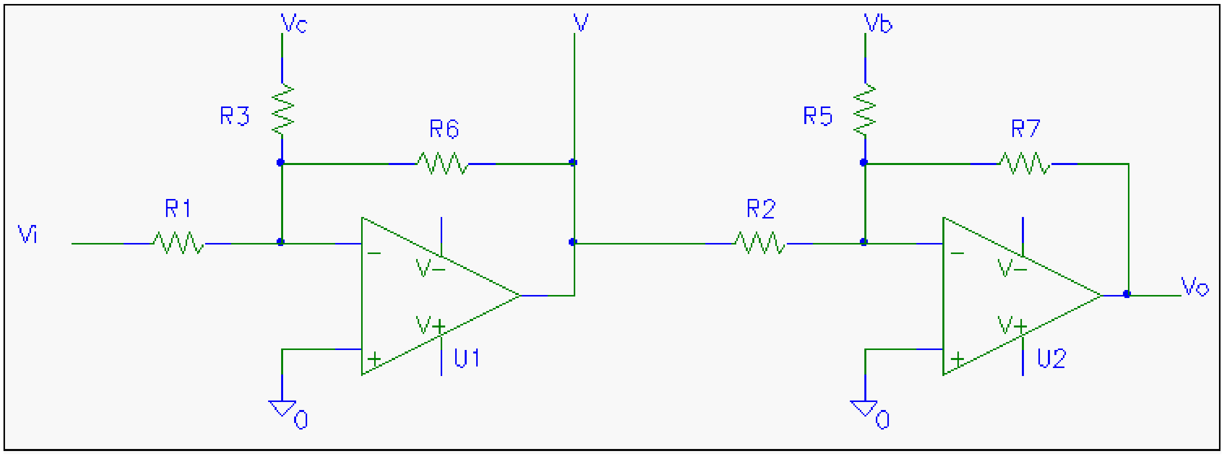

Figure 8 - Main signal flow circuit for a constant Q parametric equaliser

The above circuit forms the main signal flow path of a constant Q parametric or graphical equaliser. The output, V, is connected the the input of a bandpass filter. This bandpass filter determines the centre frequency and Q-factor of the equaliser. The output of the bandpass filter is then send to a variable gain section. The output of this section can either be sent to V_c, to cut frequencies, or V_b to boost frequencies. Note that constant Q operation is achieved by only sending a signal to V_c or V_b, but not both.

All the resistors except R3 and R5 has the same values. The value of R3 determines the cut ratio, while the value of R5 determines the boost ratio.

If $V_c = aZ(s)V$ and $V_b = bZ(s)V$ then $V_o = \frac{1+\beta Z(s)}{1+\alpha Z(s)} V_i$

where Z(s) is the transfer function of the bandpass filter. To boost the mid frequencies, a must be 0 while b is adjusted. The opposite is true to cut mid frequencies. Note that for constant-Q operation, the one which is not used, must be zero.

One way to implement the variable gain stage and distributor is two use a linear potentiometer with a centre tap. A resistor is connected from the output of the banpass filter to the wiper of the potentiometer. This resistor determines the maximum boost/cut ratio. The one end of the pot is connected to V_c, while the other end is connected to V_b.

If a centre tap potentiometer is not available, a standard dual-gang linear potentiometer can be used. The one half is used to control the gain of special symmetrical gain stage, while the other half drives a comparator which controls analogue switches which either connects the output of the gain stage to V_c or V_b. A standard comparator in combination with a SPDT analogue switch can be used to switch between the V_c and V_b inputs. It is advisable to use hysterises in the comparator.

The circuit in Figure 9 shows a circuit for a symmetrical gain stage. This gain stage gives maximum gain at the end of the boost/cut control?s travel, while giving zero gain at its centre.

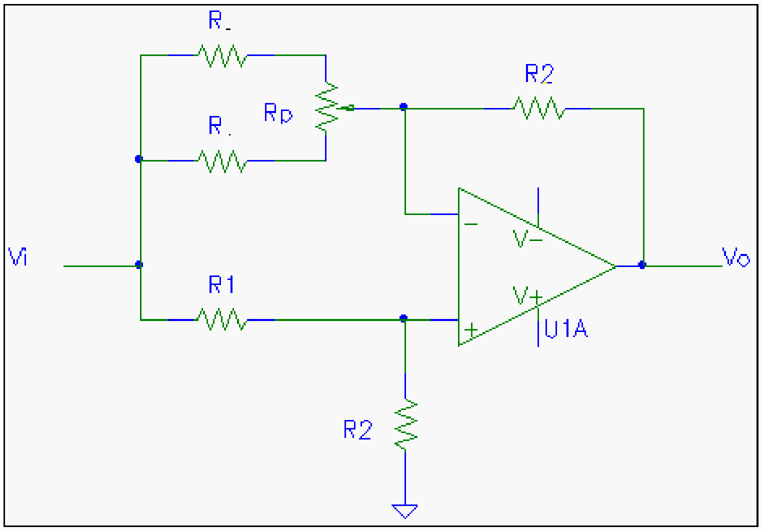

Figure 9 - Symmetrical variable gain circuit

The gain of this circuit is as follows:

$$A_v = \frac{V_o}{V_i} = -\frac{R_2(R_1 - R_T)}{R_T(R_1 + R_2)}$$

where $R_T=(R+gR_p)||(R+(1-g)R_p)$ and g the position of R_p is.

The maximum value of R_T, R_{Tmax} is when g=0.5, which would also result in A_{vmin}. The minimum value of R_T, R_{Tmin} is when g=0 or g=1, which would also result in A_{vmax}. Here is the equations for R_{Tmax} and R_{Tmin}:

$R_{Tmax} = (2R+R_p)/4$

$R_{Tmin} = (R+R_p)||R$

If g=0.5, that is, a flat response, A_v must be 0. To accomplice that R_1 must equal R_{Tmin}, therefore

$$R_1 = \frac{2R + R_p}{4}$$

For maximum linearity of the boost/cut control, R must not be too small compared to R_p. Note that the larger R is made, the smaller the maximum gain of the gain stage becomes. This circuit shows a deadband in its response about the centre of R_p. This is actually an advantage, for it gives adequate time for the analogue switches to switch from V_c to V_b and vice-versa. It is even possible to give the switches a dead-band, where the filter and gain section is totally cut-out if R_p is in its centre position.

The first step in designing this circuit is to select a value for R_p. The best choice for R is the same as R_p. This gives the most linear curve, without losing to much gain. After the values of R_p and R have been selected, the value for R_1 is calculated.

Next the values the of A_{vmax} and R_2 must be calculated. The value of R_2 is calculated as follows:

$$R_2 = \frac{A_v R_T R_1}{R_T - R_1 - A_v R_T}$$

The value of R_2 cannot be negative, therefore

$R_T-R_1-A_{vmax}R_T > 0$ or $A_{vmax} < \frac{R_1 - R_T}{R_T}$

After the value of A_{vmax} has been selected, the value of R_2 can be calculated. It is best to compose R_1 of a fixed resistor and preset resistor in series. The circuit can then be balanced to give zero gain at zero boost/cut.

Bandpass filters used in equalisers:

The filter in Figure 10 is a simple passive network. This circuit is usually used in sweepable mid sections of older mixing consoles. Note that a buffer is usually used to buffer the voltage from the wiper of R_p and also to set the correct banpass gain. Let?s call this gain a.

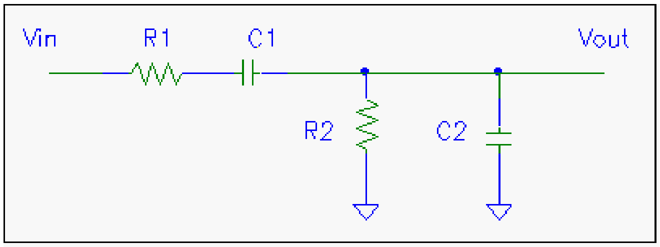

Figure 10 - Passive bandpass filter network

The following formulas describes this bandpass filter:

$$Z(s) = \frac{V_{out}}{V_{in}} = \frac{s\frac{1}{R_1C_2}}{s^2 + s\frac{C_1(R_1+R_2)+C_2R_2}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}$$

$$\omega_o = \sqrt{\frac{1}{R_1R_2C_1C_2}} \quad Q = \frac{\sqrt{R_1R_2C_1C_2}}{C_1(R_1+R_2)+C_2R_2} \quad A_{vpass} = \frac{C_1R_2}{C_1(R_1+R_2)+C_2R_2}$$

If $C_1=C_2=C$ and $R_1=R_2=R$ like in the practical circuit where R_1 and R_2 are part of a dual-gang pot for a sweepable frequency, the equations is like this:

$$Z(s) = \frac{V_{out}}{V_{in}} = \frac{s\frac{1}{RC}}{s^2 + s\frac{3}{RC} + \frac{1}{R^2C^2}} \quad \omega_o = \frac{1}{RC} \quad Q = \frac{1}{3} \quad A_{vpass} = \frac{1}{3}$$

Figure 11 shows a circuit for an active bandpass filter. This circuit is used if the centre frequency and Q-factor is fixed, like in a graphical equaliser.

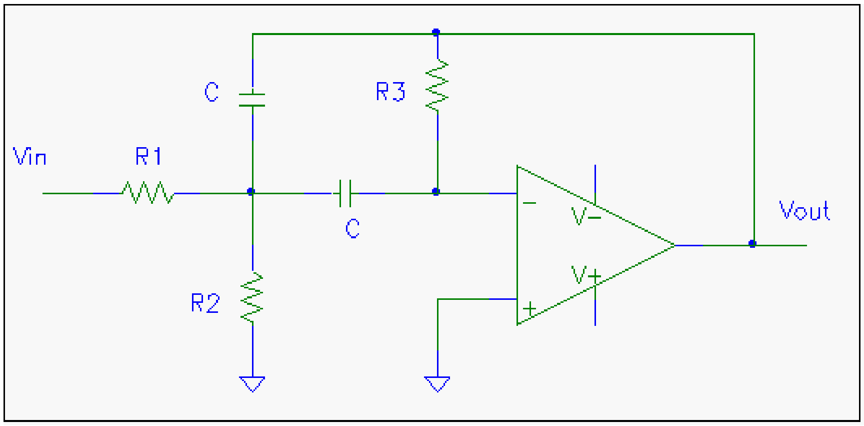

Figure 11 - Active single op-amp bandpass filter

The transfer function of this circuit is as follows:

$$Z(s) = \frac{V_{out}}{V_{in}} = -\frac{s\frac{1}{CR_1}}{s^2 + s\frac{2}{CR_3} + \frac{R_1+R_2}{C^2R_1R_2R_3}}$$

It is rather complicated to design this filter, the following equations simplify that. The input to the equations if f_c, Q, A_{vpass} and an arbitrary selection of C.

$$R_1 = \frac{Q}{2\pi f_c A_{vpass} C} \quad R_2 = \frac{Q}{2\pi f_c C(2Q^2 - A_{vpass})} \quad R_3 = 2R_1 A_{vpass}$$

The most flexable filter is a state variable filter. This type of bandpass filter's centre frequency and Q-factor can be independently adjusted. That is the reason why this type of filter is used in most modern sweepable and parametric equaliser circuits. Due to the complexity of the filter and the huge amount of op-amps, this filter have the drawback of being more noisy than the above filters. Figure 12 shows a circuit of a state-variable filter. The op-amps U1 and U2 form summing amplifiers, while U3 and U4 form two integrators. The output of U3 is the bandpass output, while the outputs of U2 and U4 is highpass and lowpass outputs respectively.

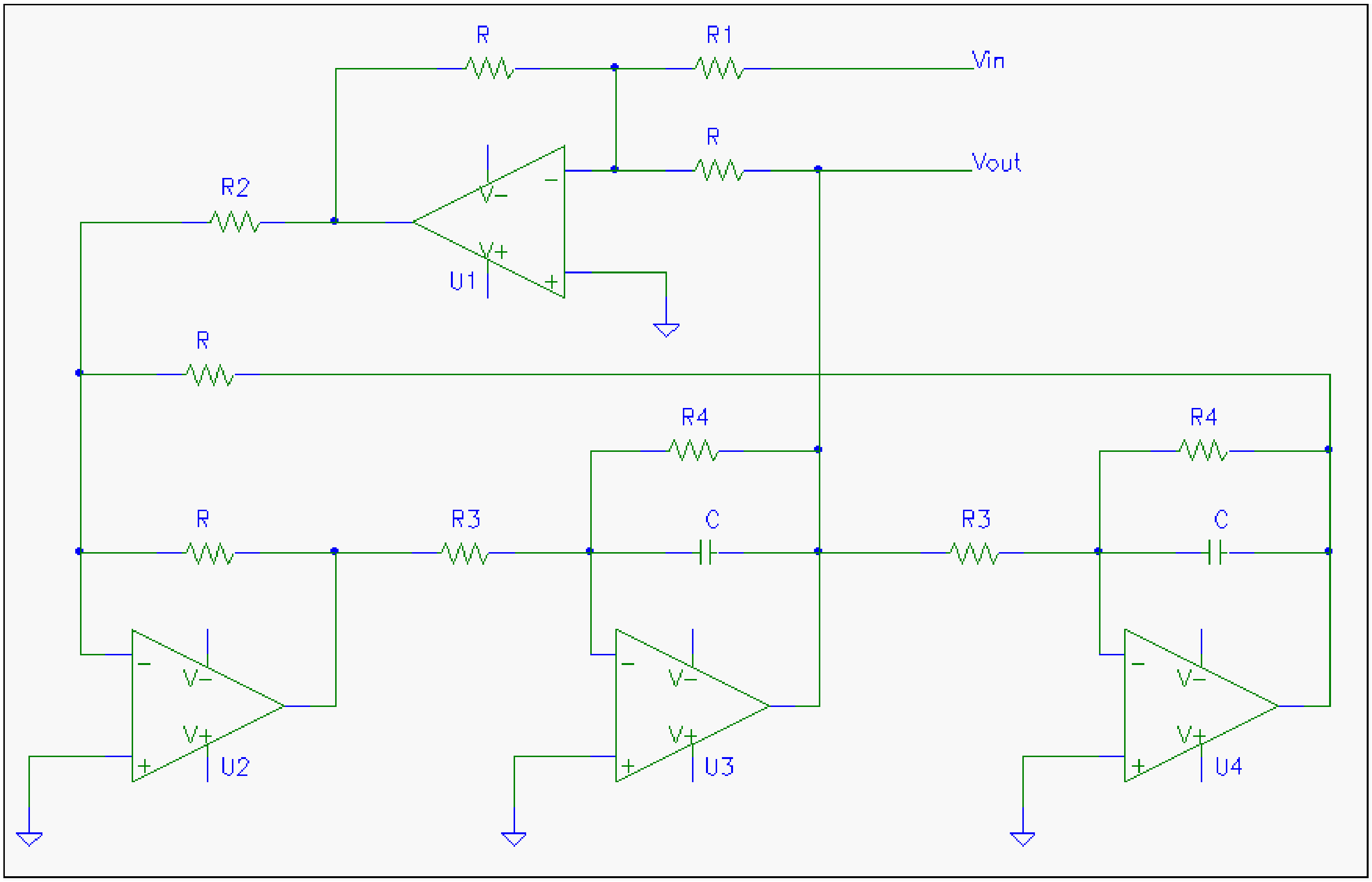

Figure 12 - State-variable active filter

The transfer function of the filter is as follows:

$$Z(s) = -\frac{s\frac{\omega_o}{Q}}{s^2 + s\frac{\omega_o}{Q} + \omega_o^{\,2}}$$

The Q-factor of the filter is adjusted by VR1, while the centre-frequency is adjusted by VR2. Note that for Q values below 1, the amplitude of the hp and lp outputs is 1/Q times higher than the bp output. If such low values of Q is needed, the input signal needs to be attenuated to prevent clipping of the hp and lp outputs. Taking this into account, it may be better to put the variable gain stage before the filter instead of after it. The drawback of putting the stages this way is that the noise generated by the filter will always be present on the output of the equaliser.

The following three equations is used to design the filter. It can be seen that the design equations is quite simple. The centre frequency is made sweepable by making R_3 a dual-gang pot, while is the Q-factor is made adjustable by making R_2 adjustable.

$$f_o = \frac{1}{2\pi R_3 C} \quad Q = \frac{R_2}{R} \quad A_{vpass} = \frac{R}{R_1}$$

By using the lowpass or highpass outputs, this filter can be used in a advanced shelving equaliser system. The only problem is that the lowpass and highpass gains is dependent on Q:

$A_{lppass} = A_{hppass} = 1/Q$

The remedy for this is to make R_2 fixed and rather make the resistor between V_{out} and the inverting input of U1 variable. Take note that for low values of Q, the lowpass and highpass outputs might clip for large signal levels. The input signal to the filter should therefore be attenuated if low values of Q is going to be used.