how to gyrate - and why

The gyration principle was suggested by theoreticians over 25 years ago, although it is rarely seen in practical circuits. It can, however, be used to simulate an inductance of (say) 10,000 H with a Q of 100 in a volume of less than one cubic inch . . . !
In this article the theoretical principles of the gyrator are discussed, and some practical circuits and applications are presented.

To be able to understand and use gyrators, a certain amount of theoretical background knowledge is necessary.
The basic circuit consists of two amplifiers (figure 1), with the input of one connected to the output of the other and vice versa. Amplifier A is an inverting and amplifier B is a non-inverting type. The slope of amplifier A is

$$s_1 = -g_1 \text{ (A/V)}$$

and the slope of amplifier B is

$$s_2 = g_2 \text{ (A/V)}.$$

This means that if amplifier A is driven with an input voltage v_1 volts, it will deliver a current of $-g_1 \cdot v_1$ amps; in other words, it will sink a current of $g_1 \cdot v_1$ (A).
Referring now to figure 1 it is clear that the voltages and currents are defined by the formulae:

$$i_2 = g_1 \cdot v_1$$

(amplifier A; the current into this amplifier is defined as positive, so that the minus sign disappears); and

$$i_1 = g_2 \cdot v_2$$

(amplifier B).
In these formulae g_1 and g_2 are so-called gyration-constants. They are very often equal ($g_1 = g_2 = g$); sometimes the phrase "gyration resistance" is used, defined by

$$R = \frac{1}{g}.$$

In figure 2 the recognised symbol for a gyrator is shown.

The next step is to connect an impedance (Z_1) across one set of terminals (dotted in figure 2). In this case the ratio of v_1 to i_1 is determined:

$$v_1 = i_1 \cdot Z_1.$$

From the gyrator formulae it is obvious that the voltage and current at the other set of terminals are defined by:

$$i_2 = g_1 \cdot v_1,$$

and

$$v_2 = \frac{i_1}{g_2}.$$

This means that the impedance "seen" across this second set of terminals is:

$$Z_2 = \frac{v_2}{i_2} = \frac{i_1/g_2}{g_1 \cdot v_1} = \frac{1}{g_1 \cdot g_2 \cdot Z_1}. \quad (1)$$

What a gyrator does

The most important application in practice is the simulation of inductors, for use in LC resonant circuits and the like.
If the impedance Z_1 in figure 2 is a pure capacitance:

$$Z_1 = \frac{1}{j\omega C},$$

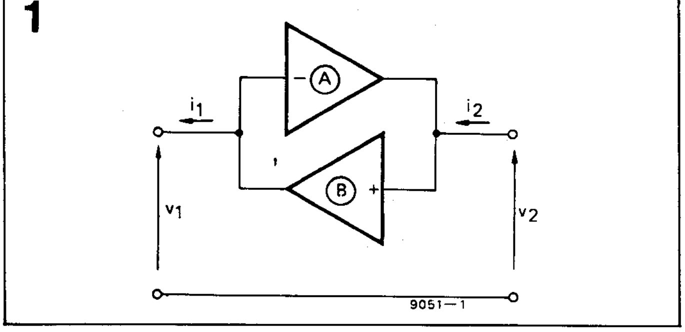
then the previous formula shows that the virtual impedance across the other set of terminals (Z_2) equals:

$$Z_2 = \frac{1}{\frac{1}{j\omega C} \cdot g_1 \cdot g_2} = j\omega \frac{C}{g_1 \cdot g_2}. \quad (2)$$

In words: if a capacitance is connected to one set of terminals, the other pair of terminals behave as if an inductance were connected between them with a value in Henries equal to the capacitance in farads divided by the product of the gyration constants. The gyration constants themselves are equal to the slopes of the two amplifiers, which leads to the interesting conclusion that a lower value for the slope leads to a higher value for the simulated inductance!
It is apparent that an LC (parallel) resonant circuit can be simulated with the circuit shown in figure 3a. In this circuit the resistors R_1 and R_2 each represent a parallel connection of the resistive components of the input impedance of one amplifier and the output impedance of the other amplifier (and the leakage resistance of the capacitor, which is usually negligible).
From the general gyrator conversion formula (1) it can be shown that this circuit is equivalent to the circuit in

Figure 1. Block diagram of the basic gyrator circuit, consisting of a non-inverting and an inverting amplifier.

Figure 2. The recognised symbol for a gyrator; the function is to "gyrate" an impedance Z_1 across one pair of terminals to a different (virtual) impedance (Z_2) across the other pair of terminals.



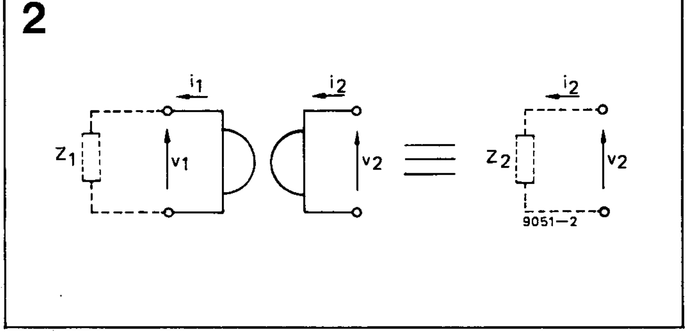


figure 3b. This, in turn, is equivalent to the circuit in figure 3c provided R_1 and R_2 in the original circuit are sufficiently large compared to the impedance of C_1 and C_2 at the operating frequency.
The components in figure 3c are derived from those in figure 3a as follows:

$$L_1 = \frac{C_1}{g_1 \cdot g_2};$$

$$R_p = \frac{C_1}{C_2} \cdot R_1;$$

$$R_2 = R_2;$$

$$C_2 = C_2.$$

From these values the resonant frequency (f_o) and the quality factor (Q) can be calculated:

$$f_o = \frac{1}{2\pi\sqrt{L_1 \cdot C_2}} = \frac{1}{2\pi\sqrt{\frac{C_1 \cdot C_2}{g_1 \cdot g_2}}};$$

$$Q = 2\pi f_o C_2 \cdot \frac{R_p \cdot R_2}{R_p + R_2} =$$

$$\frac{R_p \cdot R_2}{R_p + R_2} \cdot \sqrt{\frac{g_1 \cdot g_2 C_1}{C_2}}.$$

In practice one can usually substitute $g_1 = g_2 = g$, and $C_1 = C_2 = C$, so that the formulae simplify to:

$$f_o = \frac{g}{2\pi C} \qquad (3)$$

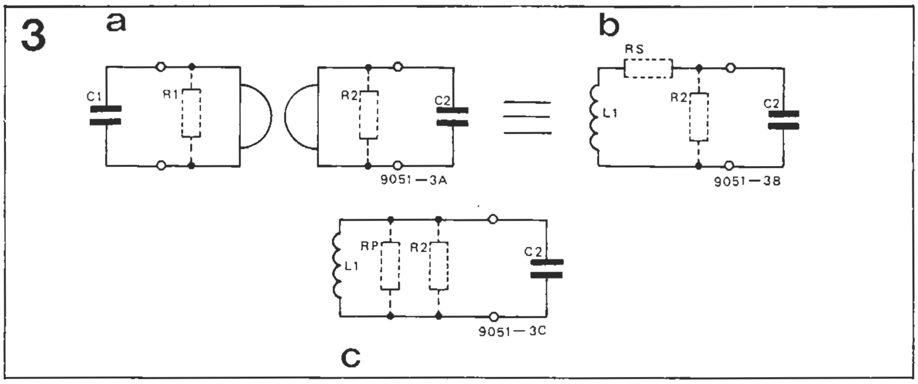
$$Q = g \cdot \frac{R_1 \cdot R_2}{R_1 + R_2}. \qquad (4a)$$

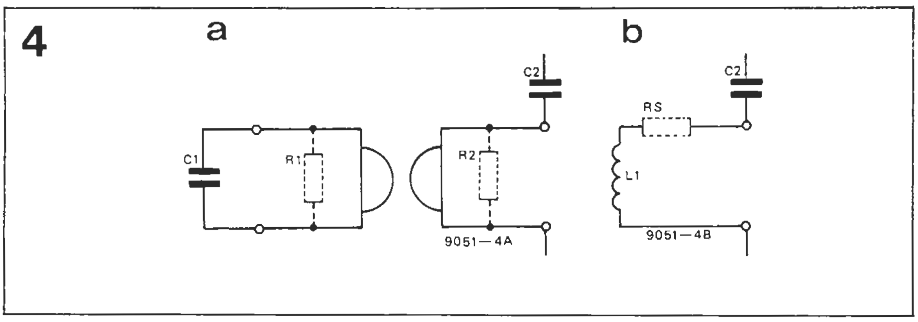
If furthermore $R_1 = R_2 = R$, 4a can be further simplified to:

$$Q = \tfrac{1}{2} g \cdot R. \qquad (4b)$$

Summary

When an impedance (Z_1) is connected across one set of terminals of a gyrator, a virtual impedance (Z_2) appears across the other set of terminals:





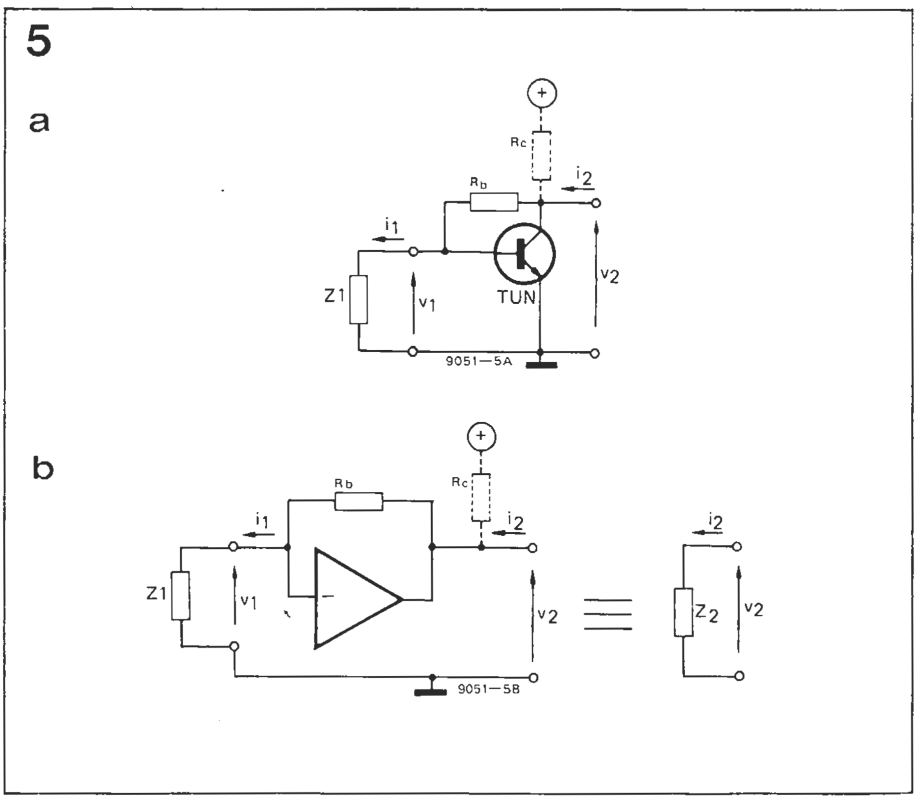


Figure 3. The most important practical application of a gyrator: simulating an LC parallel resonant circuit (figures 3B and 3C) with the aid of a gyrator and two capacitors (figure 3A).

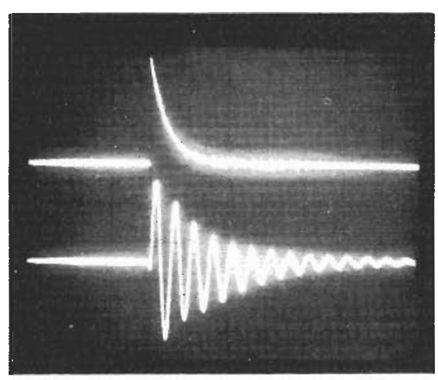
Figure 4. In the same way a series resonant circuit (4B) can be simulated with a gyrator and two capacitors (4A).

Figure 5. The basic circuit (5A) and a block diagram (5B) of the one tun gyrator.

Figure 6. A practical application of the one tun gyrator in a smoothing circuit for the supply rail of a preamplifier. The impedance of the section inside the dotted lines is equivalent to an inductance of 250 H!

Figure 7. The circuit of the gyrator which is used in the minidrum. If a short pulse is applied to input P a decaying 30 Hz sine wave appears at the output, simulating the sound of a bassdrum.

Figure 8. An oscilloscope photo of the bassdrum in operation. The upper trace is the pulse applied to the input, and the lower trace is the output. (Scale: horizontal 50ms/div., vertical 200mV/div.)



$$Z_2 = \frac{1}{g_1 \cdot g_2 \cdot Z_1} \qquad (1)$$

If the impedance Z_1 is a pure capacitance (C_1), and furthermore $g_1 = g_2 = g$, the virtual impedance (Z_2) is an inductance:

$$Z_2 = j\omega \frac{C_1}{g_1 \cdot g_2} = j\omega \frac{C_1}{g^2} \qquad (2)$$

which can also be written as:

$$L_2 = \frac{C_1}{g^2}. \qquad (2a)$$

If a second capacitor (C_2) is connected across the second set of terminals, the result is a parallel (LC) tuned circuit. If $g_1 = g_2 = g$, $C_1 = C_2 = C$ and the input and output impedances (R_1 and R_2) are equal, the resonant frequency (f_o) and quality factor (Q) are:

$$f_o = \frac{g}{2\pi C} \qquad (3)$$

$$Q = \tfrac{1}{2} g \cdot R. \qquad (4b)$$

Without further calculation it can be stated that the resonant frequency and quality factor of a series tuned circuit (figure 4) are given by the same formulae. It is obvious from the above that the input and output impedances of the amplifiers should be as high as possible to obtain a high quality factor. The slope of the amplifiers should also be high if a high quality factor is required; however this leads to a high resonant frequency unless relatively large capacitors are used. A simple calculation shows that for, say, Q = 1000 at f_o = 100 Hz the gyration constant (or slope) must be $g = 2.10^{-3}$ (if the input and output impedances in parallel are taken to be 1 MΩ) and capacitors $C_1 = C_2 \approx 30\mu F$ are needed. If these capacitors are electrolytics the equivalent leakage resistance may exceed the value assumed above (1 MΩ), so that a still higher value for g and hence for the capacitors is required, and so on... Having explained the theory of the gyrator, we can now discuss some practical circuits.

One tun gyrator

This particular circuit is used fairly regularly, although it is doubtful whether many people realise that it works as a gyrator!

The basic circuit is shown in figure 5a, and figure 5b shows the same circuit with more theoretical symbols. It is clearly an asymmetrical gyrator: the transistor is the inverting amplifier, with a gyration constant:

$$g_1 = \frac{i_2}{v_1} = S \approx 40 I_C.$$

The collector-to-base resistor (R_b) is the non-inverting "amplifier", with a gyration constant:

$$g_2 = \frac{i_1}{v_2} \approx \frac{1}{R_b}.$$

The second approximation is based on the assumption that v_2 is far greater than v_1, which is usually the case in practice. The impedance conversion is therefore defined in this case as:

$$Z_2 = \frac{1}{g_1 \cdot g_2 \cdot Z_1} \approx \frac{R_b}{S \cdot Z_1}.$$

A practical application of this gyrator is shown in figure 6. In this case the slope is approximately equal to

$$S \approx 40 I_C \approx 200 \cdot 10^{-3}\ A/V,$$

so that the virtual impedance of the section inside the dotted lines is approximately:

$$Z_2 \approx \frac{R_b}{S \cdot Z_1} \approx j\omega \cdot \frac{47 \cdot 10^3}{2 \cdot 10^2} \approx j\omega \cdot 250.$$

In other words, it behaves like a coil with an inductance of 250 H! Adding the capacitor (C_2) across the output gives a low-pass filter with a cut-off frequency of approximately 0.3 Hz. This means that it is a very useful smoothing circuit for the power supply of a preamplifier, for instance. The quality factor is very low, of course – theoretically Q ≈ 1 in this case! – so that it is usually unsuitable for other applications.

The minidrum gyrator

The gyrator used in the minidrum (elsewhere in this issue) is a rather more complicated circuit; see figure 7.

The inverting amplifier in this circuit is T_9. The collector load impedance for this transistor is a current source (T_8),

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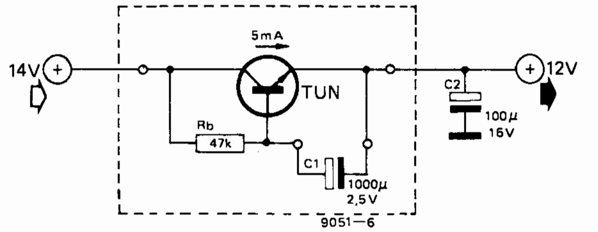

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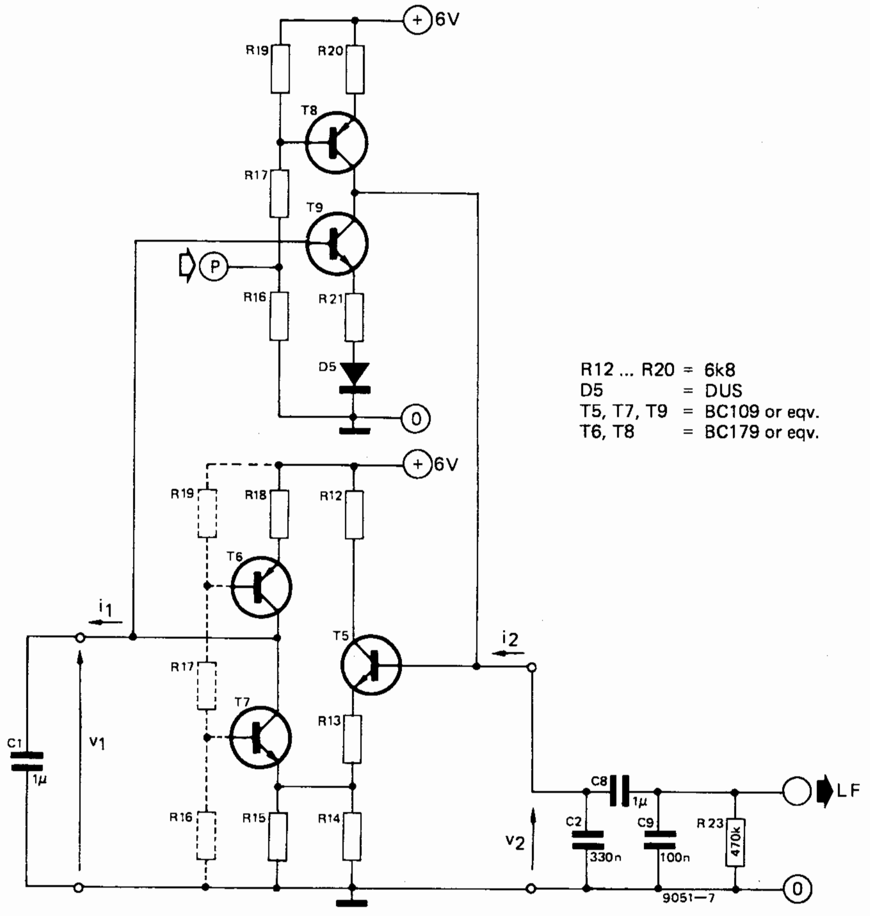

so that the gyration constant is simply:

$$g_1 = \frac{i_2}{v_1} = \frac{1}{R_{21}+r_d+r_e},$$

in which r_d and r_e are the dynamic (= AC) resistances of the diode (D_5) and the emitter of T_9 respectively. These impedances are determined by the current through T_8 and T_9, which is approximately 0.2mA, so that

$$r_e \approx r_d \approx \frac{1}{40I} \approx 125\Omega,$$

and

$$g_1 \approx \frac{1}{7 \cdot 10^3} \approx 1.4 \cdot 10^{-4}.$$

The non-inverting amplifier consists of the long-tailed pair T_5/T_7 (a differential amplifier), with the current source T_6 as collector load impedance for T_7. The gyration constant (g_2) is determined in this case by R_{13}, r_e (T_5) and r_e (T_7), which have approximately the same values as R_{21}, r_d and r_e (T_9) in the above formulae. From this it follows that

$$g_2 \approx g_1 = g \approx 1.4 \cdot 10^{-4},$$

so that a capacitor of 220μF across one pair of terminals will be "gyrated" into an inductance of 10,000 H across the other set of terminals!

This particular gyrator circuit has some outstanding characteristics. In the first place it is symmetrical ($g_1 = g_2 = g$), as shown above; furthermore the DC balance is maintained over a wide range of supply voltages without any adjustment, and the current consumption is low (approximately 2mA with a 6 V supply). Finally, the performance is mainly determined by the closeness in value of the (nominally) 6k8 resistors to one another. This means that if all resistors are, say, 5% too high in value (i.e. all are 7k1) the performance does not deteriorate. In the minidrum (bassdrum) capacitors C_1 and C_2 are added, so that a resonant circuit is obtained; when the circuit is excited by a pulse which is applied to the input marked P it delivers a decaying sine-wave, of which the frequency is:

$$f_0 \approx \frac{g}{2\pi\sqrt{C_1 C_2}} \approx 32 \text{ Hz},$$

see figure 8.

The quality factor of the resonant circuit itself depends on the current gain of the transistors used, and can vary between about 60 and 200. However, in the minidrum an extra damping resistor (R_{23}) is added which brings the Q down to approximately 10.

It is interesting to note that in this particular gyrator circuit the collector of T_5 can be used as a fairly low impedance output without influencing the quality factor, and the base of T_7 can be used as an input. Because these two points are in phase, a resistor of, say, 100k connected between them will cause the circuit to oscillate. In effect this is an LC oscillator, of which the frequency is determined by C_1 and C_2.