

# An Introduction to Gyrator Theory

*How inductors can be simulated using resistors, capacitors, and op amps.*

BY BRYANT T. MORRISON

**A** GYRATOR, believe it or not, is an inductor without any turns of wire. Although the theory behind this interesting circuit has been established for some time, only within the past few years have synthesized inductors been used on a wide scale. Before we examine the gyrator in detail, let's review some basic properties of inductors.

A pure inductance is a circuit element whose opposition to the flow of alternating current (*inductive reactance*) varies directly with frequency. At dc or zero hertz, the ideal inductor has zero ohms of resistance (a perfect conductor) and zero ohms of reactance. Therefore, we can say that it also has zero ohms of impedance—the vector sum of resistance and reactance. However, as we move into the realm of ac, the reactance of an inductor increases according to the formula  $X_L = 2\pi fL$ ; where  $X_L$  is measured in ohms;  $f$  (frequency) in hertz; and  $L$  (inductance) in henries. Its resistance remains zero ohms. At infinite frequency, the inductor has infinite reactance, and will permit no ac to flow.

So far we have been talking about an *ideal* inductor. Actually, every inductor has a certain amount of resistance and capacitance as well as inductance. As shown in Figs. 1A and 1B, an iron-core inductor can be modeled as an inductance in series with a resistance,  $R1$ ; and this combination is in parallel with a capacitance and series resistance,  $R2$ . An air-core inductor (Figs. 2A and 2B) behaves as an inductance and series resistance  $R1$  would. In both cases,  $L$  is the inductance of the coil, and  $R1$  is the resistance of the wire which comprises the coil. The iron-core inductor contains two additional elements,  $R2$  and  $C$ , which represent losses within the core. With dc, there are no core losses, and consequently, our model's  $C$  permits no current to flow through  $R2$ . At higher and higher frequencies, core losses increase. Thus, in our model, increased current flows through  $R2$  as the capacitor's reactance decreases.

**Synthesizing an Inductor.** By combining resistors and a capacitor with a

gain stage, we can create a circuit which appears to the "outside world" as a real inductor. To understand how, we will analyze the inductor models (Figs. 1B and 2B) in terms of "port admittance." A *port* is a point through which energy can enter or leave. In the case of an electrical circuit, it can consist of a pair of terminals to which a circuit element is connected. The inductors and their models in Figs. 1 and 2 are ports, and when a voltage source is connected across them, an input voltage ( $V_{IN}$ ) is applied and an input current ( $I_{IN}$ ) flows.

*Admittance*, measured in *mhos*, is the reciprocal of impedance. In other words, admittance is the ratio of current to voltage. If an element's admittance is zero mhos, no current will flow through it no matter how high the voltage is across it. Such an element is a perfect insulator or open circuit. On the other hand, an element with infinite admittance will conduct infinite current, even if a low voltage source is connected across it. It is a perfect conductor or a short circuit. Combining these two terms, port admittance is the ratio of the current flowing into the port ( $I_{IN}$ ) to the voltage across the port ( $V_{IN}$ ).

Referring to Fig. 1B, we can see that resistors  $R1$  and  $R2$  set the limits of port impedance at both very high and very low frequencies. At dc, the admittance of the inductor  $L$  is infinite (a short circuit), and only  $R1$  limits the current through it. Capacitor  $C$  behaves as an open circuit

with zero admittance, so  $R2$  is removed from the circuit. At an infinite frequency  $L$  is an open circuit and  $R1$  is removed from the circuit. However,  $C$  is a short circuit and current through it is limited only by  $R2$ . Between these frequency extremes,  $L$  will determine the port's admittance, because it is much larger than  $C$ .

The port admittance of the air-core coil at dc is simply the reciprocal of resistance  $R1$ , since  $L$  has infinite admittance. At an infinite frequency, the port admittance is zero, because the inductance acts as an open circuit, and no input current can flow.

**Analyzing the Gyrator.** Now let's apply these concepts to the gyrator circuits (Figs. 1C and 2C). As in the equivalent circuits,  $R1$  represents the ohmic resistance of the coil wire, and  $C$  and  $R2$  are core losses which increase in step with the applied frequency. However, something new has been added—a gain stage. Any active device can be used, but here we choose an op amp for its simplicity, high gain, almost infinite input impedance, and very low output impedance. The gyrator op amps are strapped for unity-gain, noninverting operation. So, within the frequency limits of the device (assume infinite bandwidth), the voltage at the output is exactly the same as that at the noninverting input.

If we apply a dc voltage across the input terminals of Fig. 1C, capacitor  $C$

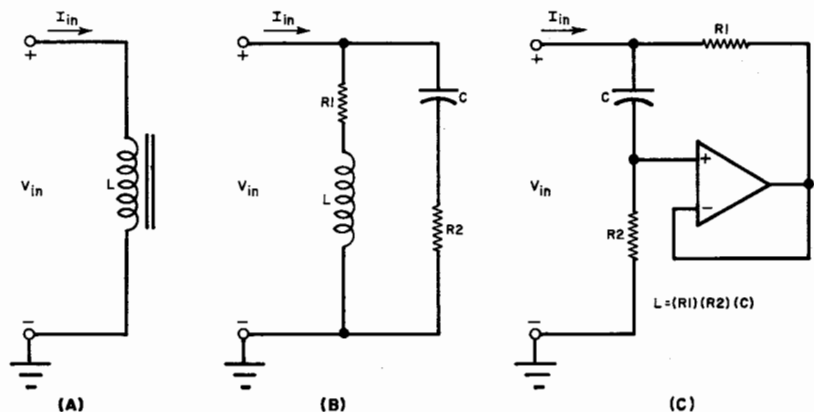


Fig. 1. Iron-core inductor (A) can be modeled as shown in (B) and simulated using the gyrator circuit in (C).

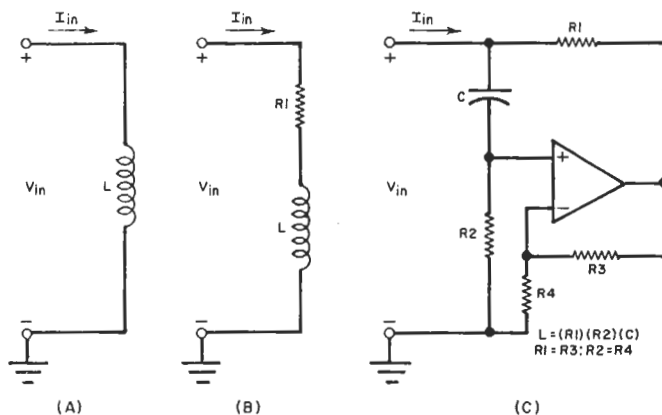


Fig. 2. An air-core coil (A) has an equivalent circuit shown in (B). Op amp gyrator (C) simulates the coil's behavior.

does not conduct, and the voltage at the noninverting input is zero. The output is also at ground potential, and because the op amp has very high output admittance (low output impedance), we can safely say that  $R1$  is connected across the port. So,  $I_{IN}$  will flow only through  $R1$ . This agrees with the behaviour of the equivalent circuit of Fig. 1B. The port admittances are maximized at dc, limited only by the values of both  $R1$ 's (assumed to be equal).

At infinite frequency,  $C$  is a short circuit, and therefore the voltage at the op amp's noninverting input (as well as that at the output) is equal to  $V_{IN}$ . Since there is no voltage drop across  $R1$ , it is effectively removed from the circuit. The only admittance path is through  $R2$  to

ground, which is the same behavior we noted in the equivalent circuit.

For frequencies between zero and infinity,  $C$  and  $R2$  act as a high-pass filter, causing less and less voltage drop across  $R1$  as frequency increases, and thus less port admittance until  $R2$ 's limiting effect comes into play. The reactive characteristics of the capacitor have successfully been inverted or gyrated so that the port behaves as an inductor. The equivalent inductance in henries is expressed by the formula  $L = (R1)(R2)(C)$ , with resistances in ohms and capacitance in farads.

With the addition of two resistors, an air-core inductor can be simulated. Air-core coils have essentially no "core" loss, and therefore have no parallel resistance in their equivalent circuits. Because of this the gyrator (Fig. 2C) uses the additional resistors to set the gain of the op amp. When the values are properly selected, they provide enough gain to compensate for  $R2$ 's losses at high frequencies. But the amount of gain must be carefully chosen—otherwise the circuit might oscillate! If  $R3$  equals  $R1$  and  $R4$  equals  $R2$ , the circuit will be stable and exhibit no parallel resistance. In practice, however, little is gained over the circuit of Fig. 1C as long as the ratio  $R2/R1$  is at least 90 to 100, because the effects of parallel resistance are negligible in most audio applications commonly encountered.

**Practical Design.** In synthesizing a useful "inductor," the same basic rules that govern the optimization of wound coils should be followed. For example, series resistance  $R1$  should be kept as small as possible and parallel resistance  $R2$  as large as possible. This corresponds to a coil wound from the heaviest wire practicable on the least lossy core available. For best performance,

$R1$  should be no lower than the op amp's minimum recommended load resistance, which falls between 100 and 2000 ohms for common op amp types. The largest acceptable value for  $R1$  is desirable, so as not to load the op amp too much, thus preventing high distortion and heating effects. To simulate a high-quality toroidally wound coil,  $R2$  should be at least 100 times greater than  $R1$ , but not so large as to become a major contributor to the op amp's input noise. As a rule of thumb, keep  $R1$  around 1000 ohms and  $R2$  between 10 kilohms and 1 megohm.

Once the values of  $R1$  and  $R2$  have been chosen, use the formula  $C = L / (R1)(R2)$  to find the required capacitance in farads. At least 100 pF should be used to avoid the detuning influences of stray capacitances.

It is important to keep the op amp functioning within acceptable circuit and signal parameters. If for any reason it begins to deviate from the role of a voltage follower, the "inductor" won't work properly. Input signals must lie within the operating bandwidth of the device, and their amplitudes must not cause the output stages to clip. In a gyrator, clipping in the gain stage is analogous to core saturation, which can cause high distortion levels.

However, this is not usually a problem with gyrators. Because they will most often be operated from the same power supplies that other audio stages use, they will not start to clip until the other amplifiers do. Unlike iron-core coils, whose saturation characteristics are functions of core material, size, number of turns, and applied current, the gyrator's saturation point is accurately predictable, and does not occur before the other active stages of the system also saturate or clip.

Using either of the gyrators we have examined will result in high-quality coils with inductances ranging from millihenries to hundreds or thousands of henries. Commonly available parts—including relatively small capacitors—can be employed. Added benefits include high magnetic field immunity and saturation characteristics, and (paradoxically) small amounts of required printed circuit board "real estate." However, there is one limitation. The gyrators we have described are single ended. That is, one side is grounded. To simulate "floating" inductors, neither side of which is connected to ground, more complex circuits using two op amps can be designed. But such gyrators are beyond the scope of this article. ◇

## PROPERTIES OF GYRATORS

### Advantages

1. Immunity to ambient magnetic fields; no coupling or crosstalk between "inductors."
2. Very small size required for large values of inductance.
3. Inexpensive, use readily available components.
4. Accurately predictable "saturation" levels.
5. Parameters can be fixed by choice of resistors.

### Disadvantages

1. Active device generates noise (can be held to low levels if proper devices are selected).
2. More complex circuits are required to simulate "floating" inductors.
3. Inductors with low series resistance and high current handling characteristics are difficult and impractical to simulate, as the circuits require high-power active devices.
4. Simulated inductors are frequency limited by their active devices' usable bandwidths and slew rates (not a problem at audio frequencies in most cases).

# The right gyrator trims the fat off active filters

Replacing inductors with gyrators creates almost perfect filters

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□ Analog filters exhibiting nearly ideal performance can be built around a gyrator—if the right configuration of this active circuit is used. In effect, the gyrator makes a capacitance behave like an inductor, freeing the filter of the problems plaguing conventional inductors, like large size, low  $Q$ , winding capacitance, nonlinearity, and magnetic susceptibility.

Yet most designers look upon the gyrator as an idealistic circuit with a “peculiar” behavior that puts it out of touch with practical applications. This attitude completely ignores its power. Unlike other active-filter circuits, the gyrator permits the designer to take advantage of the large body of data and techniques already developed for passive LC filters. He can start with a passive prototype circuit and then replace each inductor with a gyrator, substantially reducing filter size and weight for frequencies up to about 50 kilohertz.

Fortunately, too, there is one gyrator realization that works superbly. Not all of them do—in the past, different versions have suffered from drawbacks like instability, poor control of loss, sensitivity to component matching, and even excessive complexity. But the preferred version is simple and stable and simulates a high-quality inductor, permitting very high-performance filters to be realized. In addition, this gyrator, unlike other active-filter circuits, preserves the most significant advantage of coupled LC networks—their inherently low sensitivity to changes in component values (see “The strength of LC filters,” p. 116).

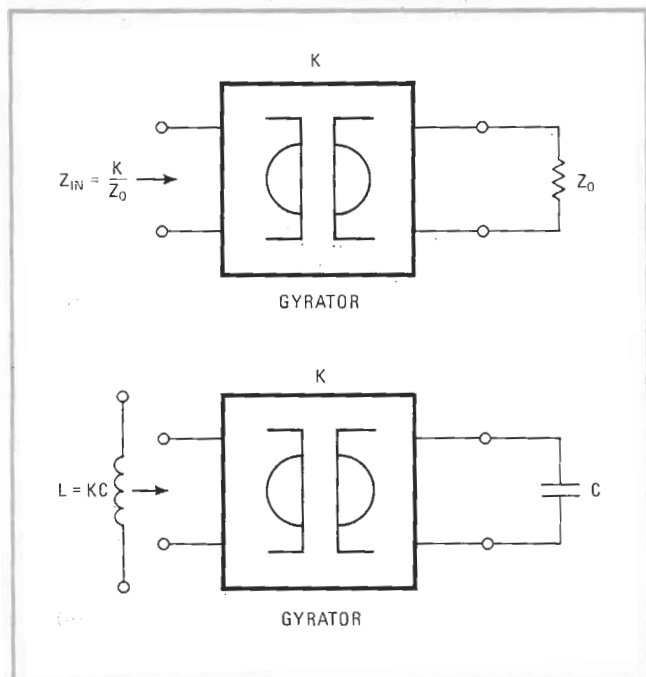
## Understanding the gyrator

Basically, the gyrator is a lossless two-port circuit (Fig. 1a) that inverts a load impedance. When used with a high- $Q$  capacitor (Fig. 1b), it simulates the virtual characteristics of a high- $Q$  inductor. The preferred realization for the gyrator requires only two amplifiers and five impedances, as shown in Fig. 2 for both the general impedance representation (a) and the practical RC implementation (b). In the latter case, the circuit simulates an inductor having a value of  $KC$ , where  $K$  is a constant determined by the resistors:

$$K = R_1 R_3 R_5 / R_2$$

At first glance, this gyrator's need for two amplifiers may seem a disadvantage. However, consider the major drawback of most single-amplifier resonators. They generally require an amplifier having a gain in excess of  $Q^2$ ; and those that do not usually are extremely sensitive to passive-element variations. On the other hand, the gyrator does not require a high-gain amplifier—in fact, stable  $Q$ s of better than 1,000 may be obtained with only 40 decibels of gain. Furthermore, unlike other active-filter circuits, the gyrator is remarkably insensitive to any amplifier parameter, so it may be built with garden-variety devices, even quad chips, as long as they are unity-gain-stable amplifiers.

Additionally, with the gyrator, amplifier phase shift enhances  $Q$ , rather than diminishing it as in other active-



1. Ideally. Coupled LC filters simulated with gyrators have characteristics approaching the ideal. In effect, the gyrator is a lossless two-port circuit (a) that inverts a load impedance. With a capacitive load impedance, the circuit (b) simulates a high-quality inductor.

## The strength of LC filters

In theory, coupled LC filters have the lowest sensitivity to component variations. These doubly terminated reactive two-ports produce a frequency response by reflecting power back to the source in the stopbands. In the passband, power transfer is maximum at natural modes (a) determined by the filter's transfer function.

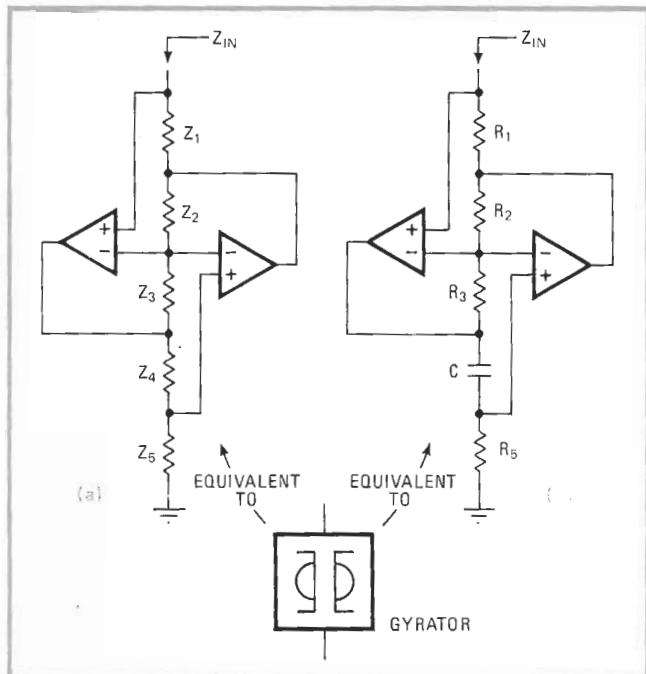
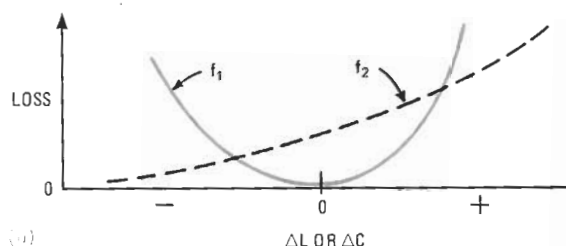
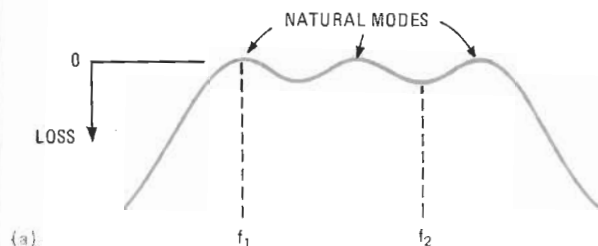
In the real world, a change in any inductor or capacitor making up the filter can only cause a loss in the load power—down from the maximum. This power loss at one of the natural modes ( $f_1$ ) increases monotonically (b), while at frequencies other than the natural modes, such as  $f_2$ , a small loss exists because of the ripple caused by reflected power. At any point within the passband, then, the change in loss has a well-behaved and slowly varying characteristic that follows the changes of any inductor or capacitor in the network.

Since no one inductor or capacitor determines a natural mode, a change in any single inductance or capacitance can only partially affect the shift in a natural-mode frequency. It follows that coupled LC filters are inherently insensitive to changes in component values. Similarly, if all of the inductors in the filter are replaced by gyrators, this insensitivity to component variations does not change,

since the gyrator is an active circuit and adds no dissipative elements to the filter.

Other active-filter circuits, however, like the biquad and state-variable or universal active filters, are developed from the state equations describing a second-order transfer function. Since these circuits duplicate the properties of a second-order (one complex pole pair) LC filter, their component sensitivity is still the same as for coupled second-order networks. On the other hand, extension to higher orders requires factorization of the transfer function into biquadratic (second-order plus lower-order) factors, each of which specifies a separate Q and natural-mode frequency. The desired transfer function is then realized by cascading biquadratic stages. Since these stages are uncoupled, changes can easily occur in the amplitude or frequency of the simulated modes, making the high-order filters built this way sensitive to component variations.

An alternative is the design approach called leap-frog. It implements the state equations of the prototype LC filter directly, using integrators and summing amplifiers. But though the resulting filter does have about the same low sensitivity as the equivalent coupled LC network, the final circuit can become very complex for high-order functions.



**2. Realistically.** Preferred gyrator realization (a) requires two amplifiers and five impedances. In practical RC implementation (b), impedance  $Z_4$  is a capacitor, and the other impedances are resistors. The gyrator may also be represented by the special symbol shown here.

filter circuits. At the ideal phase shift of  $90^\circ$ , the Q of the simulated inductor is approximately equal to that of the capacitor being used. If the phase shift is greater than  $90^\circ$ —which is usually the case—the Q becomes even higher.

Needless to say, the inductor the gyrator simulates is not perfect—the gyrator can be no better than the resistors and capacitors with which it is built. Of the two, resistors are less worrisome, for tin-oxide, metal-film, and thin-film types all perform acceptably. Capacitors, on the other hand, are the weakest link in the gyrator circuit, and there are usually two or more of them per complex pole pair. They impose the first limitation—maximum Q—in any realization, and their capacitance may change a lot with temperature. The table reviews the important characteristics of a variety of capacitor types. Generally, NPO ceramic devices are least affected by temperature, whereas polypropylene units achieve the highest Q.

### Creating a floating gyrator

One seeming limitation of the gyrator is that it is grounded at one end. But the floating inductor often needed in a filter can be simulated successfully—for example, by connecting two grounded gyrators. However, this does not necessarily mean that an extra gyrator is required for every floating inductor in a passive LC

filter. Figure 3a shows two gyrators sharing the bottom resistor,  $R_5$ , at opposite ends, so as to simulate a single floating inductor. This resistor is described in the gyrator constant:

$$L = \frac{R_1 R_3 R_5}{R_2} C$$

The equation may be rewritten as:

$$L = \frac{R_1 R_3 C}{R_2} R_5$$

which says that the simulated inductance is directly proportional to the value of  $R_5$ . Therefore, if  $R_5$  in fact becomes a loaded port for the gyrator, the simulated inductance will depend on the value of resistance connected to that port.

A cursory examination of the preferred gyrator realization (in Fig. 2b) will reveal a corollary to the above relationship. Between the top of  $R_1$  (the input port) and the bottom of  $C$ , a voltage null exists because of the amplifiers' input connections. Therefore, the port to which  $R_5$  is connected has the same voltage as the input port (although the currents are not the same, otherwise an apparent inductance could not exist). As a result, the input impedance,  $Z_{IN}$ , is also proportional to resistor  $R_5$ . This resistor could even be a network of resistors to describe the topological connections, of, say, a T or pi network of inductors, as indicated in Fig. 3b.

### Designing another floating gyrator

Indirectly, a floating inductor may be achieved in another way—one that opens up other possibilities for converting a passive LC filter to its active equivalent. When a second capacitor is added to the gyrator and one resistor removed, the circuit becomes a functionally dependent negative resistor (FDNR)—an element that appears to be a negative resistance that decreases in value with frequency.

Consider the transfer function for the basic impedance converter of Fig. 2a:

$$Z_{IN} = (Z_1 Z_3 Z_5) / (Z_2 Z_4)$$

If  $Z_1$  and  $Z_3$  are capacitors and the other impedances are resistors, then:

$$Z_1 = Z_3 = 1/sC$$

where  $s = j\omega$ . The input impedance can then be written as:

$$Z_{IN} = \text{FDNR} = \frac{1}{s^2} \frac{R_5}{C^2 R_2 R_4} = -\frac{1}{\omega^2} \frac{R_5}{C^2 R_2 R_4}$$

This element may be used to solve the problem of simulating floating inductors in low-pass filters. The technique is simple—just divide all elements by  $s$ , which is the complex variable, and then replace the  $1/s^2$  terms with an FDNR, as shown in Fig. 3c. The floating inductors become resistors having a value of  $L$ . Resistor  $R_A$  and resistor  $R_B$  simply provide bias and response for the circuit at dc.

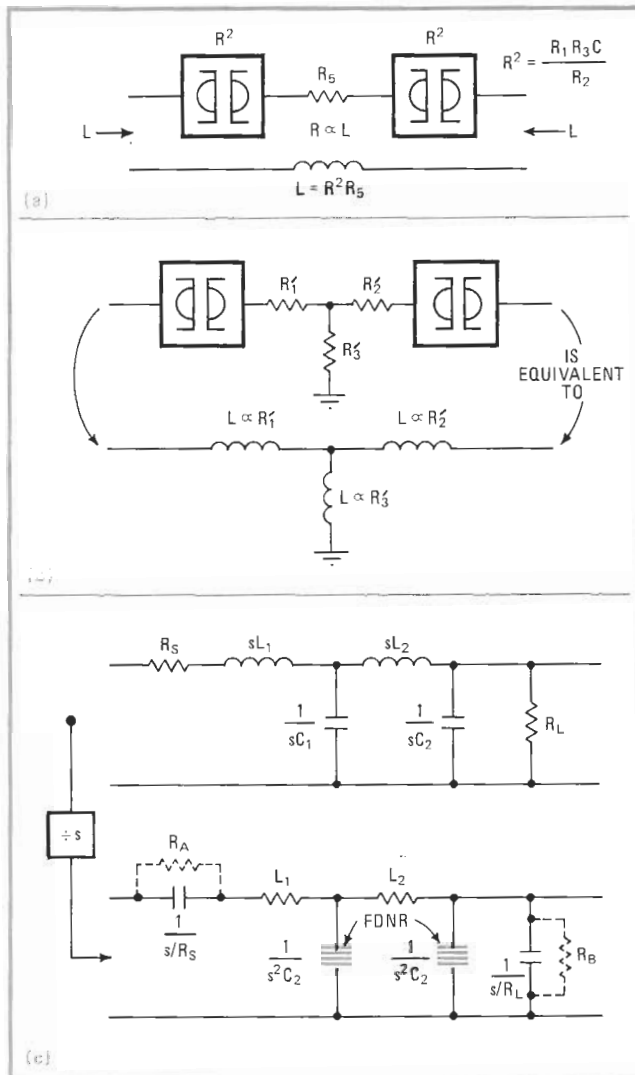
Although the gyrator is not the easiest circuit to understand, designing with it is really not that hard. Suppose the requirement is for a double-tuned bandpass

### CAPACITOR CHARACTERISTICS

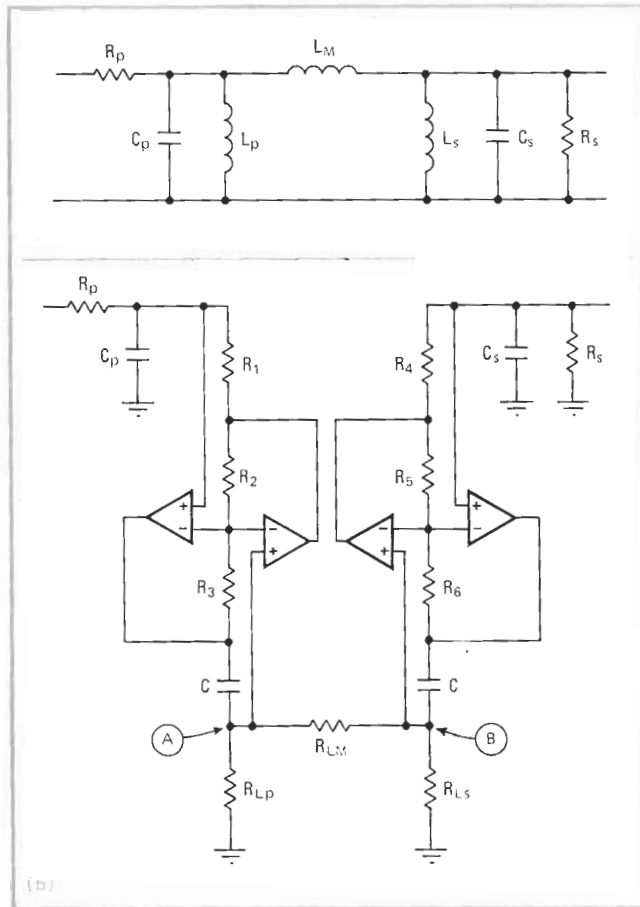
| Type            | Q (at 1 kHz) | Temperature coefficient (ppm/°C) | Temperature range (°C) |
|-----------------|--------------|----------------------------------|------------------------|
| Mica            | 600          | 1 to +70                         | -55 to +125            |
| Polystyrene     | 2,000        | -150 ±50                         | -55 to +85             |
| NPO ceramic     | 1,500        | ±30                              | -55 to +125            |
| Polypropylene   | 3,000        | -115                             | -55 to +125            |
| Glass           | 1,500        | +140 ±25                         | -55 to +125            |
| Polycarbonate** | 500          | ≈ 50*                            | -40 to +100            |
| Mylar**         | 100          | large                            | -55 to +85             |
| Polyester**     | 100          | -160*                            | -40 to +100            |
| Porcelain       | 2,500        | ±25                              | -55 to +125            |

\*0°C to 50°C

\*\*Q and C nonlinear functions of frequency and function



**3. Floating.** Lower terminal of basic gyrator (Fig. 2b) is grounded. To float the circuit, two gyrators may share the same resistor (a), or a network of resistors (b). Floating inductors may also be simulated (c) with functionally dependent negative resistors (FDNRs).



**4. Bandpass filter.** Double-tuned bandpass filter of (a) may be built with two gyrators, as in (b). Sharing resistor  $R_{LM}$ , the grounded gyrators simulate the pi network of inductors in the passive version. All of the amplifiers may be general-purpose devices.

filter, like the one drawn in Fig. 4a. The procedure is straightforward. First compile the design data required:

- $e_r$ , the desired passband ripple, expressed in peak-to-peak decibels;
- $f_0$ , the center frequency in hertz;
- $f_r$ , the ripple bandwidth in Hz;
- $C$ , the capacitance value for both  $C_p$  and  $C_s$  in farads;
- $r$ , the termination ratio of  $R_p/R_s$ .

Next, calculate these variables:

$$A = (10^{e_r/20})^{-1}$$

$$q = \left[ \frac{2(1 - A^2)^{1/2} + r + (1/r)}{2 - 2(1 - A^2)^{1/2}} \right]^{1/2}$$

$$Q' = f_0/f_r$$

$$\alpha = 2(1 - A^2)^{1/2}$$

$$X = [\alpha(1 + q^2)]^{1/2}$$

$$Q = Q'X$$

$$X' = \left\{ \frac{\alpha}{2} (1 + q^2) \left[ 1 + \frac{[\alpha^2 + 4(10^{e_r/20} - 1)]^{1/2}}{\alpha^2} \right] \right\}^{1/2}$$

Then compute the design results:

$$R_s = \frac{Q}{\omega_0 C r^{1/2}}$$

$$R_p = R_s r$$

$$L_M = \frac{R_s r^{1/2}}{\omega_0 q}$$

$$L_p = L_s = \frac{L_M}{\omega_0^2 C L_M - 1}$$

$$G_0 = \frac{1}{r^{1/2}} \left[ \frac{q}{1 + q^2} \right]$$

where  $G_0$  is the midband gain.

$$BW(3 \text{ dB}) = \frac{X'}{X} f_r$$

where  $BW(3 \text{ dB})$  is the 3-dB bandwidth.

### Building a double-tuned filter

Two gyrators replace the pi network of inductors, as shown in Fig. 4b. Now assign the same capacitance value for  $C$  as that chosen for  $C_p$  and  $C_s$ . The amplifiers may be a quad-type 4136, since choosing such a device ensures a good match between the amplifiers for optimum gyrator performance. Next, determine the gyrator resistances from:

$$L_p = (R_1 R_3 R_{Lp} C) / R_2$$

It is good practice to maintain the resistances at about the same value, so assume:

$$R_1 = R_2 = R_3 = R_{Lp} = R$$

Then:

$$R^2 = L_p / C$$

The value of  $R$  should be large enough to minimize amplifier loading and slightly smaller than the amplifier differential input impedance. For convenience, choose the closest standard value for  $R$  and let:

$$R = R_1 = R_2 = R_3$$

Then compute:

$$R_{Lp} = L_p / CR$$

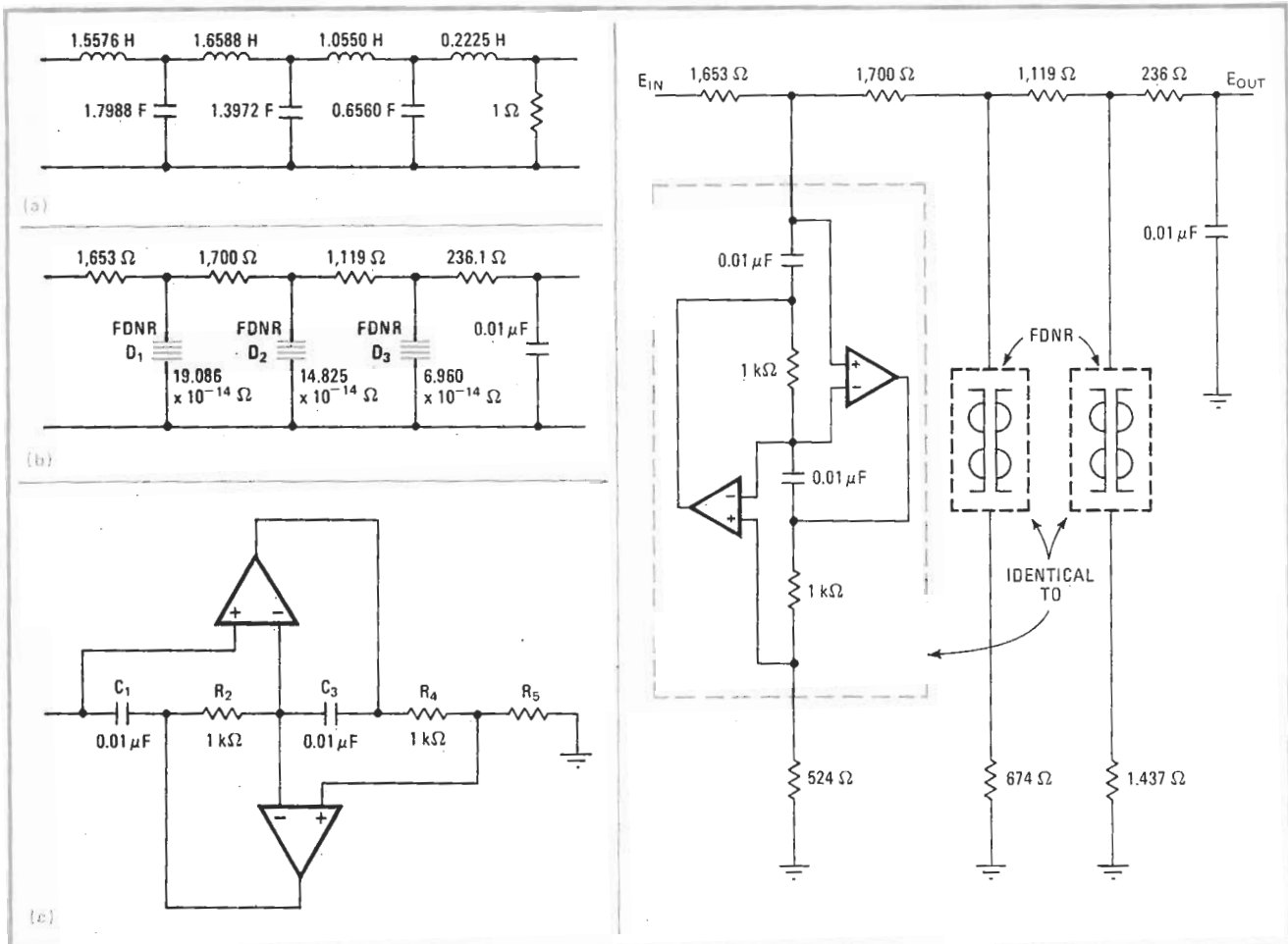
Again select a nearby standard value for  $R_{Lp}$  and scale  $R_{LM}$ :

$$R_{LM} / R_{Lp} = L_M / L_p$$

This ratio must be maintained, since it determines the coupling in the circuit. All of the component values are now known. The tolerances for  $C$  may be  $\pm 5\%$  and  $\pm 1\%$  for  $R$ , although the tolerances for  $R_{1-6}$  could be looser as long as the devices' environmental characteristics are acceptable.

### Tuning the filter

Tuning is simple. Overall  $Q$  has already been determined by the value of  $R_{LM}$ . Using point A as the output, the primary (left-hand) gyrator is set by shorting point B to ground and adjusting  $R_1$  or  $R_2$  to achieve resonance at the center frequency. Reducing  $R_1$  (by shunting it) will decrease  $L_p$ , raising the resonant frequency. The reverse holds for  $R_2$ . Tune for  $0^\circ$  phase shift relative to the



**5. Aliasing filter.** Number of floating inductors in low-pass aliasing filter (a) makes implementation with grounded gyrators difficult. Instead, FDNRs may be used (b), each of which requires two capacitors (c). The final circuit (d) employs only three of these FDNRs.

source. Next, disconnect point B from ground, and tune  $R_4$  or  $R_5$  for  $90^\circ$  phase shift (at the center frequency) at the output, or point A, which is equivalent. (Tuning with a Lissajous circle can provide accuracy to better than  $2^\circ$ .)

A second example demonstrates how to design with FDNRs. The requirement is for a low-pass aliasing filter—a seventh-order Butterworth circuit having a 1-dB corner at 15 kilohertz.

#### Using functionally dependent negative resistors

A prototype circuit (Fig. 5a) is first obtained from one of the standard tables in existing literature. Here, the number of floating inductors prevents easy implementation with the grounded gyrators, and the best approach is to use their close relative—the FDNR.

To convert the LC prototype circuit to an FDNR realization, first normalize the corner frequency to 1 radian per second. Next scale the circuit for frequency by dividing the inductors and capacitors by  $2\pi(15 \text{ kHz})$ , and then scale the impedances for a convenient capacitor value, say 0.01 microfarad or  $1/(0.01 \times 10^{-6})$  ohms, in which case multiply the inductances by  $10^8$  and divide the capacitances by  $10^8$ . Finally, dividing all of the network impedances by the complex variable,  $s$ , results in the FDNR realization of Fig. 5b.

Each FDNR is actually the basic gyrator configuration, but with two capacitors (Fig. 5c) instead of just one. In this example, all of the resistors, except  $R_5$ , are set equal to 1 kilohm, and the two capacitors to 0.01 microfarad each. Then  $R_5$  may be computed from:

$$R_5 = (R_2 R_4 C_1 C_3) / D$$

where  $D$  is the impedance value of the FDNR. Therefore, for  $D_1$ ,  $R_5 = 524$  ohms; for  $D_2$ ,  $R_5 = 674$  ohms, and for  $D_3$ ,  $R_5 = 1.437$  kilohms.

Figure 5d shows the final circuit, in which all seven capacitors have the same value, an essential point in low-cost design. The circuit may be built with 1% resistors having values closest to those computed and with NPO ceramic capacitors screened to tolerances of  $\pm 1\%$ . Again, garden-variety amplifiers, like the 4136 quad, perform well enough for the purpose.  $\square$

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