Chapter 2

Review of Circuit Theory

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2.1 Introduction

Although this book minimizes math, some algebra is germane to the understanding of analog electronics. Math and physics are presented here in the manner in which they are used later, so no practice exercises are given. For example, after the voltage divider rule is explained, it is used several times in the development of other concepts, and this usage constitutes practice.

Circuits are a mix of passive and active components. The components are arranged in a manner that enables them to perform some desired function. The resulting arrangement of components is called a circuit or sometimes a circuit configuration. The art portion of analog design is developing the circuit configuration. There are many published circuit configurations for almost any circuit task, thus all circuit designers need not be artists.

When the design has progressed to the point that a circuit exists, equations must be written to predict and analyze circuit performance. Textbooks are filled with rigorous methods for equation writing, and this review of circuit theory does not supplant those textbooks. But, a few equations are used so often that they should be memorized, and these equations are considered here.

There are almost as many ways to analyze a circuit as there are electronic engineers, and if the equations are written correctly, all methods yield the same answer. There are some simple ways to analyze the circuit without completing unnecessary calculations, and these methods are illustrated here.

2.2 Laws of Physics

Ohm's law is stated as V=IR, and it is fundamental to all electronics. Ohm's law can be applied to a single component, to any group of components, or to a complete circuit. When the current flowing through any portion of a circuit is known, the voltage dropped across that portion of the circuit is obtained by multiplying the current times the resistance (Equation 2–1).

$$V = IR \tag{2–1}$$

In Figure 2–1, Ohm's law is applied to the total circuit. The current, (I) flows through the total resistance (R), and the voltage (V) is dropped across R.

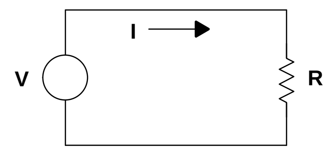

Figure 2–1. Ohm's Law Applied to the Total Circuit

In Figure 2–2, Ohm's law is applied to a single component. The current (I_R) flows through the resistor (R) and the voltage (V_R) is dropped across R. Notice, the same formula is used to calculate the voltage drop across R even though it is only a part of the circuit.

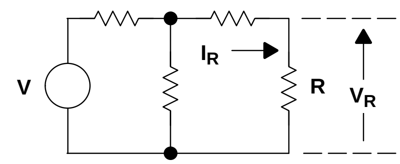

Figure 2–2. Ohm's Law Applied to a Component

Kirchoff's voltage law states that the sum of the voltage drops in a series circuit equals the sum of the voltage sources. Otherwise, the source (or sources) voltage must be dropped across the passive components. When taking sums keep in mind that the sum is an algebraic quantity. Kirchoff's voltage law is illustrated in Figure 2–3 and Equations 2–2 and 2–3.

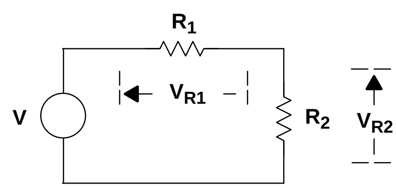

Figure 2–3. Kirchoff's Voltage Law

$$\sum V_{SOURCES} = \sum V_{DROPS} \tag{2–2}$$

$$V = V_{R1} + V_{R2} \tag{2–3}$$

Kirchoff's current law states: the sum of the currents entering a junction equals the sum of the currents leaving a junction. It makes no difference if a current flows from a current source, through a component, or through a wire, because all currents are treated identically. Kirchoff's current law is illustrated in Figure 2–4 and Equations 2–4 and 2–5.

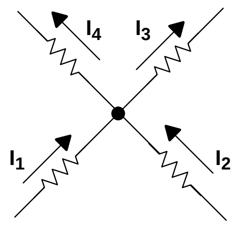

Figure 2–4. Kirchoff's Current Law

$$\sum I_{IN} = \sum I_{OUT} \tag{2–4}$$

$$I_1 + I_2 = I_3 + I_4 \tag{2–5}$$

2.3 Voltage Divider Rule

When the output of a circuit is not loaded, the voltage divider rule can be used to calculate the circuit's output voltage. Assume that the same current flows through all circuit elements (Figure 2–5). Equation 2–6 is written using Ohm's law as V = I ($R_1 + R_2$). Equation 2–7 is written as Ohm's law across the output resistor.

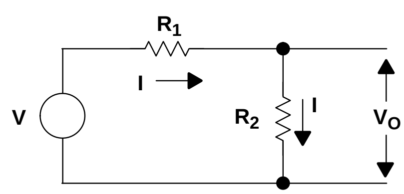

Figure 2–5. Voltage Divider Rule

$$I = \frac{V}{R_1 + R_2} \tag{2–6}$$

$$V_{OUT} = IR_2 \tag{2–7}$$

Substituting Equation 2–6 into Equation 2–7, and using algebraic manipulation yields Equation 2–8.

$$V_{OUT} = V \frac{R_2}{R_1 + R_2} \tag{2–8}$$

A simple way to remember the voltage divider rule is that the output resistor is divided by the total circuit resistance. This fraction is multiplied by the input voltage to obtain the output voltage. Remember that the voltage divider rule always assumes that the output resistor is not loaded; the equation is not valid when the output resistor is loaded by a parallel component. Fortunately, most circuits following a voltage divider are input circuits, and input circuits are usually high resistance circuits. When a fixed load is in parallel with the output resistor, the equivalent parallel value comprised of the output resistor and loading resistor can be used in the voltage divider calculations with no error. Many people ignore the load resistor if it is ten times greater than the output resistor value, but this calculation can lead to a 10% error.

2.4 Current Divider Rule

When the output of a circuit is not loaded, the current divider rule can be used to calculate the current flow in the output branch circuit (R_2). The currents I_1 and I_2 in Figure 2–6 are assumed to be flowing in the branch circuits. Equation 2–9 is written with the aid of Kirchoff's current law. The circuit voltage is written in Equation 2–10 with the aid of Ohm's law. Combining Equations 2–9 and 2–10 yields Equation 2–11.

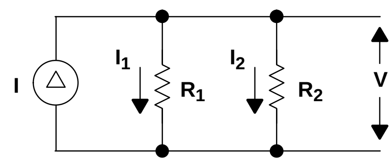

Figure 2–6. Current Divider Rule

$$I = I_1 + I_2 \tag{2–9}$$

$$V = I_1R_1 = I_2R_2 \tag{2–10}$$

$$I = I_1 + I_2 = I_2\frac{R_2}{R_1} + I_2 = I_2\left(\frac{R_1 + R_2}{R_1}\right) \tag{2–11}$$

Rearranging the terms in Equation 2–11 yields Equation 2–12.

$$I_2 = I\left(\frac{R_1}{R_1 + R_2}\right) \tag{2–12}$$

The total circuit current divides into two parts, and the resistance (R_1) divided by the total resistance determines how much current flows through R_2. An easy method of remembering the current divider rule is to remember the voltage divider rule. Then modify the voltage divider rule such that the opposite resistor is divided by the total resistance, and the fraction is multiplied by the input current to get the branch current.

2.5 Thevenin's Theorem

There are times when it is advantageous to isolate a part of the circuit to simplify the analysis of the isolated part of the circuit. Rather than write loop or node equations for the complete circuit, and solving them simultaneously, Thevenin's theorem enables us to isolate the part of the circuit we are interested in. We then replace the remaining circuit with a simple series equivalent circuit, thus Thevenin's theorem simplifies the analysis.

There are two theorems that do similar functions. The Thevenin theorem just described is the first, and the second is called Norton's theorem. Thevenin's theorem is used when the input source is a voltage source, and Norton's theorem is used when the input source is a current source. Norton's theorem is rarely used, so its explanation is left for the reader to dig out of a textbook if it is ever required.

The rules for Thevenin's theorem start with the component or part of the circuit being replaced. Referring to Figure 2–7, look back into the terminals (left from C and R_3 toward point XX in the figure) of the circuit being replaced. Calculate the no load voltage (V_{TH}) as seen from these terminals (use the voltage divider rule).

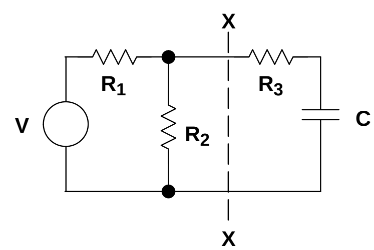

Figure 2–7. Original Circuit

Look into the terminals of the circuit being replaced, short independent voltage sources, and calculate the impedance between these terminals. The final step is to substitute the Thevenin equivalent circuit for the part you wanted to replace as shown in Figure 2–8.

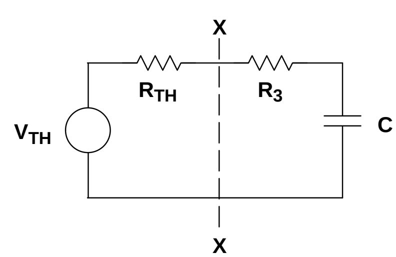

Figure 2–8. Thevenin's Equivalent Circuit for Figure 2–7

The Thevenin equivalent circuit is a simple series circuit, thus further calculations are simplified. The simplification of circuit calculations is often sufficient reason to use Thevenin's theorem because it eliminates the need for solving several simultaneous equations. The detailed information about what happens in the circuit that was replaced is not available when using Thevenin's theorem, but that is no consequence because you had no interest in it.

As an example of Thevenin's theorem, let's calculate the output voltage (V_{OUT}) shown in Figure 2–9A. The first step is to stand on the terminals X–Y with your back to the output circuit, and calculate the open circuit voltage seen (V_{TH}). This is a perfect opportunity to use the voltage divider rule to obtain Equation 2–13.

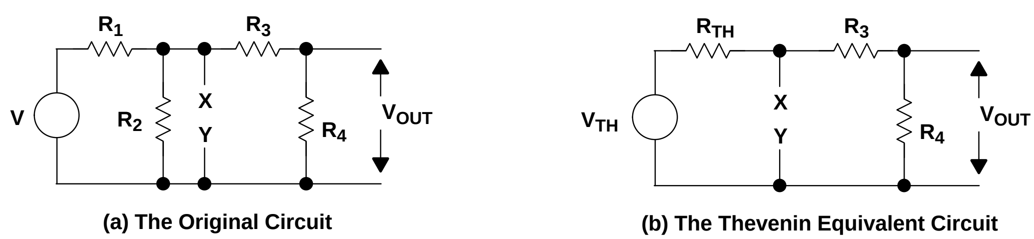

Figure 2–9. Example of Thevenin's Equivalent Circuit

$$V_{TH} = V \frac{R_2}{R_1 + R_2} \tag{2–13}$$

Still standing on the terminals X-Y, step two is to calculate the impedance seen looking into these terminals (short the voltage sources). The Thevenin impedance is the parallel impedance of R_1 and R_2 as calculated in Equation 2–14. Now get off the terminals X-Y before you damage them with your big feet. Step three replaces the circuit to the left of X-Y with the Thevenin equivalent circuit V_{TH} and R_{TH}.

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = R_1 \parallel R_2 \tag{2–14}$$

Note:

Two parallel vertical bars (||) are used to indicate parallel components as shown in Equation 2–14.

The final step is to calculate the output voltage. Notice the voltage divider rule is used again. Equation 2–15 describes the output voltage, and it comes out naturally in the form of a series of voltage dividers, which makes sense. That's another advantage of the voltage divider rule; the answers normally come out in a recognizable form rather than a jumble of coefficients and parameters.

$$V_{OUT} = V_{TH}\frac{R_4}{R_{TH} + R_3 + R_4} = V\left(\frac{R_2}{R_1 + R_2}\right)\frac{R_4}{\frac{R_1R_2}{R_1+R_2} + R_3 + R_4} \qquad (2\text{–}15)$$

The circuit analysis is done the hard way in Figure 2–10, so you can see the advantage of using Thevenin's Theorem. Two loop currents, I_1 and I_2, are assigned to the circuit. Then the loop Equations 2–16 and 2–17 are written.

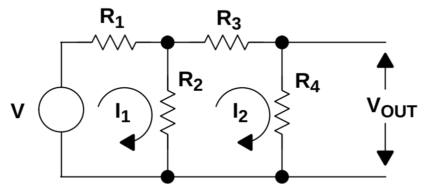

Figure 2–10. Analysis Done the Hard Way

$$V = I_1\left(R_1 + R_2\right) - I_2R_2 \qquad (2\text{–}16)$$

$$I_2\left(R_2 + R_3 + R_4\right) = I_1R_2 \qquad (2\text{–}17)$$

Equation 2–17 is rewritten as Equation 2–18 and substituted into Equation 2–16 to obtain Equation 2–19.

$$I_1 = I_2\frac{R_2 + R_3 + R_4}{R_2} \qquad (2\text{–}18)$$

$$V = I_2\left(\frac{R_2 + R_3 + R_4}{R_2}\right)\left(R_1 + R_2\right) - I_2R_2 \qquad (2\text{–}19)$$

The terms are rearranged in Equation 2–20. Ohm's law is used to write Equation 2–21, and the final substitutions are made in Equation 2–22.

$$I_2 = \frac{V}{\frac{R_2+R_3+R_4}{R_2}\left(R_1 + R_2\right) - R_2} \qquad (2\text{–}20)$$

$$V_{OUT} = I_2R_4 \qquad (2\text{–}21)$$

$$V_{OUT} = V\frac{R_4}{\frac{\left(R_2+R_3+R_4\right)\left(R_1+R_2\right)}{R_2} - R_2} \qquad (2\text{–}22)$$

This is a lot of extra work for no gain. Also, the answer is not in a usable form because the voltage dividers are not recognizable, thus more algebra is required to get the answer into usable form.

2.6 Superposition

Superposition is a theorem that can be applied to any linear circuit. Essentially, when there are independent sources, the voltages and currents resulting from each source can be calculated separately, and the results are added algebraically. This simplifies the calculations because it eliminates the need to write a series of loop or node equations. An example is shown in Figure 2–11.

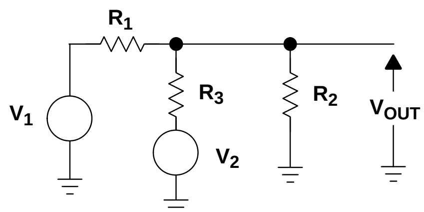

Figure 2–11. Superposition Example

When V_1 is grounded, V_2 forms a voltage divider with R_3 and the parallel combination of R_2 and R_1. The output voltage for this circuit (V_{OUT2}) is calculated with the aid of the voltage divider equation (2–23). The circuit is shown in Figure 2–12. The voltage divider rule yields the answer quickly.

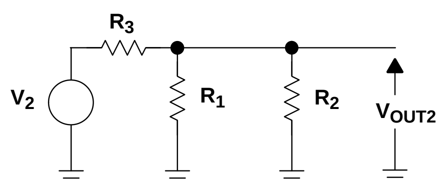

Figure 2–12. When V_1 is Grounded

$$V_{OUT2} = V_2 \frac{R_1 \parallel R_2}{R_3 + R_1 \parallel R_2} \tag{2–23}$$

Likewise, when V_2 is grounded (Figure 2–13), V_1 forms a voltage divider with R_1 and the parallel combination of R_3 and R_2, and the voltage divider theorem is applied again to calculate V_{OUT} (Equation 2–24).

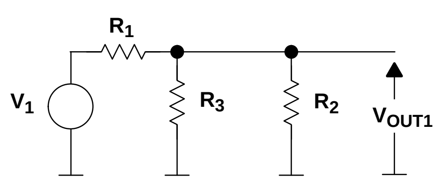

Figure 2–13. When V_2 is Grounded

$$V_{OUT1} = V_1 \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} \tag{2–24}$$

After the calculations for each source are made the components are added to obtain the final solution (Equation 2–25).

$$V_{OUT} = V_1 \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} + V_2 \frac{R_1 \parallel R_2}{R_3 + R_1 \parallel R_2} \tag{2–25}$$

The reader should analyze this circuit with loop or node equations to gain an appreciation for superposition. Again, the superposition results come out as a simple arrangement that is easy to understand. One looks at the final equation and it is obvious that if the sources are equal and opposite polarity, and when $R_1 = R_3$, then the output voltage is zero. Conclusions such as this are hard to make after the results of a loop or node analysis unless considerable effort is made to manipulate the final equation into symmetrical form.

2.7 Calculation of a Saturated Transistor Circuit

The circuit specifications are: when $V_{IN} = 12$ V, $V_{OUT} < 0.4$ V at $I_{SINK} < 10$ mA, and $V_{IN} < 0.05$ V, $V_{OUT} > 10$ V at $I_{OUT} = 1$ mA. The circuit diagram is shown in Figure 2–14.

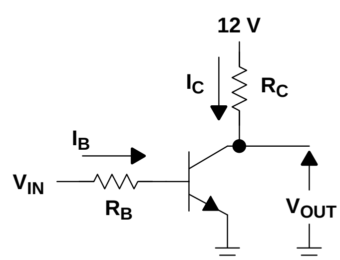

Figure 2–14. Saturated Transistor Circuit

The collector resistor must be sized (Equation 2–26) when the transistor is off, because it has to be small enough to allow the output current to flow through it without dropping more than two volts to meet the specification for a 10-V output.

$$R_C \le \frac{V_{+12} - V_{OUT}}{I_{OUT}} = \frac{12 - 10}{1} = 2\text{ k} \tag{2–26}$$

When the transistor is off, 1 mA can be drawn out of the collector resistor without pulling the collector or output voltage to less than ten volts (Equation 2–27). When the transistor is on, the base resistor must be sized (Equation 2–28) to enable the input signal to drive enough base current into the transistor to saturate it. The transistor beta is 50.

$$I_C = \beta I_B = \frac{V_{+12} - V_{CE}}{R_C} + I_L \approx \frac{V_{+12}}{R_C} + I_L \qquad (2\text{–}27)$$

$$R_B \le \frac{V_{IN} - V_{BE}}{I_B} \qquad (2\text{–}28)$$

Substituting Equation 2–27 into Equation 2–28 yields Equation 2–29.

$$R_B \le \frac{\left(V_{IN} - V_{BE}\right)\beta}{I_C} = \frac{(12 - 0.6)50\ \text{V}}{\left[\frac{12}{2} + (10)\right]\text{mA}} = 35.6\ \text{k} \qquad (2\text{–}29)$$

When the transistor goes on it sinks the load current, and it still goes into saturation. These calculations neglect some minor details, but they are in the 98% accuracy range.

2.8 Transistor Amplifier

The amplifier is an analog circuit (Figure 2–15), and the calculations, plus the points that must be considered during the design, are more complicated than for a saturated circuit. This extra complication leads people to say that analog design is harder than digital design (the saturated transistor is digital i.e.; on or off). Analog design is harder than digital design because the designer must account for all states in analog, whereas in digital only two states must be accounted for. The specifications for the amplifier are an ac voltage gain of four and a peak-to-peak signal swing of 4 volts.

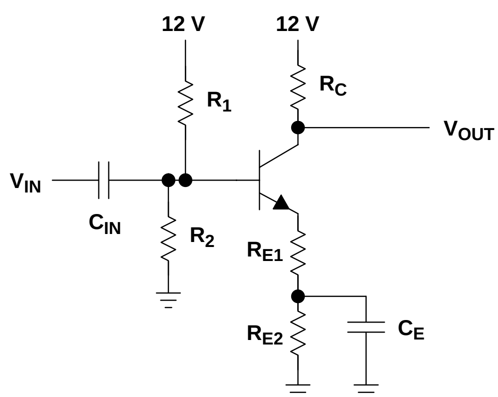

Figure 2–15. Transistor Amplifier

I_C is selected as 10 mA because the transistor has a current gain (β) of 100 at that point. The collector voltage is arbitrarily set at 8 V; when the collector voltage swings positive

2 V (from 8 V to 10 V) there is still enough voltage dropped across R_C to keep the transistor on. Set the collector-emitter voltage at 4 V; when the collector voltage swings negative 2 V (from 8 V to 6 V) the transistor still has 2 V across it, so it stays linear. This sets the emitter voltage (V_E) at 4 V.

$$R_C \le \frac{V_{+12} - V_C}{I_C} = \frac{12\ V - 8\ V}{10\ mA} = 400\ \Omega \tag{2–30}$$

$$R_E = R_{E1} + R_{E2} = \frac{V_E}{I_E} = \frac{V_E}{I_B + I_C} \cong \frac{V_E}{I_C} = \frac{4\ V}{10\ mA} = 400\ \Omega \tag{2–31}$$

Use Thevenin's equivalent circuit to calculate R_1 and R_2 as shown in Figure 2–16.

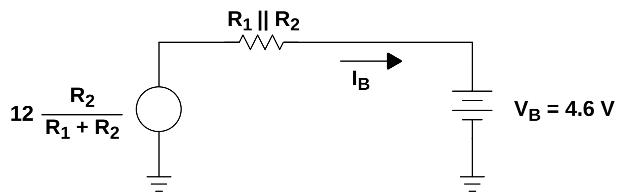

Figure 2–16. Thevenin Equivalent of the Base Circuit

$$I_B = \frac{I_C}{\beta} = \frac{10\ mA}{100} = 0.1\ mA \tag{2–32}$$

$$V_{TH} = \frac{12R_2}{R_1 + R_2} \tag{2–33}$$

$$R_{TH} = \frac{R_1R_2}{R_1 + R_2} \tag{2–34}$$

We want the base voltage to be 4.6 V because the emitter voltage is then 4 V. Assume a voltage drop of 0.4 V across R_{TH}, so Equation 2–35 can be written. The drop across R_{TH} may not be exactly 0.4 V because of beta variations, but a few hundred mV does not matter is this design. Now, calculate the ratio of R_1 and R_2 using the voltage divider rule (the load current has been accounted for).

$$R_{TH} = \frac{0.4}{0.1}k = 4\ k \tag{2–35}$$

$$V_{TH} = I_BR_{Th} + V_B = 0.4 + 4.6 = 5 = 12\frac{R_2}{R_1 + R_2} \tag{2–36}$$

$$R_2 = \frac{7}{5}\ R_1 \tag{2–37}$$

R_1 is almost equal to R_2, thus selecting R_1 as twice the Thevenin resistance yields approximately 4 K as shown in Equation 2–35. Hence, R_1 = 11.2 k and R_2 = 8 k. The ac gain is approximately R_C/R_{E1} because C_E shorts out R_{E2} at high frequencies, so we can write Equation 2–38.

$$R_{E1} = \frac{R_C}{G} = \frac{400}{4} = 100\ \Omega \tag{2–38}$$

$$R_{E2} = R_E - R_{E1} = 400 - 100 = 300\ \Omega \tag{2–39}$$

The capacitor selection depends on the frequency response required for the amplifier, but 10 μF for C_{IN} and 1000 μF for C_E suffice for a starting point.