# How to Design Analog Circuits Operational Amplifiers 

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This month we'll discuss one of the most useful integrated circuits-the operational amplifier. Even though they may seem complex, using them is not.


PERHAPS THE MOST WIDELY USED AMPLIfier circuit is the operational amplifier (op-amp). Commonly available as an integrated circuit, the op-amp can be configured with external components to perform a wide variety of circuit functions, including amplification, non-inverting amplification, addition, subtraction, differentiation, integration, etc.

An operational amplifier is characterized by high gain, high input impedance, and low output impedance. Additionally, the input stage of an opamp consists of a differential amplifier. In that type of amplifier the output signal is proportional to the difference between two input signals. Let's take a closer look at how a differential amplifier works.

## The differential amplifier

A basic differential-amplifier circuit is shown in Fig. 1. All the corresponding components in the two transistor circuits of that amplifier are assumed to be identical. That would not be a good assumption if you were using discrete components but, as the transistors used for differential amplifiers in op-amp IC's are made on the same substrate, they are essentially identical. Because of that, and because of the symmetry of the circuit, the currents through both transistors are identical when idling. But, if different
signals are present at each input, then the collector currents through each transistor will be different. Despite that, however, the total current through Q1 and Q2 will remain constant, and be approximately equal to $\mathrm{V}_{\mathrm{CC}} / \mathrm{R}_{\mathrm{E}}$. The output of the differential amplifier is the difference between the voltage developed on the collector of Q1 and the voltage developed on the collector of Q2.

## Common mode inputs

If identical signals were fed to each input of the circuit of Fig. 1 (that is referred to as common-mode operation)
there would be no output. That is because the voltages at the collectors would be equal due to the circuit symmetry. In real circuits, however, there is always some output due to dissimilarities between the two halves of the amplifier. The ratio of that output signal to the input signal is called the common-mode gain. The ratio of the normal gain (when different signals are fed to the two inputs) to the commonmode gain is referred to as the CMRR (Common Mode Rejection Ratio). The CMRR is often expressed in decibels and called the CMR (Common-Mode Rejection). $\left(\mathbf{C M R}=20 \log _{10} C M R R\right.$. $)$ It is de-


FIG. 1-IN A DIFFERENTIAL AMPLIFIER, the values of $R_{E 1}$ and $R_{E 2}$ are often equal to zero.
sirable that the CMRR be large so that the circuit will be immune to common-mode disturbances. Such common-mode disturbances include noise, changes in circuit operation due to temperature changes and power-supply voltage variations, and other factors that affect both halves of the amplifier.

## Differential mode inputs

When two different signals are applied to the input terminals ( $\mathrm{V}_{\text {IN } 1}$ is not equal to $\mathrm{V}_{\text {IN } 2}$ ), the currents through each transistor are different, and a signal will be seen across the output terminals. If we apply a signal $\mathrm{V}_{1}$ to Q 1 , the output signal will be equal to $A_{V} V_{1}$, where $A_{V}$ is the voltage gain of the amplifier. That input is referred to as the non-inverting input. If that same signal were applied to Q2, the output would be $-\mathrm{A}_{\mathrm{V}} \mathrm{V}_{1}$. That input is referred to as the inverting input.

## Op amps as devices

An op-amp can be thought of as a differential amplifier with a very large voltage gain. The symbol for an op-amp is shown in Fig. 2. The inverting input is marked with a minus sign and the noninverting input is marked with a plus sign.


FIG. 2-THE STANDARD OP-AMP SYMBOL. Note that not all of the device's leads are shown.

To analyze an operational amplifier, it is easier to first consider it as an ideal device. The analysis can then be expanded to take into account the limitations of a real one. The model we will use for the ideal operational amplifier is shown in Fig. 3. One very important assumption is that the input resistance, $\mathrm{R}_{\mathrm{IN}}$, is infinite. The voltage gain of the op-amp, $\mathrm{A}_{\mathrm{V}}$, as well as the bandwidth, are also assumed to be infinite. The output impedance, $\mathrm{R}_{\mathrm{OUT}}$, and response time can be considered to be equal to zero. These assumptions make analysis of opamp circuits relatively easy. Obviously, though, no op-amp will fulfill any of the ideal characteristics, but the closer we come to them, the better our results will be.


FIG. 3-USING THIS MODEL of an ideal op-amp will make studying the device much easier.


FIG.4-SOME EARLY OP-AMP IC's such as the 709 (shown in a) did not have as much circuitry as later IC's. That's why it had pinouts for such things as input-frequency compensation. Later IC op-amps, such as the 741 (shown in b) have that circuitry built in.


FIG. 5-THE FIVE BASIC op-amp circuits that use feedback are the inverting amplifier (a), the non-inverting amplifier (b), the voltage follower (c), the adder ( $d$ ), and the subtractor (e).

Figure 4-a shows one of the early popular op-amps, the 709. It has several leads or terminals that were not shown in the diagram of the ideal IC. First, there are leads for the positive and negative supply voltages, $+\mathrm{V}_{\mathrm{CC}}$ and $-\mathrm{V}_{\mathrm{CC}}$, respectively. Also, there is a terminal to which a frequency-compensating circuit is connected. That circuit's function is to stabilize the op-amp and keep it from oscillating. The capacitor-resistor network required is usually specified by the manufacturer of the IC. If there are noterminals for that purpose on the op-amp you are using (as on the more recent $741 \mathrm{op-amp}$ shown in Fig. 4-b), the compensation network may already be included on the chip. The 741 also has offset null or balance terminals that allow you to adjust the idling output voltage to 0 -volt DC when both inputs are grounded.

## Feedback

Feedback is often used in op-amp circuits. In feedback circuits, a portion of
the signal from the output of an amplifier is fed back to the input. If that signal causes the output level to be reduced, the circuit is said to have negative feedback. Should the opposite be true and the output level be increased, the circuit is said to have positive feedback. Negative feedback can be used to improve certain characteristics of a circuit. Positive feedback, however, can often make the circuit unstable and cause it to oscillate. Also, it can cause an output signal to be generated even when no signal is input.

## Basic op-amp circuits

If any of the op-amp circuits we will discuss are to work properly, the properties of the IC should be as close as possible to the ideal characteristics discussed above. Also, the inputs of the op-amp must be balanced when no signal is applied to the circuit. That means that the impedance between each input and ground should be identical. In the inverting amplifier shown in Fig. 5-a (we'll be
discussing that circuit in depth shortly), for example, the inverting input terminal sees a resistance equal to the parallel combination of $\mathrm{R}_{\mathrm{X}}$ and $\mathrm{R}_{\mathrm{F}}$. A resistor of that value should be wired between the noninverting terminal and ground to keep both halves of the op-amp in balance. The balancing resistor will not be shown in any of the figures that follow, but nonetheless should be included

## Voltage comparator

When an op-amp is used in its simplest form, without feedback, it can serve as a voltage comparator, as shown in Fig. 6. Because of the very high gain of the opamp, the output of that circuit will either be a positive- or negative-going spike. The value of $V_{1}$ that causes the output of the op-amp to switch polarity is equal to the voltage input to the non-inverting terminal. For example, let's assume that $\mathrm{V}_{2}$ equals zero (which sets the non-inverting input at ground potential) and that $\mathrm{V}_{1}$ is a sinewave. The signal at the inverting input $\left(V_{1}\right)$ is then compared to the signal at the non-inverting input (again in this case it is zero), and when it is greater than zero, the op-amp's output will be negative, and vice versa.

Let's now assume that $\mathrm{V}_{2}$ is at +3 volts. In that case, the output will not switch polarity and become negative until the amplitude of the sinewave at the inverting input exceeds +3 volts. Conversely, the op-amp's output will go positive when the voltage at the inverting terminal drops below +3 volts. The same principles apply no matter what voltage is applied to the non-inverting inputs, even if that voltage is chosen to be negative.

Voltage comparators can be used for a variety of applications. One of those is turning a logic circuit in a D/A (Digital-to-Analog) converter on or off.

## Inverting amplifier

In addition to the comparator, there are other basic circuits in which the op-amp is used. Five of those circuits are shown in Fig. 5. Note that, unlike the comparator, the circuits shown all use feedback.

Let's turn our attention once again to the inverting amplifier circuit shown in Fig. 5-a. There, an input signal, $\mathrm{V}_{1}$, is fed through $\mathrm{R}_{\mathrm{X}}$ to the inverting input. It is amplified and appears at the output as $\mathrm{V}_{\text {Out }}$. A portion of the output voltage is fed back through $\mathrm{R}_{\mathrm{F}}$. That is done to reduce the gain of the circuit so that the output will be of the same waveform type as the input. What we mean by that is that if the input to the circuit is a sinewave, the output, too, will be a sinewave. (You'll remember from our discussion of voltage comparators that although the input was a sinewave, the output consisted of a series of spikes.)

In the inverting amplifier, no current can flow into the inverting input (because of its assumed infinite impedance). Therefore, $\mathrm{I}_{\mathrm{IN}}$ must flow only through $\mathrm{R}_{\mathrm{X}}$


FIG. 6-IF THE INPUTS ARE TO BE BALANCED, $\mathbf{R}_{1}$ should be equal to $\mathbf{R}_{2}$.
and $\mathrm{R}_{\mathrm{F}}$ to $\mathrm{V}_{\mathrm{Out}}$ and the current through $\mathrm{R}_{\mathrm{X}}$ must equal that through $\mathrm{R}_{\mathrm{F}}$. And, since no current can flow into the inverting input, it is at the same voltage as the non-inverting input. As the non-inverting input is in this case grounded, the inverting output under these conditions is said to be at virtual ground.

The next step is to calculate the gain of the circuit. By definition, the voltage gain of any circuit is the ratio of the output voltage to the input voltage, or $\mathrm{A}_{\mathrm{V}}=$ $\mathrm{V}_{\text {OUT }} / \mathrm{V}_{\text {IN }}$. Taking that one more step, since (from Ohm's law) we know that V $=I R$, we can rewrite that previous expresion as $A_{V}=I_{\text {OUT }} R_{\text {OUT }} / I_{\text {IN }} R_{\text {IN }}$. Next, since from our previous discusion we know that $\mathrm{I}_{\mathrm{IN}}=\mathrm{I}_{\mathrm{OUT}}$, we can write $\mathrm{A}_{\mathrm{V}}=$ $\mathrm{R}_{\mathrm{OUT}} / \mathrm{R}_{\text {IN }}$. Finally, subsituting the circuit variables from Fig. 5-a for the general terms, we can write:

$$
A_{V}=-\frac{V_{\text {OUT }}}{V_{1}}=-\frac{R_{F}}{R_{X}}
$$

The negative sign that appears in the equation indicates that the circuit's output will be inverted with respect to the signal at the non-inverting input.
Now, let's look at a practical example. Suppose you have a 0.25 -volt signal at the input and you need 2.5 volts at the output. The required gain is then $2 \cdot 5 / 0.25=10$. Using the above equation, we can see that to get that gain, $\mathrm{R}_{\mathrm{F}}$ must be ten times $\mathrm{R}_{\mathrm{X}}$. A good choice would be 100,000 ohms for $\mathrm{R}_{\mathrm{F}}$ and 10,000 ohms for $\mathrm{R}_{\mathrm{X}}$.

## Non-inverting amplifier

The non-inverting amplifier circuit in Fig. 5-b is almost identical to the inverting amplifier. Again, the signal from $\mathrm{V}_{2}$ is amplified and appears at the output of the circuit as $\mathrm{V}_{\text {Out }}$. Also as before, the output waveform is of the same type as that at the input. The difference, however, is that in this case the output is not inverted.

Using Kirchoff's current-law and assuming that the voltage at the inverting input equals the voltage at the noninverting input, we can determine the voltage gain. First of all, we can state that $\mathrm{I}_{\mathrm{IN}}=\mathrm{V}_{2} / \mathrm{R}_{\mathrm{X}}$. Second, since $\mathrm{I}_{\mathrm{F}}$ must equal $\mathrm{I}_{\mathrm{IN}}$, we know that $\mathrm{V}_{\text {OUT }}=\mathrm{I}_{\mathrm{IN}}\left(\mathrm{R}_{\mathrm{X}} \mathrm{R}_{\mathrm{F}}\right)$. And finally, we can now rearrange terms and write:

$$
A_{V}=\frac{V_{\text {out }}}{V_{2}}=\frac{R_{X}+R_{F}}{R_{X}}=\frac{R_{F}}{R_{X}}+1
$$

If we short $R_{F}$ in the circuit, and omit $\mathrm{R}_{\mathrm{X}}$ we get the circuit shown in Fig. 5-c. As that circuit's gain can still be described by equation 2, we can state that the gain is equal to $R_{F} / R_{X}+1$. But, since the circuit changes we've made will cause the first term in that expression to drop out, the gain simply becomes equal to 1 . That type of circuit is called a voltage follower. Once again, the output is not inverted with respect to the input.

## Adder

Figure 5-d shows a circuit with multiple input circuits at the op-amp's inverting terminal. Each of those circuits affect the overall gain of the entire amplifier. Using the principle of superposition and what we know about the inverting amplifier, we can determine the output of the adder. (Incidently, the principal of superposition states that if several voltages are applied to a circuit or network at the same time, the current that flows is the same as the sum of the currents that would flow if the voltages were applied one at a time.) That output $\mathrm{V}_{\text {Out }}$ is equal to:

$$
-\left[\left(\frac{R_{F}}{R_{x 1}}\right) v_{x_{1}}+\left(\frac{R_{F}}{R_{x 2}}\right) v_{x_{2}}+\left(\frac{R_{F}}{R_{x_{3}}}\right) v_{x_{3}}\right]
$$

If all the $\mathrm{R}_{\mathrm{X}}$ input resistors were equal, that equation could be simplified to:

$$
v_{\text {OUT }}=-\frac{R_{F}}{R_{x}}\left(v_{x 1}+v_{x 2}+v_{x 3}\right)
$$

That last equation makes it clear that the output is proportional to the sum of the input voltages.

You'll note that the output from the adder is inverted with respect to the input. It's possible, however, to use a noninverting amplifier configuration to get an adder where the output is not inverted. Just connect the input voltages and their associated input resistors to the noninverting terminal of the IC. Now the output of the circuit will be:
$v_{\text {OUT }}=-\frac{R_{x}+R_{F}}{2 R_{x}}\left(v_{x_{1}}+v_{x 2}+v_{x 3}\right)$

That assumes, of course, that $\mathrm{R}_{\mathrm{X} 1}=\mathrm{R}_{\mathrm{X} 2}$ $=\mathrm{R}_{\mathrm{X} 3}=\mathrm{R}_{\mathrm{X}}$.

## Subtractor

Operation of the subtractor, or difference amplifier, shown in Fig. 5-e can also be understood with the help of the superposition principle. The output due to a signal at $V_{1}$ is $V_{\text {OUT }}=-A_{V} V_{1}$. The output due to $V_{2}$ is $V_{\text {Out }}=+A_{V} V_{2}$. The total output is the difference of $V_{1}$ and $V_{2}$. Referred to as $\mathrm{V}_{\text {OutD }}$, it is equal to:
$A_{V} V_{2}-A_{V} V_{1}=\left(1+\frac{R_{F}}{R_{X}}\right) V_{2}-\left(\frac{R_{F}}{R_{X}}\right) V_{1}$
$=V_{2}+\frac{R_{F}}{R_{x}}\left(V_{2}-V_{1}\right)$

As you can see, $\mathrm{V}_{\text {Outd }}$ is related to the difference between the the voltages applied to the inverting and non-inverting inputs of the op-amp.

## Filters

One of the properties of capacitors is that they pass high frequencies while attenuating low frequencies. Thus, if you place a capacitor in series with a signal source as shown in Fig. 7-a, it will let only the high frequencies pass from $\mathrm{V}_{\text {IN }}$ to $\mathrm{V}_{\text {OUT }}$. That circuit is called a high-pass filter.

Similarly, if you place a capacitor across a circuit as shown in Fig. 7-b, it will bypass, or short, the high frequency signals to ground and let only the low frequencies pass from $\mathrm{V}_{\text {IN }}$ to $\mathrm{V}_{\text {Out }}$. That circuit is called a low-pass filter.

If you properly combine both filters into one circuit, the extreme high and extreme low frequencies will be attenuated and only the mid-range frequencies will pass from $\mathrm{V}_{\text {IN }}$ to $\mathrm{V}_{\text {Out }}$. That, of course, is a bandpass filter. Conversely, if the resistor and capacitor in the circuits are chosen carefully, a circuit that will attenuate only a small band of frequencies can be designed. That circuit is referred to as a band-rejection or notch filter.

Those filters can be built using opamps as well. For instance, if only a resistor is placed in the feedback circuit, all frequencies will be passed back to the input. However, if a capacitor is used there, only high frequencies are passed. That, in turn, causes the high frequency part of the input signal to be attenuated by the op-amp itself, but allows the low frequency part to pass unattenuated from the input to the output. Thus, that circuit behaves just like a low-pass filter. If, on the other hand, a resistor is used in the feedback circuit but a capacitor is used at the input, the circuit behaves just like a highpass filter. It is possible to design many different types of active filters using resistors, capacitors, and op-amps.


FIG. 7-A PASSIVE HIGH-PASS FILTER is shown in $a$, a low-pass filter in $b$.

## Practical circuit considerations

Several op-amp characteristics must be
considered if a device is to perform properly in the circuits described. One is how much power the op-amp can dissipate safely. Another is the maximum signal and supply voltages that can be applied to the device before any breakdown occurs. Those maximum values are usually indicated in the op-amp's data sheet. We must also consider factors such as offset voltage, slew rate, and frequency response/stability. Let's now take a look at those last three factors.

## Offset voltage

When we discussed the differential amplifier we noted that if both halves of that circuit were not identical there would be a voltage at the output even with no input signal. Since an op-amp is made up mostly of differential amplifiers, that also holds true for that device. However, there are ways to overcome the problem.

Op-amps should be adjusted for a 0 volt output with no signal applied. Some op-amps have terminals for nulling out any offset voltage. If the device you are using doesn't, a voltage divider like the one in Fig. 8 can be used. Potentiometer R1 should be 10,000 ohms. The exact value of R 2 is not critical, but it is a good idea to keep it below 1000 ohms.


FIG. 8-IF OFFSET-COMPENSATION terminals are not provided, offset nulling can be done using a voltage divider. Here, that divider is connected to the non-inverting input.

## Slew rate

The maximum possible rate-of-change of an op-amp's output is called the slew rate. If the frequency of an input signal is too high, the output will not follow the
input faithfully. An example of that is shown in Fig. 9. If the maximum undistorted sinusoidal output voltage required from the op-amp is $V_{\text {Outmax }}$, the maximum sinusoidal frequency $\left(f_{\mathrm{MAX}}\right)$ it will reproduce properly is related to the op-amp's rated slew rate by the formula:

$$
f_{\text {MAX }}=\frac{\text { SLEW RATE }}{6.28 \text { V OUTMAX }}
$$



FIG. 9-DISTORTION (shown by dotted lines) will result if the input signal's frequency exceeds the op-amp's slew rate.

## Frequency response and stability

Op amps can have very narrow bandwidths. Typical open-loop frequencyresponse curves are frequently supplied by the manufacturer of the device. For example, the open-loop frequency response for a National LM1 18 op-amp can be approximated from the curve in Fig. 10. We can see that at the $110-\mathrm{dB}$ level (a voltage gain of 300,000 ) the gain is flat to 100 Hz , at least as far as the open-circuit response is concerned.

Now, let's say that we want to find the response at a different level of gain. In the inverting amplifier of Fig. 5-a, the gain can be reduced each time feedback is increased simply by reducing the resistance of $R_{F}$ with respect to the resistance of $R_{X}$. If, for example, $\mathrm{R}_{\mathrm{F}}$ were 100,000 ohms and $R_{X}$ were 10,000 ohms, voltage gain, $A_{V}$ would be 1000 or $(20 \log 1000=) 60$ dB. Returning to Fig. 10, if a horizontal line is drawn at the $60-\mathrm{dB}$ level, it will intersect the sloping line of the open-loop frequency-response curve at $30,000 \mathrm{~Hz}$. Thus, at that gain level, the frequency response of the op-amp is flat to 30,000 Hz .

The bandwidth of an op-amp circuit can also be found from the gainbandwidth product of the IC. For instance, if the op-amp's open-loop gain is 300,000 and its bandwidth is 100 Hz , its gain-bandwidth product is the product of the two numbers- $300,000 \times 100=3$ continued on page 93


FIG. 10-A FREQUENCY-RESPONSE CURVE, such as the one shown here, can be used to determine the bandwidth of an op-amp. Such curves are often provided on the device's data sheet.

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$\times 10^{7}$. If the gain is reduced to any other level, let's say to 60 dB or 1000 , the gain-bandwidth product must remain constant. That indicates that the bandwidth times 1000 must be equal to $3 \times$ $10^{7}$, or that the bandwidth is equal to $3 \times$ $10^{7} / 1000=30,000 \mathrm{~Hz}$.

To improve stability of the overall circuit, R-C networks are frequently added. However, the use of excessively large capacitors will affect the device's response. The manufacturer frequently indicates just what resistors and capacitors should be used in the circuit for stabilization purposes. Also, many op-amps already have stabilizing circuits built in.

## Power amplifiers

Small- and medium-signal amplifying devices have so far been described. One job of such a device may be to supply gain for a meter movement. In other cases, small-signal devices can fill different circuit requirements in a radio or record player. To drive a loudspeaker in a hi-fi or PA system, power devices are required, however. In the next part of this series we'll look at power amplifiers.

