OP AMP APPLICATIONS

CHAPTER 5: ANALOG FILTERS

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SECTION 5-1: INTRODUCTION

Filters are networks that process signals in a frequency-dependent manner. The basic concept of a filter can be explained by examining the frequency dependent nature of the impedance of capacitors and inductors. Consider a voltage divider where the shunt leg is a reactive impedance. As the frequency is changed, the value of the reactive impedance changes, and the voltage divider ratio changes. This mechanism yields the frequency dependent change in the input/output transfer function that is defined as the frequency response.

Filters have many practical applications. A simple, single pole, lowpass filter (the integrator) is often used to stabilize amplifiers by rolling off the gain at higher frequencies where excessive phase shift may cause oscillations.

A simple, single pole, highpass filter can be used to block DC offset in high gain amplifiers or single supply circuits. Filters can be used to separate signals, passing those of interest, and attenuating the unwanted frequencies.

An example of this is a radio receiver, where the signal you wish to process is passed through, typically with gain, while attenuating the rest of the signals. In data conversion, filters are also used to eliminate the effects of aliases in A/D systems. They are used in reconstruction of the signal at the output of a D/A as well, eliminating the higher frequency components, such as the sampling frequency and its harmonics, thus smoothing the waveform.

There are a large number of texts dedicated to filter theory. No attempt will be made to go heavily into much of the underlying math: Laplace transforms, complex conjugate poles and the like, although they will be mentioned.

While they are appropriate for describing the effects of filters and examining stability, in most cases examination of the function in the frequency domain is more illuminating.

An ideal filter will have an amplitude response that is unity (or at a fixed gain) for the frequencies of interest (called the *passband*) and zero everywhere else (called the *stopband*). The frequency at which the response changes from passband to stopband is referred to as the *cutoff frequency*.

Figure 5-1(A) shows an idealized lowpass filter. In this filter the low frequencies are in the passband and the higher frequencies are in the stopband.

The functional complement to the lowpass filter is the highpass filter. Here, the low frequencies are in the stopband, and the high frequencies are in the passband. Figure 5-1(B) shows the idealized highpass filter.

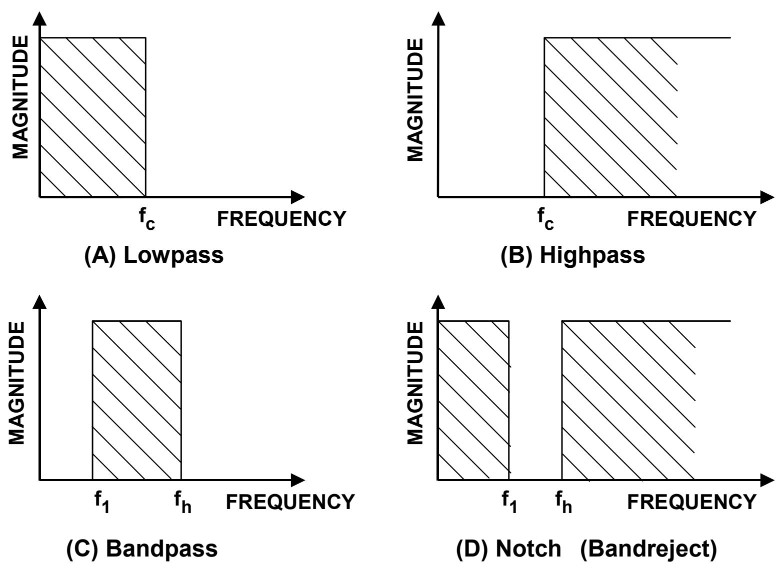

Figure 5-1: *Idealized filter responses*

If a highpass filter and a lowpass filter are cascaded, a *bandpass* filter is created. The bandpass filter passes a band of frequencies between a lower cutoff frequency, f_l, and an upper cutoff frequency, f_h. Frequencies below f_l and above f_h are in the stopband. An idealized bandpass filter is shown in Figure 5-1(C).

A complement to the bandpass filter is the *bandreject*, or *notch* filter. Here, the passbands include frequencies below f_l and above f_h. The band from f_l to f_h is in the stopband. Figure 5-1(D) shows a notch response.

The idealized filters defined above, unfortunately, cannot be easily built. The transition from passband to stopband will not be instantaneous, but instead there will be a transition region. Stop band attenuation will not be infinite.

The five parameters of a practical filter are defined in Figure 5-2, opposite.

The *cutoff frequency* (F_c) is the frequency at which the filter response leaves the error band (or the −3dB point for a Butterworth response filter). The *stopband frequency* (F_s) is the frequency at which the minimum attenuation in the stopband is reached. The *passband ripple* (A_{max}) is the variation (error band) in the passband response. The *minimum passband attenuation* (A_{min}) defines the minimum signal attenuation within the stopband. The steepness of the filter is defined as the *order* (M) of the filter. M is also the number of poles in the transfer function. A pole is a root of the denominator of the transfer function. Conversely, a zero is a root of the numerator of the transfer function. Each pole gives a –6 dB/octave or –20 dB/decade response. Each zero gives a +6dB/octave, or +20 dB/decade response.

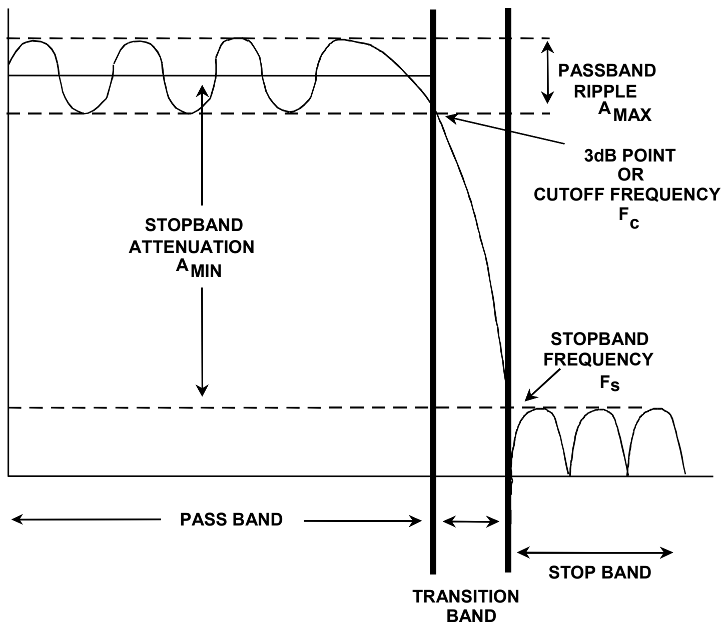

Figure 5.2: *Key filter parameters*

Note that not all filters will have all these features. For instance, all-pole configurations (i.e. no zeros in the transfer function) will not have ripple in the stopband. Butterworth and Bessel filters are examples of all-pole filters with no ripple in the passband.

Typically, one or more of the above parameters will be variable. For instance, if you were to design an antialiasing filter for an ADC, you will know the cutoff frequency (the maximum frequency that you want to pass), the stopband frequency, (which will generally be the Nyquist frequency (= ½ the sample rate)) and the minimum attenuation required (which will be set by the resolution or dynamic range of the system). You can then go to a chart or computer program to determine the other parameters, such as filter order, F_0, and Q, which determines the peaking of the section, for the various sections and/or component values.

It should also be pointed out that the filter will affect the phase of a signal, as well as the amplitude. For example, a single pole section will have a 90° phase shift at the crossover frequency. A pole pair will have a 180° phase shift at the crossover frequency. The Q of the filter will determine the rate of change of the phase. This will be covered more in depth in the next section.

NOTES:

SECTION 5-2: THE TRANSFER FUNCTION

The S-Plane

Filters have a frequency dependent response because the impedance of a capacitor or an inductor changes with frequency. Therefore the complex impedances:

$$Z_L = s\,L \qquad \text{Eq. 5-1}$$

and

$$Z_C = \frac{1}{s\,C} \qquad \text{Eq. 5-2}$$

$$s = \sigma + j\omega \qquad \text{Eq. 5-3}$$

are used to describe the impedance of an inductor and a capacitor, respectively, where σ is the Neper frequency in nepers per second (NP/s) and ω is the angular frequency in radians per sec (rad/s).

By using standard circuit analysis techniques, the transfer equation of the filter can be developed. These techniques include Ohm's law, Kirchoff's voltage and current laws, and superposition, remembering that the impedances are complex. The transfer equation is then:

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0} \qquad \text{Eq. 5-4}$$

Therefore, H(s) is a rational function of s with real coefficients with the degree of m for the numerator and n for the denominator. The degree of the denominator is the order of the filter. Solving for the roots of the equation determines the poles (denominator) and zeros (numerator) of the circuit. Each pole will provide a –6dB/octave or –20dB/decade response. Each zero will provide a +6dB/octave or +20dB/decade response. These roots can be real or complex. When they are complex, they occur in conjugate pairs. These roots are plotted on the s plane (complex plane) where the horizontal axis is σ (real axis) & the vertical axis is ω (imaginary axis). How these roots are distributed on the s plane can tell us many things about the circuit. In order to have stability, all poles must be in the left side of the plane. If we have a zero at the origin, that is a zero in the numerator, the filter will have no response at DC (highpass or bandpass).

Assume an RLC circuit, as in Figure 5-3. Using the voltage divider concept it can be shown that the voltage across the resistor is:

$$H(s) = \frac{Vo}{Vin} = \frac{RCs}{LCs^2 + RCs + 1} \qquad \text{Eq. 5-5}$$

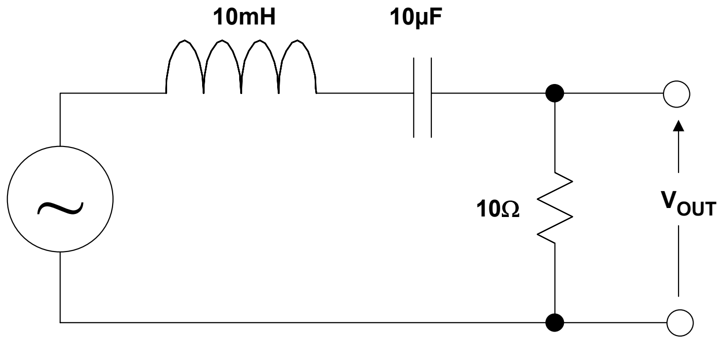

Figure 5-3: *RLC circuit*

Substituting the component values into the equation yields:

$$H(s) = 10^3 \times \frac{s}{s^2 + 10^3 s + 10^7}$$

Factoring the equation and normalizing gives:

$$H(s) = 10^3 \times \frac{s}{[\, s - (-0.5 + j\,3.122) \times 10^3] \times [\, s - (-0.5 - j\,3.122) \times 10^3]}$$

This gives a zero at the origin and a pole pair at:

$$s = (-0.5 \pm j3.122) \times 10^3$$

Next, plot these points on the s plane as shown in Figure 5-4:

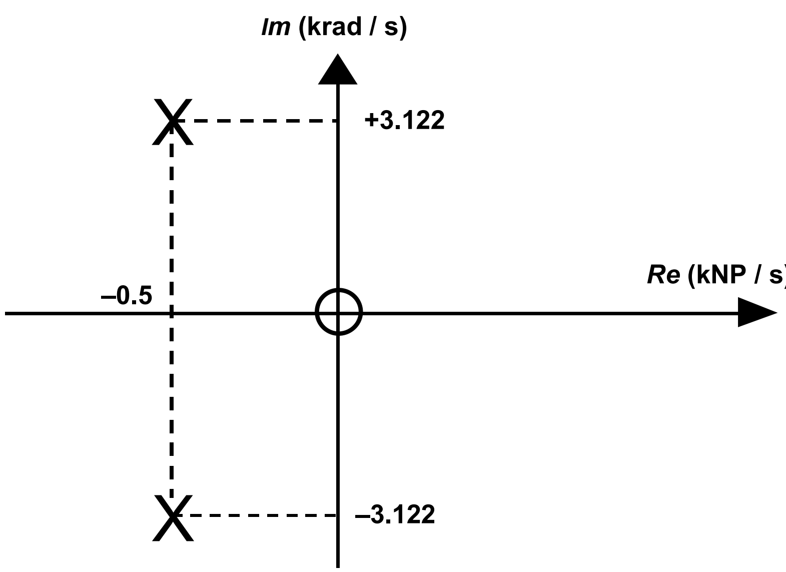

Figure 5-4: *Pole and zero plotted on the s-plane*

The above discussion has a definite mathematical flavor. In most cases we are more interested in the circuit's performance in real applications. While working in the s plane is completely valid, I'm sure that most of us don't think in terms of Nepers and imaginary frequencies.

F_o & Q

So if it is not convenient to work in the s plane, why go through the above discussion? The answer is that the groundwork has been set for two concepts that will be infinitely more useful in practice: F_o & Q.

F_o is the cutoff frequency of the filter. This is defined, in general, as the frequency where the response is down 3dB from the passband. It can sometimes be defined as the frequency at which it will fall out of the passband. For example, a 0.1dB Chebyshev filter can have its F_o at the frequency at which the response is down > 0.1dB.

The shape of the attenuation curve (as well as the phase and delay curves, which define the time domain response of the filter) will be the same if the ratio of the actual frequency to the cutoff frequency is examined, rather than just the actual frequency itself. Normalizing the filter to 1 rad/s, a simple system for designing and comparing filters can be developed. The filter is then scaled by the cutoff frequency to determine the component values for the actual filter.

Q is the "quality factor" of the filter. It is also sometimes given as α where:

$$\alpha = \frac{1}{Q} \qquad \text{Eq. 5-6}$$

This is commonly known as the *damping ratio*. ξ is sometimes used where:

$$\xi = 2\alpha \qquad \text{Eq. 5-7}$$

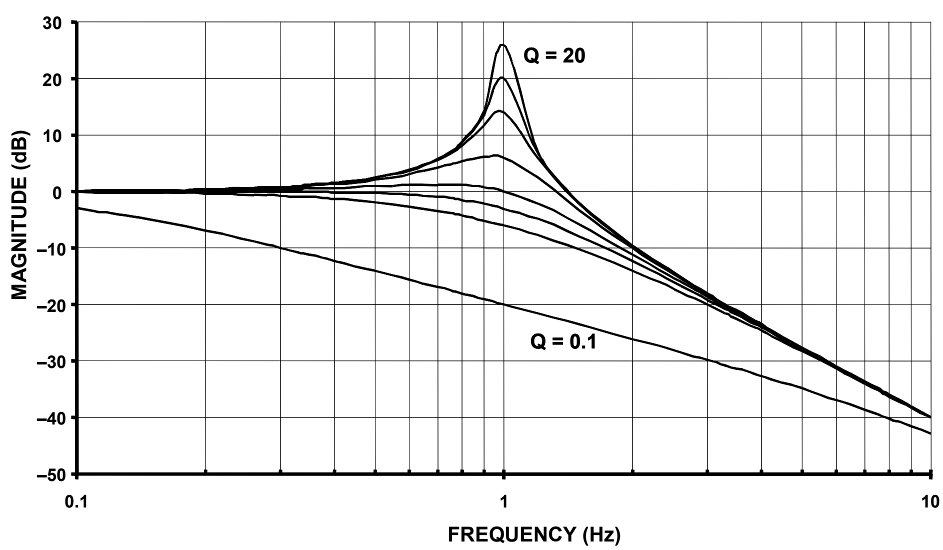

Figure 5-5: *Lowpass filter peaking versus Q*

If Q is > 0.707, there will be some peaking in the filter response. If the Q is < 0.707, rolloff at F_0 will be greater; it will have a more gentle slope and will begin sooner. The amount of peaking for a 2 pole lowpass filter vs. Q is shown in Figure 5-5.

Rewriting the transfer function H(s) in terms of ω_o and Q:

$$H(s) = \frac{H_0}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \qquad \text{Eq. 5-8}$$

where H_o is the passband gain and $\omega_o = 2\pi F_o$.

This is now the *lowpass prototype* that will be used to design the filters.

Highpass Filter

Changing the numerator of the transfer equation, H(s), of the lowpass prototype to H_0s^2 transforms the lowpass filter into a highpass filter. The response of the highpass filter is similar in shape to a lowpass, just inverted in frequency

The transfer function of a highpass filter is then:

$$H(s) = \frac{H_0 s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \qquad \text{Eq. 5-9}$$

The response of a 2-pole highpass filter is illustrated in Figure 5-6.

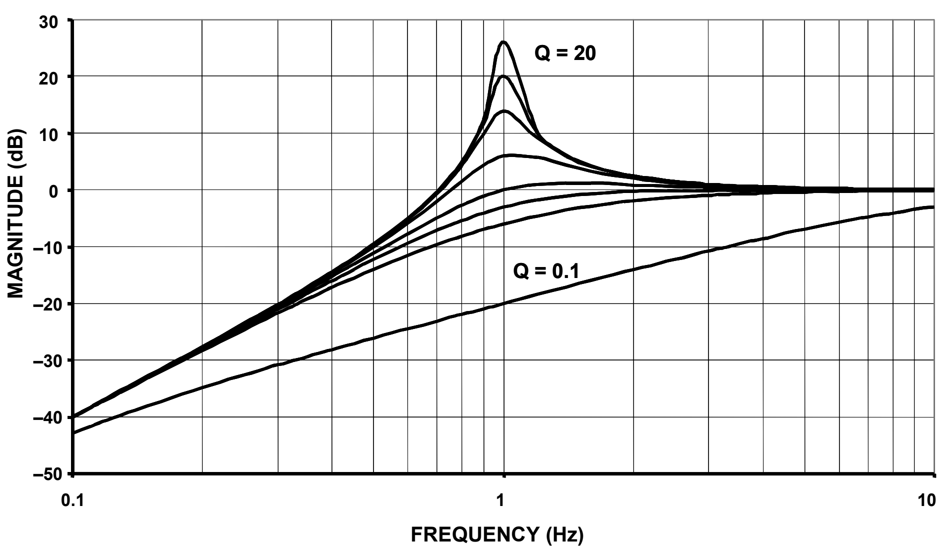

Figure 5-6:*Highpass filter peaking versus Q*

Bandpass Filter

Changing the numerator of the lowpass prototype to $H_o\omega_o^2$ will convert the filter to a bandpass function.

The transfer function of a bandpass filter is then:

$$H(s) = \frac{H_0\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \qquad \text{Eq. 5-10}$$

ω_o here is the frequency ($F_0 = 2\,\pi\,\omega_0$) at which the gain of the filter peaks.

H_o is the circuit gain and is defined:

$$H_o = H/Q. \qquad \text{Eq. 5-11}$$

Q has a particular meaning for the bandpass response. It is the selectivity of the filter. It is defined as:

$$Q = \frac{F_0}{F_H - F_L} \qquad \text{Eq. 5-12}$$

where F_L & F_H are the frequencies where the response is –3dB from the maximum.

The bandwidth (BW) of the filter is described as:

$$BW = F_H - F_L \qquad \text{Eq. 5-13}$$

It can be shown that the resonant frequency (F_0) is the geometric mean of F_L & F_H, which means that F_0 will appear half way between F_L & F_H on a logarithmic scale.

$$F_0 = \sqrt{F_H F_L} \qquad \text{Eq. 5-14}$$

Also, note that the skirts of the bandpass response will always be symmetrical around F_0 on a logarithmic scale.

The response of a bandpass filter to various values of Q are shown in Figure 5-7 (next page).

A word of caution is appropriate here. Bandpass filters can be defined two different ways. The narrowband case is the classic definition that we have shown above.

In some cases, however, if the high and low cutoff frequencies are widely separated, the bandpass filter is constructed out of separate highpass and lowpass sections. Widely separated in this context means separated by at least 2 octaves (×4 in frequency). This is the wideband case.

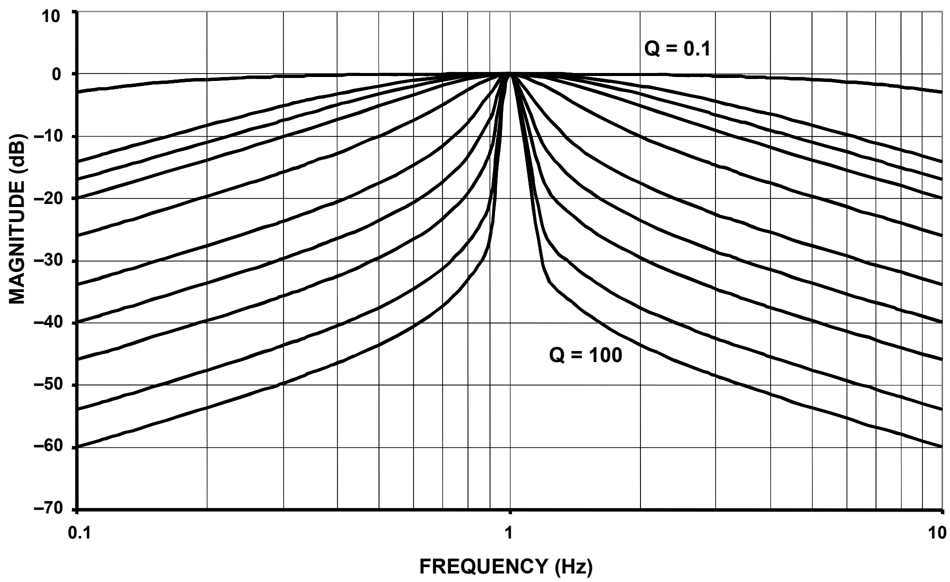

Figure 5-7: *Bandpass filter peaking versus Q*

Bandreject (Notch) Filter

By changing the numerator to $s^2 + \omega_z^2$, we convert the filter to a bandreject or notch filter. As in the bandpass case, if the corner frequencies of the bandreject filter are separated by more than an octave (the wideband case), it can be built out of separate lowpass and highpass sections. We will adopt the following convention: A narrowband bandreject filter will be referred to as a *notch* filter and the wideband bandreject filter will be referred to as *bandreject* filter.

A notch (or bandreject) transfer function is:

$$H(s) = \frac{H_0 (s^2 + \omega_z^2)}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \qquad \text{Eq. 5-15}$$

There are three cases of the notch filter characteristics. These are illustrated in Figure 5-8 (opposite). The relationship of the pole frequency, ω_0, and the zero frequency, ω_z, determines if the filter is a standard notch, a lowpass notch or a highpass notch.

If the zero frequency is equal to the pole frequency a standard notch exists. In this instance the zero lies on the $j\omega$ plane where the curve that defines the pole frequency intersects the axis.

A lowpass notch occurs when the zero frequency is greater than the pole frequency. In this case ω_z lies outside the curve of the pole frequencies. What this means in a practical sense is that the filter's response below ω_z will be greater than the response above ω_z. This results in an elliptical lowpass filter.

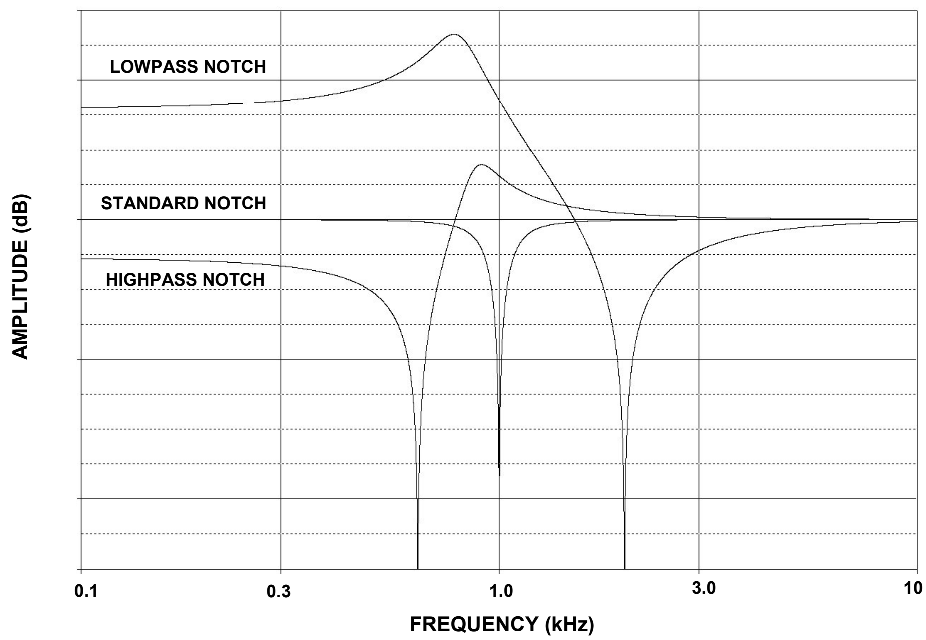

Figure 5-8: *Standard, lowpass, and highpass notches*

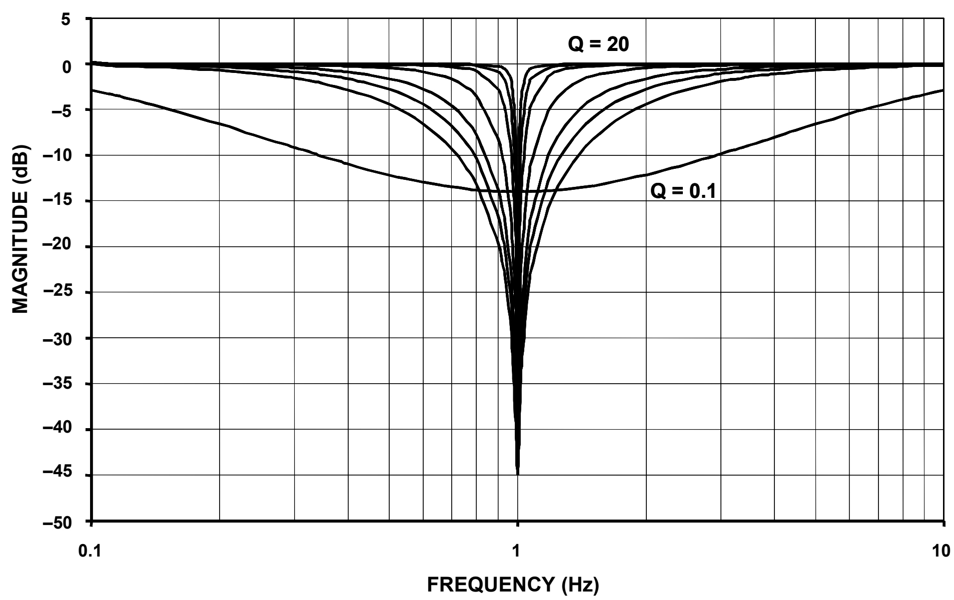

Figure 5-9: *Notch filter width versus frequency for various Q values*

A highpass notch filter occurs when the zero frequency is less than the pole frequency. In this case ω_z lies inside the curve of the pole frequencies. What this means in a practical sense is that the filters response below ω_z will be less than the response above ω_z. This results in an elliptical highpass filter.

The variation of the notch width with Q is shown in Figure 5-9.

Allpass Filter

There is another type of filter that leaves the amplitude of the signal intact but introduces phase shift. This type of filter is called an *allpass*. The purpose of this filter is to add phase shift (delay) to the response of the circuit. The amplitude of an allpass is unity for all frequencies. The phase response, however, changes from 0° to 360° as the frequency is swept from 0 to infinity. The purpose of an all pass filter is to provide phase equalization, typically in pulse circuits. It also has application in single side band, suppressed carrier (SSB-SC) modulation circuits.

The transfer function of an allpass filter is:

$$H(s) = \frac{s^2 - \frac{\omega_0}{Q} s + \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \quad \text{Eq. 5-16}$$

Note that an allpass transfer function can be synthesized as:

$$H_{AP} = H_{LP} - H_{BP} + H_{HP} = 1 - 2H_{BP}. \quad \text{Eq. 5-17}$$

Figure 5-10 (opposite) compares the various filter types.

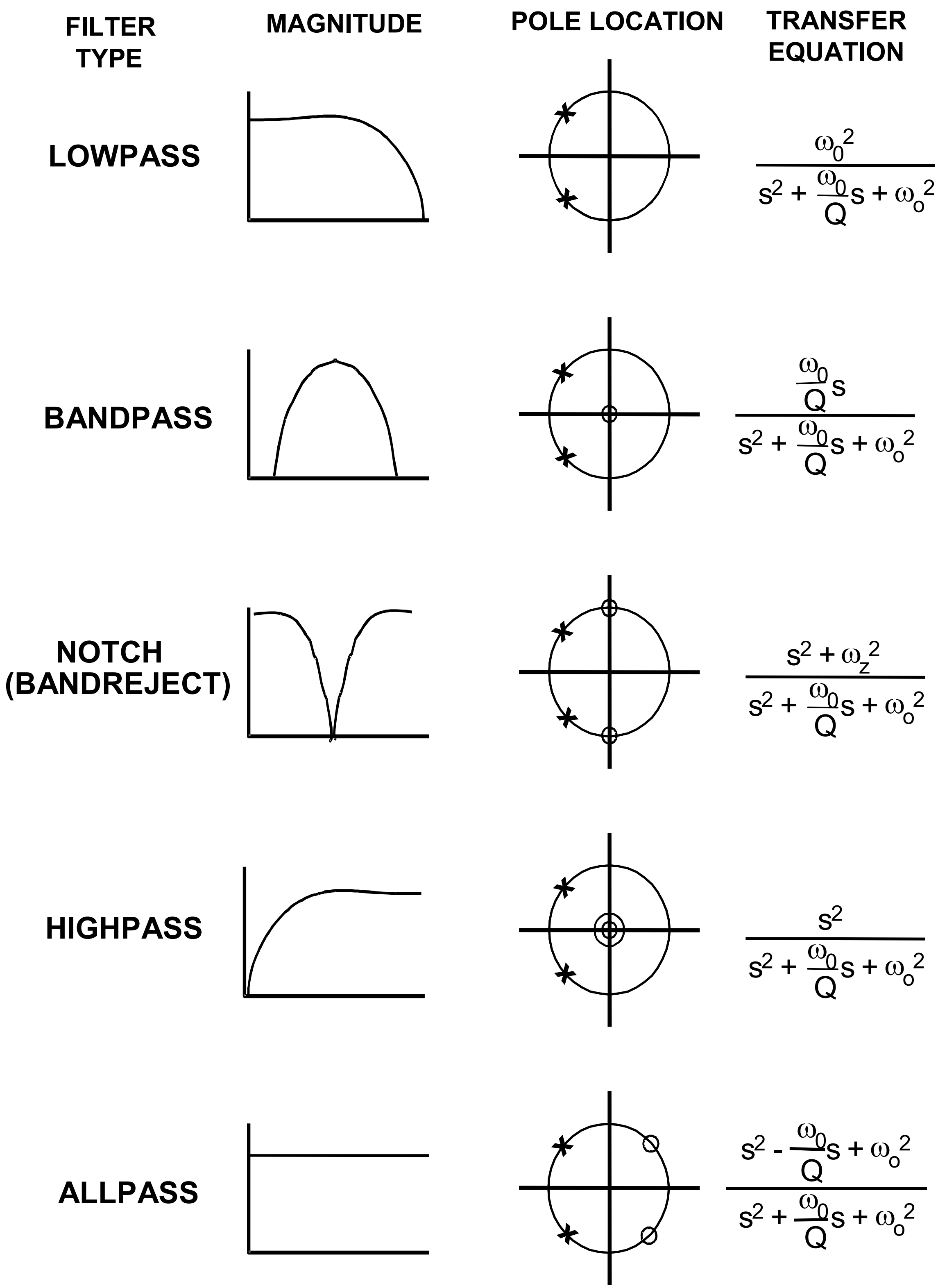

Figure 5-10: *Standard second-order filter responses*

Phase Response

As mentioned earlier, a filter will change the phase of the signal as well as the amplitude. The question is, does this make a difference? Fourier analysis indicates a square wave is made up of a fundamental frequency and odd order harmonics. The magnitude and phase responses, of the various harmonics are precisely defined. If the magnitude or phase relationships are changed, then the summation of the harmonics will not add back together properly to give a square wave. It will instead be distorted, typically showing overshoot and ringing or a slow rise time. This would also hold for any complex waveform.

Each pole of a filter will add 45° of phase shift at the corner frequency. The phase will vary from 0° (well below the corner frequency) to 90° (well beyond the corner frequency). The start of the change can be more than a decade away. In multipole filters, each of the poles will add phase shift, so that the total phase shift will be multiplied by the number of poles (180° total shift for a two pole system, 270° for a three pole system, etc.).

The phase response of a single pole, low pass filter is:

$$\phi(\omega) = -\arctan\frac{\omega}{\omega_o} \qquad \text{Eq. 5-18}$$

The phase response of a lowpass pole pair is:

$$\phi(\omega) = -\arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} + \sqrt{4-\alpha^2}\right)\right] - \arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} - \sqrt{4-\alpha^2}\right)\right] \qquad \text{Eq. 5-19}$$

For a single pole highpass filter the phase response is:

$$\phi(\omega) = \frac{\pi}{2} - \arctan\frac{\omega}{\omega_o} \qquad \text{Eq. 5-20}$$

The phase response of a highpass pole pair is:

$$\phi(\omega) = \pi - \arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} + \sqrt{4-\alpha^2}\right)\right] - \arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} - \sqrt{4-\alpha^2}\right)\right] \qquad \text{Eq. 5-21}$$

The phase response of a bandpass filter is:

$$\phi(\omega) = \frac{\pi}{2} - \arctan\left(\frac{2Q\omega}{\omega_0} + \sqrt{4Q^2 - 1}\right) - \arctan\left(\frac{2Q\omega}{\omega_0} - \sqrt{4Q^2 - 1}\right) \quad \text{Eq. 5-22}$$

The variation of the phase shift with frequency due to various values of Q is shown in Figure 5-11 (for lowpass, highpass, bandpass, and allpass) and in Figure 5-12 (for notch).

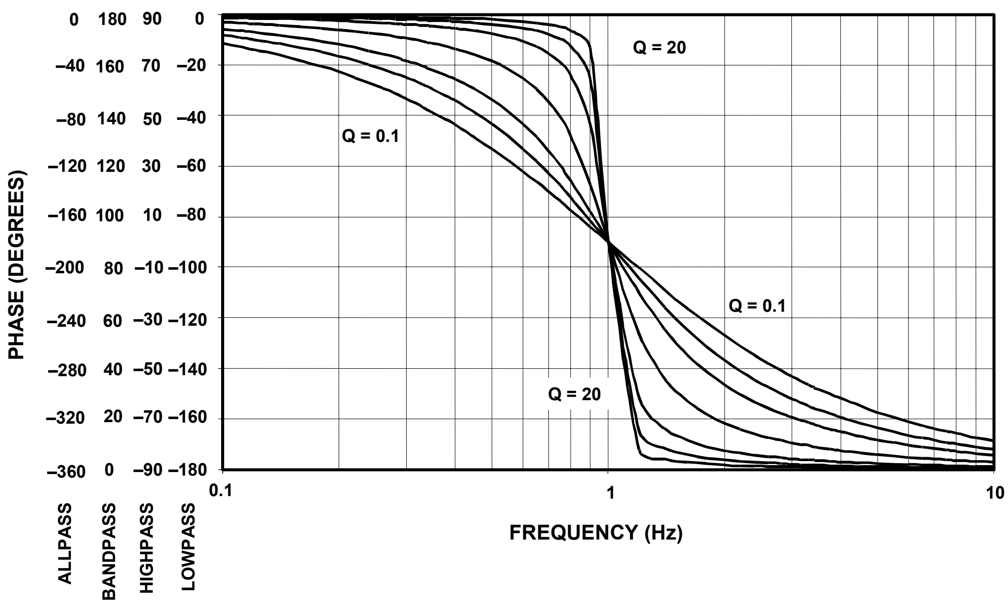

Figure 5-11: *Phase response versus frequency*

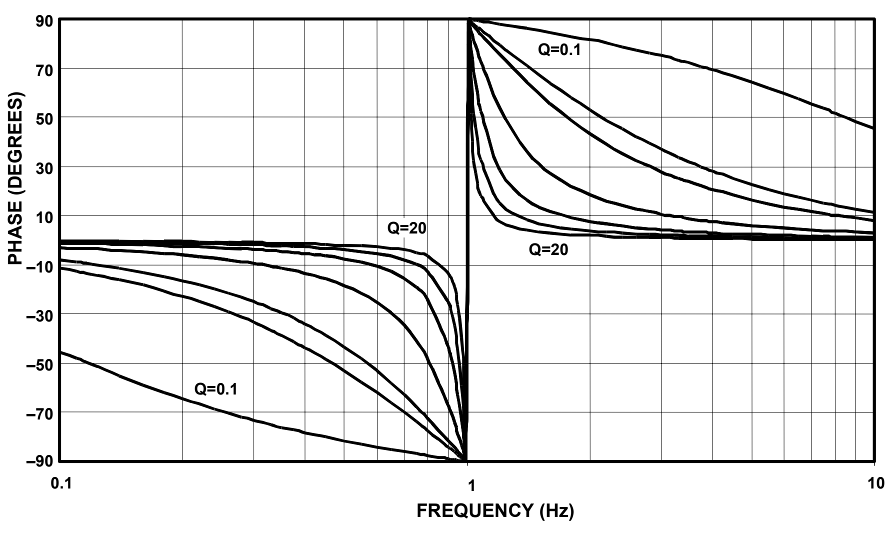

Figure 5-12: *Notch filter phase response*

It is also useful to look at the change of phase with frequency. This is the group delay of the filter. A flat (constant) group delay gives best phase response, but, unfortunately, it also gives the least amplitude discrimination. The group delay of a single lowpass pole is:

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega} = \frac{\cos^2\phi}{\omega_0} \qquad \text{Eq. 5-23}$$

For the lowpass pole pair it is:

$$\tau(\omega) = \frac{2\sin^2\phi}{\alpha\omega_0} - \frac{\sin 2\phi}{2\omega} \qquad \text{Eq. 5-24}$$

For the single highpass pole it is:

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega} = \frac{\sin^2\phi}{\omega_0} \qquad \text{Eq. 5-25}$$

For the highpass pole pair it is:

$$\tau(\omega) = \frac{2\sin^2\phi}{\alpha\omega_0} - \frac{\sin 2\phi}{2\omega} \qquad \text{Eq. 5-26}$$

And for the bandpass pole pair it is:

$$\tau(\omega) = \frac{2Q\ 2\cos^2\phi}{\alpha\omega_0} + \frac{\sin 2\phi}{2\omega} \qquad \text{Eq. 5-27}$$

The Effect of Nonlinear Phase

A waveform can be represented by a series of frequencies of specific amplitude, frequency and phase relationships. For example, a square wave is:

$$F(t) = A\left(\frac{1}{2} + \frac{2}{\pi}\sin\omega t + \frac{2}{3\pi}\sin 3\omega t + \frac{2}{5\pi}\sin 5\omega t + \frac{2}{7\pi}\sin 7\omega t + \ldots\right) \qquad \text{Eq. 5-28}$$

If this waveform were passed through a filter, the amplitude and phase response of the filter to the various frequency components of the waveform could be different. If the phase delays were identical, the waveform would pass through the filter undistorted. If, however, the different components of the waveform were changed due to different amplitude and phase response of the filter to those frequencies, they would no longer add up in the same manner. This would change the shape of the waveform. These distortions would manifest themselves in what we typically call overshoot and ringing of the output.

Not all signals will be composed of harmonically related components. An amplitude modulated (AM) signal, for instance, will consist of a carrier and 2 sidebands at ± the modulation frequency. If the filter does not have the same delay for the various waveform components, then "envelope delay" will occur and the output wave will be distorted.

Linear phase shift results in constant group delay since the derivative of a linear function is a constant.

SECTION 5-3: TIME DOMAIN RESPONSE

Up until now the discussion has been primarily focused on the frequency domain response of filters. The time domain response can also be of concern, particularly under transient conditions. Moving between the time domain and the frequency domain is accomplished by the use of the Fourier and Laplace transforms. This yields a method of evaluating performance of the filter to a non-sinusoidal excitation.

The transfer function of a filter is the ratio of the output to input time functions. It can be shown that the impulse response of a filter defines its bandwidth. The time domain response is a practical consideration in many systems, particularly communications, where many modulation schemes use both amplitude and phase information.

Impulse Response

The impulse function is defined as an infinitely high, infinitely narrow pulse, with an area of unity. This is, of course, impossible to realize in a physical sense. If the impulse width is much less than the rise time of the filter, the resulting response of the filter will give a reasonable approximation actual impulse response of the filter response.

The impulse response of a filter, in the time domain, is proportional to the bandwidth of the filter in the frequency domain. The narrower the impulse, the wider the bandwidth of the filter. The pulse amplitude is equal to ω_c/π, which is also proportional to the filter bandwidth, the height being taller for wider bandwidths. The pulse width is equal to $2\pi/\omega_c$, which is inversely proportional to bandwidth. It turns out that the product of the amplitude and the bandwidth is a constant.

It would be a nontrivial task to calculate the response of a filter without the use of Laplace and Fourier transforms. The Laplace transform converts multiplication and division to addition and subtraction, respectively. This takes equations, which are typically loaded with integration and/or differentiation, and turns them into simple algebraic equations, which are much easier to deal with. The Fourier transform works in the opposite direction.

The details of these transform will not be discussed here. However, some general observations on the relationship of the impulse response to the filter characteristics will be made.

It can be shown, as stated, that the impulse response is related to the bandwidth. Therefore, amplitude discrimination (the ability to distinguish between the desired signal from other, out of band signals and noise) and time response are inversely proportional. That is to say that the filters with the best amplitude response are the ones with the worst time response. For all-pole filters, the Chebyshev filter gives the best amplitude discrimination, followed by the Butterworth and then the Bessel.

If the time domain response were ranked, the Bessel would be best, followed by the Butterworth and then the Chebyshev. Details of the different filter responses will be discussed in the next section.

The impulse response also increases with increasing filter order. Higher filter order implies greater bandlimiting, therefore degraded time response. Each section of a multistage filter will have its own impulse response, and the total impulse response is the accumulation of the individual responses. The degradation in the time response can also be related to the fact that as frequency discrimination is increased, the Q of the individual sections tends to increase. The increase in Q increases the overshoot and ringing of the individual sections, which implies longer time response.

Step Response

The step response of a filter is the integral of the impulse response. Many of the generalities that apply to the impulse response also apply to the step response. The slope of the rise time of the step response is equal to the peak response of the impulse. The product of the bandwidth of the filter and the rise time is a constant. Just as the impulse has a function equal to unity, the step response has a function equal to 1/s. Both of these expressions can be normalized, since they are dimensionless.

The step response of a filter is useful in determining the envelope distortion of a modulated signal. The two most important parameters of a filter's step response are the overshoot and ringing. Overshoot should be minimal for good pulse response. Ringing should decay as fast as possible, so as not to interfere with subsequent pulses.

Real life signals typically aren’t made up of impulse pulses or steps, so the transient response curves don’t give a completely accurate estimation of the output. They are, however, a convenient figure of merit so that the transient responses of the various filter types can be compared on an equal footing.

Since the calculations of the step and impulse response are mathematically intensive, they are most easily performed by computer. Many CAD (Computer Aided Design) software packages have the ability to calculate these responses. Several of these responses are also collected in the next section.

SECTION 5-4: STANDARD RESPONSES

There are many transfer functions that may satisfy the attenuation and/or phase requirements of a particular filter. The one that you choose will depend on the particular system. The importance of the frequency domain response versus the time domain response must be determined. Also, both of these considerations might be traded off against filter complexity, and thereby cost.

Butterworth

The Butterworth filter is the best compromise between attenuation and phase response. It has no ripple in the passband or the stopband, and because of this is sometimes called a maximally flat filter. The Butterworth filter achieves its flatness at the expense of a relatively wide transition region from passband to stopband, with average transient characteristics.

The normalized poles of the Butterworth filter fall on the unit circle (in the *s* plane). The pole positions are given by:

$$-\sin\frac{(2K-1)\pi}{2n} + j\cos\frac{(2K-1)\pi}{2n} \qquad K=1,2....n \qquad \text{Eq. 5-29}$$

where K is the pole pair number, and n is the number of poles.

The poles are spaced equidistant on the unit circle, which means the angles between the poles are equal.

Given the pole locations, ω_0, and α (or Q) can be determined. These values can then be use to determine the component values of the filter. The design tables for passive filters use frequency and impedance normalized filters. They are normalized to a frequency of 1 rad/sec and impedance of 1Ω. These filters can be denormalized to determine actual component values. This allows the comparison of the frequency domain and/or time domain responses of the various filters on equal footing. The Butterworth filter is normalized for a –3dB response at $\omega_o = 1$.

The values of the elements of the Butterworth filter are more practical and less critical than many other filter types. The frequency response, group delay, impulse response and step response are shown in Figure 5-15. The pole locations and corresponding ω_o and α terms are tabulated in Figure 5-26.

Chebyshev

The Chebyshev (or Chevyshev, Tschebychev, Tschebyscheff or Tchevysheff, depending on how you translate from Russian) filter has a smaller transition region than the same-order Butterworth filter, at the expense of ripples in its passband. This filter gets its name because the Chebyshev filter minimizes the height of the maximum ripple, which is the Chebyshev criterion.

Chebyshev filters have 0dB relative attenuation at DC. Odd order filters have an attenuation band that extends from 0dB to the ripple value. Even order filters have a gain equal to the passband ripple. The number of cycles of ripple in the passband is equal to the order of the filter.

The poles of the Chebyshev filter can be determined by moving the poles of the Butterworth filter to the right, forming an ellipse. This is accomplished by multiplying the real part of the pole by k_r and the imaginary part by k_I. The values k_r and k_I can be computed by:

$$K_r = \sinh A \qquad \text{Eq. 5-30}$$

$$K_I = \cosh A \qquad \text{Eq. 5-31}$$

where:

$$A = \frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon} \qquad \text{Eq. 5-32}$$

where n is the filter order and:

$$\varepsilon = \sqrt{10^R - 1} \qquad \text{Eq. 5-33}$$

where:

$$R = \frac{R_{dB}}{10} \qquad \text{Eq. 5-34}$$

where:

$$R_{dB} = \text{passband ripple in dB} \qquad \text{Eq. 5-35}$$

The Chebyshev filters are typically normalized so that the edge of the ripple band is at $\omega_o = 1$. The 3dB bandwidth is given by:

$$A_{3dB} = \frac{1}{n} \cosh^{-1}\left(\frac{1}{\varepsilon}\right) \qquad \text{Eq. 5-36}$$

This is tabulated in Table 1 (opposite).

The frequency response, group delay, impulse response and step response are cataloged in Figures 5-16 to 5-20 on following pages, for various values of passband ripple (0 .01dB, 0.1dB, 0.25dB, 0.5dB and 1dB). The pole locations and corresponding ω_o and α terms for these values of ripple are tabulated in Figures 5-27 to 5-31 on following pages.

ORDER	.01dB	.1dB	.25dB	.5dB	1dB
2	3.30362	1.93432	1.59814	1.38974	1.21763
3	1.87718	1.38899	1.25289	1.16749	1.09487
4	1.46690	1.21310	1.13977	1.09310	1.05300
5	1.29122	1.13472	1.08872	1.05926	1.03381
6	1.19941	1.09293	1.06134	1.04103	1.02344
7	1.14527	1.06800	1.04495	1.03009	1.01721
8	1.11061	1.05193	1.03435	1.02301	1.01316
9	1.08706	1.04095	1.02711	1.01817	1.01040
10	1.07033	1.03313	1.02194	1.01471	1.00842

Table 1: *3dB bandwidth to ripple bandwidth for Chebyshev filters*

Bessel

Butterworth filters have fairly good amplitude and transient behavior. The Chebyshev filters improve on the amplitude response at the expense of transient behavior. The Bessel filter is optimized to obtain better transient response due to a linear phase (i.e. constant delay) in the passband. This means that there will be relatively poorer frequency response (less amplitude discrimination).

The poles of the Bessel filter can be determined by locating all of the poles on a circle and separating their imaginary parts by:

$$\frac{2}{n} \qquad \text{Eq. 5-37}$$

where n is the number of poles. Note that the top and bottom poles are distanced by where the circle crosses the jω axis by:

$$\frac{1}{n} \qquad \text{Eq. 5-38}$$

or half the distance between the other poles.

The frequency response, group delay, impulse response and step response for the Bessel filters are cataloged in Figure 5-21. The pole locations and corresponding ω_o and α terms for the Bessel filter are tabulated in Figure 5-32.

Linear Phase with Equiripple Error

The linear phase filter offers linear phase response in the passband, over a wider range than the Bessel, and superior attenuation far from cutoff. This is accomplished by letting the phase response have ripples, similar to the amplitude ripples of the Chebyshev. As the ripple is increased, the region of constant delay extends further into the stopband. This will also cause the group delay to develop ripples, since it is the derivative of the phase response. The step response will show slightly more overshoot than the Bessel and the impulse response will show a bit more ringing.

It is difficult to compute the pole locations of a linear phase filter. Pole locations are taken from the Williams book (see Reference 2), which, in turn, comes from the Zverev book (see Reference 1).

The frequency response, group delay, impulse response and step response for linear phase filters of 0.05° ripple and 0.5° ripple are given in Figures 5-22 and 5-23. The pole locations and corresponding ω_0 and α terms are tabulated in Figures 5-33 and 5-34.

Transitional Filters

A transitional filter is a compromise between a Gaussian filter, which is similar to a Bessel, and the Chebyshev. A transitional filter has nearly linear phase shift and smooth, monotonic rolloff in the passband. Above the passband there is a break point beyond which the attenuation increases dramatically compared to the Bessel, and especially at higher values of n.

Two transition filters have been tabulated. These are the Gaussian to 6dB and Gaussian to 12dB.

The Gaussian to 6dB filter has better transient response than the Butterworth in the passband. Beyond the breakpoint, which occurs at $\omega = 1.5$, the rolloff is similar to the Butterworth.

The Gaussian to 12dB filter's transient response is much better than Butterworth in the passband. Beyond the 12dB breakpoint, which occurs at $\omega = 2$, the attenuation is less than the Butterworth.

As is the case with the linear phase filters, pole locations for transitional filters do not have a closed form method for computation. Again, pole locations are taken from Williams's book (see Reference 2). These were derived from iterative techniques.

The frequency response, group delay, impulse response and step response for Gaussian to 12dB and 6dB are shown in Figures 5-24 and 5-25. The pole locations and corresponding ω_0 and α terms are tabulated in Figures 5-35 and 5-36.

Comparison of All-Pole Responses

The responses of several all-pole filters, namely the Bessel, Butterworth and Chebyshev (in this case of 0.5dB ripple) will now be compared. An 8 pole filter is used as the basis for the comparison. The responses have been normalized for a cutoff of 1Hz. Comparing Figures 5-13 and 5-14 below, it is easy to see the tradeoffs in the response types. Moving from Bessel through Butterworth to Chebyshev, notice that the amplitude discrimination improves as the transient behavior gets progressively poorer.

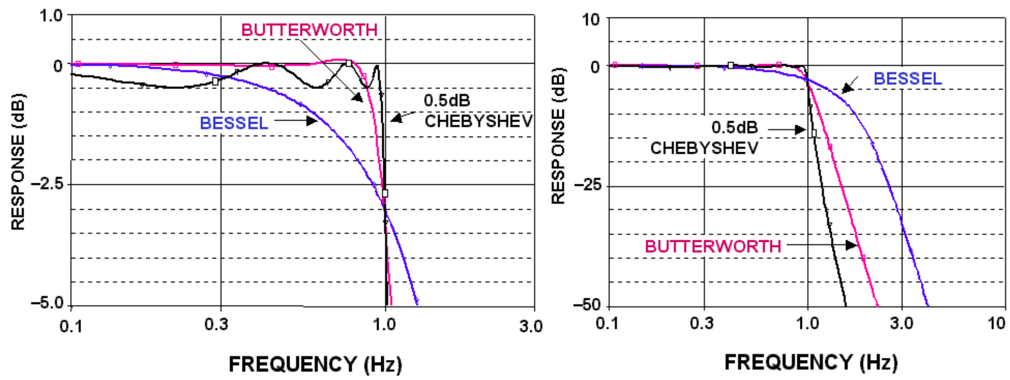

Figure 5-13: *Comparison of amplitude response of Bessel, Butterworth and Chebyshev filters*

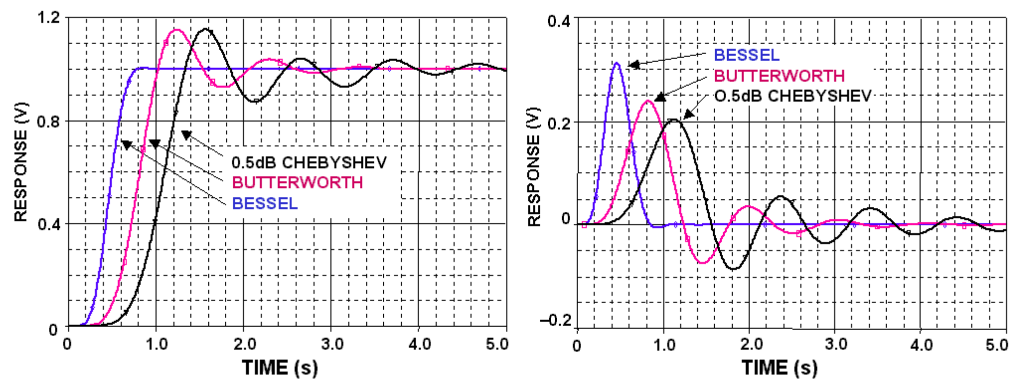

Figure 5-14: *Comparison of Step and Impulse Responses of Bessel, Butterworth and Chebyshev*

Elliptical

The previously mentioned filters are all-pole designs, which mean that the zeros of the transfer function (roots of the numerator) are at one of the two extremes of the frequency range (0 or ∞). For a lowpass filter, the zeros are at f = ∞. If finite frequency transfer function zeros are added to poles an Elliptical filter (sometimes referred to as Cauer filters) is created. This filter has a shorter transition region than the Chebyshev filter because it allows ripple in both the stopband and passband. It is the addition of zeros in the stopband that causes ripple in the stopband but gives a much higher rate of attenuation, the most possible for a given number of poles. There will be some "bounceback" of the stopband response between the zeros. This is the stopband ripple. The Elliptical filter also has degraded time domain response.

Since the poles of an elliptic filter are on an ellipse, the time response of the filter resembles that of the Chebyshev.

An Elliptic filter is defined by the parameters shown in Figure 5-2, those being A_{max}, the maximum ripple in the passband, A_{min}, the minimum attenuation in the stopband, F_c, the cutoff frequency, which is where the frequency response leaves the passband ripple and F_S, the stopband frequency, where the value of A_{max} is reached.

An alternate approach is to define a filter order n, the modulation angle, θ, which defines the rate of attenuation in the transition band, where:

$$\theta = \sin^{-1} \frac{1}{F_s} \qquad \text{Eq. 5-39}$$

and ρ which determines the passband ripple, where:

$$\rho = \sqrt{\frac{\varepsilon^2}{1 + \varepsilon^2}} \qquad \text{Eq. 5-40}$$

where ε is the ripple factor developed for the Chebyshev response, and the passband ripple is:

$$R_{dB} = -10 \log (1 - \rho^2) \qquad \text{Eq. 5-41}$$

Some general observations can be made. For a given filter order n, and θ, A_{min} increases as the ripple is made larger. Also, as θ approaches 90°, F_S approaches F_C. This results in extremely short transition region, which means sharp rolloff. This comes at the expense of lower A_{min}.

As a side note, ρ determines the input resistance of a passive elliptical filter, which can then be related to the VSWR (Voltage Standing Wave Ratio).

Because of the number of variables in the design of an elliptic filter, it is difficult to provide the type of tables provided for the previous filter types. Several CAD (Computer Aided Design) packages can provide the design values. Alternatively several sources, such as Williams's (see Reference 2), provide tabulated filter values. These tables classify the filter by

$$C\ n\ \rho\ \theta$$

where the C denotes Cauer. Elliptical filters are sometime referred to as Cauer filters after the network theorist Wilhelm Cauer.

Maximally Flat Delay With Chebyshev Stopband

Bessel type (Bessel, linear phase with equiripple error and transitional) filters give excellent transient behavior, but less than ideal frequency discrimination. Elliptical filters give better frequency discrimination, but degraded transient response. A maximally flat delay with Chebyshev stopband filter takes a Bessel type function and adds transmission zeros. The constant delay properties of the Bessel type filter in the passband are maintained, and the stopband attenuation is significantly improved. The step response exhibits no overshoot or ringing, and the impulse response is clean, with essentially no oscillatory behavior. Constant group delay properties extend well into the stopband for increasing n.

As with the elliptical filter, numeric evaluation is difficult. Williams's book (see Reference 2) tabulates passive prototypes normalized component values.

Inverse Chebyshev

The Chebyshev response has ripple in the passband and a monotonic stopband. The inverse Chebyshev response can be defined that has a monotonic passband and ripple in the stopband. The inverse Chebyshev has better passband performance than even the Butterworth. It is also better than the Chebyshev, except very near the cutoff frequency. In the transition band, the inverse Chebyshev has the steepest rolloff. Therefore, the inverse Chebyshev will meet the A_{min} specification at the lowest frequency of the three. In the stopband there will, however, be response lobes which have a magnitude of:

$$\frac{\varepsilon^2}{(1 - \varepsilon)} \qquad \text{Eq. 5-42}$$

where ε is the ripple factor defined for the Chebyshev case. This means that deep into the stopband, both the Butterworth and Chebyshev will have better attenuation, since they are monotonic in the stopband. In terms of transient performance, the inverse Chebyshev lies midway between the Butterworth and the Chebyshev.

The inverse Chebyshev response can be generated in three steps. First take a Chebyshev lowpass filter. Then subtract this response from 1. Finally, invert in frequency by replacing ω with 1/ω.

These are by no means all the possible transfer functions, but they do represent the most common.

Using the Prototype Response Curves

In the following pages, the response curves and the design tables for several of the lowpass prototypes of the all-pole responses will be cataloged. All the curves are normalized to a −3dB cutoff frequency of 1Hz. This allows direct comparison of the various responses. In all cases the amplitude response for the 2 through 10 pole cases for the frequency range of 0.1Hz. to 10Hz. will be shown. Then a detail of the amplitude response in the 0.1Hz to 2Hz. passband will be shown. The group delay from 0.1Hz to 10Hz and the impulse response and step response from 0 seconds to 5 seconds will also be shown.

To use these curves to determine the response of real life filters, they must be denormalized. In the case of the amplitude responses, this is simply accomplished by multiplying the frequency axis by the desired cutoff frequency F_C. To denormalize the group delay curves, we divide the delay axis by $2\pi F_C$, and multiply the frequency axis by F_C, as before. Denormalize the step response by dividing the time axis by $2\pi F_C$. Denormalize the impulse response by dividing the time axis by $2\pi F_C$ and multiplying the amplitude axis by the same amount.

For a highpass filter, simply invert the frequency axis for the amplitude response. In transforming a lowpass filter into a highpass (or bandreject) filter, the transient behavior is not preserved. Zverev (see Reference 1) provides a computational method for calculating these responses.

In transforming a lowpass into a narrowband bandpass, the 0Hz axis is moved to the center frequency F_0. It stands to reason that the response of the bandpass case around the center frequency would then match the lowpass response around 0Hz. The frequency response curve of a lowpass filter actually mirrors itself around 0Hz, although we generally don't concern ourselves with negative frequency.

To denormalize the group delay curve for a bandpass filter, divide the delay axis by πBW, where BW is the 3dB bandwidth in Hz. Then multiply the frequency axis by BW/2. In general, the delay of the bandpass filter at F_0 will be twice the delay of the lowpass prototype with the same bandwidth at 0Hz. This is due to the fact that the lowpass to bandpass transformation results in a filter with order 2n, even though it is typically referred to it as having the same order as the lowpass filter from which it is derived. This approximation holds for narrowband filters. As the bandwidth of the filter is increased, some distortion of the curve occurs. The delay becomes less symmetrical, peaking below F_0.

The envelope of the response of a bandpass filter resembles the step response of the lowpass prototype. More exactly, it is almost identical to the step response of a lowpass filter having half the bandwidth. To determine the envelope response of the bandpass filter, divide the time axis of the step response of the lowpass prototype by πBW, where BW is the 3dB bandwidth. The previous discussions of overshoot, ringing, etc. can now be applied to the carrier envelope.

The envelope of the response of a narrowband bandpass filter to a short burst of carrier (that is where the burst width is much less than the rise time of the denormalized step response of the bandpass filter) can be determined by denormalizing the impulse response of the low pass prototype. To do this, multiply the amplitude axis and divide the time axis by πBW, where BW is the 3dB bandwidth. It is assumed that the carrier frequency is high enough so that many cycles occur during the burst interval.

While the group delay, step and impulse curves cannot be used directly to predict the distortion to the waveform caused by the filter, they are a useful figure of merit when used to compare filters.

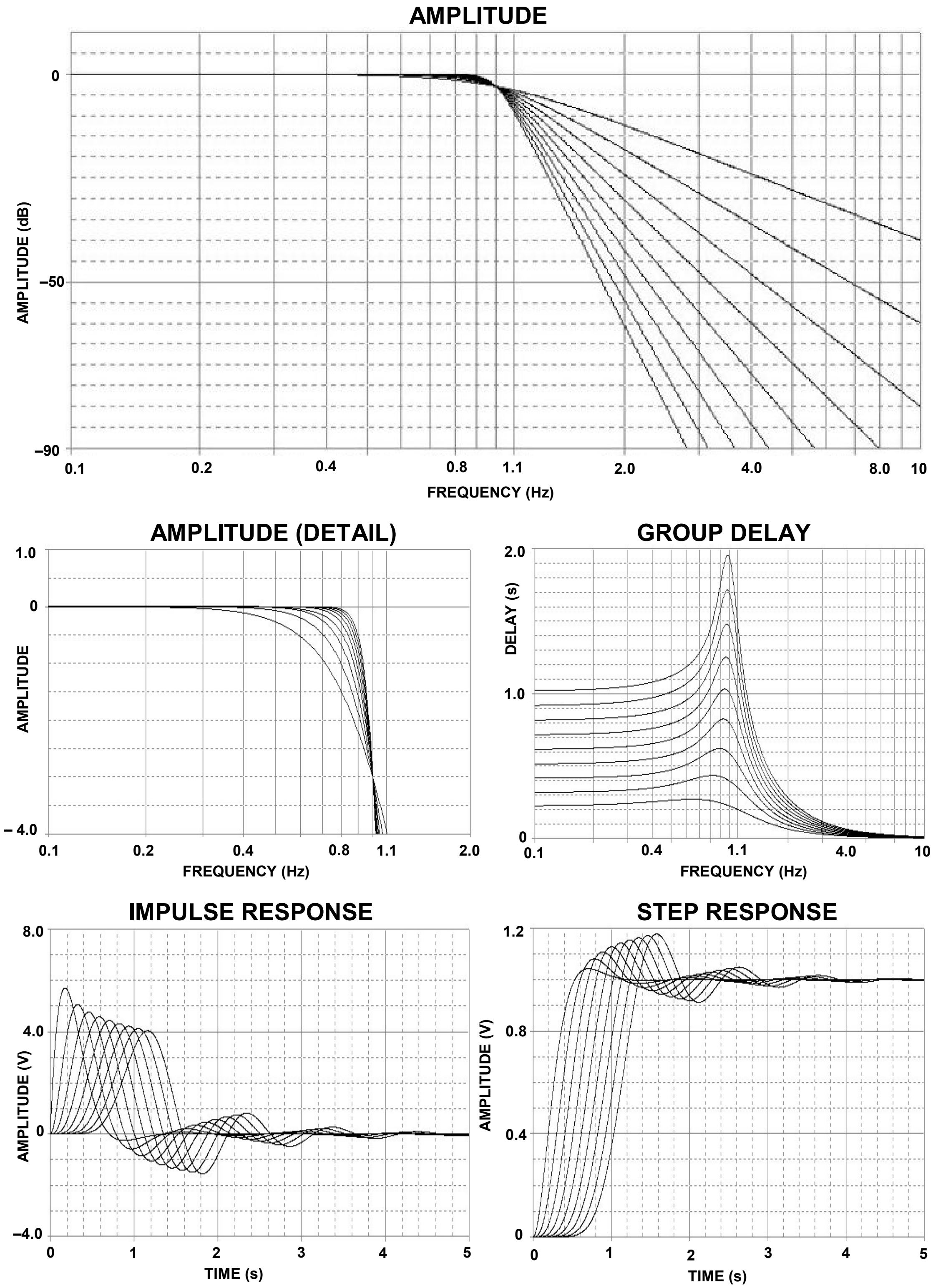

Figure 5-15: *Butterworth response*

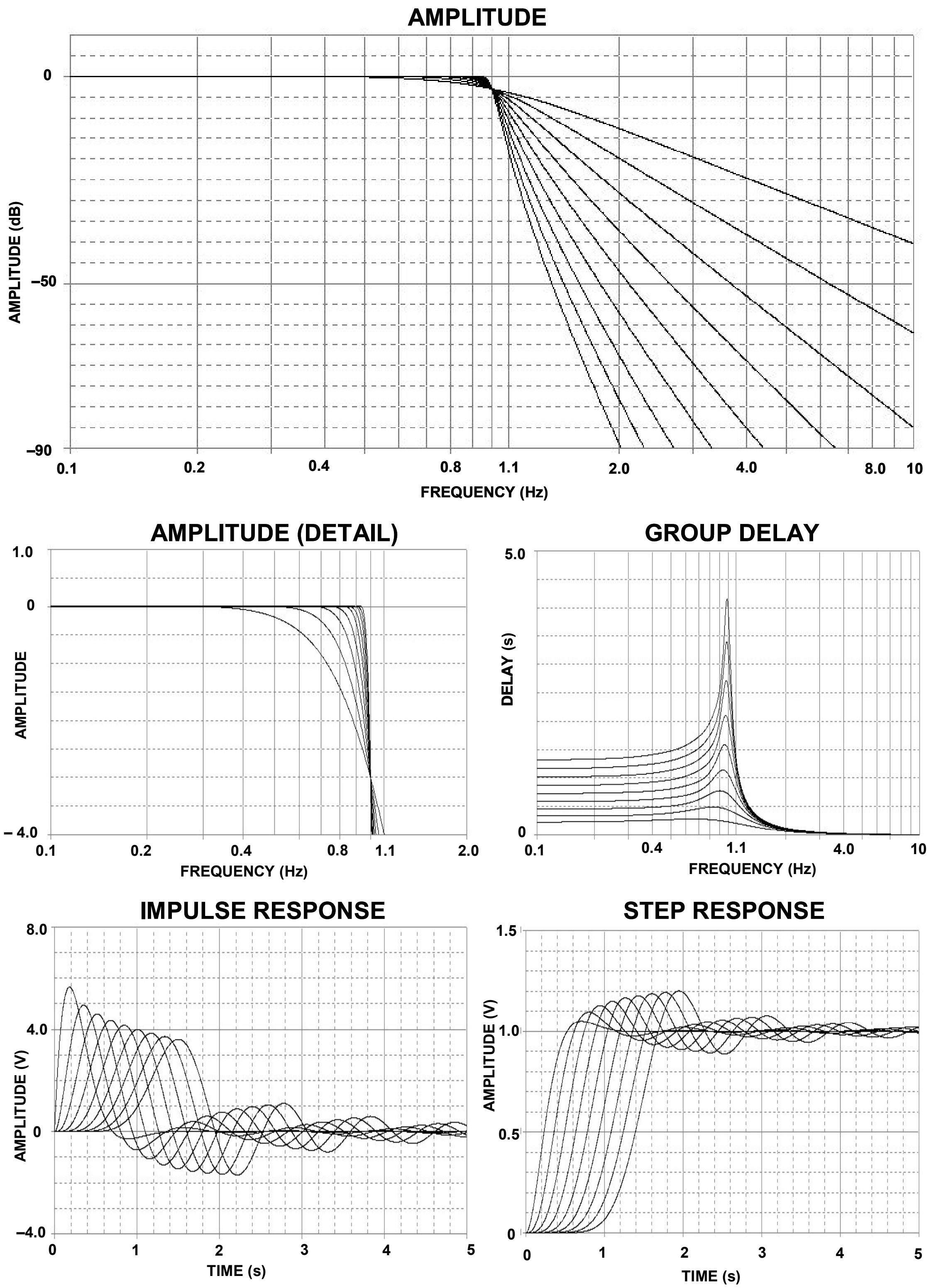

Figure 5-16: *0.01dB Chebyshev Response*

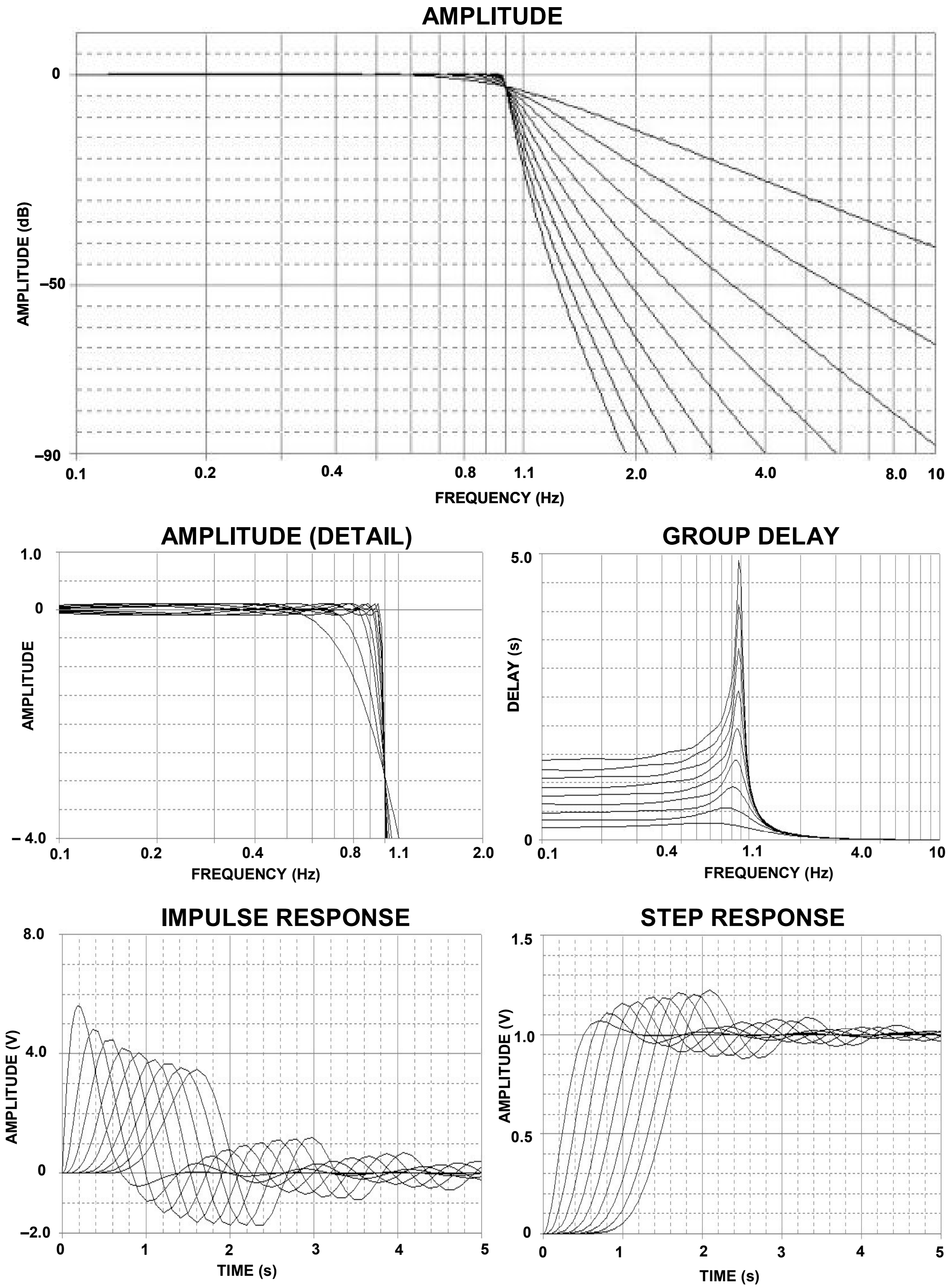

Figure 5-17: *0.1dB Chebyshev Response*

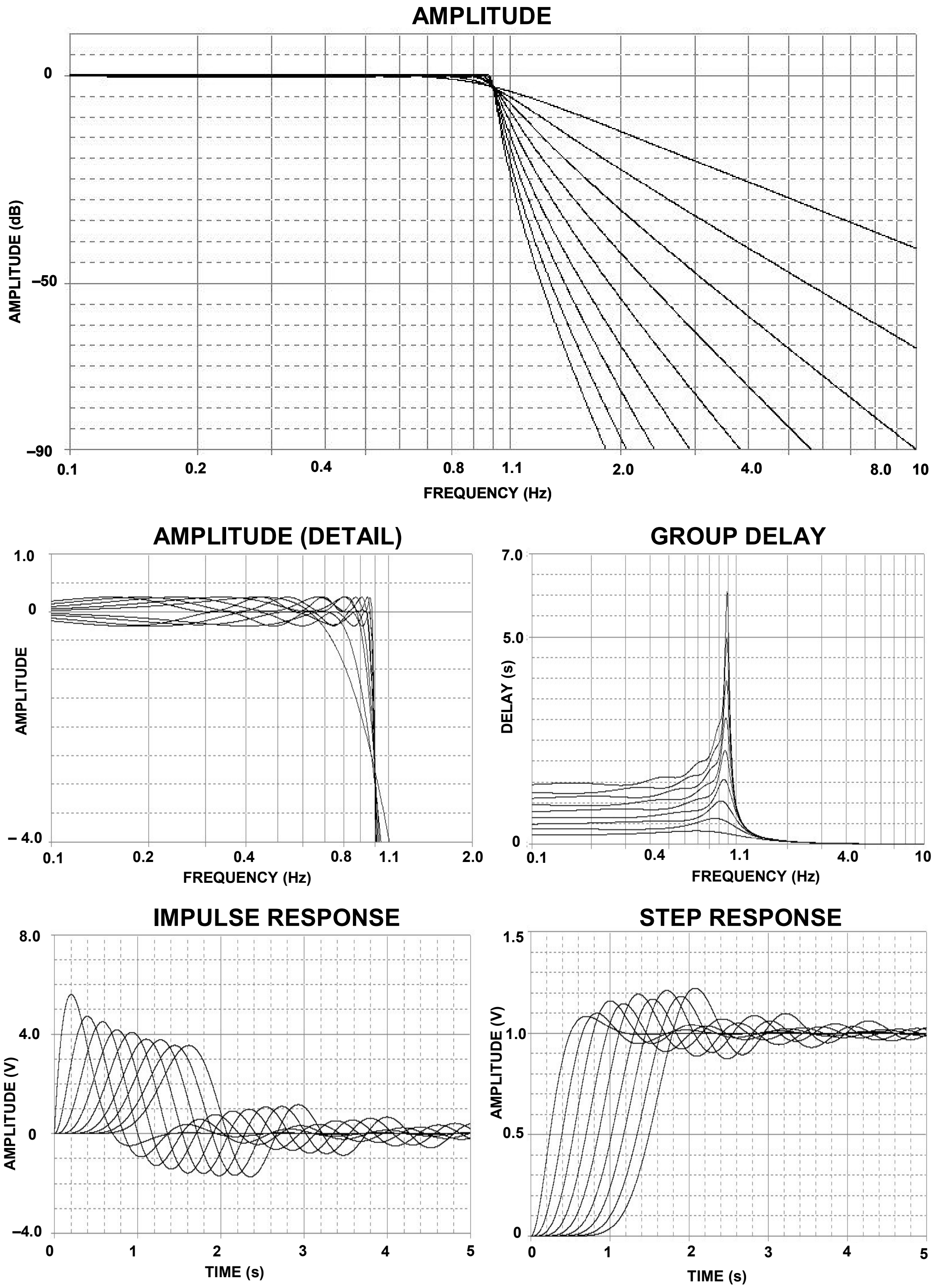

Figure 5-18: *0.25dB Chebyshev Response*

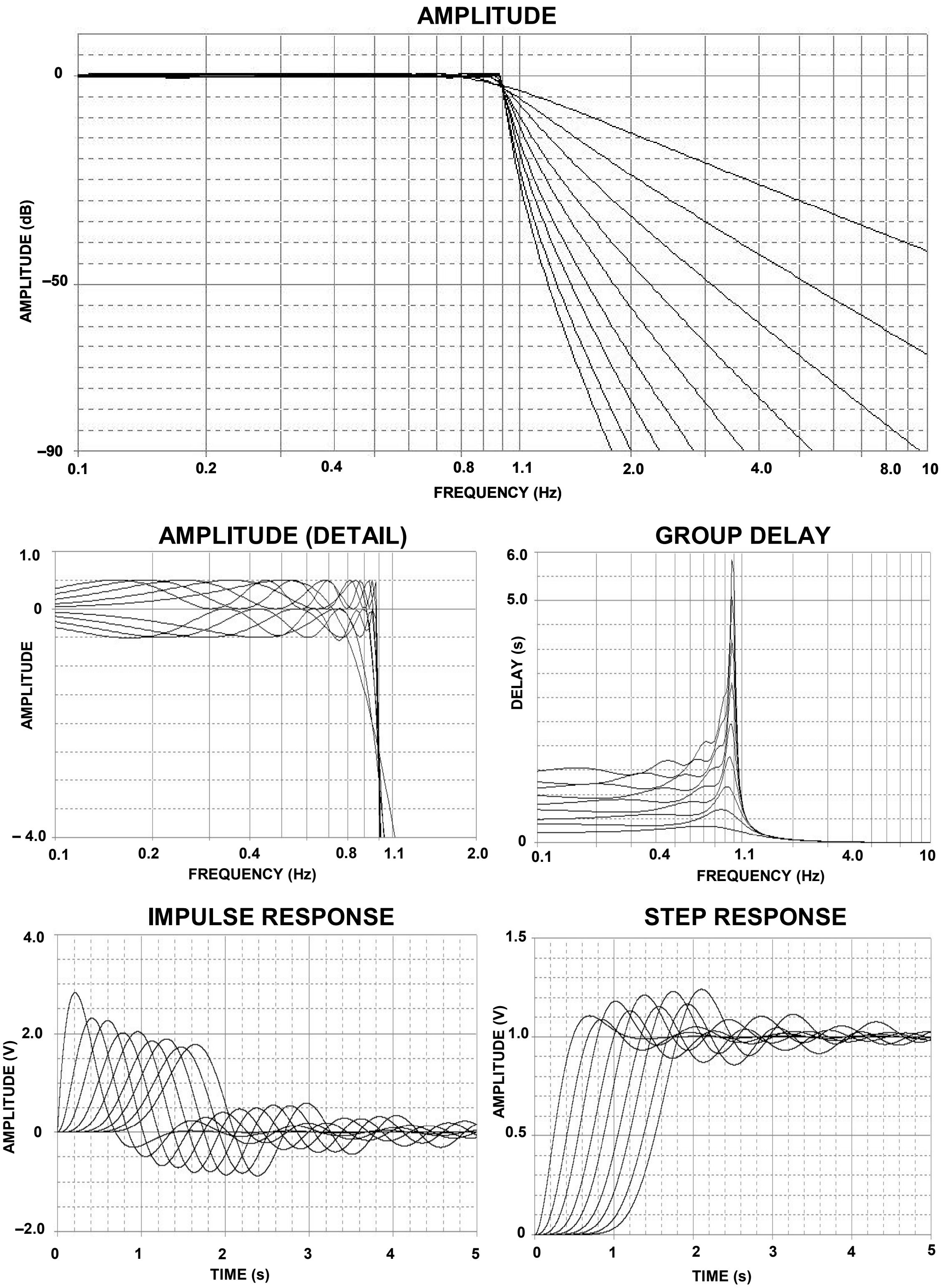

Figure 5-19: *0.5dB Chebyshev Response*

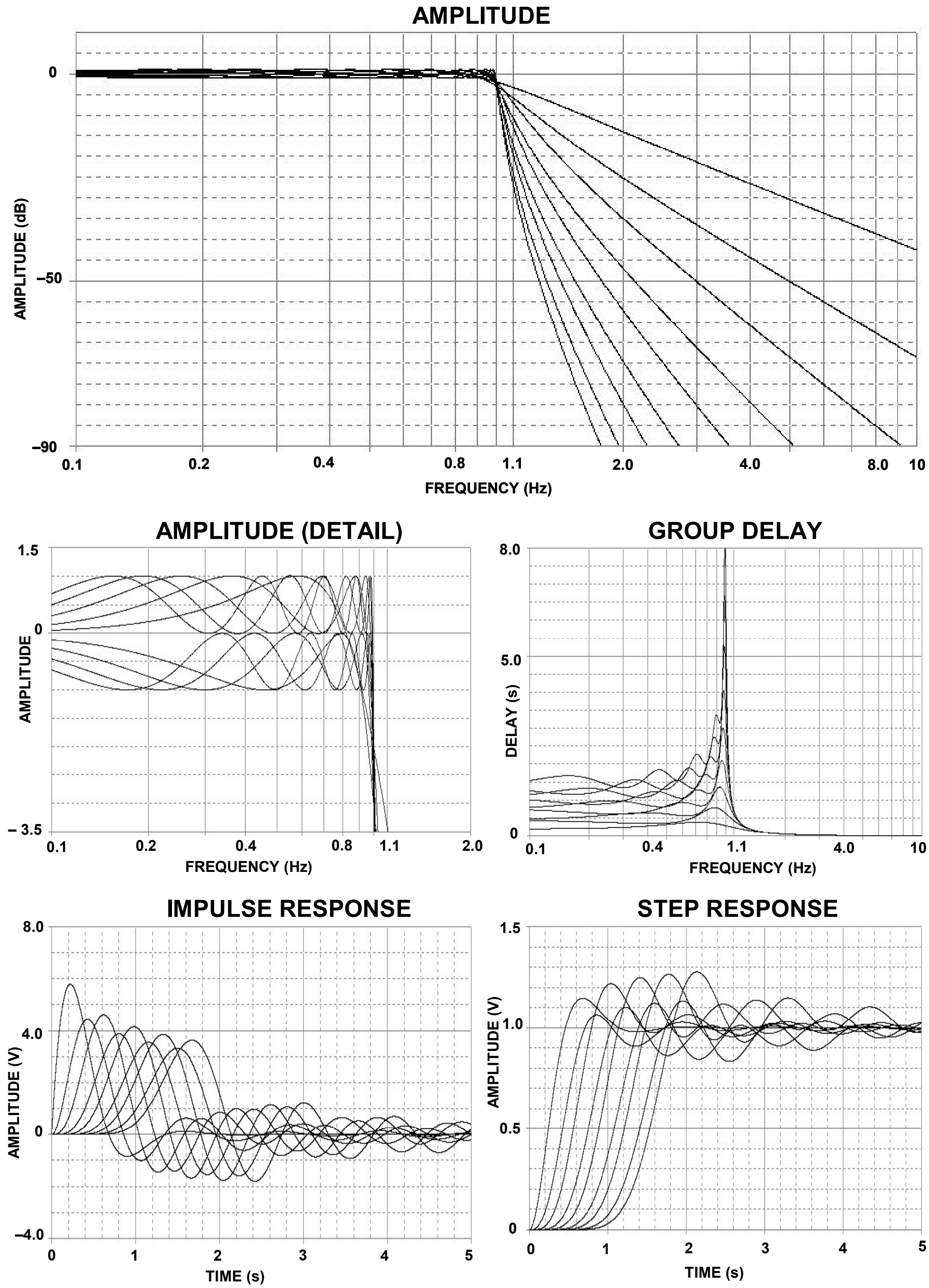

Figure 5-20: *1dB Chebyshev Response*

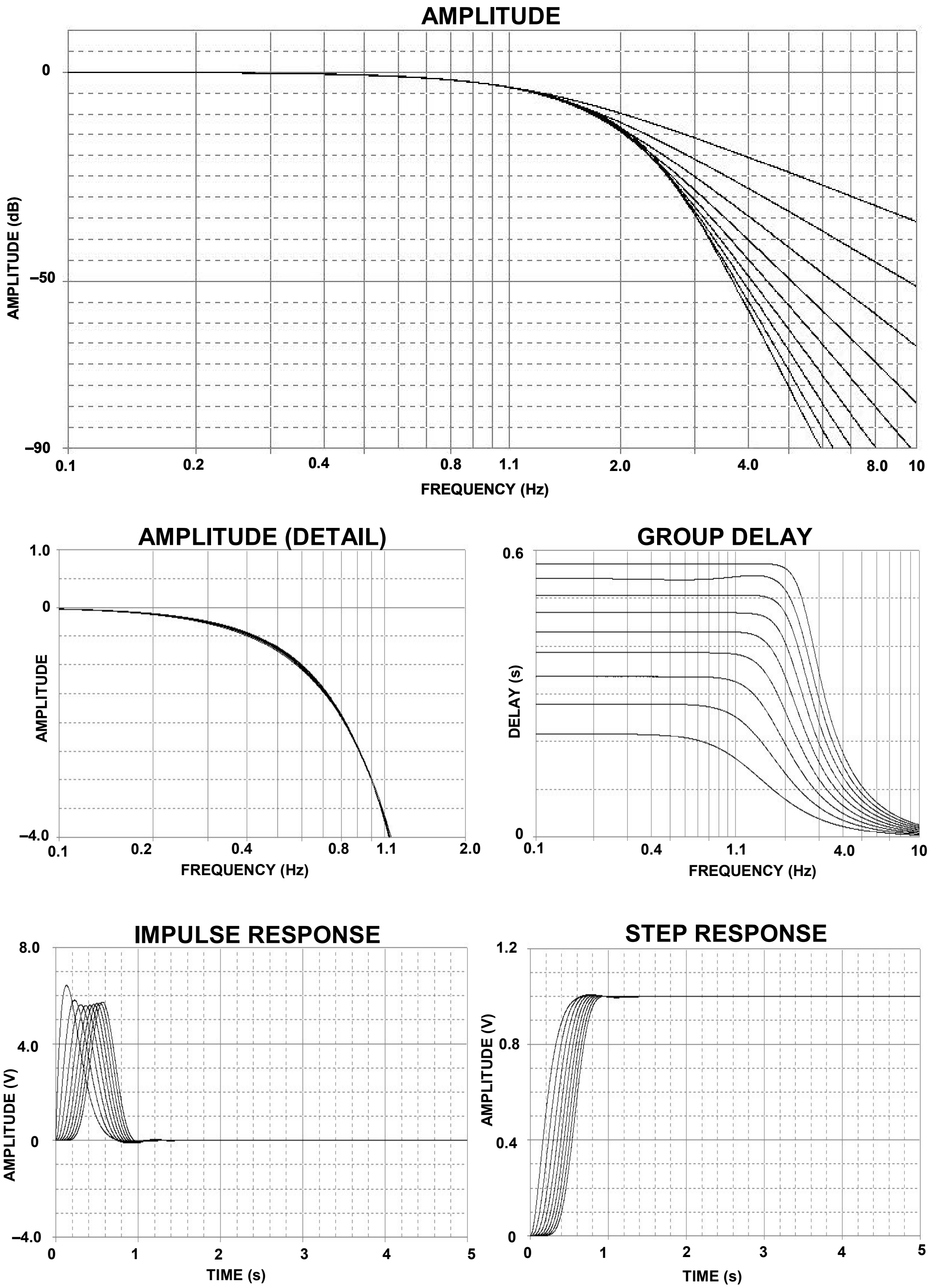

Figure 5-21: *Bessel Response*

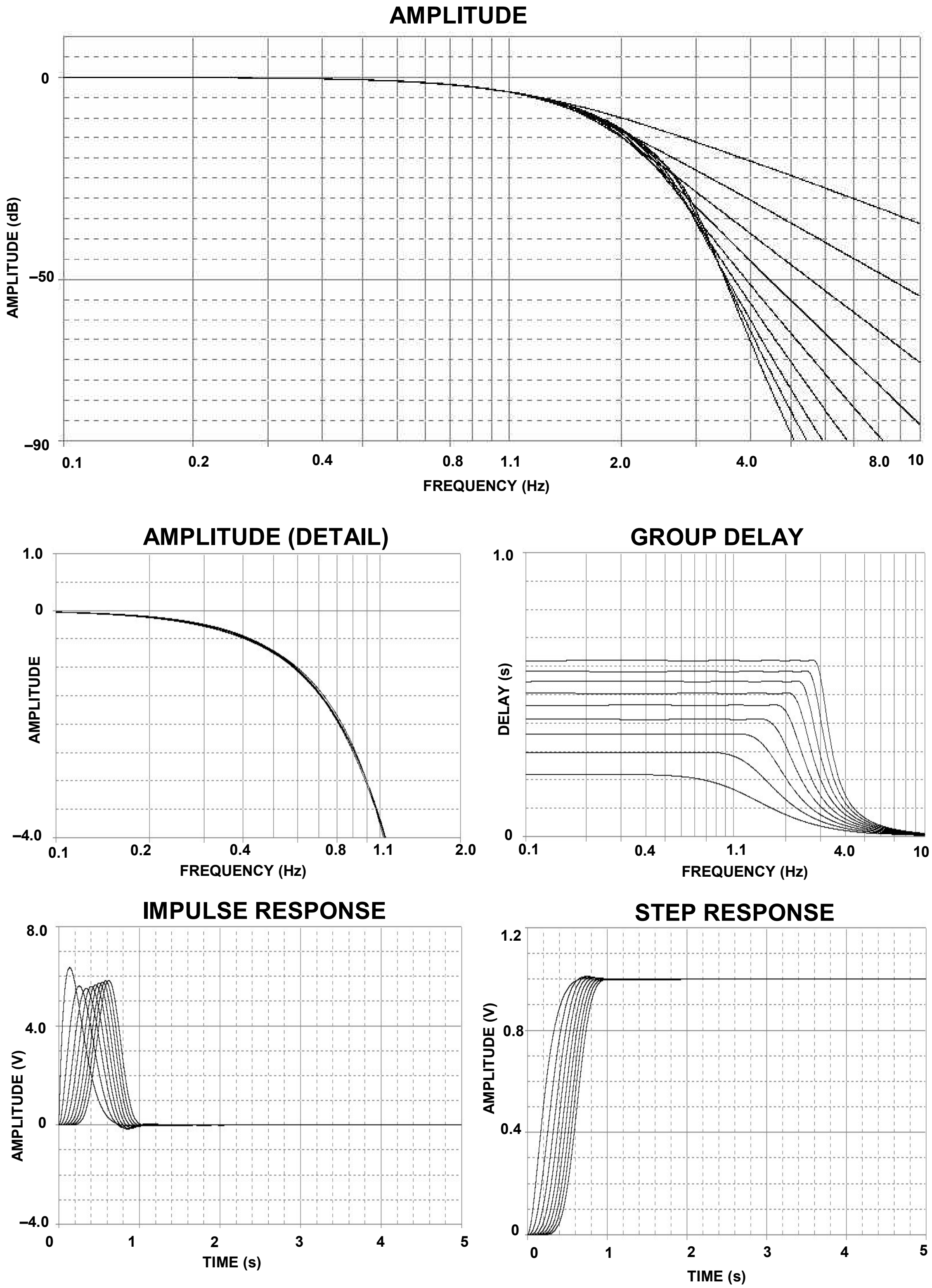

Figure 5-22: *Linear Phase Response with Equiripple Error of 0.05°*

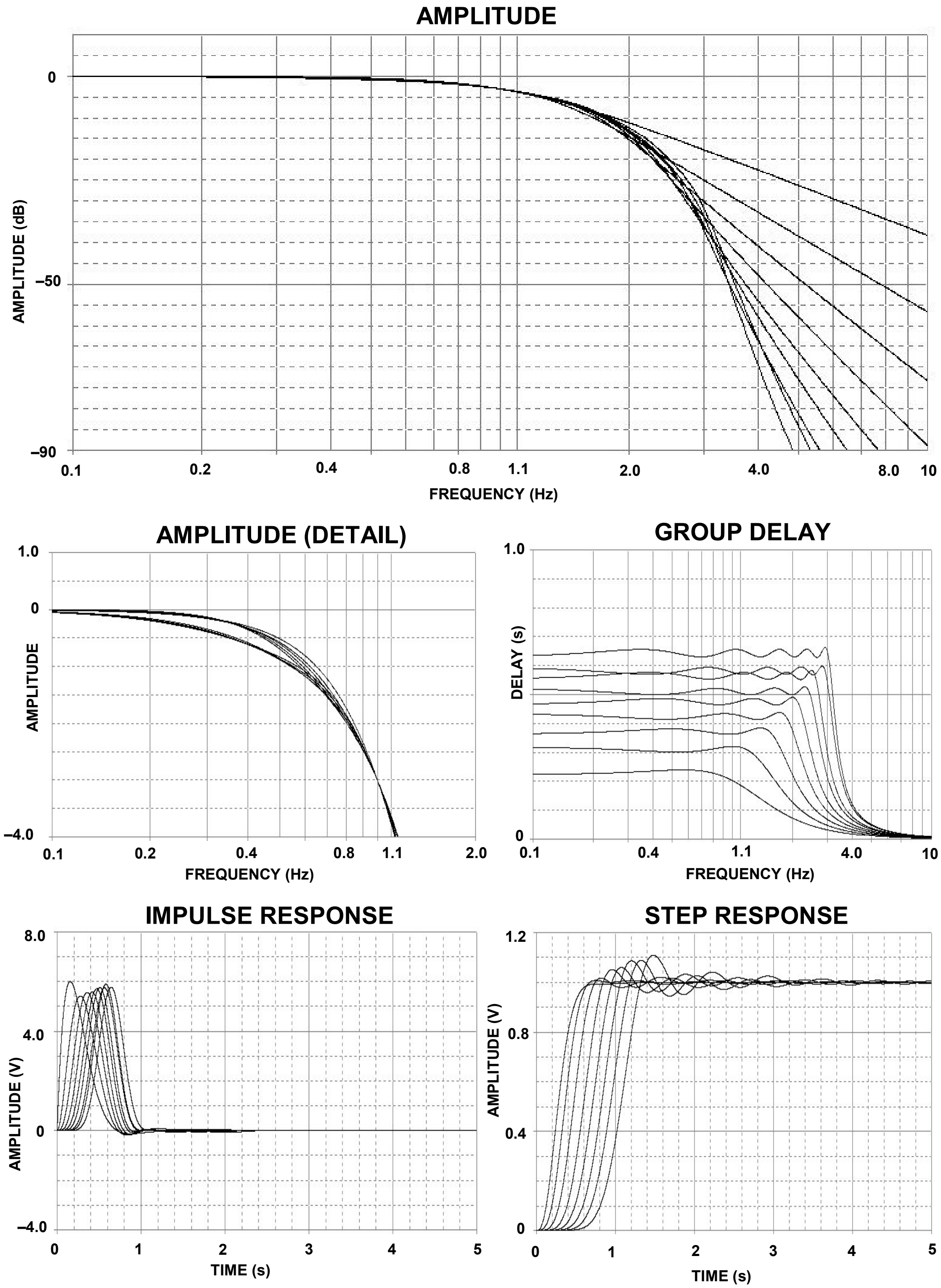

Figure 5-23: *Linear Phase Response with Equiripple Error of 0.5°*

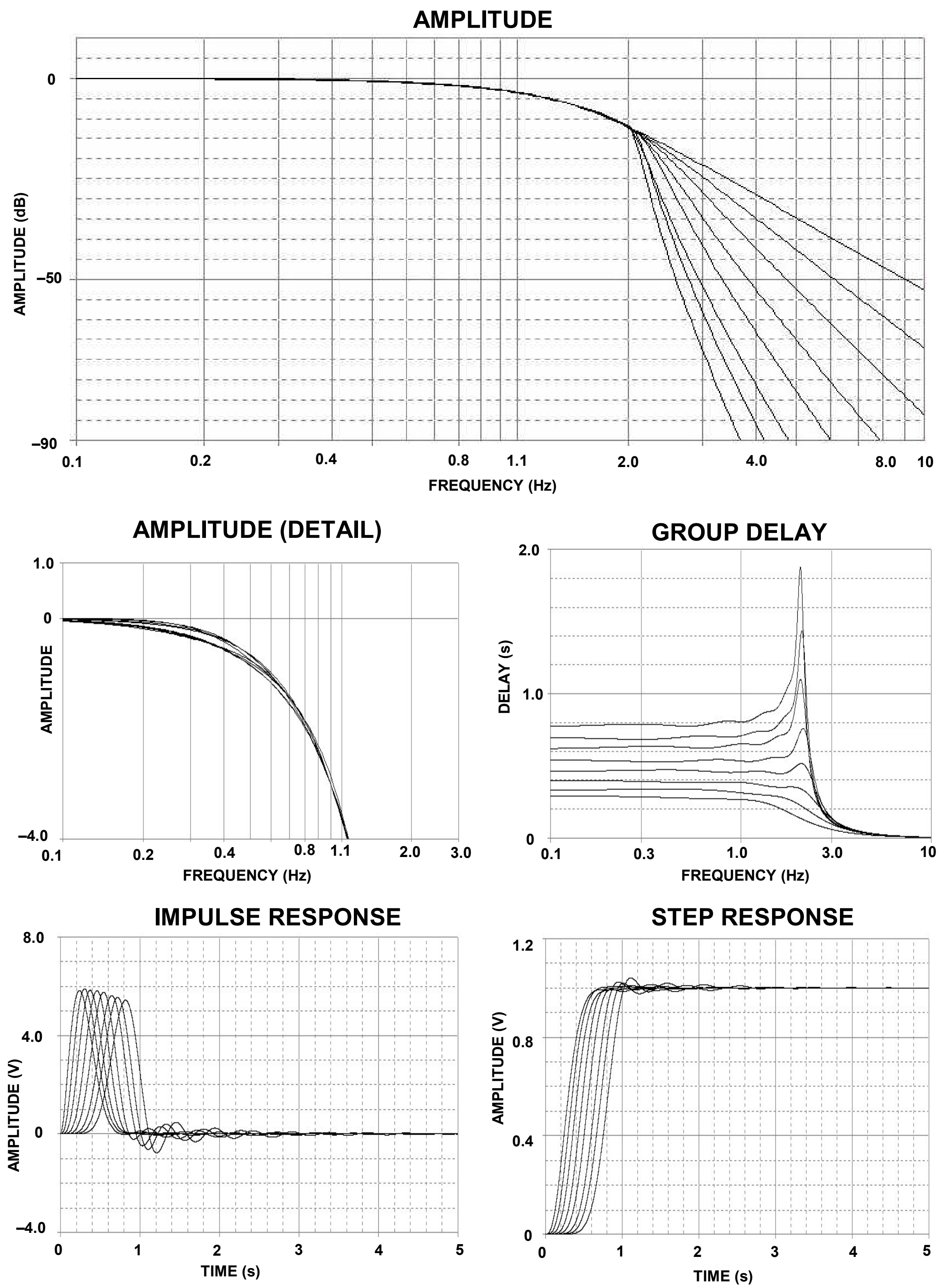

Figure 5-24: *Gaussian to 12 dB Response*

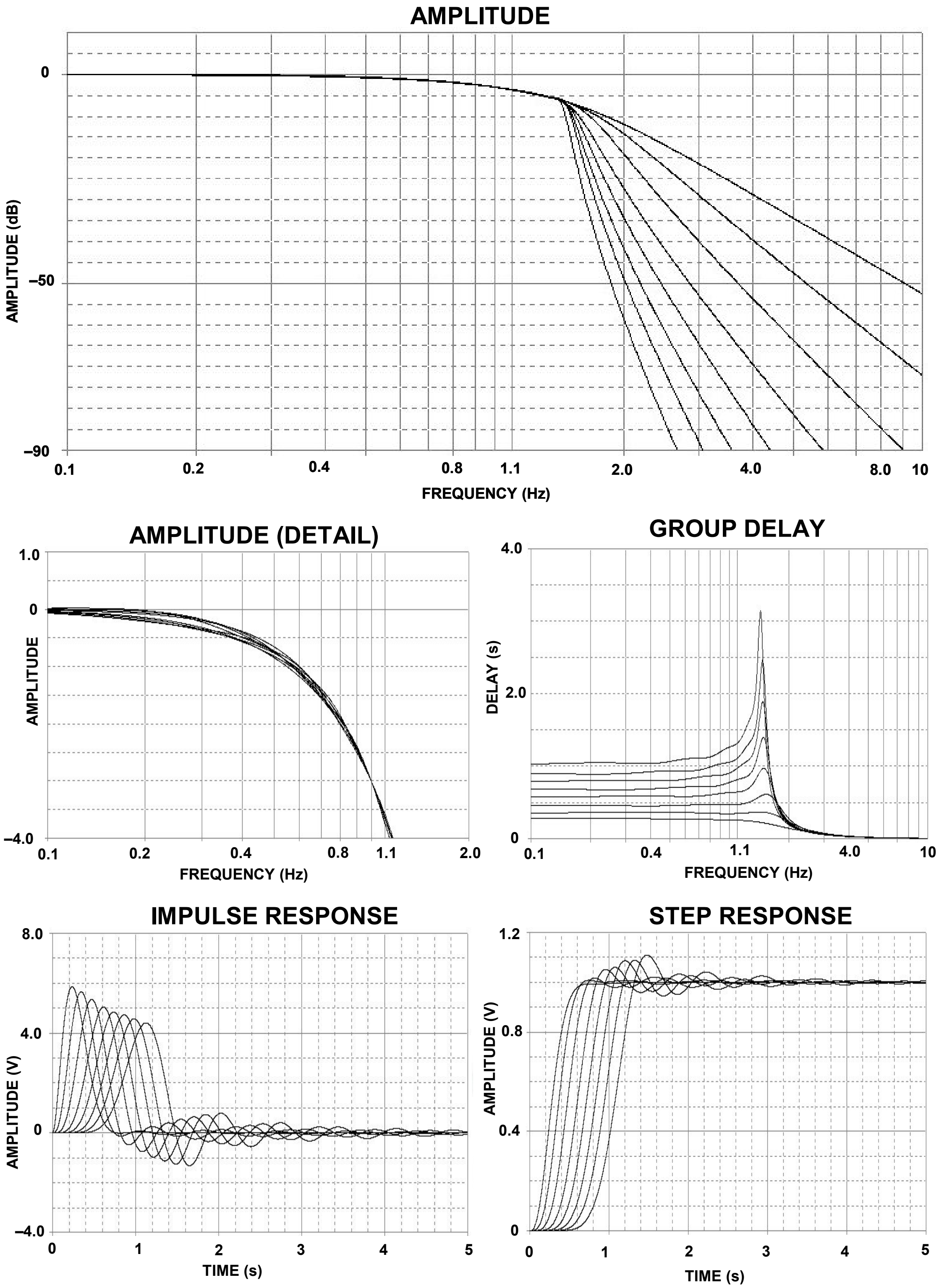

Figure 5-25: *Gaussian to 6dB Response*

		REAL	IMAGINARY				-3 dB	PEAKING	PEAKING
ORDER	SECTION	PART	PART	F_0	α	Q	FREQUENCY	FREQUENCY	LEVEL
2	1	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
3	1	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	2	1.0000		1.0000			1.0000		
4	1	0.9239	0.3827	1.0000	1.8478	0.5412	0.7195		
	2	0.3827	0.9239	1.0000	0.7654	1.3065		0.8409	3.0102
5	1	0.8090	0.5878	1.0000	1.6180	0.6180	0.8588		
	2	0.3090	0.9511	1.0000	0.6180	1.6182		0.8995	4.6163
	3	1.0000		1.0000			1.0000		
6	1	0.9659	0.2588	1.0000	1.9319	0.5176	0.6758		
	2	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	3	0.2588	0.9659	1.0000	0.5176	1.9319		0.9306	6.0210
7	1	0.9010	0.4339	1.0000	1.8019	0.5550	0.7449		
	2	0.6235	0.7818	1.0000	1.2470	0.8019		0.4717	0.2204
	3	0.2225	0.9749	1.0000	0.4450	2.2471		0.9492	7.2530
	4	1.0000		1.0000			1.0000		
8	1	0.9808	0.1951	1.0000	1.9616	0.5098	0.6615		
	2	0.8315	0.5556	1.0000	1.6629	0.6013	0.8295		
	3	0.5556	0.8315	1.0000	1.1112	0.9000		0.6186	0.6876
	4	0.1951	0.9808	1.0000	0.3902	2.5628		0.9612	8.3429
9	1	0.9397	0.3420	1.0000	1.8794	0.5321	0.7026		
	2	0.7660	0.6428	1.0000	1.5320	0.6527	0.9172		
	3	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	4	0.1737	0.9848	1.0000	0.3474	2.8785		0.9694	9.3165
	5	1.0000		1.0000			1.0000		
10	1	0.9877	0.1564	1.0000	1.9754	0.5062	0.6549		
	2	0.8910	0.4540	1.0000	1.7820	0.5612	0.7564		
	3	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	4	0.4540	0.8910	1.0000	0.9080	1.1013		0.7667	1.8407
	5	0.1564	0.9877	1.0000	0.3128	3.1970		0.9752	10.2023

Figure 5-26: *Butterworth Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.6743	0.7075	0.9774	1.3798	0.7247		0.2142	0.0100
3	1	0.4233	0.8663	0.9642	0.8780	1.1389		0.7558	2.0595
	2	0.8467		0.8467			0.8467		
4	1	0.6762	0.3828	0.7770	1.7405	0.5746	0.6069		
	2	0.2801	0.9241	0.9656	0.5801	1.7237		0.8806	5.1110
5	1	0.5120	0.5879	0.7796	1.3135	0.7613		0.2889	0.0827
	2	0.1956	0.9512	0.9711	0.4028	2.4824		0.9309	8.0772
	3	0.6328		0.6328			0.6328		
6	1	0.5335	0.2588	0.5930	1.7995	0.5557	0.4425		
	2	0.3906	0.7072	0.8079	0.9670	1.0342		0.5895	1.4482
	3	0.1430	0.9660	0.9765	0.2929	3.4144		0.9554	10.7605
7	1	0.4393	0.4339	0.6175	1.4229	0.7028	0.6136		
	2	0.3040	0.7819	0.8389	0.7247	1.3798		0.7204	3.4077
	3	0.1085	0.9750	0.9810	0.2212	4.5208		0.9689	13.1578
	4	0.4876		0.4876			0.4876		
8	1	0.4268	0.1951	0.4693	1.8190	0.5498	0.3451		
	2	0.3168	0.5556	0.6396	0.9907	1.0094		0.4564	1.3041
	3	0.2418	0.8315	0.8659	0.5585	1.7906		0.7956	5.4126
	4	0.0849	0.9808	0.9845	0.1725	5.7978		0.9771	15.2977
9	1	0.3686	0.3420	0.5028	1.4661	0.6821	0.4844		
	2	0.3005	0.6428	0.7096	0.8470	1.1807		0.5682	2.3008
	3	0.1961	0.8661	0.8880	0.4417	2.2642		0.8436	7.3155
	4	0.0681	0.9848	0.9872	0.1380	7.2478		0.9824	17.2249
	5	0.3923		0.3923			0.3923		
10	1	0.3522	0.1564	0.3854	1.8279	0.5471	0.2814		
	2	0.3178	0.454	0.5542	1.1469	0.8719		0.3242	0.5412
	3	0.2522	0.7071	0.7507	0.6719	1.4884		0.6606	3.9742
	4	0.1619	0.891	0.9056	0.3576	2.7968		0.8762	9.0742
	5	0.0558	0.9877	0.9893	0.1128	8.8645		0.9861	18.9669

Figure 5-27: *0.01dB Chebyshev Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.6104	0.7106	0.9368	1.3032	0.7673		0.3638	0.0999
3	1	0.3490	0.8684	0.9359	0.7458	1.3408		0.7952	3.1978
	2	0.6970		0.6970			0.6970		
4	1	0.2177	0.9254	0.9507	0.4580	2.1834		0.8994	7.0167
	2	0.5257	0.3833	0.6506	1.6160	0.6188	0.5596		
5	1	0.3842	0.5884	0.7027	1.0935	0.9145		0.4457	0.7662
	2	0.1468	0.9521	0.9634	0.3048	3.2812		0.9407	10.4226
	3	0.4749		0.4749			0.4749		
6	1	0.3916	0.2590	0.4695	1.6682	0.5995	0.3879		
	2	0.2867	0.7077	0.7636	0.7509	1.3316		0.6470	3.1478
	3	0.1049	0.9667	0.9724	0.2158	4.6348		0.9610	13.3714
7	1	0.3178	0.4341	0.5380	1.1814	0.8464		0.2957	0.4157
	2	0.2200	0.7823	0.8126	0.5414	1.8469		0.7507	5.6595
	3	0.0785	0.9755	0.9787	0.1604	6.2335		0.9723	15.9226
	4	0.3528		0.3528			0.3528		
8	1	0.3058	0.1952	0.3628	1.6858	0.5932	0.2956		
	2	0.2529	0.5558	0.6106	0.8283	1.2073		0.4949	2.4532
	3	0.1732	0.8319	0.8497	0.4077	2.4531		0.8137	7.9784
	4	0.0608	0.9812	0.9831	0.1237	8.0819		0.9793	18.1669
9	1	0.2622	0.3421	0.4310	1.2166	0.8219		0.2197	0.3037
	2	0.2137	0.6430	0.6776	0.6308	1.5854		0.6064	4.4576
	3	0.1395	0.8663	0.8775	0.3180	3.1450		0.8550	10.0636
	4	0.0485	0.9852	0.9864	0.0982	10.1795		0.9840	20.1650
	5	0.2790		0.2790			0.2790		
10	1	0.2493	0.1564	0.2943	1.6942	0.5902	0.2382		
	2	0.2249	0.4541	0.5067	0.8876	1.1266		0.3945	1.9880
	3	0.1785	0.7073	0.7295	0.4894	2.0434		0.6844	6.4750
	4	0.1146	0.8913	0.8986	0.2551	3.9208		0.8839	11.9386
	5	0.0395	0.9880	0.9888	0.0799	12.5163		0.9872	21.9565

Figure 5-28: *0.1dB Chebyshev Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.5621	0.7154	0.9098	1.2356	0.8093		0.4425	0.2502
3	1	0.3062	0.8712	0.9234	0.6632	1.5079		0.8156	4.0734
	2	0.6124		0.6124			0.6124		
4	1	0.4501	0.3840	0.5916	1.5215	0.6572	0.5470		
	2	0.1865	0.9272	0.9458	0.3944	2.5356		0.9082	8.2538
5	1	0.3247	0.5892	0.6727	0.9653	1.0359		0.4917	1.4585
	2	0.1240	0.9533	0.9613	0.2580	3.8763		0.9452	11.8413
	3	0.4013		0.4013			0.4013		
6	1	0.3284	0.2593	0.4184	1.5697	0.6371	0.3730		
	2	0.2404	0.7083	0.7480	0.6428	1.5557		0.6663	4.3121
	3	0.0880	0.9675	0.9715	0.1811	5.5205		0.9635	14.8753
7	1	0.2652	0.4344	0.5090	1.0421	0.9596		0.3441	1.0173
	2	0.1835	0.7828	0.8040	0.4565	2.1908		0.7610	7.0443
	3	0.0655	0.9761	0.9783	0.1339	7.4679		0.9739	17.4835
	4	0.2944		0.2944			0.2944		
8	1	0.2543	0.1953	0.3206	1.5862	0.6304	0.2822		
	2	0.2156	0.5561	0.5964	0.7230	1.3832		0.5126	3.4258
	3	0.1441	0.8323	0.8447	0.3412	2.9309		0.8197	9.4683
	4	0.0506	0.9817	0.9830	0.1029	9.7173		0.9804	19.7624
9	1	0.2176	0.3423	0.4056	1.0730	0.9320		0.2642	0.8624
	2	0.1774	0.6433	0.6673	0.5317	1.8808		0.6184	5.8052
	3	0.1158	0.8667	0.8744	0.2649	3.7755		0.8589	11.6163
	4	0.0402	0.9856	0.9864	0.0815	12.2659		0.9848	21.7812
	5	0.2315		0.2315			0.2315		
10	1	0.2065	0.1565	0.2591	1.5940	0.6274	0.2267		
	2	0.1863	0.4543	0.4910	0.7588	1.3178		0.4143	3.0721
	3	0.1478	0.7075	0.7228	0.4090	2.4451		0.6919	7.9515
	4	0.0949	0.8915	0.8965	0.2117	4.7236		0.8864	13.5344
	5	0.0327	0.9883	0.9888	0.0661	15.1199		0.9878	23.5957

Figure 5-29: *0.25dB Chebyshev Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.5129	0.7225	1.2314	1.1577	0.8638		0.7072	0.5002
3	1	0.2683	0.8753	1.0688	0.5861	1.7061		0.9727	5.0301
	2	0.5366		0.6265			0.6265		
4	1	0.3872	0.3850	0.5969	1.4182	0.7051	0.5951		
	2	0.1605	0.9297	1.0313	0.3402	2.9391		1.0010	9.4918
5	1	0.2767	0.5902	0.6905	0.8490	1.1779		0.5522	2.2849
	2	0.1057	0.9550	1.0178	0.2200	4.5451		1.0054	13.2037
	3	0.3420		0.3623			0.3623		
6	1	0.2784	0.2596	0.3963	1.4627	0.6836	0.3827		
	2	0.2037	0.7091	0.7680	0.5522	1.8109		0.7071	5.5025
	3	0.0746	0.9687	1.0114	0.1536	6.5119		1.0055	16.2998
7	1	0.2241	0.4349	0.5040	0.9161	1.0916		0.3839	1.7838
	2	0.1550	0.7836	0.8228	0.3881	2.5767		0.7912	8.3880
	3	0.0553	0.9771	1.0081	0.1130	8.8487		1.0049	18.9515
	4	0.2487		0.2562			0.2562		
8	1	0.2144	0.1955	0.2968	1.4779	0.6767	0.2835		
	2	0.1817	0.5565	0.5989	0.6208	1.6109		0.5381	4.5815
	3	0.1214	0.8328	0.8610	0.2885	3.4662		0.8429	10.8885
	4	0.0426	0.9824	1.0060	0.0867	11.5305		1.0041	21.2452
9	1	0.1831	0.3425	0.3954	0.9429	1.0605		0.2947	1.6023
	2	0.1493	0.6436	0.6727	0.4520	2.2126		0.6374	7.1258
	3	0.0974	0.8671	0.8884	0.2233	4.4779		0.8773	13.0759
	4	0.0338	0.9861	1.0046	0.0686	14.5829		1.0034	23.2820
	5	0.1949		0.1984			0.1984		
10	1	0.1736	0.1566	0.2338	1.4851	0.6734	0.2221		
	2	0.1566	0.4545	0.4807	0.6515	1.5349		0.4267	4.2087
	3	0.1243	0.7078	0.7186	0.3459	2.8907		0.6968	9.3520
	4	0.0798	0.8919	0.8955	0.1782	5.6107		0.8883	15.0149
	5	0.0275	0.9887	0.9891	0.0556	17.9833		0.9883	25.1008

Figure 5-30: *0.5dB Chebyshev Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.4508	0.7351	0.8623	1.0456	0.9564		0.5806	0.9995
3	1	0.2257	0.8822	0.9106	0.4957	2.0173		0.8528	6.3708
	2	0.4513		0.4513			0.4513		
4	1	0.3199	0.3868	0.5019	1.2746	0.7845		0.2174	0.1557
	2	0.1325	0.9339	0.9433	0.2809	3.5594		0.9245	11.1142
5	1	0.2265	0.5918	0.6337	0.7149	1.3988		0.5467	3.5089
	2	0.0865	0.9575	0.9614	0.1800	5.5559		0.9536	14.9305
	3	0.2800		0.2800			0.2800		
6	1	0.2268	0.2601	0.3451	1.3144	0.7608		0.1273	0.0813
	2	0.1550	0.7106	0.7273	0.4262	2.3462		0.6935	7.6090
	3	0.0608	0.9707	0.9726	0.1249	8.0036		0.9688	18.0827
7	1	0.1819	0.4354	0.4719	0.7710	1.2971		0.3956	2.9579
	2	0.1259	0.7846	0.7946	0.3169	3.1558		0.7744	10.0927
	3	0.0449	0.9785	0.9795	0.0918	10.8982		0.9775	20.7563
	4	0.2019		0.2019			0.2019		
8	1	0.1737	0.1956	0.2616	1.3280	0.7530		0.0899	0.0611
	2	0.1473	0.5571	0.5762	0.5112	1.9560		0.5373	6.1210
	3	0.0984	0.8337	0.8395	0.2344	4.2657		0.8279	12.6599
	4	0.0346	0.9836	0.9842	0.0702	14.2391		0.9830	23.0750
9	1	0.1482	0.3427	0.3734	0.7938	1.2597		0.3090	2.7498
	2	0.1208	0.6442	0.6554	0.3686	2.7129		0.6328	8.8187
	3	0.0788	0.8679	0.8715	0.1809	5.5268		0.8643	14.8852
	4	0.0274	0.9869	0.9873	0.0555	18.0226		0.9865	25.1197
	5	0.1577		0.1577			0.1577		
10	1	0.1403	0.1567	0.2103	1.3341	0.7496		0.0698	0.0530
	2	0.1266	0.4548	0.4721	0.5363	1.8645		0.4368	5.7354
	3	0.1005	0.7084	0.7155	0.2809	3.5597		0.7012	11.1147
	4	0.0645	0.8926	0.8949	0.1441	6.9374		0.8903	16.8466
	5	0.0222	0.9895	0.9897	0.0449	22.2916		0.9893	26.9650

Figure 5-31: *1dB Chebyshev Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	1.1050	0.6368	1.2754	1.7328	0.5771	1.0020		
3	1	1.0509	1.0025	1.4524	1.4471	0.6910	1.4185		
	2	1.3270		1.3270			1.3270		
4	1	1.3596	0.4071	1.4192	1.9160	0.5219	0.9705		
	2	0.9877	1.2476	1.5912	1.2414	0.8055		0.7622	0.2349
5	1	1.3851	0.7201	1.5611	1.7745	0.5635	1.1876		
	2	0.9606	1.4756	1.7607	1.0911	0.9165		1.1201	0.7768
	3	1.5069		1.5069			1.5069		
6	1	1.5735	0.3213	1.6060	1.9596	0.5103	1.0638		
	2	1.3836	0.9727	1.6913	1.6361	0.6112	1.4323		
	3	0.9318	1.6640	1.9071	0.9772	1.0234		1.3786	1.3851
7	1	1.6130	0.5896	1.7174	1.8784	0.5324	1.2074		
	2	1.3797	1.1923	1.8235	1.5132	0.6608	1.6964		
	3	0.9104	1.8375	2.0507	0.8879	1.1262		1.5961	1.9860
	4	1.6853		1.6853			1.6853		
8	1	1.7627	0.2737	1.7838	1.9763	0.5060	1.1675		
	2	0.8955	2.0044	2.1953	0.8158	1.2258		1.7932	2.5585
	3	1.3780	1.3926	1.9591	1.4067	0.7109		0.2011	0.0005
	4	1.6419	0.8256	1.8378	1.7868	0.5597	1.3849		
9	1	1.8081	0.5126	1.8794	1.9242	0.5197	1.2774		
	2	1.6532	1.0319	1.9488	1.6966	0.5894	1.5747		
	3	1.3683	1.5685	2.0815	1.3148	0.7606		0.7668	0.0807
	4	0.8788	2.1509	2.3235	0.7564	1.3220		1.9632	3.0949
	5	1.8575		1.8575			1.8575		
10	1	1.9335	0.2451	1.9490	1.9841	0.5040	1.2685		
	2	1.8467	0.7335	1.9870	1.8587	0.5380	1.4177		
	3	1.6661	1.2246	2.0678	1.6115	0.6205	1.7848		
	4	1.3648	1.7395	2.2110	1.2346	0.8100		1.0785	0.2531
	5	0.8686	2.2994	2.4580	0.7067	1.4150		2.1291	3.5944

Figure 5-32: *Bessel Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	1.0087	0.6680	1.2098	1.6675	0.5997	0.9999		
3	1	0.8541	1.0725	1.3710	1.2459	0.8026		0.6487	0.2232
	2	1.0459		1.0459			1.0459		
4	1	0.9648	0.4748	1.0753	1.7945	0.5573	0.8056		
	2	0.7448	1.4008	1.5865	0.9389	1.0650		1.1864	1.6286
5	1	0.8915	0.8733	1.2480	1.4287	0.6999	1.2351		
	2	0.6731	1.7085	1.8363	0.7331	1.3641		1.5703	3.3234
	3	0.9430		0.9430			0.9430		
6	1	0.8904	0.4111	0.9807	1.8158	0.5507	0.7229		
	2	0.8233	1.2179	1.4701	1.1201	0.8928		0.8975	0.6495
	3	0.6152	1.9810	2.0743	0.5932	1.6859		1.8831	4.9365
7	1	0.8425	0.7791	1.1475	1.4684	0.6810	1.1036		
	2	0.7708	1.5351	1.7177	0.8975	1.1143		1.3276	1.9162
	3	0.5727	2.2456	2.3175	0.4942	2.0233		2.1713	6.3948
	4	0.8615		0.8615			0.8615		
8	1	0.8195	0.3711	0.8996	1.8219	0.5489	0.6600		
	2	0.7930	1.1054	1.3604	1.1658	0.8578		0.7701	0.4705
	3	0.7213	1.8134	1.9516	0.7392	1.3528		1.6638	3.2627
	4	0.5341	2.4761	2.5330	0.4217	2.3713		2.4178	7.6973
9	1	0.7853	0.7125	1.0604	1.4812	0.6751	1.0102		
	2	0.7555	1.4127	1.6020	0.9432	1.0602		1.1937	1.6005
	3	0.6849	2.0854	2.1950	0.6241	1.6024		1.9697	4.5404
	4	0.5060	2.7133	2.7601	0.3667	2.7274		2.6657	8.8633
	5	0.7983		0.7983			0.7983		
10	1	0.7592	0.3413	0.8324	1.8241	0.5482	0.6096		
	2	0.7467	1.0195	1.2637	1.1818	0.8462		0.6941	0.4145
	3	0.7159	1.6836	1.8295	0.7826	1.2778		1.5238	2.8507
	4	0.6475	2.3198	2.4085	0.5377	1.8598		2.2276	5.7152
	5	0.4777	2.9128	2.9517	0.3237	3.0895		2.8734	9.9130

Figure 5-33: *Linear Phase with Equiripple Error of 0.05° Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.8590	0.6981	1.1069	1.5521	0.6443	1.0000		
3	1	0.6969	1.1318	1.3292	1.0486	0.9536		0.8918	0.9836
	2	0.8257		0.8257			0.8257		
4	1	0.7448	0.5133	0.9045	1.6468	0.6072	0.7597		
	2	0.6037	1.4983	1.6154	0.7475	1.3379		1.3713	3.1817
5	1	0.6775	0.9401	1.1588	1.1693	0.8552		0.6518	0.4579
	2	0.5412	1.8256	1.9041	0.5684	1.7592		1.7435	5.2720
	3	0.7056		0.7056			0.7056		
6	1	0.6519	0.4374	0.7850	1.6608	0.6021	0.6522		
	2	0.6167	1.2963	1.4355	0.8592	1.1639		1.1402	2.2042
	3	0.4893	2.0982	2.1545	0.4542	2.2016		2.0404	7.0848
7	1	0.6190	0.8338	1.0385	1.1922	0.8388		0.5586	0.3798
	2	0.5816	1.6455	1.7453	0.6665	1.5004		1.5393	4.0353
	3	0.4598	2.3994	2.4431	0.3764	2.6567		2.3549	8.6433
	4	0.6283		0.6283			0.6283		
8	1	0.5791	0.3857	0.6958	1.6646	0.6007	0.5764		
	2	0.5665	1.1505	1.2824	0.8835	1.1319		1.0014	2.0187
	3	0.5303	1.8914	1.9643	0.5399	1.8521		1.8155	5.6819
	4	0.4148	2.5780	2.6112	0.3177	3.1475		2.5444	10.0703
9	1	0.5688	0.7595	0.9489	1.1989	0.8341		0.5033	0.3581
	2	0.5545	1.5089	1.6076	0.6899	1.4496		1.4033	3.7748
	3	0.5179	2.2329	2.2922	0.4519	2.2130		2.1720	7.1270
	4	0.4080	2.9028	2.9313	0.2784	3.5923		2.8740	11.1925
	5	0.5728		0.5728			0.5728		
10	1	0.5249	0.3487	0.6302	1.6659	0.6003	0.5215		
	2	0.5193	1.0429	1.1650	0.8915	1.1217		0.9044	1.9598
	3	0.5051	1.7264	1.7988	0.5616	1.7806		1.6509	5.3681
	4	0.4711	2.3850	2.4311	0.3876	2.5802		2.3380	8.3994
	5	0.3708	2.9940	3.0169	0.2458	4.0681		2.9709	12.2539

Figure 5-34: *Linear Phase with Equiripple Error of 0.5° Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
3	1	0.9360	1.2168	1.5352	1.2194	0.8201		0.7775	0.2956
	2	0.9360		0.9360			0.9360		
4	1	0.9278	1.6995	1.9363	0.9583	1.0435		1.4239	1.5025
	2	0.9192	0.5560	1.0743	1.7113	0.5844	0.8582		
5	1	0.8075	0.9973	1.2832	1.2585	0.7946		0.5853	0.1921
	2	0.7153	0.2053	0.7442	1.9224	0.5202	0.5065		
	3	0.8131		0.8131			0.8131		
6	1	0.7019	0.4322	0.8243	1.7030	0.5872	0.6627		
	2	0.6667	1.2931	1.4549	0.9165	1.0911		1.1080	1.7809
	3	0.4479	2.1363	2.1827	0.4104	2.4366		2.0888	7.9227
7	1	0.6155	0.7703	0.9860	1.2485	0.8010		0.4632	0.2168
	2	0.5486	1.5154	1.6116	0.6808	1.4689		1.4126	3.8745
	3	0.2905	2.1486	2.1681	0.2680	3.7318		2.1289	11.5169
	4	0.6291		0.6291			0.6291		
8	1	0.5441	0.3358	0.6394	1.7020	0.5876	0.5145		
	2	0.5175	0.9962	1.1226	0.9220	1.0846		0.8512	1.7432
	3	0.4328	1.6100	1.6672	0.5192	1.9260		1.5507	5.9962
	4	0.1978	2.0703	2.0797	0.1902	5.2571		2.0608	14.4545
9	1	0.4961	0.6192	0.7934	1.2505	0.7997		0.3705	0.2116
	2	0.4568	1.2145	1.2976	0.7041	1.4203		1.1253	3.6221
	3	0.3592	1.7429	1.7795	0.4037	2.4771		1.7055	8.0594
	4	0.1489	2.1003	2.1056	0.1414	7.0704		2.0950	17.0107
	5	0.5065		0.5065			0.5065		
10	1	0.4535	0.2794	0.5327	1.7028	0.5873	0.4283		
	2	0.4352	0.8289	0.9362	0.9297	1.0756		0.7055	1.6904
	3	0.3886	1.3448	1.3998	0.5552	1.8011		1.2874	5.4591
	4	0.2908	1.7837	1.8072	0.3218	3.1074		1.7598	9.9618
	5	0.1136	2.0599	2.0630	0.1101	9.0802		2.0568	19.1751

Figure 5-35: *Gaussian to 12dB Design Table*

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
3	1	0.9622	1.2214	1.5549	1.2377	0.8080		0.7523	0.2448
	2	0.9776	0.5029	1.0994	1.7785	0.5623	0.8338		
4	1	0.7940	0.5029	0.9399	1.6896	0.5919	0.7636		
	2	0.6304	1.5407	1.6647	0.7574	1.3203		1.4058	3.0859
5	1	0.6190	0.8254	1.0317	1.1999	0.8334		0.5460	0.3548
	2	0.3559	1.5688	1.6087	0.4425	2.2600		1.5279	7.3001
	3	0.6650		0.6650			0.6650		
6	1	0.5433	0.3431	0.6426	1.6910	0.5914	0.5215		
	2	0.4672	0.9991	1.1029	0.8472	1.1804		0.8831	2.2992
	3	0.2204	1.5067	1.5227	0.2895	3.4545		1.4905	10.8596
7	1	0.4580	0.5932	0.7494	1.2223	0.8182		0.3770	0.2874
	2	0.3649	1.1286	1.1861	0.6153	1.6253		1.0680	4.6503
	3	0.1522	1.4938	1.5015	0.2027	4.9328		1.4860	13.9067
	4	0.4828		0.4828			0.4828		
8	1	0.4222	0.2640	0.4979	1.6958	0.5897	0.4026		
	2	0.3833	0.7716	0.8616	0.8898	1.1239		0.6697	1.9722
	3	0.2678	1.2066	1.2360	0.4333	2.3076		1.1765	7.4721
	4	0.1122	1.4798	1.4840	0.1512	6.6134		1.4755	16.4334
9	1	0.3700	0.4704	0.5985	1.2365	0.8088		0.2905	0.2480
	2	0.3230	0.9068	0.9626	0.6711	1.4901		0.8473	3.9831
	3	0.2309	1.2634	1.2843	0.3596	2.7811		1.2421	9.0271
	4	0.0860	1.4740	1.4765	0.1165	8.5804		1.4715	18.6849
	5	0.3842		0.3842			0.3842		
10	1	0.3384	0.2101	0.3983	1.6991	0.5885	0.3212		
	2	0.3164	0.6180	0.6943	0.9114	1.0972		0.5309	1.8164
	3	0.2677	0.9852	1.0209	0.5244	1.9068		0.9481	5.9157
	4	0.1849	1.2745	1.2878	0.2871	3.4825		1.2610	10.9284
	5	0.0671	1.4389	1.4405	0.0931	10.7401		1.4373	20.6296

Figure 5-36: *Gaussian to 6dB Design Table*

NOTES: