Log Converters

One of the most predictable non-linear elements commonly available is the bipolar transistor. The relationship between collector current and emitter base voltage is precisely logarithmic from currents below one picoamp to currents above one milliamp. Using a matched pair of transistors and integrated circuit operational amplifiers, it is relatively easy to construct a linear to logarithmic converter with a dynamic range in excess of five decades.

The circuit in *Figure 1* generates a logarithmic output voltage for a linear input current. Transistor Q_1 is used as the non-linear feedback element around an LM108 operational amplifier. Negative feedback is applied to the emitter of Q_1 through divider, R_1 and R_2, and the emitter base junction of Q_2. This forces the collector current of Q_1 to be exactly equal to the current through the input resistor. Transistor Q_2 is used as the feedback element of an LM101A operational amplifier. Negative feedback forces the collector current of Q_2 to equal the current through R_3. For the values shown, this current is 10 μA. Since the collector current of Q_2 remains constant, the emitter base voltage also remains constant. Therefore, only the V_{BE} of Q_1 varies with a change of input current. However, the output voltage is a function of the difference in emitter base voltages of Q_1 and Q_2:

$$E_{OUT} = \frac{R_1 + R_2}{R_2} (V_{BE_2} - V_{BE_1}). \qquad (1)$$

For matched transistors operating at different collector currents, the emitter base differential is given by

$$\Delta V_{BE} = \frac{kT}{q} \log_e \frac{I_{C_1}}{I_{C_2}} \qquad (2)$$

where k is Boltzmann's constant, T is temperature in degrees Kelvin and q is the charge of an electron. Combining these two equations and writing the expression for the output voltage gives

$$E_{OUT} = \frac{-kT}{q} \left[\frac{R_1 + R_2}{R_2} \right] \log_e \left[\frac{E_{IN} R_3}{E_{REF} R_{IN}} \right] \qquad (3)$$

for $E_{IN} \geq 0$. This shows that the output is proportional to the logarithm of the input voltage. The coefficient of the log term is directly proportional to absolute temperature. Without compensation, the scale factor will also vary directly with temperature. However, by making R_2 directly proportional to temperature, constant gain is obtained. The temperature compensation is typically 1% over a temperature range of -25°C to 100°C for the resistor specified. For limited temperature range applications, such as 0°C to 50°C, a 430Ω sensistor in series with a 570Ω resistor may be substituted for the 1k resistor, also with 1% accuracy. The divider, R_1 and R_2, sets the gain while the current through R_3 sets the zero. With the values given, the scale factor is 1V/decade and

$$E_{OUT} = - \left[\log_{10} \left| \frac{E_{IN}}{R_{IN}} \right| + 5 \right] \qquad (4)$$

where the absolute value sign indicates that the dimensions of the quantity inside are to be ignored.

Log generator circuits are not limited to inverting operation. In fact, a feature of this circuit is the ease with which non-inverting operation is obtained. Supplying the input signal to A_2 and the reference current to A_1 results in a log output that is not inverted from the input. To achieve the same 100 dB dynamic range in the non-inverting configuration, an LM108 should be used for A_2, and an LM101A for A_1. Since the LM108 cannot use feedforward compensation, it is frequency compensated with the standard 30 pF capacitor.

The only other change is the addition of a clamp diode connected from the emitter of Q_1 to ground. This prevents damage to the logging transistors if the input signal should go negative.
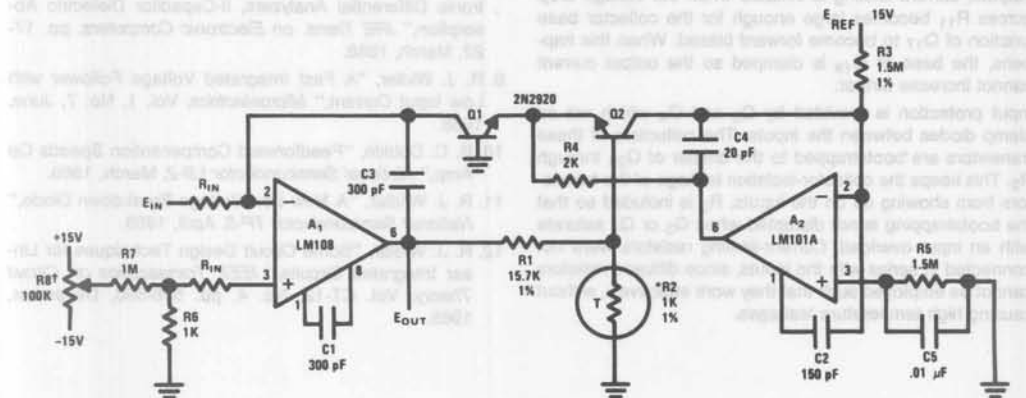
*Tel Labs Type Q_{81} Manchester, N.H.

†Offset Voltage Adjust

FIGURE 1. Log Generator with 100 dB Dynamic Range

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The log output is accurate to 1% for any current between 10 nA and 1 mA. This is equivalent to about 3% referred to the input. At currents over 500 μA the transistors used deviate from log characteristics due to resistance in the emitter, while at low currents, the offset current of the LM108 is the major source of error. These errors occur at the ends of the dynamic range, and from 40 nA to 400 μA the log converter is 1% accurate referred to the input. Both of the transistors are used in the grounded base connection, rather than the diode connection, to eliminate errors due to base current. Unfortunately, the grounded base connection increases the loop gain. More frequency compensation is necessary to prevent oscillation, and the log converter is necessarily slow. It may take 1 to 5 ms for the output to settle to 1% of its final value. This is especially true at low currents.

The circuit shown in *Figure 2* is two orders of magnitude faster than the previous circuit and has a dynamic range of 80 dB. Operation is the same as the circuit in *Figure 1*, except the configuration optimizes speed rather than dynamic range. Transistor Q_1 is diode connected to allow the use of feedforward compensation[1] on an LM101A operational amplifier. This compensation extends the bandwidth to 10 MHz and increases the slew rate. To prevent errors due to the finite h_{FE} of Q_1 and the bias current of the LM101A, an LM102 voltage follower buffers the base current and input current. Although the log circuit will operate without the LM102, accuracy will degrade at low input currents. Amplifier A_2 is also compensated for maximum bandwidth. As with the previous log converter, R_1 and R_2 control the sensitivity; and R_3 controls the zero crossing of the transfer function. With the values shown the scale factor is 1V/decade and

$$E_{OUT} = -\left[log_{10} \left| \frac{E_{IN}}{R_{IN}} \right| + 4 \right] \quad (5)$$

from less than 100 nA to 1 mA.

Anti-log or exponential generation is simply a matter of rearranging the circuitry. *Figure 3* shows the circuitry of the log converter connected to generate an exponential output from a linear input. Amplifier A_1 in conjunction with transistor Q_1 drives the emitter of Q_2 in proportion to the input voltage. The collector current of Q_2 varies exponentially with the emitter-base voltage. This current is converted to a voltage by amplifier A_2. With the values given

$$E_{OUT} = 10^{-[E_{IN}]}. \quad (6)$$

Many non-linear functions such as $X^{1/2}$, X^2, X^3, $1/X$, XY, and X/Y are easily generated with the use of logs. Multiplication becomes addition, division becomes subtraction and powers become gain coefficients of log terms. *Figure 4* shows a circuit whose output is the cube of the input. Actually, any power function is available from this circuit by changing the values of R_9 and R_{10} in accordance with the expression:

$$E_{OUT} = E_{IN}^{\frac{16.7 R_9}{R_9 + R_{10}}}. \quad (7)$$

Note that when log and anti-log circuits are used to perform an operation with a linear output, no temperature compensating resistors at all are needed. If the log and anti-log transistors are at the same temperature, gain changes with temperature cancel. It is a good idea to use a heat sink which couples the two transistors to minimize thermal gradients. A 1°C temperature difference between the log and anti-log transistors results in a 0.3% error. Also, in the log converters, a 1°C difference between the log transistors and the compensating resistor results in a 0.3% error.

Either of the circuits in *Figures 1* or *2* may be used as dividers or reciprocal generators. Equation 3 shows the outputs of the log generators are actually the ratio of two currents:
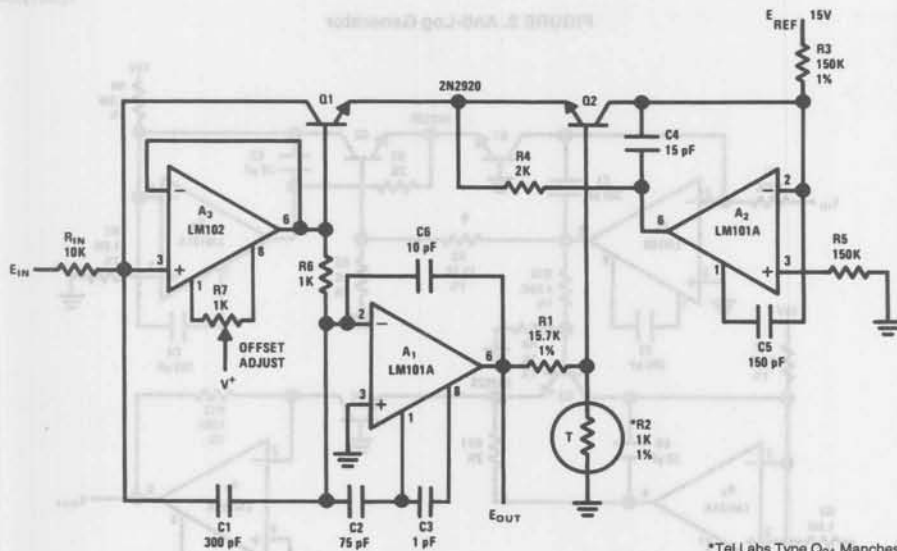

FIGURE 2. Fast Log Generator

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the input current and the current through R_3. When used as a log generator, the current through R_3 was held constant by connecting R_3 to a fixed voltage. Hence, the output was just the log of the input. If R_3 is driven by an input voltage, rather than the 15V reference, the output of the log generator is the log ratio of the input current to the current through R_3. The anti-log of this voltage is the quotient. Of course, if the divisor is constant, the output is the reciprocal.

A complete one quadrant multiplier/divider is shown in *Figure 5*. It is basically the log generator shown in *Figure 1* driving the anti-log generator shown in *Figure 3*. The log generator output from A_1 drives the base of Q_3 with a voltage proportional to the log of E_1/E_2. Transistor Q_3 adds a voltage proportional to the log of E_3 and drives the anti-log transistor, Q_4. The collector current of Q_4 is converted to an

output voltage by A_4 and R_7, with the scale factor set by R_7 at $E_1 E_3/10E_2$.

Measurement of transistor current gains over a wide range of operating currents is an application particularly suited to log multiplier/dividers. Using the circuit in *Figure 5*, PNP current gains can be measured at currents from 0.4 μA to 1 mA. The collector current is the input signal to A_1, the base current is the input signal to A_2, and a fixed voltage to R_5 sets the scale factor. Since A_2 holds the base at ground, a single resistor from the emitter to the positive supply is all that is needed to establish the operating current. The output is proportional to collector current divided by base current, or h_{FE}.

In addition to their application in performing functional operations, log generators can provide a significant increase in
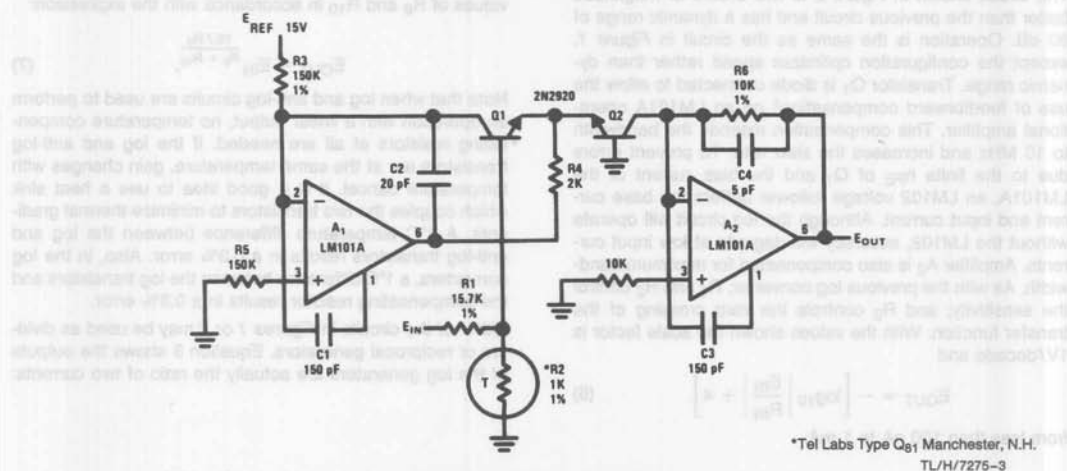

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FIGURE 3. Anti-Log Generator

FIGURE 4. Cube Generator

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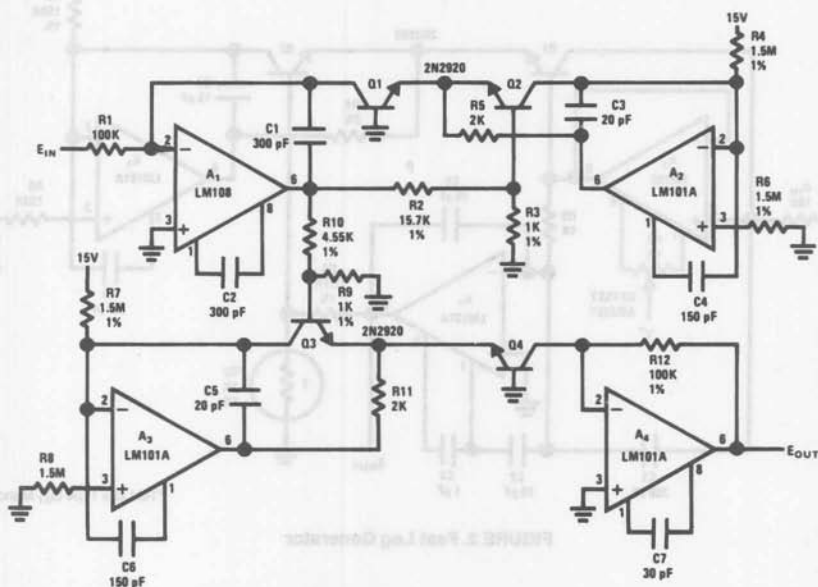
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the dynamic range of signal processing systems. Also, unlike a linear system, there is no loss in accuracy or resolution when the input signal is small compared to full scale. Over most of the dynamic range, the accuracy is a percent-of-signal rather than a percent-of-full-scale. For example, using log generators, a simple meter can display signals with 100 dB dynamic range or an oscilloscope can display a 10 mV and 10V pulse simultaneously. Obviously, without the log generator, the low level signals are completely lost.

To achieve wide dynamic range with high accuracy, the input operational amplifier necessarily must have low offset voltage, bias current and offset current. The LM108 has a maximum bias current of *3 nA* and offset current of *400 pA* over a −55°C to 125°C temperature range. By using equal source resistors, only the offset current of the LM108 causes an error. The offset current of the LM108 is as low as many FET amplifiers. Further, it has a low and constant temperature coefficient rather than doubling every 10°C. This results in greater accuracy over temperature than can be achieved with FET amplifiers. The offset voltage may be

zeroed, if necessary, to improve accuracy with low input voltages.

The log converters are low level circuits and some care should be taken during construction. The input leads should be as short as possible and the input circuitry guarded against leakage currents. Solder residues can easily conduct leakage currents, therefore circuit boards should be cleaned before use. High quality glass or mica capacitors should be used on the inputs to minimize leakage currents. Also, when the +15V supply is used as a reference, it must be well regulated.

REFERENCES

1. R. C. Dobkin, *"Feedforward Compensation Speeds Op Amp"*, National Semiconductor Corporation, Linear Brief 2, April, 1969.

2. R. J. Widlar, *"Monolithic Operational Amplifiers—The Universal Linear Component"*, National Semiconductor Corporation, AN-4, April, 1968.
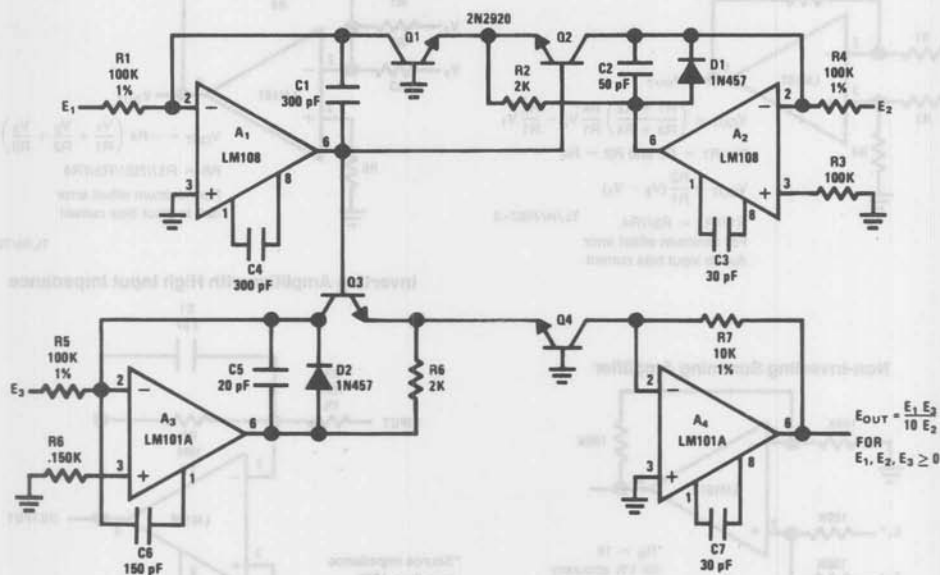
FIGURE 5. Multiplier/Divider

$$E_{OUT} = \frac{E_1 E_3}{10 E_2}$$ FOR $E_1, E_2, E_3 \geq 0$

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