Analog square-root circuit handles wide input range

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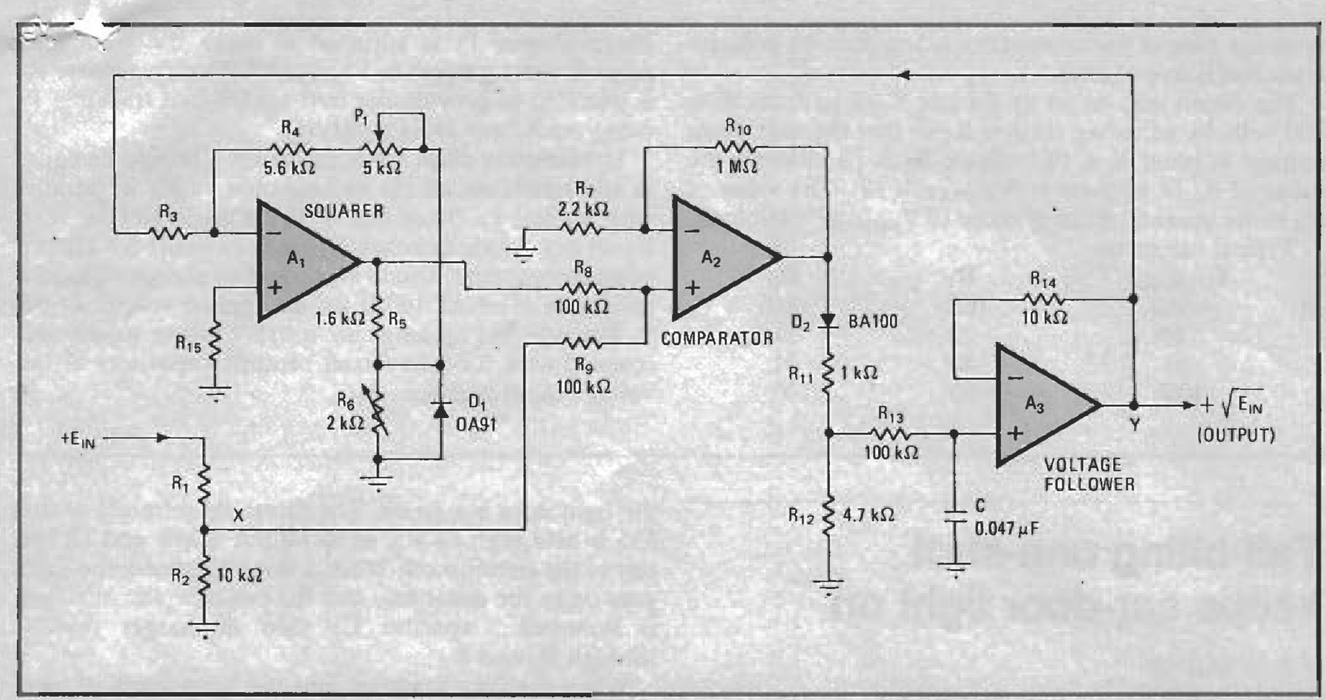
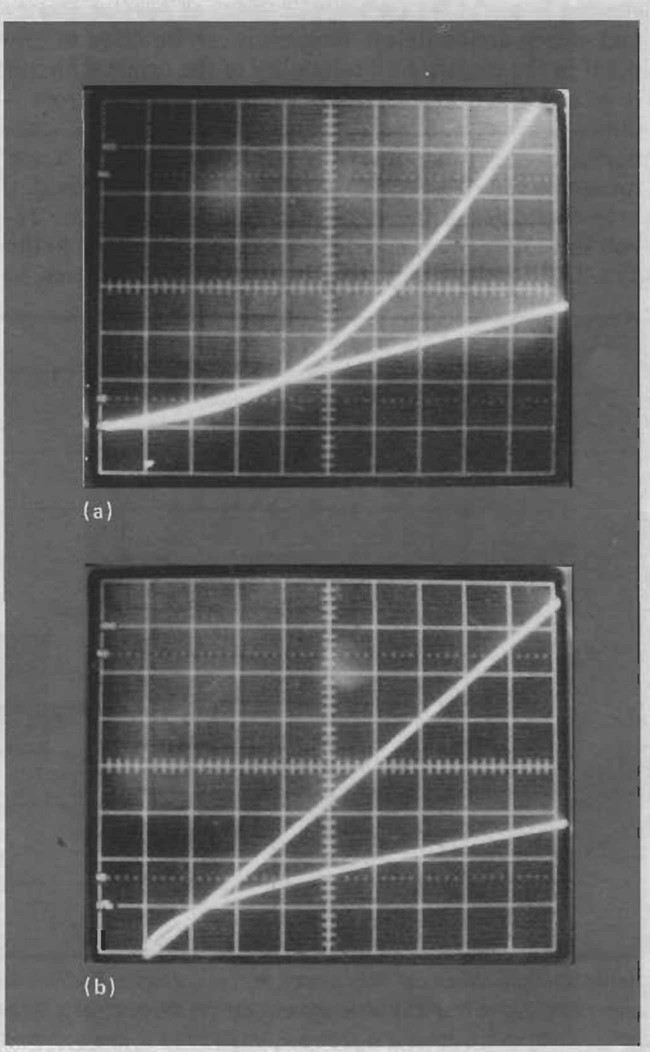
A square-root circuit is frequently needed for linearizing the output from transducers that have a square-law response. It also finds many applications in analog computations. The design described here produces a square root with accuracy within 1% for input voltages in the range from 0 to +100 volts.

As shown in Fig. 1, the circuit has three operational-amplifier stages: a squarer using op amp A_1, a compara-



1. Getting to the root. Output from three-op-amp circuit is square root of input. Comparator A_2 balances input with square of output to produce the root. Accuracy within 1% is achieved through good square-law characteristic of diode D_1 in feedback loop of squarer A_1, plus adjustment of scaling and tracking controls P_1 and R_6. Voltage follower A_3 buffers voltage across capacitor.



2. Two views of operation. In (a) the upper trace is a square-law input, and the lower trace is the linear output from the circuit of Fig. 1. In (b) the upper trace is a linear input, and the lower trace is the square-root output. Crossovers in photos occur at input of 1 volt.

tor using op amp A_2, and a voltage follower using op amp A_3. It operates by comparing the scaled input voltage to the square of the voltage that is fed back from the output of the circuit. When the two are equal, the output voltage must be the root of the input.

The positive input voltage at point X is applied to the noninverting terminal of the comparator through resistor R_9. The comparator's output goes positive and charges the capacitor C; the capacitor voltage is buffered by the voltage follower and applied to the inverting input of the squarer by means of the scaling resistor R_3.

Because of the approximate square-law characteristics of diode D_1 (modified in curvature by R_6) in the feedback loop of the squarer, the output from A_1 is the square of the voltage at its inverting input; it is negative because of the inversion. This negative output drives the noninverting input of the comparator through R_8. When the magnitude of this voltage equals that from point X, the comparator output goes negative, and C discharges through R_{13} and R_{12}. (The function of diode D_2 is to prevent the comparator from putting negative charge on C.)

Thus the comparator automatically modifies the voltage on C to maintain the output of the follower and the input to the squarer at $+V_{IN}^{1/2}$ (Fig. 2). It is connected as a noninverting high-gain amplifier that responds to voltage changes very rapidly without producing any sawtooth components at the output. Swept voltage tests made with an oscilloscope show that the response time of the comparator is less than 10 milliseconds and is free of jitter.

The circuit may be set up for any $V_{IN(MAX)}$ from 10 to 100 volts by adjusting resistor R_1 so that the maximum voltage at point X is 10 v. Since R_2 is 10 kilohms, the value of R_1 in kilohms is $(V_{IN(MAX)} - 10)$. The value of R_3 in the squarer circuit is made 10 $V_{IN(MAX)}{}^{1/2}$ kilohms.

Typical values are:

$V_{IN(MAX)}$ (volts)	R_1 (kΩ)	R_3 (kΩ)
10	0	31.6
50	40	71
100	90	100

Potentiometer P_1 is adjusted to make the voltage at point Y exactly equal to $V_{IN(MAX)}{}^{1/2}$. Finally, resistor R_6 is trimmed to provide the best square-root tracking, P_1 being readjusted as R_6 is varied.

To minimize offset error, resistor R_{15} should be equal to the resistance of the combination of R_3 in parallel with R_4 and P_1. Since $(R_4 + P_1)$ is much smaller than R_3 on any range, however, R_{15} may be made 6.8 kΩ as a good compromise. Diode D_1 should be chosen to have a resistance of about 160 Ω for an applied voltage of 0.8 v. The type 741 op amps use a ±15-v power supply, decoupled with 0.1-microfarad ceramic capacitors at the voltage-input points. □

LINEAR TO LOG

Can you show me a simple IC-based circuit which would convert a linear-voltage input into a logarithmic output that could drive a meter? It would be very useful for extending the range of VU and S meters.—J. Cable, Lehigh Acres, FL.

Once upon a time, logarithmic amps were common circuit elements, but as digital stuff started to take over, analog log amps were used less and less. That's really a shame because an analog log amp is a simple one-IC solution to a lot of circuit problems. You're quite right that it's a perfect addition to metering circuitry and, if you get into it, you'll also find that it's great in audio-signal processing as well. It used to be that every compressor and limiter on the market was built around a log amp, but digital signal processing has shown up in that area as well. But enough nostalgia.

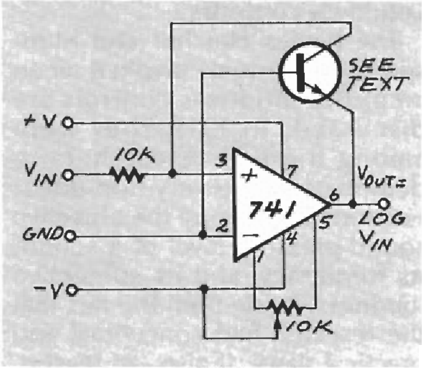


FIG. 2

The circuit in Fig. 2 is a basic log amp built around a single op-amp. The configuration is often referred to as a "transdiode" circuit, since the output of the op-amp is equal to the base-emitter voltage of the transistor. The current in the feedback loop of the op-amp is equal to the current flow at the input of the op-amp. Since the input current is proportional to the voltage across the input resistor, it's also proportional to the collector current in the transistor. The base-emitter voltage of the transistor is related logarithmically to the collector current so the output of the op-amp will vary logarithmically with the op-amp's input voltage.

The circuit is built around a 741 but you can use any op-amp you want. The transistor, however, should be a high-gain type, capable of handling the power; since you're only using it to drive a meter, you can probably get by with something like a 2N3391.

I'm sure you know that whenever you build a meter amp, getting the circuit working is only half the battle—you also have to calibrate it. In a straightforward linear amp that isn't much of a problem, but log amps make it a bit more difficult. You can use the brute-force approach of putting known signals at the input and then padding the output, but regardless of the method you use, you have to take into account the offset introduced by the op-amp. That's the purpose of the potentiometer across the offset adjustment pins of the op-amp.

Since the log of one is zero, you should feed the amp with one unit of positive signal and tweak the potentiometer to get zero out of the op-amp. The amount of accuracy you get depends on the gain, the temperature, and the level of the input.

If you really want to get into this, you'll find that there's a lot of math involved in calculating the circuit parameters and there's just not enough room here to go through all the gory details. It's safe to say, therefore, that the success you're going to have with log amps in general is directly proportional to the number of hours you spend doing research. Good luck. **R-E**

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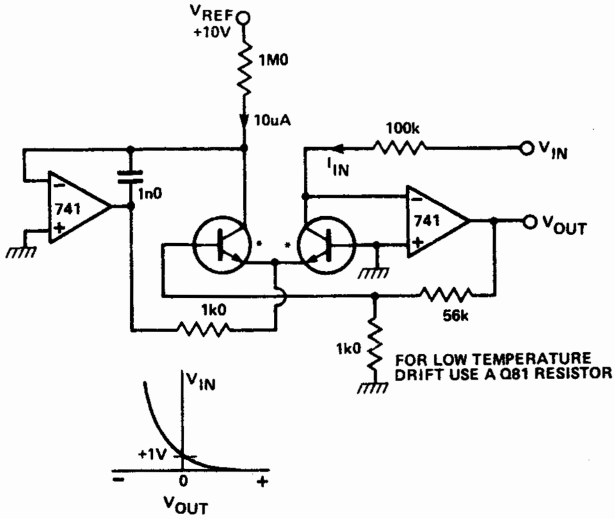
Log Converter

V_{OUT} changes by 1 V for every octave change of the I_{IN} current
*The matched transistors can be two 2N3905 in thermal contact, or a dual transistor (LM394), or pat of an array (CA3046).



LOGARITHMIC RESPONSE

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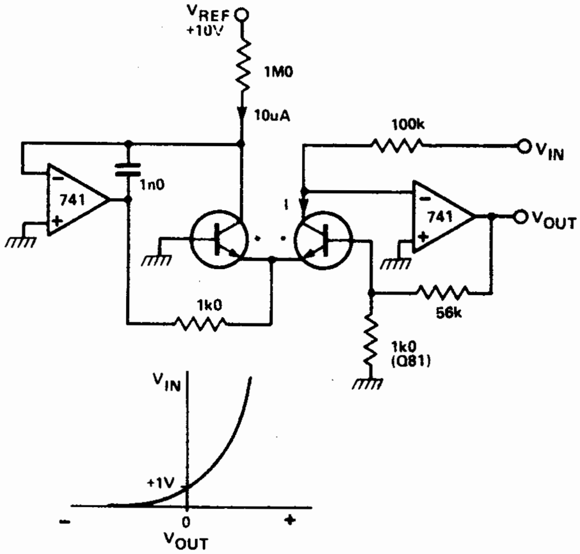
Antilog (Exponential) Converter

$V_{OUT} = I \times 100k$
The current I doubles for every 1 V increase of V_{IN}
When $V_{IN} = 0$ V, I = 10 uA



EXPONENTIAL RESPONSE